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The asymptotic stability of a fractional epidemiological model "Covid 19 Variant Anglais" with Caputo derivative

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Abstract

We have all been injured by corona and its mutations, not just us but the whole world. The global impact of coronavirus (COVID-19) has been profound and the public health threat it represents is the most serious seen in a respiratory virus since 1918. This paper is concerned with a fractional order $S_N S_C IR$ model involving the Caputo fractional derivative. The effective methods to solve the fractional epidemic models we introduced to construct a simple and effective analytical technique that can be easily extended and applied to other fractional models and can help guide the concerned bodies in preventing or controlling, even predicting the infectious disease outbreaks. The equilibrium points and the basic reproduction number are computed. An analysis of the local asymptotic stability at the disease-free equilibrium is given; Next, we study the stability of the equilibrium points in the sense of Mittag-Leffler. Moreover, some numerical simulations are included to verify the theoretical achievement. These results provide good evidence for the implications of the theoretical results corresponding to the model.

Keywords: mittag-Leffler; global stability; lyapunov function
2020 MSC: 33E12, 34D23, 37L45

1 Introduction

The main goal of mathematical models in epidemiology is to understand the behavior of a particular infectious disease, such as the prevalence and the duration of the epidemic, and its impact in the population.

Mathematical models are used to describe reality, but usually they are simplifications because it is almost impossible to make computations with a large set of input parameters.

Recently, fractional derivatives have been used to describe epidemiological models and they have proven to be more accurate in some cases, when compared to the classical ones. We find in the literature different models described by fractional derivatives, like the MSEIR model [1].

In [4], the autors present a model describing a transmissible and infectious disease in epidemiology called Covid-19 british variant and its transmission factors, In our paper, we propose a fractional $S_N S_C IR$ model, where the spread of the disease is described by a system of fractional-order differential equations. It is worthwhile mentioning that fractional derivatives are non-local operators, and thus may be more suitable for modeling systems dependent on past

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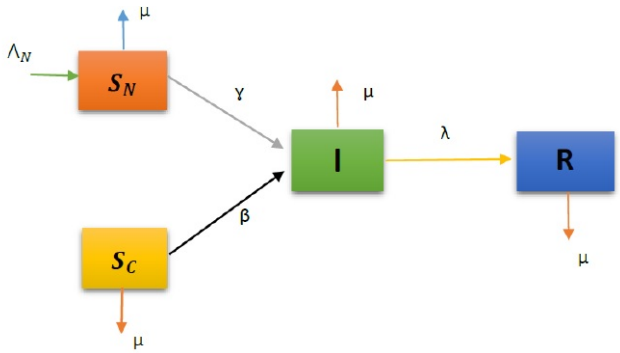
history (memory). Also, since the fractional order can be any positive real α, we can choose the one that better fits the data. Therefore, we can adjust the model to real data and, thus, better predict the evolution of the disease.

Mathematical Infectious Disease Modeling is a tool to study how diseases spread, predict the future trajectory of an outbreak, help guide public health planning and infectious disease control. This is what we apply in our work, we bring reality to life in a mathematical model to study the stability of the disease. First, we use the SIR model, on compartment S (Susceptible) we divide it into two parts S_N (those susceptible who have not had Covid 19) and S_C (those susceptible who have already had Covid 19) more compartment I (individuals infected with the English variant) and finally compartment R (recovered).

2 Diagram transmission of british variant Covid-19 between humans:

Some of the emergence of Corona disease and the infection of many people in the whole world with it and for a short time, then a mutated appearance of this disease called 'British Variant'. Moreover, the new idea in our article is to divide the people who are exposed to the disease into two categories, a category that has previously contracted the normal corona virus, and the other type has not been sick with corona.



Description of biological parameters:.

