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Application of data envelopment analysis in determining the efficiency of management and company

Omid Dalir[a], Taghi Torabi[b,*], Mahnaz Rabiei[c], Yeganeh Mousavi Jahromi[d]

[a]*Department of Finance, Faculty of Management and Economics, Science and Research Branch, Islamic Azad University, Tehran, Iran*

[b]*Department of Economics,Faculty of Management and Economics, Science and Research Branch, Islamic Azad University, Tehran, Iran*

[c]*Department of Economics, Research Center for Modeling and Optimization in Science and Engineering, South Tehran Branch, Islamic Azad University, Tehran, Iran*

[d]*Department of Economics, Faculty of Management and Economics, Payam-e-Noor University, Tehran, Iran*

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Abstract

Data Envelopment Analysis (DEA) is one of the widely used methods in measuring the technical efficiency of organizations. Data envelopment analysis is a classical non-parametric technique based on mathematical programming, which is used to compare the evaluation of the efficiency of a set of similar decision-making units. Its significant advantage is that it does not need to determine parametric specifications (such as the production function) to get performance points. Data envelopment analysis is known as a valid and stable tool used in performance evaluation, which provides a single measure of performance for each unit relative to its peers. Even though the number of data envelopment analysis models is constantly increasing and each one has a specialized aspect, the basis of all is a number of main models, among which we can refer to the "Charnes, Cooper and Rhodes" models (1978) as CCR, in which the assumption of constant returns to scale (CRS) has been used in the analysis. There are different approaches to determining the effective factors on efficiency and productivity and how to measure the productivity of production factors. Among the investigated companies in the years 2019 to 2019, it was found that the technical efficiency of the companies decreased, the technological changes of the companies increased, the managerial efficiency of the companies decreased, the efficiency of the scale of the companies decreased, and the overall productivity of the companies faced a decrease.

Keywords: data envelopment analysis, efficiency, management efficiency, performance evaluation
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1 Introduction

The efficient market hypothesis states that the market is sensitive to new information and is quickly affected, and the price of securities is a reflection of information and shows its real value at any time. According to this hypothesis, informed investors show a rational reaction to this information. Several recent studies support the theory that capital markets tend to be quite sensitive to financial news. That means, after a large increase or a large decrease in stock

*Corresponding author
Email addresses: omiddalir614@yahoo.com (Omid Dalir), taghi.torabi100@gmail.com (Taghi Torabi), mahnaz.rabiei@yahoo.com (Mahnaz Rabiei), y.m.jahromi@gmail.com (Yeganeh Mousavi Jahromi)

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prices in the market, reverse price movements are visible. Overreaction is one of the anomalies of the capital market, and it occurs when the stock price changes more than it should change due to new information, which in the long run is associated with the return of the stock price. The role of the stock market to boost the economy of countries like Iran, which are faced with a huge amount of wandering capital on the one hand and a lack of investment resources on the other hand, is significant. Therefore, knowing the factors affecting the behavior of the stock market can be considered an effective step in the direction of capital in the Iranian economy [11, 13].

2 CCR pattern

The name of this pattern is taken from the first letters of the names of the three researchers who created it, namely Charnes, Cooper and Rhodes, and is considered as a basis for the formation of other patterns in data envelopment analysis [10]. This model has a fixed scale efficiency and tries to increase the efficiency fraction of the unit under investigation by choosing the optimal weights for the input and output variables in such a way that the efficiency of other units does not exceed the limit of one [7].

2.1 CCR ratio model and fractional programming

In the relative measurement of units, Farrell focused on the weighted sum of units to make a virtual unit and proposed the following relationship as a common measurement tool for measuring the technical efficiency:

$$\text{Balanced set of inputs/Balanced set of outputs} = \text{efficiency}$$

If the goal is to examine the efficiency of n units, each of which has m inputs and S outputs, the efficiency of the jth unit is calculated as follows:

$$\text{Management} = \frac{\sum_{r=1}^{s} u_r y_{rj}}{\sum_{i=1}^{m} v_i x_{ij}}$$

x_{ij}: I^{th} input rate for unit $(I = 1, 2, ..., m)j$

y_{rj}: r^{th} output rate for j^{th} unit $(r = 1, 2, ..., s)$

u_r: the weight given to the r^{th} output (r^{th} output value)

v_i: the weight given to the I^{th} input (i^{th} input cost)

The important thing in the above relation is that this efficiency measurement tool needs a set of weights that are used for all the units under investigation. In this regard, two points should be considered. First, the value of inputs and outputs can be different and it is difficult to measure them, and on the other hand, different units may organize their operations in such a way that they provide outputs with different values. Therefore, they need different weights in measuring the efficiency [9].

