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Effect of energetic electrons on dynamic motion of ion-acoustic waves in the presence of external periodic force

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Abstract

The dynamic motions of ion-acoustic (IA) waves in plasma are studied by considering the Kappa-Cairns electron distribution. The effect of external periodic force is also considered, here. Using the reductive perturbation technique, a Korteweg-de Vries (K-dV) equation is obtained. The quasiperiodic motions of IA waves are investigated by considering two-dimensional phase portraits and time-series analysis. The effects of non–thermal parameters (i.e., κ, α) and the strength (f_0) of the external periodic force on nonlinear travelling wave structures are also discussed.

Keywords: Ion-acoustic waves, dynamic motions, external periodic force, energetic electrons
2020 MSC: 47E05, 35G25

1 Introduction

Usually, in plasmas far from thermal equilibrium, the velocity distribution function of the charged particles is commonly assumed as non-Maxwellian. The existence of a population of highly energetic electrons characterized in such plasmas. Generally, the Kappa distribution [11] and the Cairns distribution [3] are two widely used non-Maxwellian models for energetic electrons. These distributions are a generalization of the Maxwellian (thermal) distribution and in certain limits, they will reduce to a Maxwellian distribution. However, there are many space plasma environments that the Kappa or the Cairns distribution functions cannot be used for the investigation of nonlinear waves. In these cases, the Combined Kappa-Cairns distribution is a more generalized distribution which may be relevant to solve this problem. Many authors have studied the properties of the combined Kappa-Cairns velocity distribution function [1, 2, 5]. They have investigated the structure of ion-acoustic solitary waves in a plasma with the Kappa-Cairns distributed electrons. However, the dynamic motion of ion-acoustic waves (IAWs) in a plasma with the Kappa-Cairns distributed electrons has not been studied so far.

There is special attention for the study of nonlinear traveling waves in plasmas and in this work, we will focus on the traveling wave. It is noted that the traveling wave properties can be affected by damping and external periodic perturbations.

Many authors investigated the dynamic behaviors of IA waves in different plasma systems using the bifurcation theory [4, 6]. For example, Saha et al., [8] studied the dynamic behavior of IA waves in (e-p-i) magneto-plasmas with Kappa distributed electrons and positrons. Selim et al., [9] were considered characteristics of nonlinear IA traveling waves in a multi–component system consisting of positive and negative ions and superthermal electrons. In addition,

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bifurcation and quasiperiodic behaviors of IA waves in a magnetized electron–ion plasma with the combined Cairns–Tsallis distribution electrons were investigated in [10]. To the best of our knowledge, there is no work in literature to study the dynamic behaviors of IA waves in plasmas with non–thermal electrons featuring Kappa–Cairns distribution in the presence of the external periodic force. Therefore, in this study, we investigate the nonlinear properties of IA waves (through a perturbative approach) in a two-component plasma with the Kappa–Cairns distributed electrons.

The manuscript is organized as follows. The dynamical equations and the derivation of the K-dV equation are presented in Sec. 2. Control of dynamic motions is demonstrated in Sec. 3. Finally, a brief conclusion of our numerical results is given in Sec. 4.

2 Basic equations and derivation of the k-dv equation

We consider a collisionless plasma consists of cold ions and energetic electrons with the Kappa–Cairns' distribution function. The normalized basic set of equations for the dynamics of the ion particles are given as

$$\frac{\partial n}{\partial t} + \frac{\partial}{\partial x}(nu) = 0, \tag{2.1}$$

