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# Stability analysis and adaptive tracking control for a class of switched nonlinear systems based on a nonlinear disturbance observer

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**Abstract**

This paper is concerned with developing an adaptive method on the basis of a nonlinear disturbance observer (NDO) in order to control a switched nonlinear system in the presence of unknown functions and external disturbances, and under arbitrary switching signals. The proposed approach employs an adaptive backstepping technique, NDO, a fuzzy logic system (FLS), and the particle swarm optimization (PSO) algorithm. Based on a common Lyapunov function (CLF), the adaptive backstepping technique is used to design a nonlinear state-feedback controller. Also, NDO and FLS are stated to estimate the disturbances and the unknown nonlinear functions, respectively. In addition, to improve the performance of the closed-loop system, the PSO algorithm is used to optimize the controller parameters. Finally, simulation examples are taken into account to demonstrate the effectiveness of the proposed strategy.

Keywords: switched systems, tracking control, adaptive backstepping technique, disturbance observer
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## 1 Introduction

Switched systems are an important kind of hybrid systems that consist of several subsystems and a signal that establishes switching behavior among them. The motivation for studying such systems arises mainly from two aspects. Firstly, switching behavior can be found in a wide range of physical and engineering systems, including electrical circuits, power systems, network control and aircrafts [5, 11, 18, 23]. Secondly, some complex systems can be modelled as switched systems with multiple subsystems, which is convenient for study and analysis.

So, the stability analysis of switched systems has more difficulties and has attracted the interest of many scientists in recent years [15, 31]. Meanwhile, when dealing with the stability and stabilization of switched systems, multiple Lyapunov functions [29, 33] and common Lyapunov function (CLF) [1, 2, 17] can often be considered as useful tools.

Adaptive control can help manage structural and environmental uncertainties and improve the system performance [13, 16, 21]. So, along with the great development of the switched systems, various tracking control methods are studied for engineering applications and dynamic processes [14, 32]; but only a few useful investigations on adaptive tracking control problem of switched systems have been obtained [26, 27]. By considering switched system in non-strict feedback

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form, the adaptive tracking problem for a class of uncertain switched nonlinear systems is investigated in [26]. In [27], the problem of adaptive tracking control is focused for a class of switched strict-feedback nonlinear systems with unknown time-varying delays and asymmetric saturation actuators under arbitrary switching. Backstepping control can also be designed for some types of switched nonlinear systems [19, 20, 34]. Furthermore, neural networks, fuzzy systems and backstepping can be utilized to design adaptive closed-loop systems ensure that their tracking error and signals are bounded [9, 28].

Over the last few years, various effective methods and techniques have been employed to attenuate or eliminate disturbances in both switched and non-switched systems [3, 6, 7, 24, 30]. A general NDO is designed for a nonlinear system with an unknown fast time-varying disturbance [24]. An NDO-based control approach is proposed in [7] for spacecraft formation flying by enhancing the disturbance attenuation ability and performance robustness of the asymptotic tracking control. In [6], a novel composite disturbance observer-based adaptive controller with adaptive laws is proposed for a tracking problem of an uncertain switched linear system.

This paper proposed the nonlinear adaptive control approach based on CLF and NDO for nonlinear switched systems in the presence of external disturbances and unknown nonlinear functions under arbitrary switching. By proposing an efficient method based on the common Lyapunov function, the output tracking control using the state-feedback controller is investigated. The control strategy is performed using adaptive backstepping where the Mamdani fuzzy logic system is employed to estimate the unknown functions.

Thus, compared with the existing results, the main contribution of this article is as follows:

1. Introducing the adaptive NDO to estimate external disturbances in nonlinear switched systems with arbitrary switching signals;
2. Estimating unknown functions of switched nonlinear systems using a fuzzy logic system;
3. Developing an adaptive nonlinear controller with a single adaptive law to avoid over parameterization and an adaptive weighted PSO to optimize the controller parameters at the same time.

The rest of this paper is organized as follows. In Section 2, problem formulation and some definitions are assumed. The design of NDO and the implementation of fuzzy rules are expressed in Section 3; furthermore, the adaptive weighted PSO algorithm is proposed for optimizing the nonlinear adaptive controller in this section. In section 4, stability analysis of the proposed method based on CLF is stated. Some simulation examples are provided to illustrate the effectiveness of the proposed approach in section 5. Eventually, conclusions are addressed in the last section (6).

**Notation:** In this article, the notations are considered standard. $R^2$ illustrate the n-dimensional Euclidean space, and $\| \bullet \|$ denotes the Euclidean vector norm. For positive integer $1 \leq i \leq n$, it also denotes $\psi_{i,\min} = \min\{\psi_{i,j}, 1 \leq j \leq m\}$ and $\psi_{i,\max} = \max\{\psi_{i,j}, 1 \leq j \leq m\}$. $L_2(0,\infty)$ is the space of square integrable functions on $(t_0,\infty)$ and $t_0$ is the initial time.

## 2 Problem formulation and preliminaries

Consider a class of switched nonlinear systems with disturbances as follows:

$$\begin{aligned} \dot{x}_i &= \hbar_{i,\sigma(t)} x_{i+1} + f_{i,\sigma(t)}(\bar{x}_i) \\ &\vdots \\ \dot{x}_n &= \hbar_{n,\sigma(t)} u_{\sigma(t)} + f_{n,\sigma(t)}(\bar{x}_n) + d_{\sigma(t)} \\ y &= x_1 \end{aligned} \tag{2.1}$$

where $\bar{x}_i = [x_1, ..., x_i]^T \in R^n \quad i = 1, 2, ..., n$ represents the system states and $y \in R$ the system output, where $n$ is the number of the states; the function $\sigma(t) = k : [0, +\infty) \rightarrow M = \{1, 2, ..., m\}$ is the switching signal, and $m$ is the number of the subsystems; also, $d_{\sigma(t)} \in R^m$ and $u_{\sigma(t)} \in R^m$ are the bounded external disturbances which belong to $L_2(0,\infty)$ and the control input of the $k^{th}$ subsystem, respectively; besides, $\hbar_{i,\sigma(t)}$ and $f_{i,\sigma(t)}(\bar{x}_i)$ are positive constants and unknown nonlinear functions, respectively.

