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A two-stage stochastic programming model for the CCS-EOR planning problem

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Abstract

Carbon-capture-and-storage (CCS) is an important technology to reduce CO_2 emissions. A commercial method to establish the CCS on a large scale is to sequestrate CO_2 in depleted oil reservoirs and to combine it with enhanced oil recovery (EOR) operations. In this way, the CO_2 emission is reduced, as well as the oil production increases. The joint CCS-EOR planning problem determines the optimum allocation of existing CO_2 to depleted reservoirs and the scheduling of the EOR operations. This paper presents a deterministic MIP model which is a modification of an existing model in the literature. Then, this model is extended to a two-stage stochastic model in which the parameters expressing the initial oil yields and the periodic depletion factor of oil yields associated with reservoirs are uncertain, and the uncertainty is realized as soon as the operation of the reservoir is started. Our stochastic model is computationally more efficient than the existing model in the literature, due to the reduction of binary variables, as well as the absence of "non-anticipativity constraints". Instead, our stochastic model is less realistic. The proposed models are examined over two case studies taken from the literature. The obtained results confirm the higher effectiveness of our stochastic model.

Keywords: Enhanced-oil-recovery, Carbon-capture-and-storage, Two-stage stochastic programming, Computational effectiveness
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1 Introduction

According to the report of the International Energy Agency (IEA, 2015) the world's energy demand will increase by 37% until 2040. Currently, a large part of energy is provided by fossil fuels, and due to the industrialization of developing countries, the world's oil demand will grow by 20% until 2040. Consequently, it is predicted that the CO_2 emissions, which is one of the main factors of environmental pollution, will increase about 80% during the next 50 years [22].

One way to reduce CO_2 emissions is to use the carbon-capture-and-storage (CCS) technology. CCS involves removing CO_2 from the flue gas, transferring the captured CO_2 through the pipelines, and storing it in underground sites [1]. Depleted crude oil reservoirs are of great interest among potential CO_2 storage sites, because in this way, the stored CO_2 can be used in enhanced oil recovery (EOR) operations in order to increase reservoir exploitation [9]. EOR refers to the transfer of oil remaining in the reservoir by injecting materials other than what typically exists

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in the reservoir. Miscible EOR is one type of EOR operations in which CO_2 is injected to enhance the oil recovery by dissolving, swelling, and reducing oil viscosity [1]. Nowaday, most of the world's produced oil comes from mature oil fields in the second half of their life. Replacing existing oil fields with new discoveries is difficult, since extraction operations are costly and time-consuming. On the other hand, growing oil demand encourages oil companies to put EOR methods in their plan.

In this paper the joint CCS-EOR planning problem is formulated as a two-stage stochastic programming (SP) model. Stochastic programming is an approach to model optimization problems in which some of the parameters are affected by uncertainty. Jonsbraten [13] classified the uncertainty in multistage SP problems into two groups, namely exogenous and endogenous. In a situation with exogenous uncertainty, optimization decisions cannot affect the underlying stochastic process. In contrast, in SP problems with endogenous uncertainty, decisions affect the underlying stochastic process by changing the probability distribution (the first type) or determining the times at which the uncertainty is realized (the second type). In the second type of endogenous uncertainty, the scenario tree describing the uncertainty realization is decision-dependent; hence, there are complexities in satisfying the non-anticipativity (NA) constraints [6, 10].

In the continuation of this section, the CCS-EOR planning literature is reviewed in short and then, the innovations of this paper are specified.

1.1 A review of the literature on the CCS-EOR planning

CCS is an important technology to reduce CO_2 emissions. Middleton and Bielicki [17] and Middleton, et al. [18] presented MILP models based on Geographic Information System (GIS) data to design a CCS pipeline network. Their models decide where and how much CO_2 to capture and store, and where to construct pipelines to minimize the annualized costs of CO_2 sequestering. Elkamel, et al. [4] presented an MILP model to co-optimize the power generation and the CO_2 capture from generating stations. Some studies focused on the best source-destination adaptation for CCS networks. Tan, et al. [20, 21] presented a MINLP model for CO_2 source-sink matching in CCS. Tapia, et al. [23] optimized the CO_2 capture, utilization, and storage system given multiple CO_2 sources and sinks. In practice, the estimation of the storage site capacity and injection rate limit is affected by uncertainty. furthermore, new information may be obtained, or unexpected events may occur, due to long planning horizons of CCS systems; therefore, the risk management is an important task. In this regard, Tapia and Tan [24] developed a fuzzy MILP model for multi-period planning of CCS systems, and He, et al., [8] proposed a robust two-stage stochastic MILP model. Zhang, et al., [28] presented an MILP model to develop a CCS-EOR supply chain superstructure by selecting CO_2 sources, capture sites, CO_2 pipelines, and utilization and storage places.

