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Evaluation and comparison of portfolio optimization with the degree of stock risk adjustment based on the performance measurement model based on the hybrid meta-heuristic algorithm and gray wolf optimization algorithm

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Abstract

The investment portfolio optimization process including allocation of assets allocated capital percentage to each asset, risk management, and creating a new portfolio with a certain level of risk and return based on investors' expectations has always been an attractive and controversial issue in the field of financial decision making. The objective of this research is to evaluate and compare portfolio optimization with the degree of stock risk adjustment based on the performance measurement model based on the hybrid meta-heuristic algorithm and gray wolf optimization algorithm. The statistical population of this research is the research statistical population which is all the listed companies in Tehran Stock Exchange for 7 years from 2014 to 2020. Based on the limitations imposed on the statistical population, the active companies in Tehran Stock Exchange have been investigated as the research sample. The obtained results from the tests show that a hybrid meta-heuristic algorithm improves the adjusted risk.

Keywords: portfolio optimization, degree of stock risk adjustment, performance measurement model, hybrid meta-heuristic algorithm, gray wolf optimization algorithm
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1 Introduction

Social and political activities and uncertainty about the future have increased with the spread of various risks and adverse events in the world, which is partly caused by the increase in the economy. Risk is one of the basic concepts in financial markets. Noticeably, the most important perception of risk is the feeling of financial loss. In other words,

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the risk is the possibility of occurring unpleasant events. This concept has a special complexity and its difficulty in the measurement and monitoring phase has brought to the fore the growing need of financial market participants to control and manage all types of risks [15].

Risk is an inherent part of the capital market whose origin can be from the general state of the economy to corporate events and margins. A part of the risk that is affected by the general state of the economy and affects the entire market is called systematic risk, and the rest is called unsystematic risk or diversification risk, or residual risk. The famous proverb "Don't put all your eggs in one basket" shows the idea of using a stock portfolio for risk management well. The theory of stock portfolio seeks to protect it against part of the risk known as an unsystematic risk by diversifying the contents of the portfolio. An individual must first measure the risk to reach a quantitative model of stock portfolio selection. Various indexes have been defined to measure risk [14]. The value at risk has opened its place to measure types of risk and this measure can be used to measure types of risk regarding the amount and types of risks faced by financial markets [18]. Value at risk is a statistical standard for measuring losses and measures risk quantitatively and conceptually. Therefore, it is included in the category of undesired risk indexes [5].

It is assumed in the single-round portfolio selection models that its contents remain unchanged until the end of the investment horizon after selecting the portfolio. Multi-round stock portfolio models allow the investor to review the contents of the portfolio at regular intervals and adjust it according to new information by removing this limitation. Therefore, multi-period models are more aligned with reality [10].

2 Theoretical bases

The issue of selecting an investment portfolio is one of the classic problems of the financial world, which was first raised by Markowitz (1959) and includes two main and inseparable components of return and risk. The main goal of this problem is to maximize the expected return at a certain level of risk or to minimize the expected risk at a certain level of return. The Markowitz model established the foundation of the single-period investment portfolio selection model. In the real world, an investor can revise his investment portfolio in any period, for this reason, investment portfolio management strategies are usually considered in several rounds [12].

Multi-period stock portfolio refers to a stock portfolio whose contents are reviewed by the investor and adjusted according to the new conditions after its formation at regular time intervals. The main problem in the multi-period stock portfolio theory is to find the optimal buying and selling policy in the direction of the change in the portfolio at the beginning of periods [4]. This effect becomes significant mostly when considering transaction costs making the selection of optimal portfolio in more realistic conditions especially when the number of transactions or transaction value increases [8]. The concept of value at risk as a new model of performance measurement was first proposed by Bamol in 1963. Since the early 1990s, it has been widely used as a tool for risk measurement. The reason for the popularity and the generality of this method was its simplicity in creating a summary statistical form of potential losses and a certain time horizon. Value-at-Risk truth was designed to provide a specific number containing information about the portfolio's risk to the analyst. This measure is an estimate of the level of loss on a portfolio or investment portfolio, which is predicted to be equal to or exceed it with a small probability. Value at risk, unlike traditional risk measures, provides a general and comprehensive view of portfolio risk. As a result, value at risk is a forward-looking performance measurement that is effective for all types of financial documents. The value-at-risk model includes three main factors of time horizon, confidence level, and capital value [3].