The objective is to design a state-feedback controller with a NDO in a way that the system output tracks $y_d(t)$ as a pre-specified time-varying signal; in addition, all the closed-loop signals of the switched nonlinear system remain bounded. So, the following assumptions and lemmas are considered according to system (2.1).

**Assumption 1:** The system's disturbances $d_{\sigma(t)}$, the tracking target $y_d(t)$ and their derivatives up to their $n^{th}$ order are continuous and bounded.

## 3 Method strategy

As mentioned before, to design an appropriate controller, three steps are proposed in this paper:

step 1: reduce effect of external disturbances by using NDO;

step 2: estimate unknown functions of switched system (2.1) by employing FLS;

step 3: reduce the effect of over parameterization and optimize the approach's parameters in the controller design procedure by using adaptive PSO as well as the method with only one adaptive law, which is proposed in the next section to improve this problem.

In the below figure, the structure of the proposed approach is illustrated.

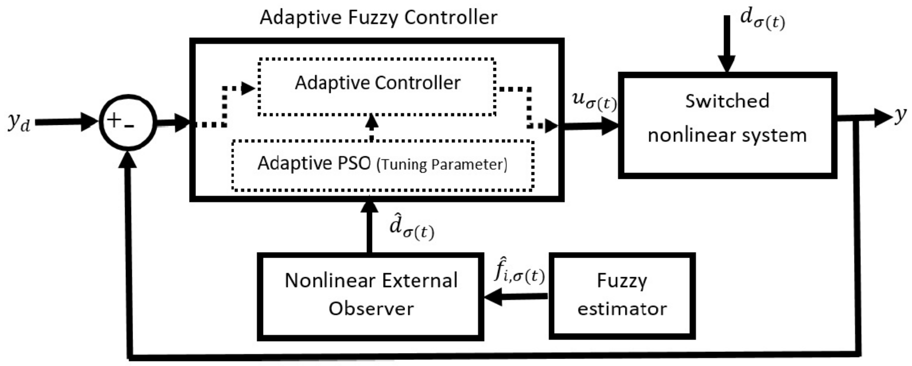


Figure 1: Structure of the proposed approach

So, in the following, all steps are expressed in detail and they will be used in section 4.

### 3.1 Nonlinear disturbance observer

In this subsection, the nonlinear disturbance observer using the state information and the system's input has been proposed to estimate the external disturbances $d_{\sigma(t)}$. In the following, the reference model equation of the switched system (2.1) with disturbances is expressed:

$$\dot{x}_n = \hbar_{n,\sigma(t)} u_{\sigma(t)} + f_{n,\sigma(t)}(\bar{x}_n) + d_{\sigma(t)} \tag{3.1}$$

According to the NDO structure [6, 7, 10] the net disturbance vector is estimated as follows ($\delta(t) = k, k = 1, ..., m$):

$$\dot{\eta}_k = -l_{d_k}\eta_k - l_{d_k}[f_{n,k}(\bar{x}_n) + \hbar_{n,k}(x_n)u_k + p_k(x_n)] \tag{3.2}$$

$$\hat{d}_k = \eta_k + p_k(x_n) \tag{3.3}$$

where $\eta_k \in R^m$ is the internal state vector of the nonlinear disturbance observation. $\hat{d}_k \in R^m$ is the estimated disturbances, $p_k(x_n)$ is called the auxiliary of disturbance observer, and $l_{d_k} \in R^m$ is the gain of nonlinear disturbances.

Considering the disturbance estimation error as below:

$$e_{d_k} = \hat{d}_k - d_k \tag{3.4}$$

Using (3.1) and (3.2) and substituting them in the derivative of (3.3) leads to:

$$\begin{aligned}\dot{\hat{d}} &= -l_{d_k}\eta_k - l_{d_k}[f_{n,k}(\bar{x}_n) + \hbar_{n,k}(x_n)u_k + p_k(x_n)] + \dot{p}_k(x_n)\\ &= -l_{d_k}(\hat{d}_k - p_k(x_n)) - l_{d_k}[\dot{x}_n - d_k + p_k(x_n)] + \dot{p}_k(x_n)\\ &= -l_{d_k}\hat{d}_k - l_{d_k}[\dot{x}_n - d_k] + \dot{p}_k(x_n)\end{aligned} \tag{3.5}$$

By taking the derivative of (3.4) and substituting (3.5) into it, the tracking error dynamics are obtained:

$$\dot{e}_{d_k} = -l_{d_k}\hat{d}_k - l_{d_k}[\dot{x}_n - d_k] + \dot{p}_k(x_n) - \dot{d}_k = -l_{d_k}e_{d_k} - l_{d_k}\dot{x}_n + \dot{p}_k(x_n) - \dot{d}_k \tag{3.6}$$

Then, by assuming the observer gain $l_{d_k} = \partial p_k(x_n)/\partial x_n$, dynamics of the disturbance estimation error are expressed as follows

$$\dot{e}_{d_k} = -l_{d_k} e_{d_k} - \dot{d}_k \tag{3.7}$$

Also, the differential equation of the nonlinear disturbance observer is computed as

$$\dot{\hat{d}} = -l_{d_k}\hat{d}_k + l_{d_k}\dot{x}_n - l_{d_k}(\hbar_{n,k}u_k + f_{n,k}(\bar{x}_n)) \tag{3.8}$$

It is supposed that there exists a positive constant $\tau$ such that $|\dot{d}(t)| \leq \tau$ for all $t > 0$; therefore, it can be easily shown that the disturbance estimation error will tend to zero, and consequently, external disturbances can be estimated by the nonlinear disturbance observer $\hat{d}_k$.

### 3.2 Unknown function estimation by fuzzy logic systems

In the controller design and stability analysis procedure, FLSs will be used to estimate the unknown functions. Therefore, the following useful concept and lemma are first recalled.