Charnes, Cooper and Rhodes [3] recognized the above problem and in order to solve it in their model, they assigned different weights to inputs and outputs and proposed units that can accept weights that are more suitable and enlightening for them in comparison with other units. Under these conditions, to evaluate the unit under investigation, which from now on will be called the zero unit, they use the solution of linear programming, which is called the CCR ratio model [12].

To make the model, suppose there are n units and the goal is to evaluate the performance of the unit under investigation (zero unit or decision-making unit) that consumes inputs $x_1, x_2, ..., x_m$ to produce outputs $y1, y2, ..., y_m$. If the allocation weights given to the outputs (or the price of the outputs) are denoted by $u_1, u_2, ..., u_s$ and the allocation weights given to the inputs (or the purchase cost of the inputs) are shown by $v_1, v_2, ..., v_m$, then CCR ratio model will be as follows [15]:

$$\begin{aligned} MAZ_0 &= \frac{\sum_{r=1}^{s} u_r y_{ro}}{\sum_{i=1}^{m} v_i x_{io}} \\ st: & \quad (j = 1, 2, ..., n) \\ \frac{\sum_{r=1}^{s} u_r y_{rj}}{\sum_{i=1}^{m} v_i x_{ij}} &\leq 1 \\ u_1, v_i \geq 0 & \quad (i = 1, 2, ..., m), (r = 1, 2, ..., s) \end{aligned}$$

The variables of the above problem are the weights, and the solution of the problem provides the most suitable and favorable values for zero unit weights and measures its efficiency. In the above model, if ur_s are very big and vis are very small, the value of the ratios expressing the limitations will be infinite and unlimited. To prevent such a problem, all the ratios (efficiency of the units) are considered smaller or equal to one and enter the model as a limitation [15].

According to the objective function, it is recognized that this model is a non-linear and non-convex model, which solves the value of v_i, u_r variables to measure the efficiency of the unit under investigation. The problem that exists in the above modeling is that this model has infinite answers. In order to avoid the infinity of the answer, the above model has been changed to the following linear programming model, for which fractional programming model and CCR linearization method can be used.

$$\max Z_0 = \sum_{r=1}^{s} y_{rj}\alpha_r$$

$$st: \quad \sum_{r=1}^{s} y_{rj}\alpha_r - \sum x_{ij}w_i \leq 0$$

$$\sum_{r=1}^{s} x_{ij}w_i = 1$$

$$\alpha_r, w_i \geq 0$$

To obtain the above relation, two times of variable change has been used.

$$tu_r = \alpha_r, \qquad tv_i = w_i$$

2.2 Input-oriented multiplicative CCR model

As mentioned, data envelopment analysis models are generally divided into two groups, input-oriented and output-oriented, and you will get to know this concept in different models. To convert the CCR ratio model into a linear programming model, consider the method used by Charnes and Cooper. In this method, it is argued that in order to maximize the value of a fractional expression, it is sufficient to consider the denominator of the fraction as equal to a fixed number and to maximize the numerator.

Based on this, the denominator of the fraction is set equal to one and a new model is obtained as follows. This model is called multiplicative form. Note that although the latest model has similarities with the variables and parameters of the previous model, it is a different and new model:

$$\max Z_0 = \sum_{r=1}^{s} u_r, y_{ro}$$

$$st: \quad \sum_{i=1}^{m} v_i x_{io} = 1$$

$$\sum_{r=1}^{s} y_{rj}\alpha_{rj} - \sum x_{ij}v_i \leq 1$$

$$u_r, v_i \geq 0 \qquad (j = 1, 2, ..., n)$$

Input-oriented models are models that reduce the inputs by keeping the outputs constant [1]. This concept will be clearly understood in the envelopment model, the envelopment model is a secondary problem of the multiplicative model [5].