- S_C : The susceptible individuals who already had the covid-19.
- S_N : The susceptible individuals who not already had the covid-19.
- I : Infected individuals by the british variant covid-19.
- R : The individuals withdrawn (healed or dead).
- β The rate of individuals who become infected by the british variant covid-19 who had already contacted covid-19.
- γ : The rate of individuals who become infected by the british variant covid-19.
- λ : Recovery rate.
- μ : Natural mortality rate.
- Λ_N : Birth rate.

This diagram can translated mathematically by the following system of differential equations:

$$\begin{cases} \dfrac{dS_N(t)}{dt} = \Lambda_N - \gamma S_N(t)I(t) - \mu S_N(t) \\ \dfrac{dS_C(t)}{dt} = -\beta S_C(t)I(t) - \mu S_C(t) \\ \dfrac{dI(t)}{dt} = \gamma S_N(t)I(t) + \beta S_C(t)I(t) - \lambda I(t) - \mu I(t) \\ \dfrac{dR(t)}{dt} = \lambda I(t) - \mu R(t) \end{cases} \tag{2.1}$$

The system (2.1) is provided with the initial conditions:

$$S_N(0) = S_{N_0} > 0,\ S_C(0) = S_{C_0} > 0,,\ I(0) = I_0 > 0,,\ R(0) = R_0 > 0.$$

and,

$$N = S_{N_0} + S_{C_0} + I_0 + R_0.$$

Motivated by system (2.1), we present here our fractional model. First, we replace each ordinary derivative in the system by the Caputo fractional derivative of order α, where α is an arbitrary real belonging to the interval $[0, 1]$. Then, each parameter $*$ is replaced by $*^\alpha$, in order to both sides of the equations have the same dimension, with the exception of N, that is dimensionless [5]. Therefore, our proposal model is given by the following system of nonlinear fractional differential equations:

$$\begin{cases} {}^cD^\alpha S_N(t) = \Lambda_N^\alpha - \gamma^\alpha S_N(t)I(t) - \mu^\alpha S_N(t) \\ {}^cD^\alpha S_C(t) = -\beta^\alpha S_C(t)I(t) - \mu^\alpha S_C(t) \\ {}^cD^\alpha I(t) = \gamma^\alpha S_N(t)I(t) + \beta^\alpha S_C(t)I(t) - \lambda^\alpha I(t) - \mu^\alpha I(t) \\ {}^cD^\alpha R(t) = \lambda^\alpha I(t) - \mu^\alpha R(t) \end{cases} \tag{2.2}$$

The system (2.2) is provided with the initial conditions:

$$S_N(0) = S_{N_0} > 0,\ S_C(0) = S_{C_0} > 0,,\ I(0) = I_0 > 0,,\ R(0) = R_0 > 0.$$

and,

$$N = S_{N_0} + S_{C_0} + I_0 + R_0,$$

${}_{t_0}^{c}D_t^\alpha$ is the caputo fractional derivative. Let $f \in C^n([a, +\infty])$ and

$${}_{t_0}^{c}D_t^\alpha f(t) = \frac{1}{\Gamma(1-\alpha)} \int_{t_0}^{t} \frac{f'(\tau)}{(t-\tau)^\alpha} d\tau, \qquad \alpha \in [0, 1].$$

Lemma 2.1. ([5].) Suppose that $f(t) \in [a, b]$ and ${}^cD^\alpha f(t) \in C[a, b]$, for $0 < \alpha \leq 1$. If ${}^cD^\alpha f(t) \geq 0$, for all $t \in [a, b]$, then $f(t)$ is non-decreasing for each $t \in [a, b]$. If ${}^cD^\alpha f(t) \leq 0$, for all $t \in (a, b)$, then $f(t)$ is non-decreasing for each $t \in [a, b]$.