The fuzzy logic systems comprise some "if-then" rules as follows:

$$R_i : If\ x_i\ is\ F_1^i\ and...and\ x_n\ is\ F_n^i\ then\ y\ is\ B^i \tag{3.9}$$

where $x = [x_1, x_2, ..., x_n]^T \in R^n$ and $y \in R$ represent the input and output of the FLS, respectively. Also, $F_1^i, F_2^i, ..., F_n^i$ and $B^i$ are fuzzy sets in R. By using the strategy of singleton fuzzification, product inference, and center-average defuzzification, the fuzzy logic system can be formulated as:

$$y(x) = \frac{\sum_{i=1}^{N} w_i \prod_{J=1}^{n} \mu F_J^l(x_J^i)}{\sum_{i=1}^{N}\left[\prod_{J=1}^{n} \mu F_J^l(x_j)\right]} \tag{3.10}$$

where $N$ represents the number of "if-then" rules, $w_i$ are the fuzzy membership functions at $\mu_{B^i}(w_i) = 1$; Let

$$T_i(x) = \frac{\prod_{J=1}^{n} \mu F_J^l(x_J^i)}{\sum_{i=1}^{N}\left[\prod_{J=1}^{n} \mu F_J^L(x_j)\right]} \tag{3.11}$$

$$T(x) = [T_1(x)...T_N(x)]^T\ and\ W = [w_1, w_2, ..., w_N]^T \tag{3.12}$$

The fuzzy logic system can be rewritten as follows:

$$y = W^T T(x) \tag{3.13}$$

**Lemma 3.1.** [22] Let $f(x)$ be a continuous function defined on a compact set $\Omega$. Then, for a given desired level of accuracy $\varepsilon > 0$, there exists a fuzzy logic system (3.13) such that

$$\sup_{x\in\Omega} |f(x) - W^T T(x)| \leq \varepsilon \tag{3.14}$$

**Remark 3.2.** According to lemma 3.1, which played a key role in the performance of the controller, all the membership functions should be chosen as Gaussian functions. In fact, within a bounded error $\varepsilon$, the linear combination of the basis function vector $T(x)$ represents the real continuous function $f(x) \in R$ that means $f(x) = W^T T(x) + \delta(\varepsilon)$, $|\delta(\varepsilon)| \leq \varepsilon$. It is noted that $0 < T^T T \leq 1$.

### 3.3 Adaptive weighted PSO algorithm

As a metaheuristic algorithm, PSO generates solutions to complicated mathematical problems based on the concept of swarm intelligence [4]. In other words, with the help of PSO, simplified simulations are performed based on the social behaviors of animals. This algorithm begins by randomly generating the so-called swarm as the initial population, and each member of the swarm, known as a particle, moves through the search space. Then, a fitness function is evaluated for all the particles, and its minimum is recorded as the optimal position. Finally, the particles optimize their position and velocity by communicating their position in each iteration to one another. In fact, the velocity is adjusted based on the historical behavior of the particles. So, it is expressed as follows:

$$v_{i+1}(t) = w_\ell v_i(t) + \upsilon\vartheta_2[x_{gbest} - x_i(t)] \tag{3.15}$$

$$x_{i+1}(t) = x_i(t) + v_{i+1}(t) \tag{3.16}$$

where $w_\ell$ is the inertia weight factor, $\upsilon$ is a positive acceleration coefficient, $\vartheta_1$ and $\vartheta_2$ are random functions on [0, 1], $x_i$ is position of the $i^{th}$ particle, $x_{pbest}$ is the best previous position of the $i^{th}$ particle, $x_{gbest}$ the best position achieved in the entire population, and $v_i$ is velocity of the $i^{th}$ particle. The current position of each particle is calculated using (3.15)–(3.16).

Adaptive weighted PSO is introduced to improve the searching potential of PSO. The acceleration coefficient $\upsilon$ in (3.15) is expressed as follows:

$$\upsilon = \upsilon_0 + \frac{t}{K_t} \tag{3.17}$$

where $K_t$ denotes the number of iterations and $t$ is the current generation, whereas $\upsilon_0$ lies on [0.5, 1] in every iteration. Equation (3.18) is used to modify the inertia weight $w_\ell$ which is a positive constant and it lies on [0, 1].

$$w_\ell = w_{\ell 0} + \vartheta_3(1 - w_{\ell 0}) \tag{3.18}$$

where $\vartheta_3$ is a random function on [0,1] and $0.5 \leq w_{\ell 0} \leq 1$. The controller parameters are chosen through conventional trial-and-error using the normal adaptive fuzzy backstepping method based on the nonlinear disturbance observer. The selected parameters cannot be guaranteed to be optimized. Adaptive weighted PSO can be used to tune the switched nonlinear controller and thus optimize the parameters. In other words, PSO can help optimize the parameters by tuning the adaptive fuzzy backstepping based on NDO. After adjusting the controller parameters (the swarm 9-dimensional particles, i.e. $r, \xi_{1,1}, \xi_{1,2}, \xi_{2,1}, \xi_{2,2}, c_1, c_2, B$) which are initialized as positive values, its performance is improved and the speed of the system response is dynamically promoted. Table 1 presents the search space of PSO.

Table 1: Adaptive fuzzy backstepping-based NDO: range of particles

| **Parameter** | **Position vector** | **Velocity vector** |
|---|---|---|
| $r$ | 0 to 100 | -1 to +1 |
| $\xi_{1,1}$ | 0 to 60 | -1 to +1 |
| $\xi_{1,2}$ | 0 to 60 | -1 to +1 |
| $\xi_{2,1}$ | 0 to 60 | -1 to +1 |
| $\xi_{2,2}$ | 0 to 60 | -1 to +1 |
| $c_1$ | 0 to 70 | -1 to +1 |
| $c_2$ | 0 to 70 | -1 to +1 |
| $B$ | 0 to 1 | -1 to +1 |

The fitness function is defined as the integral of time-weighted absolute value of error (ITAE) as follows:

$$ITAE = \int_0^T Te(t) \tag{3.19}$$

## 4 Adaptive backstepping control method

In this section, the adaptive fuzzy backstepping scheme based on NDO is proposed, in which the parameters of the control law are optimized with the adaptive weighted PSO. To achieve this purpose, first, one adaptive law is presented for a controller design approach. Then, by considering an appropriate common Lyapunov function $V$, common virtual function $\alpha_i$ will be designed; and finally, according to common virtual control law and NDO, the control law $u_k$ will be designed.

Considering the closed-loop system (2.1), the adaptive control law $u_k$ is chosen as follows:

$$u_k = \frac{1}{\hbar_{n,k}}\left(\frac{\hat{\theta}}{2\xi_{i,\min}^2} z_n + \lambda_n z_n + z_n + \hat{d}_k\right) \tag{4.1}$$

where $\hat{\theta}$ represents an estimate of $\theta = \sum_{i=1}^n \|W_{i,\max}\|^2$ and $z_n$ is internal controller parameter; $\xi_{n,k}$ and $\lambda_n$ are also positive constants which $\xi_{n,\min} = \min\{\xi_{n,k}, k \in M\}$. Denote $\lambda_n = \hbar_{n,\max} + c_n$ where $c_n$ is a positive design parameter and $\hbar_{n,\max} = \max\{\hbar_{n,k} : k \in M\}$.