Various studies have been conducted to optimize multi-period stock portfolios. Geraviyan [6] has conducted research entitled "stable optimization and simulation of multi-period stock portfolio". In this research, the multi-period stock selection problem is modeled and solved by a family of robust models with uncertainty in stock prices and taking into account the transaction cost. The quality of the solution of alternative models in the problem is evaluated and compared. The simulation method is used to make these comparisons. The result of evaluating the answers of stable models is shown with the help of a numerical example. In this example, criteria such as average and variance of total income values, the possible amount of income with different probabilities, capital at risk, and capital at conditional risk have been calculated to compare the models.

In several recent decades, researchers modeled the behaviors of groups of organisms that have appropriate patterns to solve optimization problems. These algorithms are usually inspired by animal behaviors and physical phenomena with all their simplicity. Metaheuristic algorithms usually perform better than logical and mathematical methods to solve optimization problems with high dimensions. That is why their use is growing and every year newer algorithms such as humpback whale and sine cosine are developed to achieve speed and accuracy.

Dividing the search process into two steps of identification and utilization is a common feature among meta-

heuristic methods. The identification step is a random search of the entire search space to achieve the most desirable parts. The utilization phase is the more detailed search of the identified desirable parts of the search space. One of the challenging issues of meta-innovative methods is to find the right balance between these two steps.

Gray wolf algorithm (GWO) is a metaheuristic algorithm that uses a hieratical hierarchical structure inspired by the social life and hunting of gray wolves of four types of wolves to simulate leadership hierarchies. This algorithm is population-based, has a simple process, and can easily be generalized to large-scale problems. Gray wolves are considered apex predators, which are at the top of the food chain pyramid. Gray wolves prefer to live in a group and each group has an average of 5-12 members. Gray wolves have a very strict social hierarchy. The leaders of the group are one male and one female, who are called alphas. Alpha is mainly responsible to make decisions about hunting, sleeping place, waking up time, etc. Alpha's decisions are dictated to the group. However, a type of democratic behavior has been observed in which the alpha follows the other wolves in the group. In a group gathering, the entire group recognizes the alpha by holding its tail down. Since Alpha's orders must be followed by the group, alpha wolves are the only ones allowed to choose a mate in the group. Interestingly, the alpha is not necessarily the strongest member of the group, but the best member in terms of group management. This shows that the organization and order of a group are much more important than its power [9]. The second level in the hierarchy of gray wolves is beta. Greg Beta is Alpha's advisor and group organizer. Beta executes Alpha's commands throughout the group and reports back to Alpha. Omega has the lowest rank among gray wolves. Omega has the role of a victim. They are the last group of wolves that are allowed to eat. If a wolf is not alpha, beta, or omega, it is called subordinate (or delta in some references). The delta wolf must report to the alpha and beta but dominates the omega. In addition to the social hierarchy, hunting gray wolves has three stages: tracking, chasing and approaching the prey. We consider alpha as the best answer and beta and delta as the second and third best solutions to model the social hierarchy of wolves. We consider the rest of the candidate solutions as Omega. Optimization is guided by alpha, beta, and delta and the fourth group follows these three groups [7]. According to the mentioned cases, this research tries to answer the following question:

How is portfolio optimization with a degree of stock risk adjustment based on a performance measurement model based on a hybrid metaheuristic algorithm and gray wolf optimization algorithm?