Lemma 2.2. ([5].) Assume that the vector function $f : \mathbb{R}^+ \times \mathbb{R}^3 \longrightarrow \mathbb{R}^3$ satisfies the following conditions:

1. Function $f(t, x(t))$ is Lebesgue measurable with respect to $t \in \mathbb{R}^3$.
2. Function $f(t, x(t))$ is continuous with respect to $X(t)$ on $\mathbb{R}^4$.
3. $\frac{df(t,x)}{dx}$ is continuous with respect to $X(t)$ on $\mathbb{R}^4$.
4. $||f(t, x)|| \leq \omega + \lambda||x||$, $\forall t \in \mathbb{R}^+$, $X \in \mathbb{R}^4$.

Here ω , λ are two positive constants.

Theorem 2.3. be $0 < \alpha < 1$ and $\lambda \in \mathbb{R}$.The solution to the initial value problem

$$\begin{cases} {}^cD_t^\alpha x(t) = \lambda x(t) \\ x(t_0) = x_0 \end{cases} \tag{2.3}$$

is given by:

$$x(t) = x_0 E_\alpha(\lambda(t - t_0)^\alpha), \quad t \geq 0.$$

The initial value problems have a unique solution.

3 The existence and uniqueness of positive solution:

Theorem 3.1. The fractional system (2.2) has a unique solution, which remains in $\mathbb{R}^4_+$ and the closed set

$$\Omega = \left\{(S_N, S_C, I, R) \in \mathbb{R}^4_+, 0 \leq S_C + S_N + I + R \leq \frac{\Lambda_N^\alpha}{\mu^\alpha}\right\}$$

is a positive invariant set of system (2.2).

Proof . Firstly, we prove that $\forall (S_N(0), S_C(0), I(0), R(0))^T \in \mathbb{R}^4_+$, system (2.2) has a unique solution. Obviously, vector function f of system (2.2) satisfies conditions (2.1)-(2.3) of Lemma 2.2. Following, we prove system $x_1(t) = S_N(t)$, $x_2(t) = S_C(t)$, $x_3(t) = I(t)$, $x_4(t) = R(t)$.

$x_1(0) = S_N(0)$, $x_2(0) = S_C(0)$, $x_3(0) = I(0)$, $x_4(0) = R(0)$.

$$X(t) = \begin{pmatrix} x_1(t) \\ x_2(t) \\ x_3(t) \\ x_4(t) \end{pmatrix} \qquad K = \begin{pmatrix} \Lambda_N^\alpha \\ 0 \\ 0 \\ 0 \end{pmatrix}$$

$$A_1 = \begin{pmatrix} -\mu^\alpha & 0 & 0 & 0 \\ 0 & -\mu^\alpha & 0 & 0 \\ 0 & 0 & -(\lambda+\mu) & 0 \\ 0 & 0 & \lambda^\alpha & -\mu^\alpha \end{pmatrix}, \quad A_2 = \begin{pmatrix} -\gamma^\alpha & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}, \quad A_3 = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & -\beta^\alpha & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

$$A_4 = \begin{pmatrix} \gamma^\alpha & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \qquad A_5 = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & \beta^\alpha & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}.$$

$${}^cD^\alpha x(t) = A_1 x(t) + x_3(t)A_2 x(t) + x_3(t)A_3 x(t) + x_3(t)A_4 x(t) + x_3(t)A_5 x(t).$$

$$\begin{aligned} ||f(t,x(t))|| =& ||A_1 x(t) + x_3(t)A_2 x(t) + x_3(t)A_3 x(t) + x_3(t)A_4 x(t) + x_3(t)A_5 x(t) + \ K|| \\ \leq& ||k|| + ||A_1||||x(t)|| + ||A_2||||x(t)|| + ||A_3||||x(t)|| + \ ||A_4||||x(t)|| + \ ||A_5||||x(t)|| \\ \leq& \omega + (||A_1|| + ||A_2|| + ||A_3|| + ||A_4|| + ||A_5||)||x(t)|| \\ =& \omega + \lambda||x(t)||. \end{aligned}$$