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} = -\frac{\partial \phi}{\partial x}, \tag{2.2}$$

$$\frac{\partial^2 \phi}{\partial x^2} = n_e - n. \tag{2.3}$$

The normalized kappa-Cairns distributed electron number density n_e is given by the relation [3-5]

$$n_e = \left(1 - \Gamma_1\phi + \Gamma_2\phi^2\right)\left(1 - \frac{\phi}{\kappa - 3/2}\right)^{-\kappa+\frac{1}{2}}, \tag{2.4}$$

where

$$\Gamma_1 = \Gamma\left(1 + \frac{3}{2(\kappa - 5/2)}\right), \tag{2.5}$$

$$\Gamma_2 = \Gamma\left(1 + \frac{1}{(\kappa - 3/2)} + \frac{3}{4(\kappa - 3/2)(\kappa - 5/2)}\right), \tag{2.6}$$

$$\Gamma = \frac{4\alpha}{1 + 3\alpha\frac{(\kappa-3/2)}{(\kappa-5/2)}}. \tag{2.7}$$

It should be noted that the Kappa–Cairns distribution function reduces to the well–known Kappa distribution function [11] for $\alpha = 0$, the Cairns non–thermal velocity distribution [3] for $\kappa \to \infty$, and the Maxwellian distribution function for $\alpha = 0$ and $\kappa \to \infty$. On the other hand, it is found that this new distribution function must satisfy the conditions $\kappa > 3/2$ and $\alpha < 1$. In the above set of dynamic equations, $n(n_e)$ is normalized to n_0, ion fluid velocity u normalized to ion–acoustic speed $C_0 = (k_B T_e/m_i)^{1/2}$, electrostatic potential ϕ is normalized by $(k_B T_e/e)$. Space and time variables are normalized by the Debye length $\lambda_D = \sqrt{k_B T_e/4\pi e^2 n_0}$ and the ion plasma period $\omega_{pi} = \sqrt{4\pi e^2 n_0/m_i}$, respectively.

To study the ion–acoustic nonlinear waves with small but finite amplitude and derive the K-dV equation, we apply the reductive perturbation technique (RPT) [12]. For this aim, the independent variables are stretched in the following form:

$$\xi = \varepsilon^{\frac{1}{2}}(x - \lambda t) \qquad \text{and} \qquad \tau = \varepsilon^{3/2} t \tag{2.8}$$

where λ (normalized to C_0) is the phase velocity of the ion–acoustic waves and $\varepsilon \ll 1$ is a small parameter measuring the strength of nonlinearity. The dependent plasma variables n, u and ϕ, are expanded about their equilibrium values as power series of ε as

$$n = 1 + \varepsilon\, n^{(1)} + \varepsilon^2 n^{(2)} + \ldots, \tag{2.9}$$

$$u = 0 + \varepsilon\, u^{(1)} + \varepsilon^2 u^{(2)} + \ldots, \tag{2.10}$$

$$\phi = 0 + \varepsilon\, \phi^{(1)} + \varepsilon^2 \phi^{(2)} + \ldots, \tag{2.11}$$

Substituting Eqs. (2.8)-(2.11) into basic equations (2.1)-(2.4) and equating the coefficients of similar powers of ε, one may obtain the lowest order of ε as

$$n^{(1)} = A_1\phi^{(1)}, u^{(1)} = \lambda A_1\phi^{(1)}, \tag{2.12}$$

with $A_1 = \left(\frac{\kappa-1/2}{\kappa-3/2} - \beta_1\right)$. We obtain the phase velocity as

$$\lambda = \sqrt{\frac{1}{\left(\frac{\kappa-1/2}{\kappa-3/2} - \beta_1\right)}}. \tag{2.13}$$

For the next order of ε, by eliminating the second-order perturbed quantities ($n^{(2)}, u^{(2)}$ and $\phi^{(2)}$), and with the help of Eq. (2.12), we finally get an evolution equation for IAWs as

$$\frac{\partial\phi^{(1)}}{\partial\tau} + \mathrm{A}\phi^{(1)}\frac{\partial\phi^{(1)}}{\partial\xi} + \mathrm{B}\frac{\partial^3\phi^{(1)}}{\partial\xi^3} = 0, \tag{2.14}$$

where the nonlinear and dispersion coefficients, respectively, are defined as

$$\mathrm{A} = \frac{3\lambda A_1}{2} - \frac{A_2}{\lambda A_1^2}, \tag{2.15}$$

$$\mathrm{B} = \frac{1}{2\lambda A_1^2}, \tag{2.16}$$

where $A_2 = \left\{\frac{(\kappa-1/2)(\kappa+1/2)}{2(\kappa-3/2)^2} + \Gamma_2 - \Gamma_1\left(\frac{\kappa-1/2}{\kappa-3/2}\right)\right\}$.