3 Literature Review

Abdi et al. [1] They claimed based on the presented hybrid model of stock portfolio optimization based on price prediction with LSTM recurrent neural network with cardinality restrictions and multi-criteria decision-making methods (Tehran security stock case study) that stock price forecasting plays an important role in creating an efficient strategy with high returns because of the volatile nature of the market and the dynamics of the price movement. In addition, prediction results are a prerequisite to make a stock portfolio with an optimal structure. Therefore, the objective of this research is to provide a hybrid model to help investors in choosing the optimal stock portfolio. Thus, the top ten industries are selected based on the criteria affecting the value of the industries from among the active industries in the Tehran security exchange using the improved hierarchical analysis process. Then, the stock prices of active companies are predicted during the period from the beginning of May 2016 to the beginning of May 2021 in the desired time horizons by a recurrent neural network of short-term persistent memory. In the next step, three stock portfolios with short-term, medium-term, and long-term time horizons are selected using the hybrid solution method. Finally, optimum weights are determined and the efficient frontier is drawn using the mixed integer nonlinear programming method with the branch and cut algorithm based on Markowitz's limited property model. The results of the research show that the presented model gives more returns to investors in terms of risk in forming stock portfolios with specific time horizons than traditional methods.

Davoodi and Sadri [2] investigated and compared innovative algorithms in providing the optimal multi-period stock portfolio model based on the value-at-risk criterion. The multi-period stock portfolio allows the investor to review the contents of the portfolio at specific time intervals to provide an optimal multi-period stock portfolio selection model based on the value at risk despite transaction costs and adjust it according to new information. A sample of ten five-share portfolios was randomly selected from the listed companies in the Tehran Stock Exchange in 2009-2014, which has an average quarterly return higher than 0.1 including the annual risk-free return (0.20). The presented model has been optimized using two continuous and cumulative genetic algorithms of particles. The value-at-risk criterion was used to measure the effectiveness of the results of the two algorithms. The result of the research shows the higher efficiency of the obtained results from the cumulative algorithm of particles compared to the genetic algorithm.

Zhang et al., [17] investigated the reduction of dimensions in stock portfolio optimization based on mean variance. In this research, they showed that reducing the dimensions of asset prices can increase the overall efficiency of stock

portfolio optimization according to the frontier efficiency of the market.

Yan et al., [16] investigated the company's stock portfolio management in the public sector. The objective of this research is to relatively fill the gap in the research literature by identifying and discussing some of the key points that public sector organizations may need to consider when they use CPM. The research design - research methodology - research approach intends to answer a comprehensive research question instead of proposing and comparing specific and limited hypotheses which asks what are the key points that public sector organizations may need when selecting the organization portfolio management. Therefore, this research has selected the interpretative qualitative research model. The findings are based on the conducted empirical research about a large public research organization in Australia. The potential application of the organization's portfolio management was repeatedly and continuously investigated in a reference group that included 15 middle management representatives and several members of the senior leadership group for one year. Findings, the evaluation indicators that are usually used in the management of the organization's portfolio (for example, the growth potential of the market share) are usually not applicable in public sector organizations.

Jalota et al., [11] investigated a support system for portfolio optimization. In this research, they investigated the optimum models using fuzzy numbers and fuzzy incremental processes, and smart hybrid algorithms in 2008-2013. Then, they investigated the validation of the model in 2015-2013. The results showed that the intelligent hybrid algorithm can be used as a decision support system.

4 Methodology

The research method consists of all the tools and steps of systematic data collection and their logical analysis method to achieve a specific goal, which is generally to find facts [13]. Based on the nature and method of scientific research, it can be divided into five groups: historical, descriptive, correlational, causal (post-event), and empirical (experimental) research. The method of this research is descriptive and analytical because at the beginning, a picture of the current situation is presented, and it is applied based on the research type. This is quantitative research in terms of the type of research data and correlation in terms of the relationship between variables. Therefore, the unit of analysis is the company according to the nature and the topic investigated in the current research, and since the investigated society is the companies present in the security exchange.

4.1 Population and Statistical Samples

The statistical population of this research is all the listed companies on Tehran Stock Exchange. Due to the large size of the statistical community and the existence of some inconsistencies among the members of the community, the following conditions have been considered for the selection of the statistical sample as follows: 1. The companies' stock should have been traded in the stock market during each year of the research period. 2. The end of the financial year of the companies should be the end of last winter month, while the companies that changed the financial year are removed from the sample. 3. These companies should not stop operations or change the financial period during the review period. 4. The examined companies should not be investment companies. Better to say that the research period is 7 years from 2014 to 2020 which was investigated as the research sample of the existing listed companies.