By Lemma 2.2 system (2.2) has a unique solution. Secondly, we prove the solution of system (2.2) is always non-negative. Based on system (2.2), we have:

$$\begin{aligned} &{}^cD^\alpha S_N|_{S_N=0} = \Lambda_N^\alpha \geq 0 \\ &{}^cD^\alpha S_C|_{S_C=0} = 0 \geq 0 \\ &{}^cD^\alpha I(t)|_{I=0} = 0 \geq 0 \\ &{}^cD^\alpha R(t)|_{R=0} = \lambda^\alpha I(t) \geq 0 \end{aligned}$$

According to Lemme2.1 1, we have $S_N(t), S_C(t), R(t) \geq 0$ for any $t \geq 0$. In order to prove the conclusion, we can assume there exists a constant t_1, such that

$$\begin{cases} I(t_1) < 0 \\ I(t) \geq 0 \qquad \forall t \in [0, t_1] \end{cases} \tag{3.1}$$

By the third equation of system (2.2), we have,

$${}^cD^\alpha I \geq -(\lambda^\alpha + \mu^\alpha)I, t \in [0, t_{t_1}].$$

The solution is

$$I(t) \geq I(0)E_\alpha(-(\lambda^\alpha + \mu^\alpha)t^\alpha) \quad t \in [0, t_1].$$

So, $I(t_1) > 0$, which contradicts the suppose. So $I(t) \geq 0$ for any $t \geq 0$. Finally, it can be seen, by adding three equations of system (2.2), that,

$$^cD^\alpha N \leq \Lambda_N^\alpha - \mu^\alpha N.$$

Solving this equation, we have:

$$N(t) \leq \left(\frac{-\Lambda_N^\alpha}{\mu^\alpha} + N(0)\right) E_0(-\mu^\alpha t^\alpha) + \frac{\Lambda_N^\alpha}{\mu^\alpha}.$$

Since $E_\alpha(-\mu^\alpha t^\alpha) \geq 0$, when $N_0 \leq \frac{\Lambda_N^\alpha}{\mu^\alpha}$, we have, $N(t) = S_N(t) + S_C(t) + I(t) + R(t) \leq \frac{\Lambda_N^\alpha}{\mu^\alpha}$. Hence, $\Omega = \left\{(S_N(t), S_C(t), I(t), R(t)) \in \mathbb{R}_+^4 | 0 \leq S_N + S_C + I + R \leq \frac{\Lambda_N^\alpha}{\mu^\alpha}\right\}$. □

4 Equilibrium:

4.1 Disease free equilibrium (DFE):

We search $\overline{S_N} \geq 0$, $\overline{S_C} \geq 0$ et $\overline{R} \geq 0$ satisfying:

$$\begin{cases} 0 &= \Lambda_N^\alpha - \gamma^\alpha \overline{S_N} I - \mu^\alpha \overline{S_N}, \\ 0 &= -\beta^\alpha \overline{S_C} I - \mu^\alpha \overline{S_C}, \\ 0 &= \gamma^\alpha \overline{S_N} I + \beta^\alpha \overline{S_C} I - \lambda^\alpha \overline{I} - \mu^\alpha \overline{I}, \\ 0 &= \lambda^\alpha \overline{I} - \mu^\alpha \overline{R} \end{cases}$$

with, $\overline{I} = 0$ We obtain: $\overline{S_N} = \frac{\Lambda_N^\alpha}{\mu^\alpha}$, $\overline{S_C} = 0$ et $\overline{R} = 0$. Therefore,

$$P_F = (\frac{\Lambda_N^\alpha}{\mu^\alpha}, 0, 0, 0)$$

4.2 Calcul of R_0: (Method of van den Driessche watmough):

We denote by:

- $\mathcal{F}_j(S_N, S_C, I, R)$ the rate of newly infected in the compartment j.
- $\mathcal{V}_j(S_N, S_C, I, R)$ the transfer rate of an individual from one compartment to another everywhere average.