By defining the adaptive law $\dot{\hat{\theta}}$ as the solution to the following differential equation (for $1 \le i \le n$) [34]:

$$\dot{\hat{\theta}} = \sum_{i=1}^{n} \frac{r}{2\xi_{i,\min}^2} z_i^2 - B\hat{\theta} \tag{4.2}$$

and using the nonlinear disturbance observer to update the estimation of external disturbance as $\hat{d}_k$ (for $1 \le k \le m$):

$$\dot{\hat{d}}_k = -l_{d_k}\hat{d}_k + l_{d_k}\dot{x}_n - l_{d_k}(f_{n,k}(\bar{x}_n) + \hbar_{n,k}(x_n)u_k) \tag{4.3}$$

Then, the common virtual control function $\alpha_i$ is defined to be the following form (for $1 \le i \le n-1$) [34]:

$$\alpha_i(X_i) = -\frac{1}{\hbar_{i,\min}}\left(\frac{\hat{\theta}}{2\xi_{i,\min}^2} + \lambda_i + \frac{1}{2}\right) z_i \tag{4.4}$$

where $\hbar_{i,\min} = \min\{\hbar_{i,k} : k \in M\}$ and $\lambda_i = \hbar_{i,\max} + c_i$ is a positive design parameter. The state $X_i$ is defined $[x_i^T, \hat{\theta}, \bar{y}_d^{(i)}]$ where $\bar{y}_d^{(i)} = [y, \dot{y}_d, ..., y_d^i]$ and $y_d^i$ being the $i^{th}$ derivate of $y_d$. Also, denote $z_1 = x_1 - y_d$ and $z_{i+1} = x_{i+1} - \alpha$ for $1 \le i \le n-1$.

**Theorem 4.1.** consider the switched nonlinear system (2.1) with the adaptive laws (4.2) and the proposed controllers (4.1). For $1 \le i \le n$ and $k \in M = \{1, ..., m\}$ there exist $W_{i,k}^T T_{i,k}(x)$ such that $\sup_{x\in\Omega} |\hat{f}_{i,k}(x) - W_{i,k}^T T_{i,k}(x)| \le \varepsilon_{i,k}$ where the approximation error $\varepsilon_{i,k}$ is bounded, and the initial value of $\hat{\theta}$ satisfies $\hat{\theta}(0) \ge 0$; then, the tracking error and closed-loop signals are bounded.

**Proof .** To analyze stability, the common Lyapunov function is candidate as

$$V = \frac{1}{2}\sum_{i=1}^{n} z_i^2 + \frac{1}{2r}\tilde{\theta}^2 + \frac{1}{2}e_{d_k}^2 \tag{4.5}$$

where $e_{d_k} = \hat{d}_k - d_k$ and $\tilde{\theta} = \theta - \hat{\theta}$ represents as the difference between $\theta$ and its estimate $\hat{\theta}$ So, the time derivative of $V$ is expressed as follows

$$\begin{aligned}
\dot{V} &= z_1(\hbar_{1,k}\alpha_1 + \hbar_{1,k}z_2 + f_{1,k} - \dot{y}_d) + \sum_{i=2}^{n-1} z_i(\hbar_{i,k}\alpha_i + \hbar_{i,k}z_{i+1} + f_{i,k} - \dot{\alpha}_{i-1}) \\
&\quad + z_n(\hbar_{n,k}u_k + f_{n,k} + d_k - \dot{\alpha}_{n-1}) - \frac{1}{r}\tilde{\theta}\dot{\hat{\theta}} - e_{d_k}^T(l_{d_k}e_{d_k} + \dot{d}_k) \\
&= z_1(\hbar_{1,k}\alpha_1 + \hbar_{1,k}z_2 + \hat{f}_{1,k}) + \sum_{i=2}^{n-1} z_i(\hbar_{i,k}\alpha_i + \hbar_{i,k}z_{i+1} + \hat{f}_{i,k}) + z_n(\hbar_{n,k}u_k + d_k + \hat{f}_{n,k}) - \frac{1}{r}\tilde{\theta}\dot{\hat{\theta}} - e_{d_k}(l_{d_k}e_{d_k} + \dot{d}_k)
\end{aligned} \tag{4.6}$$

where $\hat{f}_{1,k} = f_{1,k} - \dot{y}_d$, $\hat{f}_{i,k} = f_{i,k} - \dot{\alpha}_{i-1}$ $(2 \le i \le n)$ and the derivative of the common virtual control function is $\dot{\alpha}_{i-1} = \sum_{l=1}^{i-1} \frac{\delta\alpha_{i-1}}{\delta x_l}\dot{x}_l + \frac{\delta\alpha_{i-1}}{\delta\hat{\theta}}\dot{\hat{\theta}} + \sum_{l=0}^{i-1} \frac{\delta\alpha_{i-1}}{\delta y_d^{(l)}} y_d^{(l+1)}$. By using Lemma 3.1, the following equation is obtained for $1 \le i \le n$

$$\hat{f}_{i,k} = W_{i,k}^T T_{i,k} + \delta_{i,k}(x_i) \text{ and } |\delta_{i,k}(x_i)|\varepsilon_{i,k} \tag{4.7}$$

Thus, (4.6) is rewritten as below

$$\begin{aligned}
\dot{V} &= \sum_{i=1}^{n-1} z_i(\hbar_{i,k}\alpha_i + \hbar_{i,k}z_{i+1} + W_{i,k}^T T_{i,k} + \delta_{i,k}(x_i)) + z_n(\hbar_{n,k}u_k + d_k + W_{n,k}^T T_{n,k} + \delta_{n,k}(x_i)) - \frac{1}{r}\tilde{\theta}\dot{\hat{\theta}} - e_{d_k}(l_{d_k}e_{d_k} + \dot{d}_k) \\
&\le \sum_{i=1}^{n} z_i(W_{i,k}^T T_{i,k} + \varepsilon_{i,k}) + \sum_{i=1}^{n-1} z_i(\hbar_{i,k}\alpha_i + \hbar_{i,k}z_{i+1}) + z_n(\hbar_{n,k}u_k + d_k) - \frac{1}{r}\tilde{\theta}\dot{\hat{\theta}} - l_{d_k}e_{d_k}^2 + e_{d_k}\dot{d}_k
\end{aligned} \tag{4.8}$$