3 Control of dynamic motions

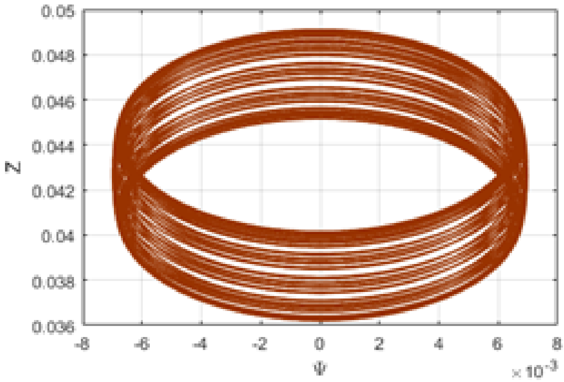
There are standard techniques to study the control of dynamic motion for IAWs with the effect of an external periodic force, namely, sensitivity analysis, time-series analysis and phase portrait analysis. Here the K-dV equation (2.14) is reduced a dynamical system employing the alteration $\chi = \xi - \mathrm{U}\tau$. Substituting the latter transformation in the equation (2.14), we get the following expression

$$-\mathrm{U}\frac{d\phi^{(1)}}{d\chi} + \mathrm{A}\phi^{(1)}\frac{d\phi^{(1)}}{d\chi} + \mathrm{B}\frac{d^3\phi^{(1)}}{\partial\chi^3} = 0. \tag{3.1}$$

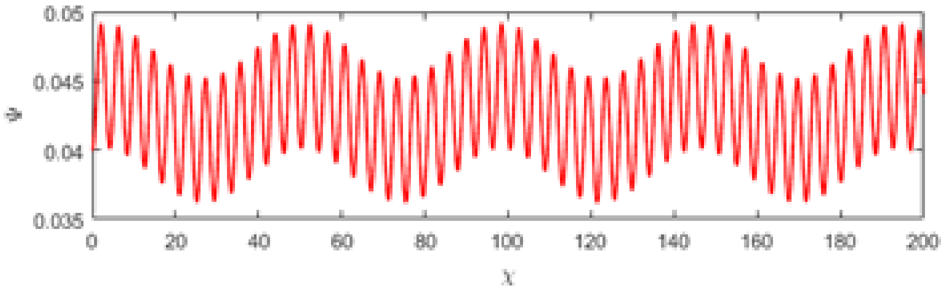
Now, introducing an external periodic force $f_0\cos(\omega\chi)$ in the system (3.1), the following perturbed dynamical structure is obtained as

$$\begin{cases} \frac{d\Psi}{d\chi} = Z, \\ \frac{dZ}{d\chi} = \frac{1}{B}\left(U\Psi - \frac{1}{2}A\Psi^2\right) f_0\cos(\omega\chi), \end{cases} \tag{3.2}$$