The studies reviewed above deal with just CO_2 capture and storage planning. However, a commercial way to deploy CCS on a large scale is to combine it with EOR operations. The main advantage of the joint CCS-EOR is that the CO_2 emission is reduced, and the oil recovery increases [2]. Jiang, et al. [12] assessed the technical and economical performance of CCS-EOR with respect to more than 40 historical related projects. The combination of CCS and EOR has been addressed in a few studies. Safarzadeh and Motahhari [19] presented a genetic algorithm to co-optimize CO_2 storage and oil production. Ettehad [5] utilized the Monte-Carlo simulation for CCS-EOR planning, where the annual captured CO_2 from the power plant and CO_2 injection cost in the oilfield are assumed to be uncertain. Jahangiri and Zhang [11] proposed an ensemble-based optimization algorithm to co-optimize oil extraction and CO_2 sequestration. Middleton [16] presented a two-phase algorithm for modeling of the CCS infrastructure and planning the EOR operations assuming that there is only one oil reservoir. Kamali and Cinar [14] addressed the CCS-EOR planning with simultaneous water and gas injection. Ampomah, et al., [3] addressed an artificial neural network (ANN) optimization approach for the CO_2 storage and EOR planning problem under uncertainty. You, et al., [27] coupled ANN models with the particle swarm optimization (PSO) algorithm to co-optimize the CCS-EOR decisions. You, et al., [26] coupled ANN models with the multi-objective PSO algorithm to design water-alternating-CO_2 injection projects considering multiple objectives including maximizing the oil recovery and the amount of CO_2 storage. Wang, et al. [26] addressed the optimization of CCS-EOR decisions while considering the impact of decided policies on the oil price and CO_2 subsidy. Guo et al. [7] presented a novel framework for CCS-EOR project to systematically evaluate the CCS network design and the EOR utilization procedure. Kashkooli, et al. [15] addressed the CO_2 storage and EOR planning problem by considering the dynamic well flow settings as the optimization variables.

To the best of our knowledge, a few works are addressing the joint CCS-EOR planning problem from the mathematical optimization modeling viewpoint [22, 1] . Tapia, et al., [22] assumed that one CO_2 source and multiple oil reservoirs are available. The aim is to design a network to transfer CO_2 from source to reservoirs and schedule the EOR operations. They formulated the problem as an MILP model. Abdoli, et al., [1] resolved the weaknesses of the model of Tapia, et al., [22] and then extended it to a SP model under endogenous uncertainty of second type. In this paper, we introduce a new two-stage SP model for the CCS-EOR planning problem, to get ride of the complexities in satisfying the NA constraints in the multi stage SP model of Abdoli, et al., [1].

1.2 The contribution and the organization of this paper

The main contributions of this paper are as follows: First, an equivalent reformulation of the deterministic model of Abdoli, et al., [1] is developed, that in which the planning time horizon is no longer divided into equally length sections. Then, the deterministic model is extended to a two-stage SP model that the uncertainty of the oil yield of reservoirs and their depletion factors is incorporated in it. The nature of uncertainty in this model is exogenous and the model is two-stage instead of multi stage. Thus, we get ride of the complexities in satisfying the NA constraints in the multi stage SP model of Abdoli, et al., [1], that leads to more computationally effectiveness, confirmed with computational experiments over two case-studies taken from Abdoli, et al., [1].

The remainder of this paper is organized as follows: Section 2 provides a detailed description of the CCS-EOR planning problem, taken from Abdoli, et al., [1], and presents the corresponding deterministic model. Section 3 presents the two-stage SP model. Section 4 examines the proposed models by two case-studies. Finally, Section 5 concludes and offers directions for future research.

2 Problem description and mathematical formulation

2.1 Problem description

Following Abdoli, et al., [1], consider a CCS-EOR system consisting of one CO_2 common source and a set $\mathbb{I} = \{1, \ldots, n\}$ of depleted oil reservoirs (indexed by i). Suppose that the planning time horizon involves T time periods of equal length and let $\mathbb{T} = \{1, 2, \ldots, T\}$ (indexed by t) be the set of time periods.. The time-dependent parameter $\bar{F}_t$ is used to show the maximum CO_2 flow that can be sent from the CO_2 source in time period t (in Mt); this is because the injection of CO_2 from the common source may increase over time as the companies are developed. The aim is to design a CCS network, connecting the common CO_2 source to a subset of reservoirs via a branching point and deciding the amount of CO_2 transferred throughout the network. The objective is to maximize the total revenue obtained by the recovered oil plus the credits received from CO_2 storage minus the fixed costs of CCS network design and the variable costs of transferring CO_2 throughout the network.