By using the control law (4.1), adaptive law (4.2) and the common virtual control function (4.4), we have

$$\dot{V} \le \sum_{i=1}^{n} z_i(W_{i,k}^T T_{i,k} + \varepsilon_{i,k}) + \sum_{i=1}^{n-1} \hbar_{i,k} z_i z_{i+1} - \sum_{i=1}^{n-1} z_i^2 \left( \frac{\hat{\theta}}{2\xi_{i,\min}^2} + \lambda_i + \frac{1}{2} \right) + \frac{B}{r}\tilde{\theta}\hat{\theta} - z_n \left( \frac{\hat{\theta}}{2\xi_{i,\min}^2} z_n + \lambda_n z_n + z_n + e_{d_k} \right)$$
$$- \frac{1}{r}\tilde{\theta} \sum_{i=1}^{n} \frac{r}{2\xi_{i,\min}^2} z_i^2 - (l_{d_k} e_{d_k}^2 + e_{d_k} \dot{d}_k)$$
$$\le \sum_{i=1}^{n} z_i(W_{i,k}^T T_{i,k} + \varepsilon_{i,k}) + \sum_{i=1}^{n-1} \hbar_{i,k} z_i z_{i+1} - \sum_{i=1}^{n} z_i^2 \left( \frac{\theta}{2\xi_{j,\min}^2} + \lambda_i + \frac{1}{2} \right) - z_n e_{d_k} + \frac{B}{r}\tilde{\theta}\hat{\theta} - l_{d_k} e_{d_k}^2 - e_{d_k} \dot{d}_k \tag{4.9}$$

One can also easily show that (note that $2mn < m^2 + n^2$)

$$\sum_{i=1}^{n-1} \hbar_{i,k} z_i z_{i+1} \le \hbar_{i,\max} \sum_{i=1}^{n} z_i^2 \tag{4.10}$$

$$\tilde{\theta}\hat{\theta} = \tilde{\theta}(\theta - \tilde{\theta}) \le \frac{1}{2}\theta^2 - \frac{1}{2}\tilde{\theta}^2 - z_n e_{d_k} - e_{d_k}\dot{d}_k \le \frac{1}{2} z_n^2 + \frac{1}{2}\tau_k^2 + e_{d_k}^2 \tag{4.11}$$

where the constant $\tau_k$ is upper bound of $\dot{d}_k$. After some simplifying, (4.9) is changed as follows

$$\dot{V} \le \sum_{i=1}^{n} z_i^2 \left\{ \|W_{i,k}\|^2 - \frac{\theta}{2\xi_{i,\min}^2} \right\} + \sum_{i=1}^{n} \left\{ -c_i z_i^2 + \frac{\xi_{i,k}^2 + \varepsilon_{i,k}^2}{2} \right\} + \frac{B}{2r}\theta^2 - \frac{B}{2r}\tilde{\theta}^2 + (1 - l_{d_k}) e_{d_k}^2 + \frac{\tau_k^2}{2} \tag{4.12}$$

By choosing an appropriate $\xi_{i,k}$ and knowing $\theta = \sum_{i=1}^{n} \|W_{i,\max}\|^2$, we have

$$\dot{V} \le \sum_{i=1}^{n} \left\{ -c_i z_i^2 + \frac{\xi_{i,k}^2 + \varepsilon_{i,k}^2}{2} \right\} + \frac{B}{2r}\theta^2 - \frac{B}{2r}\tilde{\theta}^2 + (1 - l_{d_k}) e_{d_k}^2 + \frac{\tau_k^2}{2} \tag{4.13}$$

Finally, inequality (4.13) can be rewritten as below

$$\dot{V} \le -\left\{ \sum_{i=1}^{n} c_i z_i^2 + \frac{B}{2r}\tilde{\theta}^2 + (l_{d_k} - 1) e_{d_k}^2 \right\} + \frac{B}{2r}\theta^2 + \frac{\tau_k^2}{2} + \sum_{i=1}^{n} \left\{ \frac{\xi_{i,\max}^2 + \varepsilon_{i,\max}^2}{2} \right\} \tag{4.14}$$

Let $a_0 = \min\{2c_i, B, 2(l_{d_k} - 1), 1 \le i \le n\}, b_0 = \frac{B}{2r}\theta^2 + \frac{\tau_k^2}{2} + \sum_{i=1}^{n} \frac{\xi_{i,\max}^2 + \varepsilon_{i,\max}^2}{2}$. Then,

$$\dot{V}_n \le -a_0 V_n + b_0 \tag{4.15}$$

$$V_n \le \left( V_n(0) - \frac{b_0}{a_0} \right) e^{-a_0 t} + \frac{b_0}{a_0} \qquad t \ge 0 \tag{4.16}$$

Thus, by holding inequality (4.17), all the signals of the closed-loop system are bounded.

$$\lim_{t \to \infty} |z_1| \le \sqrt{\frac{2b_0}{a_0}} \tag{4.17}$$

The proof is completed. □

## 5 Numerical examples

In this section, the proposed method is applied to some examples to demonstrate the feasibility and effectiveness of the proposed approach as well as to show that how the proposed approach is worked.

**Example 5.1.** Consider two-dimensional switched nonlinear system as follows:

$$\begin{aligned}\dot{x}_1 &= \hbar_{1,\sigma(t)}x_2 + f_{1,\sigma(t)}(\bar{x}_i)\\ \dot{x}_2 &= \hbar_{2,\sigma(t)}u_{\sigma(t)(t)} + f_{2,\sigma(t)}(\bar{x}_n) + d_{\sigma(t)}\\ y &= x_1, \quad y_d = \sin(t)\\ d_1 &= d_2 = 0.5\sin(0.8t), \text{ and } \sigma(t) = k \in \{1,2\}\end{aligned} \tag{5.1}$$

where $\hbar_{1,k} = [1,1], \hbar_{2,k} = [2,1], f_{1,k} = [x_1, \cos x_1], f_{2,k} = [x_1^2 x_2, x_1 x_2]$. The initial conditions are defined as $x_1(0) = 0.06, x_2(0) = 0.06, \hat{\theta}(0) = 2$ and $\hat{d}_1(0) = \hat{d}_2(0) = 0$. The NDO gains are also defined as $I_d = [0.1 \quad 0.25]$ for the disturbance in the first and second subsystems. For the adaptive controller, suboptimal parameters are first considered as follows:

$r = 1, c_1 = 2, c_2 = 2, B = 0.02, \xi_{1,k} = [0.025, 3.25]$ and $\xi_{2,k} = [0.55, 1.8]$. Then, adapted weighted PSO is employed to optimize the parameters as per Table 2

Table 2: Optimized parameters of the controller

| **Parameter** | **Velocity vector** |
|---|---|
| $r$ | 3.34 |
| $\xi_{1,1}$ | 0.15 |
| $\xi_{1,2}$ | 3.24 |
| $\xi_{2,1}$ | 1.55 |
| $\xi_{2,2}$ | 1.71 |
| $c_1$ | 1.88 |
| $c_2$ | 2.25 |
| $B$ | 0.011 |

For the first and second subsystems, the objective is to design the adaptive controller in a way that $\hat{d}_k$ could estimate the system disturbance $d_k$ which helps the system's output $y$ to track $y_d$ as the desired trajectory under an arbitrary switching system. Adaptive law $\hat{\theta}$, control laws $u_1$ and $u_2$, and NDO are given respectively as

$$\begin{aligned}\hat{\theta} &= \frac{r}{2\xi_{1,1}^2}z_1^2 + \frac{r}{2\xi_{2,1}^2}z_2^2 - B\hat{\theta}\\ \dot{\hat{d}}_1 &= -l_{d_1}(x)\hat{d} + l_{d_1}(x)(\dot{x}_2 - l_{d_1}(x)(f_{2,1}(x) + \hbar_{2,1}(x)u)\\ \dot{\hat{d}}_2 &= -l_{d_2}(x)\hat{d} + l_{d_2}(x)(\dot{x}_2 - l_{d_2}(x)(f_{2,2}(x) + \hbar_{2,2}(x)u)\\ u_1 &= -\frac{1}{\hbar_{2,1}}\left(\frac{\hat{\theta}}{2\xi_{2,1}^2}z_2 + \lambda_2 z_2 + z_2 + \hat{d}_1\right), u_2 = -\frac{1}{\hbar_{2,2}}\left(\frac{\hat{\theta}}{2\xi_{2,2}^2}z_2 + \lambda_2 z_2 + z_2 + \hat{d}_2\right)\end{aligned}$$

where $z_1 = x_1 - y_d, z_2 = x_2 - \alpha_1$ and $\lambda_2 = c_2 + \hbar_{2,1}$. The virtual control function $\alpha_1$ is defined as $\alpha_1 = -\frac{1}{\hbar_{1,2}}\left(\frac{\hat{\theta}}{2\xi_{1,1}^2}z_1 + \lambda_1 z_1 + \frac{z_1}{2}\right)$, where $\lambda_1 = c_1 + \hbar_{1,2}$. Figs 2 shows the simulation results of the designed controller. Fig. 2 depicts the system output and reference signal $y_d$ under the arbitrary switching signal. Figs. 3 and 4 show the tracking error $y - y_d$ and the trajectory of the nonlinear adaptive law, respectively.

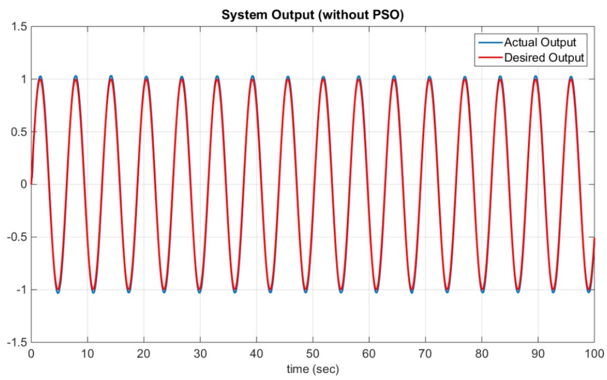


Figure 2: System output without PSO for example 5.1

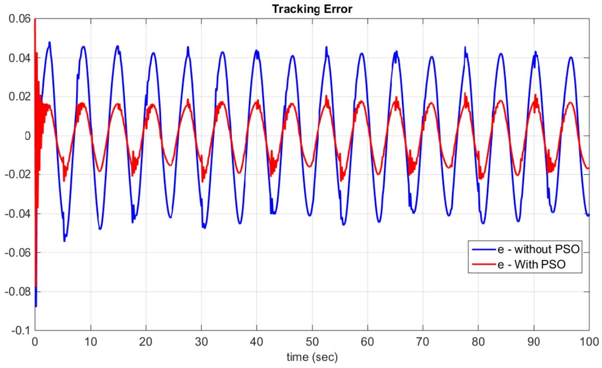


Figure 3: Tracking error of system with and without PSO for example 5.1

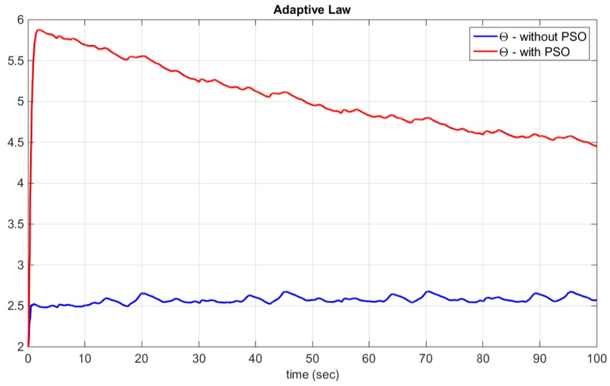


Figure 4: Adaptive law with and without PSO for example 5.1

As it is shown from the figures 2–4, the tracking error of the proposed method with adaptive weighted PSO has better performance than tracking error without it, but it has a slight difference in adaptive law. Fig. 5 illustrates the disturbance estimation error. As it is observed in this figure, the tracking error of the proposed controller without optimized parameters increases over time, which leads to better convergence of another controller to zero.