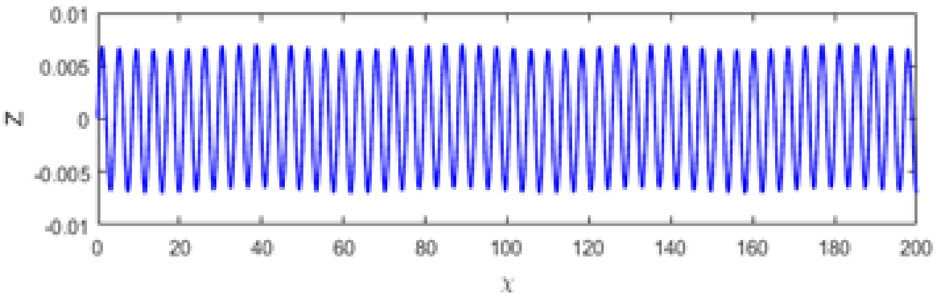
where $\Psi(\chi) = \phi^{(1)}$. Here, ω denotes frequency and f_0 denotes intensity of the external periodic force implemented in the system. We should note that the system Eq. (3.2) is a planar dynamical system and the phase orbits defined by the vector fields of Eq. (3.2) will determine all traveling wave solutions of Eq. (2.14). We study the bifurcations of phase portraits of (3.2) in the $(\phi^{(1)}, Z)$ phase plane when the parameters A, B, and U are clear for us. The coefficients A and B are functions from the parameters κ and α, and hence, it is difficult to explore the system for a complete range of parametric space. Therefore, we investigate our numerical analysis for some fixed values of these parameters. For investigation of the control of dynamic motions of the structure (3.2), different techniques, such as (i) phase portrait and (ii) time-series analysis can be used [4, 7]. A geometric structure of the trajectories of a three-dimensional dynamical system is shown by analyzing a 2D phase portrait in phase space. In the phase portrait, each set of initial conditions is represented by a different curve or point. It consists of a plot of the trajectories in the state space. This gives the information about whether there is an attractor, a repellor, or a limit cycle for a set of parameter values. On the other hand, a time-series analysis shows a series of data points indexed in time series. In fact, nonlinear time-series analysis allows to extract from the measured time series the physical properties of the system that generated them.



(a)



(b)



(c)

Figure 1: 2D plot of the phase orbits and (b)-(c) time-series analysis of Ψ and Z vs. χ of the system (3.2) for initial conditions: $(\Psi,\, Z) = (0.04,\, 0.0001)$ with κ=5, U_0=0.01, α=0.076, f_0=0.01 and ω=1.5.

Figures 1(a)-(c), present the phase portrait, the behavior of potential Ψ, and electric field Z in a collisionless nonthermal plasma with the Kappa-Cairns distributed electrons, respectively. Figure 1(a) displays that the ion trajectories lie on the surface of the torus which is a sign of the quasiperiodic solution. In other words, this figure confirms that the system (3.2) exhibits a quasiperiodic behavior when an external periodic force is considered. In Figs. 1(b)- (c), time-series analysis of Ψ and Z versus χ are presented. A multi-frequency but finite oscillation is observed in Figs. 1(b)-(c).

Figure 2, represents the variation of the potential of nonlinear waves for different values of κ in the presence of an external periodic force with $f_0 = 0.01$ and $\omega = 1.5$. Here, red and blue curves are plotted for $\kappa = 5$ and $\kappa = 8$, respectively. It is observed that by increasing the spectral index κ, the amplitude of the nonlinear ion–acoustic waves will be increased.

In Fig. 3, the effect of the increase in the density of the energetic electrons via parameter α in the presence of an external periodic force is studied. It is seen that the amplitude of nonlinear waves decreases as parameter α increases. Therefore, from the comparison of Figs. 2 and 3, we obtain that non–thermal Kappa-Cairns distributed electrons have significant effects on the evolution of the IA traveling waves.

Now, the effect of the strength of external periodic force f_0 is investigated in Fig. (2.4). It is found that if the strength of the external force gradually increases, the amplitude of the periodic waves will be increased. It happens because, increasing of strength of external force enhancements the IAW potential energy.