The matrices $\mathcal{F}$ and $\mathcal{V}$ are represented by:

$$\mathcal{F} = \begin{pmatrix} 0 \\ 0 \\ \gamma^\alpha S_N I + \beta^\alpha S_C I \\ 0 \end{pmatrix} \quad \text{and} \quad \mathcal{V} = \begin{pmatrix} \Lambda_N^\alpha - \gamma^\alpha S_N I - \mu^\alpha S_N \\ -\beta^\alpha S_C I - \mu^\alpha S_C \\ -(\lambda^\alpha + \mu^\alpha) I \\ \lambda^\alpha I - \mu^\alpha R \end{pmatrix}.$$

The calculation of their resprctive Jacobian at the disease free equilibrium, point $P_F = (\frac{\Lambda_N^\alpha}{\mu^\alpha}, 0, 0, 0)$ given:

$$\mathcal{F}(P_F) = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & \frac{\gamma^\alpha \Lambda_N^\alpha}{\mu^\alpha} & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}, \qquad \mathcal{V}(P_F) = \begin{pmatrix} -\mu^\alpha & 0 & -\frac{\gamma^\alpha \Lambda_N^\alpha}{\mu^\alpha} & 0 \\ 0 & -\mu^\alpha & 0 & 0 \\ 0 & 0 & -(\lambda^\alpha + \mu^\alpha) & 0 \\ 0 & 0 & \lambda^\alpha & -\mu^\alpha \end{pmatrix}$$

Consider F and V the matrices given by:

$$F = \begin{pmatrix} \frac{\gamma^\alpha \Lambda_N^\alpha}{\mu^\alpha} & 0 \\ 0 & 0 \end{pmatrix} \quad \text{and} \quad V = \begin{pmatrix} -(\lambda^\alpha + \mu^\alpha) & 0 \\ \lambda^\alpha & -\mu^\alpha \end{pmatrix}$$

The basic reproduction rate is the spectral radius of the matrix $-FV^{-1}$ the calculation given:

$$R_0 = \frac{\gamma^\alpha \Lambda_N^\alpha}{\mu^\alpha(\lambda^\alpha + \mu^\alpha)}.$$

4.2 Endemic equilibrium (EE):

For $\overline{I} > 0$. Using the second equation of the system we get:

$$\overline{S_C} = 0.$$

By considering the third equation of the system we obtain:

$$\overline{S_N} = \frac{\lambda^\alpha + \mu^\alpha}{\gamma^\alpha}.$$

Using the first equation of the system we get:

$$\overline{I} = \frac{\mu^\alpha}{\gamma^\alpha}\left(\frac{\gamma^\alpha \Lambda_N^\alpha}{\mu^\alpha(\lambda^\alpha + \mu^\alpha)} - 1\right).$$

According to the fourth equations we obtain:

$$\overline{R} = \frac{\lambda^\alpha}{\gamma^\alpha}\left(\frac{\gamma^\alpha \Lambda_N^\alpha}{\mu^\alpha(\lambda^\alpha + \mu^\alpha)} - 1\right).$$

Therefore, the endemic equilibrium point given by:

$$E = \left(\frac{\lambda^\alpha + \mu^\alpha}{\gamma^\alpha}, 0, \frac{\mu^\alpha}{\gamma^\alpha}(R_0 - 1), \frac{\lambda^\alpha}{\gamma^\alpha}(R_0 - 1)\right).$$

5 Local stability of the disease free equilibrium:

Theorem 5.1. The disease free equilibrium point of system (2.2) is locally asymptoticaly stable (LAS) if $R_0 < 1$ and unstable if $R_0 > 1$.