2.3 Input-oriented CCR envelopment model

To make data envelopment analysis model, Charnes, Cooper and Rhodes [3] have reached an empirical relationship about the number of evaluated units and the number of inputs and outputs as follows:

$$(\text{Number of inputs} + \text{number of outputs}) \geq 3 \text{ the number of evaluated units}$$

Failure to practically use the above relationship causes a large number of units to be placed on the efficient border, and in other words, have an efficiency score of one, so the model's separation power is reduced. From there, a constraint must be written for each unit to obtain the linear programming model in which the number of constraints is more than the number of variables, and since the volume of operations in simplex solving is more dependent on the number of constraints than variables, therefore, solving the secondary problem of the multiplicative model will require less operations. If the variable corresponding to the constraint $\sum_{i=1}^{m} v_i x_{io} = 1$ in the secondary problem is θ and the variable corresponding to the constraints $\sum_{r=1}^{m} y_{rj} u_r - \sum_{i=1}^{n} x_{ij} v_i \leq 0$ is expressed by λ_j, the secondary model is obtained with a little change which is known as envelopment model:

$$
\begin{aligned}
& \min y_0 = \theta \\
st: \quad & \sum_{j=1}^{n} \lambda_j y_{rj} \geq y_{ro}, \quad (r = 1, 2, ..., s) \\
& \sum_{j=1}^{n} \lambda_j x_{ij} \leq \theta x_{io}, \quad (i = 1, 2, ..., m) \\
& \lambda_j \geq 0, \qquad \theta, (j = 1, 2, ..., n)
\end{aligned}
$$

Consider that in the primary model there were m input and s output and n free units in the free sign, according to which the primary problem has $(n+1)$ variables, the number of constraints of the secondary model is less than the primary problem and its solution requires less volume of operations. It should be mentioned that it shows a ratio of inputs and outputs of all units that are mixed together and make a virtual unit. θ is a coefficient that is included in the objective function and the model seeks to minimize it, and in cases where its value is smaller than one, it minimizes the number on the right side and provides less input resources to the virtual unit.

2.4 Output-oriented CCR models

Efficiency can be studied from two perspectives focusing on inputs and outputs. Charnes, Cooper and Rhodes [4] defined the efficiency according to these two perspectives as follows [2]:

2.5 Output-oriented multiplicative CCR model

$$
\begin{aligned}
& \max Z_0 = \sum_{i=1}^{m} x_{io} v_i \\
st: \quad & \sum_{r=1}^{s} y_{ro} u_r = 1 \\
& \sum_{r=1}^{s} y_{rj} u_r - \sum_{i=1}^{m} x_{ij} v_i \leq 0 \quad (j = 1, 2, ..., n) \\
& u_r, v_i \geq 0
\end{aligned}
$$

Output-oriented envelopment CCR model

$$
\begin{aligned}
& \max y_0 = \theta \\
st: \quad & \sum_{j=1}^{n} \lambda_j y_{rj} \geq \theta y_{ro} \qquad (r = 1, 2, ..., s) \\
& \sum_{j=1}^{n} x_j \lambda_j \leq x_{io} \qquad (i = 1, 2, ..., m) \\
& \lambda_j \geq 0, \qquad (j = 1, 2, ..., n)
\end{aligned}
$$

Here, θ is the variable corresponding to the first constraint and λ_j is the variable corresponding to the other constraints of the primary model. The model objective is to obtain the maximum output value. In this model, we have $\theta^* \geq 1$ and $\frac{1}{\theta^*}$ shows the level of efficiency [14].

3 Determination of input and output variables

The efficiency of companies is calculated by using data envelopment analysis (DEA). Data envelopment analysis measures the relative efficiency of units that have similar inputs and outputs. The efficiency or inefficiency of any decision-making unit depends on the performance of that unit in transferring inputs to its outputs compared to other units in a particular domain. This method creates an efficiency frontier between zero and 1 for companies. Companies with an efficiency score of 1 are very efficient, and companies with an efficiency score of less than 1 are below the efficiency limit and must reach the efficiency limit by reducing costs or increasing the revenues [6].

In this study, measuring the company efficiency with data envelopment analysis is derived from the model of Demirjan et al. [6]. To express the output of the company, the company sales variable has been used, and for the input, fixed assets, intangible assets, cost, and expenses have been used.