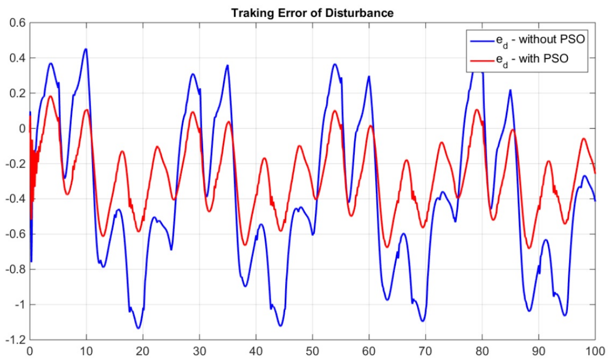


Figure 5: Tracking error of disturbance observer with and without PSO for example 5.1

Finally, the below figure depicts the switching signal.

According to Figs. 2 to 5, the performance of the system under disturbance observation is guaranteed; therefore, it can be derived that adaptive weighted PSO could decrease the disturbance estimation error as well as the output tracking error compared to the corresponding levels obtained without it; nevertheless, all the signals of the closed-loop system remained bounded using both methods.

**Example 5.2.** Consider a continuous stirred tank reactor with two modes feed stream in [25], the mathematical

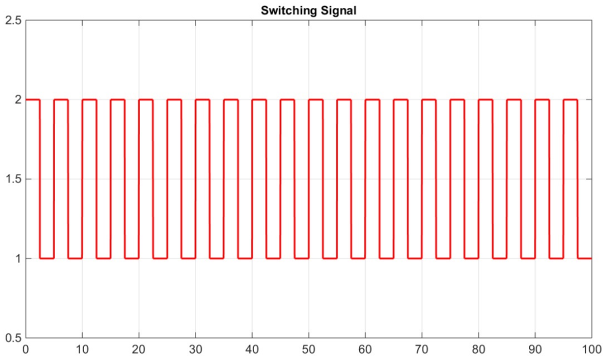


Figure 6: Switching signal for example 5.1

model can be written as

$$\begin{cases} \dot{C}_A = \frac{q_\sigma}{V}(C_{Af_a} - C_A) - a_0 e^{-\frac{E}{RT}} C_A \\ \dot{T} = \frac{q_\sigma}{V}(T_{f_a} - T) - a_1 e^{-\frac{E}{RT}} C_A - \frac{UA}{V_\rho C_\rho}(T_c - T) \end{cases} \tag{5.2}$$

The physical meaning of the above system's parameters can be found in [25] which is depicted in fig. 7.

Using the method in [12], the system can be expressed as the following switched system consisting of two subsystems:

$$\begin{cases} \dot{x}_1(t) = h_{1,\sigma} x_2(t) + f_{1,\sigma}(x_1(t)) \\ \dot{x}_2(t) = h_{2,\sigma} u(t) + f_{2,\sigma}(x_1(t)) + d_\sigma(t) \end{cases} \tag{5.3}$$

where system parameters are $h_1 = [1\ 1], h_2 = [1\ 1], f_1 = [0.5x_1\ 2x_1], f_2 = [0\ 0]$, and external disturbances are presumed to be $d_1 = d_2 = 0.2\cos(0.5t)$.

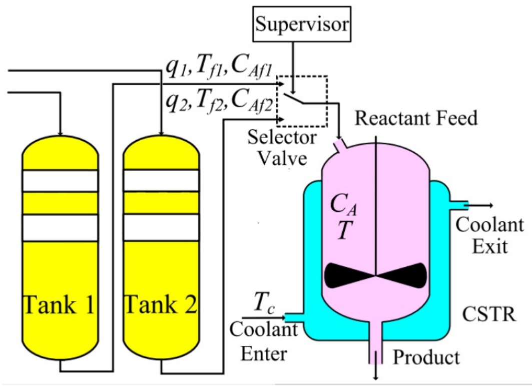


Figure 7: Schematic diagram of the process [25] for example 5.2

By considering initial conditions as $x_1(0) = 0.2, x_2(0) = 0.2, \hat{\theta}(0) = 0, \hat{d}_1(0) = \hat{d}_2(0) = 0$, periodic mismatched delays $\tau_s + \Delta\tau_s = (10 + 4\sin(3\pi t))$ and $\tau_c + \Delta\tau_c = (7 + 3\cos(\pi t))$ milliseconds, and also the NDO gains are defined as $I_d = [1.5\ 1.1]$, then the designing parameters are considered as follows:

$r = 0.3, c_1 = 1, c_2 = 8, B = 0.01$ and $\xi_{1,k} = [0.2, 0.4]$.

The performance of the stirred tank reactor is simulated by the proposed method and the results are compared with ref. [8]. Figures 8–12 depict the comparison of both methods.

As it is shown from the above figures, the proposed method with delay has better performance in both states, as well as the proposed approach is robust against external disturbances. Figure 12 shows that the tracking error of the disturbance decreases over time.

**Example 5.3.** In this example, switched nonlinear system in [34] is taken into account. In order to compare both

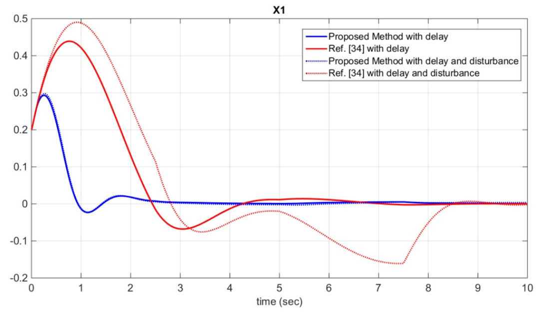


Figure 8: State response of system (x1) simulated by proposed method and Ref. [8] with mismatched delay and disturbance for example 5.2

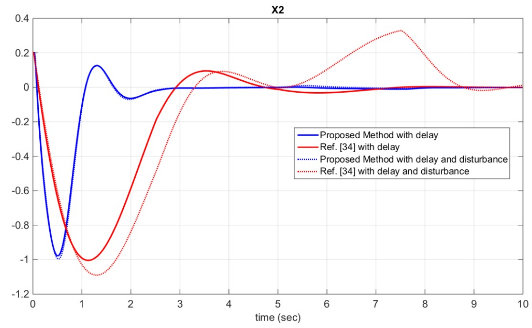


Figure 9: State response of system (x2) simulated by proposed method and Ref. [8] with mismatched delay and disturbance for example 5.2