4.1.1 Variables and Model

The Capital Asset Pricing Model (CAPM) was formed based on Capital Market Theory (CMT). However, the Markowitz model is a normative model that shows how investors should behave, and Capital Market Theory (CMT) is proof. In fact, general equilibrium theories such as Capital Market Theory (CMT), market line security (SML), and CAPM help us understand market behavior.

Our variables in this research include risk and return that each of which will be explained later. CAPM and CMT as equilibrium models have practical advantages in preparing:

1) A systematic risk scale,

2) An evaluating security scale,

3) To be a standard for performance scale.

Briefly, CMT deals with how assets should be priced if investors behave as suggested by Markowitz. CAPM uses the results of CMT to derive the relationship between the expected return and the systematic risk of each portfolio stock.

CAPM is a main paradigm in the financial field based on Markowitz's two-parameter portfolio analytical model. One of the necessary assumptions of this theory is homogeneous expectations, a complete market, the existence of the same borrowing rate, and risk-free lending. After considering these assumptions, CML can be obtained and decomposition theory can be proved. The result of this theory is that every investor will choose his optimal stock portfolio from the combination of two portfolios.

One has risk-free assets and another has a market stock portfolio. The evaluation of each stock in this collection leads to the clarification of the fact that the expected yield of the stock is a positive linear function of the (covariance) of that stock with the market portfolio. This relationship is called CAMP.

CAPM is known based on the concept of CML. Two important aspects of CML are decomposition theory and market stock portfolio theory. The main equation of the CAPM capital asset pricing model is as follows:

$$R_e = r_f + (r_M - r_f)\,\beta. \tag{4.1}$$

It can be said to explain gray wolf algorithm (GWO) that this algorithm includes 3 main steps:

1. Observing hunting, tracking and approaching.
2. Approaching, encircling circling the prey, and misleading (pursing and encircling) it until it stops moving
3. Attacking.

The mathematical model of blockade behavior is shown in the following equations. In which, the relations below t is the current iteration, A and C are the coefficient vectors, X_p is the position vector of the prey, and X is the position vector of the gray wolf.

$$\vec{D} = \left|\vec{C} \cdot \vec{X}_p(t) - \vec{X}(t)\right| \tag{4.2}$$

$$\vec{X}(t+1) = \vec{X}_p(t) - \vec{A} \cdot \vec{D}. \tag{4.3}$$

Vectors A and C are calculated as follows:

$$\vec{A} = 2\vec{a} \cdot \vec{r_1} - \vec{a} \tag{4.4}$$

$$\vec{C} = 2\vec{r_2} \tag{4.5}$$

In the mentioned relationships, the variable a decreases linearly from 2 to 0 during iterations, and r_1, r_2 are random vectors in the interval of [0, 1].

$$\vec{D}_\alpha = \left|\vec{C}_1 \cdot \vec{X}_\alpha - \vec{X}\right|, \quad \vec{D}_\beta = \left|\vec{C}_2 \cdot \vec{X}_\beta - \vec{X}\right|, \quad \vec{D}_\delta = \left|\vec{C}_3 \cdot \vec{X}_\delta - \vec{X}\right| \tag{4.6}$$

$$\vec{X}_1 = \vec{X}_\alpha - \vec{A}_1 \cdot \vec{D}_\alpha, \quad \vec{X}_2 = \vec{X}_\beta - \vec{A}_2 \cdot \vec{D}_\beta, \quad \vec{X}_3 = \vec{X}_\delta - \vec{A}_3 \cdot \vec{D}_\delta \tag{4.7}$$

$$\vec{X}(t+1) = \frac{\vec{X}_1 + \vec{X}_2 + \vec{X}_3}{3}. \tag{4.8}$$

4.2 Data collection method

The method of theoretical study and library is mainly used to study the literature on the subject and examine the background of the research, for the studies and views that have existed on the subject under research, and to create a suitable framework for examining the subject. Therefore, the theoretical section and the subject literature have been studied and reviewed by referring to library resources including books, weekly and monthly magazines, periodicals, publications of research and research centers, academic theses and related research theses, seminars and conferences, and searching in electronic databases. This research is based on the figures and real data of the stock market and financial statements of the active companies in the Tehran Stock Exchange. In this research, financial information was obtained from the financial statements and accompanying notes relating to the companies under study and with Tehran Stock Exchange organization compact discs, and finally, data were analyzed using statistical analysis software. Data collection tools in this research can be divided as follows according to the description of the data collection methods section: Rahavard-e-Novin software types of Mabna Company: provide the values of analyzed variables of research.