Two types of pipelines can be utilized in the network designing: the primary pipeline connecting the CO_2 source to the branching point and the secondary pipelines that are extended from the branching point to the reservoirs. The lengths of the primary pipeline and the secondary pipeline associated with reservoir i are denoted by d and d'_i (in km), respectively. The primary and secondary pipes are divided into several types depending on their diameter, length and material. Let $\mathbb{L}$ (indexed by ℓ) and $\mathbb{K}$ (indexed by k) be the sets of different types that can be selected for the primary and secondary pipelines, respectively. Following Abdoli, et al., [1], the other required parameters are described as follows:

$\underline{f}_i$, $\bar{f}_i$ The lower and upper bounds on the amount of CO_2 that is allowed to be injected into reservoir i during each period of its utilization (in Mt), respectively

$\underline{u}_\ell$,$\bar{u}_\ell$ The minimum and the maximum amounts of flow that can be transferred through the primary pipeline of type ℓ in each period (in Mt), respectively

$\underline{w}_k$, $\overline{w}_k$ The minimum and the maximum amounts of flow that can be transferred through the secondary pipeline of type k in each period (in Mt), respectively

g_ℓ, $g'_{i,k}$ The fixed costs of establishing a primary pipeline of type ℓ and a secondary pipeline of type k associated with reservoir i (in $M\$$), respectively

h_ℓ, $h'_{i,k}$ The variable cost associated with transferring CO_2 through a primary pipeline of type ℓ (in $\dfrac{M\$}{Mt \times km}$), and the variable cost of transferring CO_2 through a secondary pipeline of type k associated with reservoir i (in $\dfrac{M\$}{Mt \times km}$)

c_i The total capacity of reservoir i (in Mt)

θ_i^{max} The amount of oil that can be recovered per megaton of CO_2 injection at the beginning of the utilization of reservoir i (in million barrels)

m_i The rate of decrease in the yield of reservoir i from one period to the next one (unit-less)

v_i The value of oil recovered from reservoir i (in $\frac{M\$}{Mbbls}$)

α_i The portion of the total injected CO_2 in reservoir i that is sequestered in it (unit-less)

b The credit of CO_2 sequestered in reservoirs (in $\frac{M\$}{Mt}$)

a_i The earliest period at which the exploitation of reservoir i is allowed to start

a'_i The latest time period at which the exploitation of reservoir i is allowed to start

e_i The duration of the operation of reservoir i (expressed in the number of periods)

β_t The coefficient used in calculating the net present value of one dollar in time period t (considering ρ as the interest rate in each time period, we have $\beta_t = \frac{1}{(1+\rho)^t}$)

Following Abdoli, et al., [1] with some difference, we assume that the primary pipeline as well as the secondary pipelines corresponding to the reservoirs for which we decided to do the operation, are ready at the beginning of the first period. In other words, we ignore the construction time and assume that the CO_2 transferring through the pipelines can be started whenever needed.

2.2 Mathematical model

Concerning the problem description and defined sets and indices, the decision variables are defined as follows:

δ_ℓ The binary variable that is 1 if the primary pipeline of size ℓ is established; otherwise 0

$\delta'_{i,k}$ The binary variable that is 1 if the secondary pipeline of type k is established for reservoir i, otherwise 0

$\eta_{i,t,k}$ The binary variable that is 1 if the reservoir i is utilized in period t and the secondary pipeline of type k is used for it, otherwise 0

y_i Nonnegative continuous variable indicating the time period that utilization of reservoir i is started; otherwise 0

$u_{t,\ell}$ Nonnegative continuous variable indicating the amount of CO_2 flow on the primary pipeline in time period t if the primary pipeline of type ℓ is exploited for the operations, otherwise is 0 (in Mt)

w_i Nonnegative continuous variable indicating the amount of CO_2 flow transferred through the secondary pipeline associated with reservoir i in each time period of its utilization (in Mt)

$q_{i,t,k}$ The nonnegative continuous variable representing the amount of flow on the secondary pipeline associated with reservoir i in time period t, which is equal to w_i if the reservoir i is utilized in time period t and the secondary pipeline of type k is used for it, otherwise it is 0 (in Mt)

$x_{i,t}$ The nonnegative continuous variable representing the amount of oil recovered by reservoir i in time period t (in $Mbbls$)