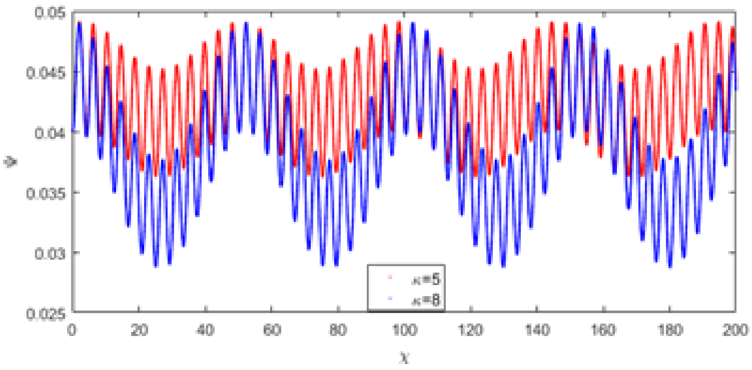


Figure 2: Profile of traveling waves potential for (a) $\kappa = 5$ (red curve) and (b) κ=8 (blue curve). Other plasma parameters are the same as Fig. 1

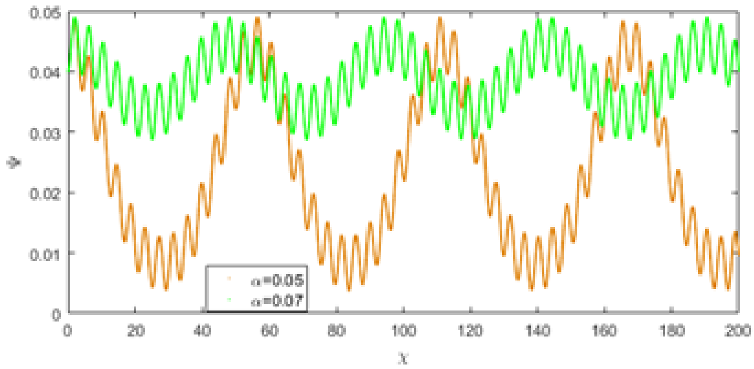


Figure 3: Profile of traveling waves potential for (a) α=0.05 (brown curve) and (b) α=0.07 (green curve). Other plasma parameters are the same as Fig. 1

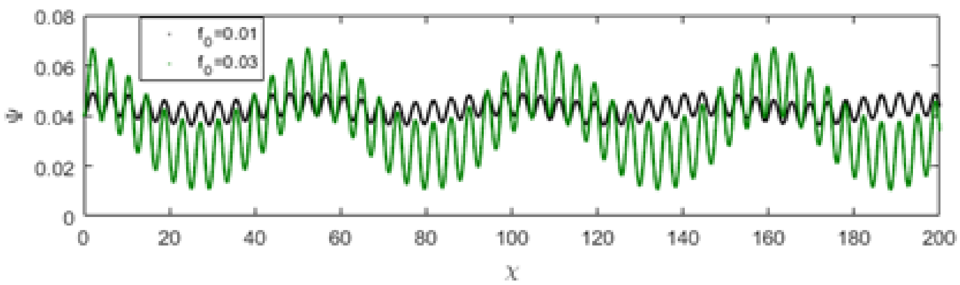


Figure 4: Variations of the potential of the nonlinear traveling waves vs. χ for different values of the amplitude of the external force (a) f_0=0.01 (black curve) and (b) f_0=0.03 (green curve). Other plasma parameters are the same as Fig. 1.

4 Conclusions

The structures and dynamic motions of the nonlinear ion-acoustic waves are studied in a non-Maxwellian plasma containing cold ions and Kappa-Cairns distributed electrons, in the presence of an external periodic force. Using RPT, the K-dV equation for ion-acoustic waves is derived. The effects of non–thermal parameters (i.e., κ, α) and the amplitude f_0 of the periodic force on IA wave structures were discussed through numerical simulations. It is observed that these parameters have remarkable effects on the nonlinear structure of the IA waves in non-Maxwellian plasmas. In other words, they play a crucial role in the control of the dynamic motions of the system (3.2).

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