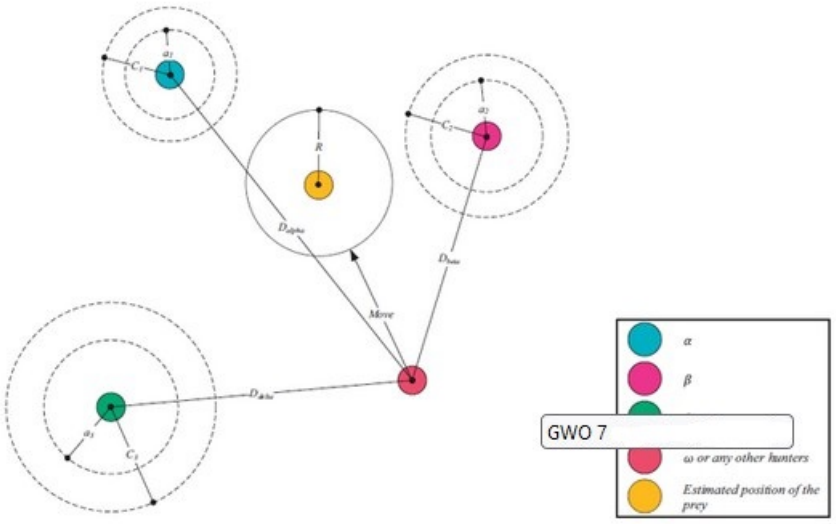

Figure 1: Figure 1. GWO7 Algorithm

4.3 Data Analysis Method

We analyzed data in this research after collecting it by the mentioned methods in the data collection part. We need tools to implement the process of data analysis. Since the data are from mathematical programming models, numerical methods of MATLAB software are used to solve them.

Since the return on assets is a random variable that is impressed by various market conditions, at first, the literature on the subject has been investigated to check the approach alignment, to model the relevant risk, and to predict the risk in the Tehran security exchange in this research. It can be understood by this specific method which market conditions are more risky and which portfolio is more volatile and which model is better.

In the second step, these conditions are integrated with portfolio optimization models. This approach provides the combination of the model to examine the characteristics of the portfolio in the Tehran Stock Exchange in specific market conditions. Practically, a portfolio optimization model to estimate the dependent structure that shows the linear and non-linear relationship between stock market variables is to minimize the volatility of the portfolio in this research.

In the third step, the portfolio optimization approach is implemented from several angles. The problem for investors is its occurrence using model weights in this approach. Based on the literature, the lack of risk management shows the amount of productivity of each portfolio or company, and its increase shows the efficiency of using portfolios or virtual machines. This increase in efficiency is directly proportional to risk. The higher this value is, the less free space in the portfolios is. Therefore, reducing the free space automatically increases the amount of risk management because there is not much free space in the portfolios.

The relative change percentage variable, compared to the best solution, can show the correctness of the performance of the hybrid metaheuristic algorithm. This parameter is the percentage of normalization of changes compared to the best answer and is displayed as a percentage with the adjusted risk parameter. In the calculation of this parameter, $F(h)$ is displayed as a metaheuristic value and $F(o)$ as an optimal value.

$$100 = (F(h) - F(o))/F(o) \times \text{adjusted risk.}$$

This variable shows that the difference between the approximate answer and the optimal answer does not increase significantly by data increase. This criterion shows the performance of the algorithm with data increase. It shows that the proposed method shows good performance in large data. Table 2 shows the relative percentage changes in the proposed algorithm.

The average relative percentage changes increase in the experimental data by data increase properly.