Proof . To analyse the stability of disease free equilibrium point P_F, the associated Jacobien of the model is evaluted at P_F, this gives:

$$J(P_F) = \begin{pmatrix} -\mu^\alpha & 0 & -\frac{\gamma^\alpha \Lambda_N^\alpha}{\mu^\alpha} & 0 \\ 0 & -\mu^\alpha & 0 & 0 \\ 0 & 0 & \frac{\gamma^\alpha \Lambda_N^\alpha}{\mu^\alpha} - (\lambda^\alpha + \mu^\alpha) & 0 \\ 0 & 0 & \lambda^\alpha & -\mu^\alpha \end{pmatrix}$$

the characteristic equation of $J(P_F)$ is

$$\begin{aligned} P_{J(P_F)}(X) &= det(XI - J(P_F)) \\ &= (X + \mu^\alpha)^3 (X - \tfrac{\gamma^\alpha \Lambda_N^\alpha}{\mu^\alpha} + (\lambda^\alpha + \mu^\alpha)) \end{aligned}.$$

with eigenvalues,

$$\lambda_1 = -\mu^\alpha \quad \text{and} \quad \lambda_2 = \frac{\gamma^\alpha \Lambda_N^\alpha}{\mu^\alpha} - (\lambda^\alpha + \mu^\alpha).$$

It is clear that the eigenvalues λ_1 is negative, while λ_2, is negative if and only if

$$R_0 = \frac{\gamma^\alpha \Lambda_N^\alpha}{\mu^\alpha(\lambda^\alpha + \mu^\alpha)} < 1.$$

Hence, P_F is locally asymptotically stable. □

6 Global stability of the disease free equilibrium:

As the composant $R(t)$ does not appear in the first three equations of (2.1). We can restrict ourselves to the system (2.2)

$$\begin{cases} {}^cD^\alpha S_N(t) = \Lambda_N^\alpha - \gamma^\alpha S_N(t)I(t) - \mu^\alpha S_N(t), \\ {}^cD^\alpha S_C(t) = -\beta^\alpha S_C(t)I(t) - \mu^\alpha S_C(t), \\ {}^cD^\alpha I(t) = \gamma^\alpha S_N(t)I(t) + \beta^\alpha S_C(t)I(t) - \lambda^\alpha I(t) - \mu^\alpha I(t). \end{cases} \tag{6.1}$$

Theorem 6.1. Be $\overline{x} = 0$ a point of equilibrium of the system with t_0 and $D \in \mathbb{R}^n$ a neighborhood of the origin. That is $V(t, x(t)) : [0, \infty) \times D \longrightarrow \mathbb{R}$ a continuously differentiable and locally Lipschitzian function with respect to x such that:

$$\alpha_1 ||X||^a \leq V(t, x(t)) \leq \alpha_2 ||X||^{ab}$$
$${}^cD_0^\beta V(t, x(t)) \leq -\alpha_3 ||x||^{ab},$$

where, $t \geq 0$, $x \in D$, $0 < \beta < 1$, α_1, α_2, α_3, a and b are strictly positive constants. So the point of equilibrium $\overline{x} = 0$ is Mittag-Leffler stable. If, $\overline{x} = 0$ is globally Mittag-Leffler stable.

Proof . We consider the following Lyapunov function:

$$V(S_N, S_c, I) = S_N + I + S_c$$

$$\begin{aligned} {}^cD^\alpha V(S_N, S_c, I) &= {}^c D^\alpha S_N + {}^c D^\alpha I + {}^c D^\alpha S_c \\ &= \Lambda_N^\alpha - \mu^\alpha S_N - \mu^\alpha S_C - \lambda^\alpha I - \mu^\alpha I \\ &= -\mu^\alpha \left[S_N + S_c + I + \frac{\lambda^\alpha}{\mu^\alpha} I - \frac{\Lambda_N^\alpha}{\mu^\alpha} \right] \\ &\leq -\mu^\alpha \left[S_N + S_c + I + \frac{\lambda^\alpha}{\mu^\alpha} I \right] \\ &\leq -\mu^\alpha \left[3 \max(S_N, S_c, I) + \frac{\lambda^\alpha}{\mu^\alpha} \max(S_N, S_c, I) \right] \end{aligned}$$