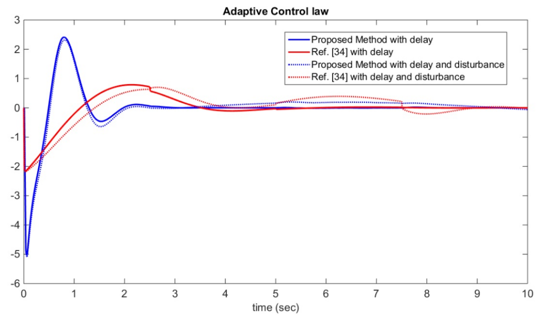


Figure 10: Adaptive control law simulated by proposed method and Ref. [8] with mismatched delay and disturbance for example 5.2

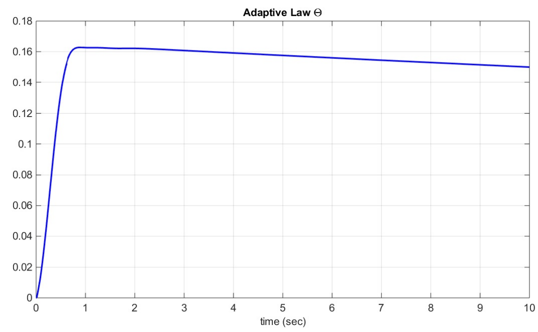


Figure 11: Adaptive law with zero initial value for example 5.2

methods (proposed and Ref. [34]), the design parameters of both methods are considered as [34]. Thus

$$
\begin{aligned}
\dot{x}_1 &= \hbar_{1,\sigma(t)} x_2 + f_{1,\sigma(t)}(\bar{x}_i) \\
\dot{x}_2 &= \hbar_{2,\sigma(t)} u_{\sigma(t)(t)} + f_{2,\sigma(t)}(\bar{x}_n) + d_{\sigma(t)} \\
y &= x_1, y_d = \sin(t), \\
d_1 &= 0.2\sin(0.8t), d_2 = 0.5\sin(0.3t), \text{ and } \sigma(t) = k \in \{1, 2\}
\end{aligned}
$$


Figure 12: Tracking error of disturbance observer for example 5.2

where $\hbar_{1,k} = [2,1], \hbar_{2,k} = [2,1], f_{1,k} = [x_1, \sin x_1], f_{2,k} = [x_1 x_2, x_1 x_2^2]$. The initial conditions are defined as $x_1(0) = 0.05, x_2(0) = 0.05, \hat{\theta}_1(0) = \hat{\theta}_2(0) = 0.5$. The common parameters are $\xi_{1,k} = [0.25, 3], \xi_{2,k} = [1.5, 1.8], c_1 = 2, c_2 = 2, r = 12$ and $B = 0.025$. Also, parameters of NDO are $\hat{d}_1(0) = \hat{d}_2(0) = 0$ and $I_d = [0.1\ 0.25]$. In the next, the results will be shown directly to avoid repeating the solving process. In Fig. 13, the adaptive laws of proposed method and Ref. [34] are depicted. In addition, both tracking errors are shown in Fig. 14. For comparison, some important criteria (including integrals of square error: $ISE = \int_0^\infty [e(t)]^2 dt$, absolute value of error: $IAE = \int_0^\infty |e(t)| dt$, the time weighted absolute value of the Error: $ITAE = \int_0^\infty t|e(t)| dt$ and the time weighted square value of the error: $ITSE = \int_0^\infty te^2(t) dt$) are computed to show better performance of the proposed approach. All the results are gathered in the following table.

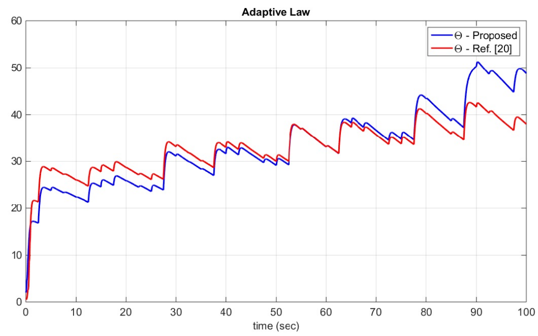


Figure 13: Adaptive laws of proposed method and Ref. [34] for example 5.3

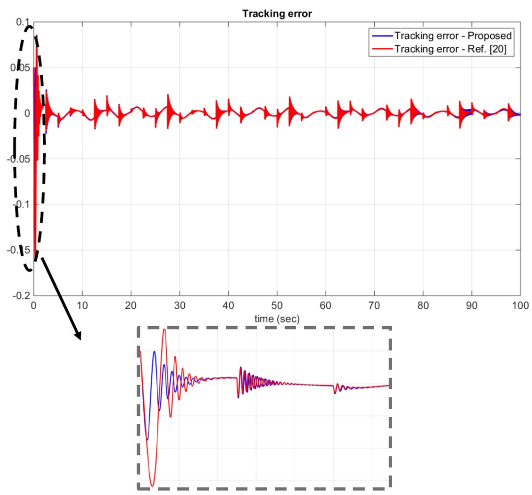


Figure 14: Tracking errors of proposed method and Ref. [34] for example 5.3

As it is demonstrated from Table 3, the proposed approach has better performance than Ref. [34]. Furthermore, the proposed method can reduce the negative effects of the disturbance on the system very well.

Table 3: Some important criteria for Example 3

| Criteria | Method | Tracking Error (e) |
|---|---|---|
| ISE | Proposed | 2.79 |
| | Ref. [34] | 8.24 |
| IAE | Proposed | 324.34 |
| | Ref. [34] | 371.65 |
| ITSE | Proposed | 13291 |
| | Ref. [34] | 14131 |
| ITAE | Proposed | 56.71 |
| | Ref. [34] | 64.69 |

## 6 Conclusion

In the present study, the adaptive controller comprising the nonlinear disturbance observer for dealing with the output tracking problem of the switched nonlinear system with unknown functions under arbitrary switching signals has been developed. To improve the performance of the proposed controller, the adaptive fuzzy backstepping technique is applied to this switched system, which uses adaptive weighted PSO to optimize the controller parameters. Moreover, the controller does not include the fuzzy basis function, so the computational burden of the controller is greatly reduced. Therefore, the structure of the proposed approach helped to improve the tracking speed and disturbance rejection of switched systems with unknown functions. In addition, the stability analysis of the designed controller indicated that under arbitrary switching signals, all the closed-loop signals remain bounded as well as ensured output tracking error close to zero, and consequently, system output proportionally tracked the reference signal with the minimum error.

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