Equation (3.1) and model (1) have been used to measure the efficiency of the company.

$$\max_{y}^{\theta} = \frac{Sales_{it}}{V_1 COGS_{it} + V_2 SG\&A_{it} + V_3 PPE_{it} + V_4 OTHER\ INTAN_{it}} \tag{3.1}$$

In this model:

Sales is the revenue from sales for company i in year t

COGS is the cost of sold goods

SG&A is the general and administrative and selling expenses

PPE is the net fixed assets

Other intan is the intangible assets

MAX is the company efficiency

The efficiency of the companies is affected by two factors of specific corporate characteristics and the manager's ability. The following model shows this relationship, in which after removing the specific corporate characteristics, the remaining value expresses the manager's abilities.

The data and raw information needed for the research were collected by referring to the websites of the Securities and Exchange Organization, the information base of the Iranian capital market, the website of Iran's financial data analysis and by referring to their financial statements.

4 Change measurement in the productivity of all companies

When the total productivity change is greater than one, it indicates an increase in productivity and performance improvement, and if it is smaller than one, it indicates a decrease in productivity and deterioration of performance over time. In addition, when the total productivity changes are equal to one, it shows that there was no change in productivity during the period under investigation.

The results of total productivity changes indicate that 62.5 percent of companies have productivity changes of more than one, which means increasing the productivity growth and improving the performance, and 56.5 percent of companies have productivity changes of less than one, which means decreasing the productivity growth and deterioration of performance.

4.1 Measuring average changes in total productivity in companies

After measuring the total productivity components of companies, the changes in the productivity of the companies in 1400 compared to 2019 are examined.

The results of the table of the average changes in the productivity of companies during the years 2019 and 2019 show that the changes in technical efficiency, the changes in management efficiency and the changes in scale efficiency in 2019 are lower than in 2019, and this shows the decrease in the efficiency of companies from a technical, management and scale point of view. Also, the results show that the technological changes of companies have increased in 2019 compared to 2014. The results of changes in total productivity also indicate that the companies had a weaker performance in 2019 compared to 2014.

Table 1: Changes of total productivity

	Changes in total productivity (TFPCH)
DMU01	0.911
DMU02	0.852
DMU03	3.190
DMU04	0.879
DMU05	0.832
DMU06	1.511
DMU07	0.725
DMU08	0.842
DMU09	1.341
DMU10	0.764
DMU11	1.031
DMU12	1.379
DMU13	2.321
DMU14	0.919
DMU15	1.597
DMU16	1.068
DMU17	0.713
DMU18	3.098
DMU19	3.132
DMU20	0.919
DMU21	1.597
DMU22	1.011
DMU23	0.815
DMU24	3.176
DMU25	3.212

Table 2: Average changes in companies

Row	**Year**	**Total Efficiency Changes**	**Technological changes (TECHCH)**	**Management efficiency changes (PECH)**	**Scale efficiency changes (SECH)**	**Total efficiency changes (TFPCH)**
1	1399	1.436	1.217	1.2265	1.056	1.512
2	1400	0.840	1.232	0.684	0.843	0.885

5 Discussion and conclusions

In order to evaluate the performance, data envelopment analysis method is used as a tool to evaluate the performance of decision-making units. Because the data envelopment analysis tool has shown its capability in evaluating the performance of decision-making units and accounting for their relative efficiency in hospitals, universities, and manufacturing companies.

Using an evaluation system, if at the very beginning, we get the most output from the input and move the inefficient units towards the efficiency limit, we have created a more suitable platform to increase our share in knowledge production. Productivity is considered one of the important concepts in economics that shows how production factors are used in the product production. In the general sense, productivity is the ratio of outputs to inputs. Productivity promotion is considered as one of the important sources of providing economic growth of countries and increasing the company's competitiveness. Advanced and successful developing countries have achieved a significant part of their economic growth in this way. There are different approaches to determine the factors impact on efficiency and productivity and how to measure the productivity of production factors.

There are different approaches to determine the factors impact on efficiency and productivity and how to measure the productivity of production factors.

Among the investigated companies in the years 2019 to 2019, it was found that the technical efficiency of the companies decreased, the technological changes of the companies increased, the managerial efficiency of the companies decreased, the efficiency of the scale of the companies decreased, and the productivity of the entire companies faced a decrease.

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