(With, $S_N \leq \max(S_N, S_c, I)$, $S_c \leq \max(S_N, S_c, I)$, $I \leq \max(S_N, S_c, I)$.)

$$\begin{aligned} &\leq -\mu^\alpha \left[3 \, ||X||_\infty + \frac{\lambda^\alpha}{\mu^\alpha} \, ||X||_\infty \right] \\ &\leq -\mu^\alpha \left[(3 + \frac{\lambda^\alpha}{\mu^\alpha}) \, ||X||_\infty \right] \end{aligned}$$

Now we go on to verify the second relation of the theorem. We have,

$$S_N \leq S_N + I + S_c, \; S_c \leq S_N + I + S_c, \; I \leq S_N + I + S_c.$$

So, $\max(S_N, S_c, I) \leq S_N + I + S_c$. Therefore,

$$||X||_\infty \leq V(S_N, S_c, I) = S_N + I + S_c \leq 3 \, ||X||_\infty.$$

□

7 Numerical simulations

To illustrate the theoretical results obtained in the previous sections, we provide a numerical scheme for finding the solution of the fractional-order. First, we choose, $R_0 = 2.2$; $N = 47000000$; $\mu = 0.05$; $\beta = 0.07$; $\lambda = 0.06$; In this case; According to the theorem the equilibrium points is globally asymptotically stable.

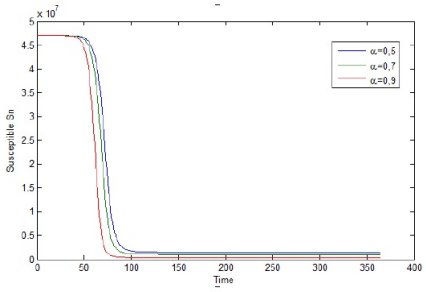


Figure 1: The initial condition is set to be $S_N(0) = 0$ for $\alpha = 0,5$, $\alpha = 0,7$, $\alpha = 0,9$. The susceptible population who not already had the covid-19 converges asymptotically to $S_N = \frac{\lambda^\alpha + \mu^\alpha}{\gamma^\alpha} = 0,3$

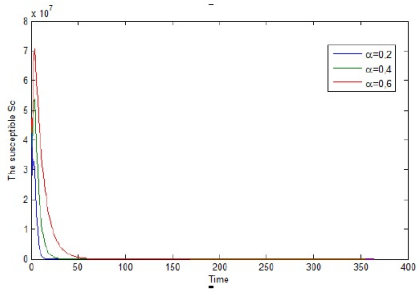


Figure 2: The initial condition is set to be $S_C(0) = 0$ for $\alpha = 0,2$, $\alpha = 0,4$, $\alpha = 0,06$. The susceptible population who already had the covid-19 converges asymptotically to $S_C = 0$.

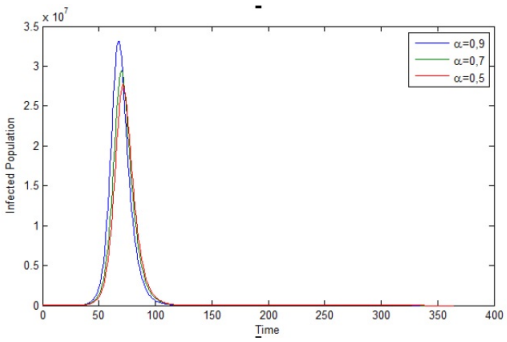


Figure 3: The initial condition is set to be $I(0) = 0$ for $\alpha = 0,5$, $\alpha = 0,7$, $\alpha = 0,9$. Infected population by the British variant covid-19 converges asymptotically to $I = 0$.