Data set	Work no.	No.	Nflb- TLBO	Nflb-GWO	Nflb- proposal method
Company1-U250_00	250	150	0.0145	0.0145	0.0145
Company1-U250_01	250	150	0.0195	0.0195	0.0193
Company2-U250_00	250	150	0.0145	0.0145	0.0138
Company2-U250_01	250	150	0.0195	0.0195	0.0178
Company3-U500_00	500	150	0.017	0.0163	0.0159
Company3-U500_01	500	150	0.0155	0.0147	0.0143
Company4-U1000_00	1000	150	0.0113	0.0103	0.0097
Company4-U1000_01	1000	150	0.014	0.0127	0.0119
Company-T60_00	60	100	0.1304	0.1304	0.1304
Company-T60_01	60	100	0.1304	0.1304	0.1304
Company6-T120_00	120	100	0.1111	0.1111	0.1023
Company6-T120_01	120	100	0.1111	0.1111	0.1013
Compny7-T249_00	249	100	0.117	0.109	0.101
Compny7-T249_01	249	100	0.1263	0.1226	0.1203
Compny8-T501_00	501	100	0.1211	0.1139	0.1107
Compny8-T501_01	501	100	0.1257	0.1231	0.1219

Table 1: Comparing the on-management of risk between the suggested hybrid metaheuristic algorithm and other algorithms

Data set	Work no.	No.	MFO	Optimum value	Adjusted risk
Company1-U250_00	250	150	100	99	1.99
Company1-U250_01	250	150	101	100	1.100
Company2-U250_00	250	150	100	99	1.99
Company2-U250_01	250	150	101	100	1.100
Company3-U500_00	500	150	201	198	3.198
Company3-U500_01	500	150	204	201	3.201
Company4-U1000_00	1000	150	403	399	4.399
Company4-U1000_01	1000	150	411	406	5.406
Company-T60_00	60	100	23	20	3.20
Company-T60_01	60	100	23	20	3.20
Company6-T120_00	120	100	45	40	5.40
Company6-T120_01	120	100	45	40	5.40
Compny7-T249_00	249	100	94	83	11.83
Compny7-T249_01	249	100	95	83	12.83
Compny8-T501_00	501	100	190	167	23.167
Compny8-T501_01	501	100	191	167	22.167

Table 2: The adjusted risk index in the performance of the suggested method.

5 Conclusion

Investors use the standard stock portfolio selection problem such as single-time. We assume that information about the future behavior of stocks is available individually. Based on this stock information, the objective of the standard stock portfolio selection problem is to maximize the rate of return and minimize the risk of the stock portfolio for a certain period. The important assumption in investing is that the investor forms the stock portfolio in a specific and predetermined period. This period can be a day, a week, a month, etc. Noticeably, decision-making is based on an analysis of the future behavior of stocks for a certain period with risk and return criteria.
The most significant issues for investors in the capital markets are to make the decision for the right security for investment and the formation of a stock optimized portfolio, which is done through risk and return evaluation. On the other hand, in the discussion of stock portfolios, if the return of the assets has a normal distribution, variance, and standard deviation are used to calculate the risk. However, the return on assets is not necessarily normal in the real world. In addition, it is sometimes significantly different from the normal distribution.
Selecting an appropriate stock portfolio and how to invest in it is one of the important and key topics that is discussed in the capital market and should be considered by investors. In this regard, the investigation and study of investors should be based on selecting the best investment portfolio according to the amount of risk and its return. Therefore, risk measurement is an important issue in investing in a stock portfolio.

One of the most important concerns of investors in the capital market is to select the optimum portfolio regarding profitability. Therefore, the variety of stock portfolio selection methods in investment and the complexity of decision-making have been greatly developed in recent decades. Traditional methods in stock portfolio selection and optimization do not have the necessary efficiency. Thus, using innovative algorithms has received more attention.

In this research, the stock portfolio selection model was presented based on adjusted risk measurement criteria using a hybrid metaheuristic algorithm. The results of the tests show that the size and number of the stock portfolio increase

dramatically by increasing the number of investors. The significant volume of investments shows the inappropriate performance of metaheuristic methods whose main reason is inappropriate local behavior in the allocation of investors in the stock portfolio. Furthermore, using a hybrid metaheuristic algorithm has improved the adjusted risk because these algorithms perform better in a dynamic environment. However, the percentage of error changes decreases from the optimum solution appropriately by increasing investors.

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