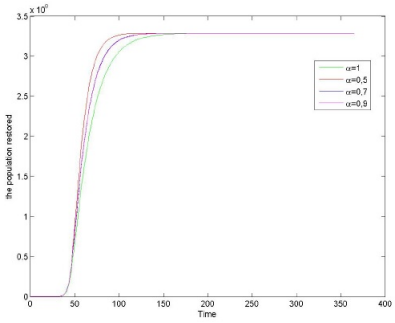


Figure 4: The initial condition is set to be $I(0) = 0$ for $\alpha = 0,5$, $\alpha = 0,7$, $\alpha = 0,9$. Infected population by the British variant covid-19 converges asymptotically to R.

References

[1] R. Almeida, A.M.C. Brito da Cruz, N. Martins, M. Teresa, and T. Monteiro, *An epidemiological MSEIR model described by the Caputo fractional derivative*, Int. J. Dyn. Control **7** (2019), 776–784.

[2] R.M. Anderson and R.M. May, *Infectious Diseases of Humans: Dynamics and Control*, Oxford University Press, Oxford, 1991.

[3] P. Debnath, H.M. Srivastava, P. Kumam, and B. Hazarika, *Fixed Point Theory and Fractional Calculus: Recent Advances and Applications*, Springer, Singapore, 2022.

[4] B. Derdei, *Study of epidemiological models: Stability, observation and estimation of parameters*, University of Lorraine, 2013.

[5] K. Diethelm, *A fractional calculus based model for the simulation of an outbreak of dengue fever*, Nonlinear Dyn. **71** (2013), no. 4, 613–619.

[6] W.O. Kermack and A.G. McKendrick, *Contributions to the mathematical theory of epidemics, II—the problem of endemicity*, Proc. R. Soc. Lond. A **138** (1932), 55–83.

[7] W.O. Kermack and A.G. McKendrick, *Contributions to the mathematical theory of epidemics, III—further studies of the problem of endemicity*, Proc. R. Soc. Lond. A **141** (1933), 94–122.

[8] A.E. Gorbalenya, S.C. Baker, and R.S. Baric, *The species Severe acute respiratory syndrome-related coronavirus: Classifying 2019-nCoV and naming it SARS-CoV-2*, Nature Microbio. **5** (2020), no. 4, 536–544.

[9] W.O. Kermack and A.G. McKendrick, *A contribution to the mathematical theory of epidemics*, Proc. R. Soc. Lond. A **115** (1927), 700—721.

[10] A. Korobeinikov, *Lyapunov functions and global stability for SIR and SIRS epidemiological models with non-linear transmission*, Bull. Math. Biol. **68** (2006), no. 3, 615—626.

[11] Y. Li, Y.Q. Chen, and I. Podlubny, *Stability of fractional-order nonlinear dynamic systems: Lyapunov direct method and generalized Mittag-Leffler stability*, Comput. Math. Appl. **59** (2010), no. 5, 1810–1821.

[12] M.Y. Li, J.R. Graef, L. Wang, and J. Karsai, *Global dynamics of a SEIR model with varying total population size*, Math. Biosci. **160** (1999), no. 2, 191–213.

[13] S. Momani and S. Hadid, *Lyapunov stability solutions of fractional integrodifferential equations*, Int. J. Math. Math. Sci. **47** (2004), 25032507.

[14] F. Ndaïrou, I. Area, J.J. Nieto, and D.F.M. Torres, *Mathematical modeling of COVID-19 transmission dynamics with a case study of Wuhan*, Chaos Solitons Fractals **135** (2020), 109846.

[15] X. Wang, Z. Wang, H. Shen, Dynamical analysis of a discrete-time SIS epidemic model on complex networks, Appl. Math. Lett. **94** (2019) 292–299.

[16] L. Zhang, J. Li, and G. Chen, *Extension of Lyapunov second method by fractional calculus*, Pure Appl. Math. **3** (2005), 1008–5513.