With respect to the above notations, we can formulate the problem in the deterministic case. The deterministic model, consists of objective function (2.1) together with constraints (2.2)–(2.18) as listed below:

$$\max z = -\sum_{\ell\in\mathbb{L}} g_\ell \delta_\ell - \sum_{i\in\mathbb{I}}\sum_{k\in\mathbb{K}} g'_{i,k}\delta'_{i,k} - \sum_{t\in\mathbb{T}} \beta_t \left(\sum_{\ell\in\mathbb{L}} dh_\ell u_{t,\ell} + \sum_{k\in\mathbb{K}}\sum_{i\in\mathbb{I}} d'_i h'_{i,k} q_{i,t,k} \right)$$

$$+ \sum_{t\in\mathbb{T}} \beta_t \sum_{i\in\mathbb{I}} v_i x_{i,t} + \sum_{t\in\mathbb{T}} \beta_t \sum_{k\in\mathbb{K}}\sum_{i\in\mathbb{I}} b\alpha_i q_{i,t,k} \tag{2.1}$$

Subject to:

$$\sum_{\ell\in\mathbb{L}} \delta_\ell \le 1 \tag{2.2}$$

$$\sum_{k\in\mathbb{K}} \delta'_{i,k} \le \sum_{\ell\in\mathbb{L}} \delta_\ell \qquad \forall i\in\mathbb{I} \tag{2.3}$$

$$\sum_{k\in\mathbb{K}} \eta_{i,t,k} \le 1 \quad \forall i\in\mathbb{I}, t\in\mathbb{T} \tag{2.4}$$

$$\sum_{t\in\mathbb{T}} \eta_{i,t,k} = e_i \delta'_{i,k} \qquad \forall\ i\in\mathbb{I},\ \forall k\in\mathbb{K} \tag{2.5}$$

$$\sum_{\substack{t'\in\mathbb{T}\\ t'\ge t+2}} \eta_{i,t',k} \le e_i\left(1-\eta_{i,t,k}+\ \eta_{i,t+1,k}\right) \qquad \forall i\in\mathbb{I}, \forall t\in\mathbb{T}: t\le T-2,\ \forall k\in\mathbb{K} \tag{2.6}$$

$$\eta_{i,1,k} \le y_i \le 1 + T\left(1-\eta_{i,1,k}\right) \qquad \forall i\in\mathbb{I},\ \forall k\in\mathbb{K} \tag{2.7}$$

$$-t\left(\eta_{i,t-1,k}-\eta_{i,t,k}\right) \le y_i \le t + T\left(1+\eta_{i,t-1,k}-\eta_{i,t,k}\right) \qquad \forall i\in\mathbb{I}, t\in\mathbb{T}: t\ge 2,\ \forall k\in\mathbb{K} \tag{2.8}$$

$$a_i \sum_{k\in\mathbb{K}} \delta'_{i,k} \le y_i \le a'_i \sum_{k\in\mathbb{K}} \delta'_{i,k} \qquad \forall i\in\mathbb{I} \tag{2.9}$$

$$\sum_{k\in\mathbb{K}} \max(\underline{f}_i, \underline{w}_k)\delta'_{i,k} \le w_i \le \sum_{k\in\mathbb{K}} \min(\bar{f}_i, \overline{w}_k)\delta'_{i,k} \quad \forall i\in\mathbb{I} \tag{2.10}$$

$$w_i - \bar{f}_i\left(1-\eta_{i,t,k}\right) \le q_{i,t,k} \le w_i \quad \forall i\in\mathbb{I}, t\in\mathbb{T},\ \forall k\in\mathbb{K} \tag{2.11}$$

$$q_{i,t,k} \le \bar{f}_i \eta_{i,t,k} \quad \forall i\in\mathbb{I}, t\in\mathbb{T},\ \forall k\in\mathbb{K} \tag{2.12}$$

$$\sum_{t\in\mathbb{T}} \alpha_i q_{i,t,k} \le c_i \qquad \forall i\in\mathbb{I},\ \forall k\in\mathbb{K} \tag{2.13}$$

$$\sum_{i\in\mathbb{I}}\sum_{k\in\mathbb{K}} q_{i,t,k} = \sum_{\ell\in\mathbb{L}} u_{t,\ell} \le \bar{F}_t \qquad \forall t\in\mathbb{T} \tag{2.14}$$

$$\underline{u}_\ell \delta_\ell \le u_{t,\ell} \le \overline{u}_\ell \delta_\ell \qquad \forall t\in\mathbb{T},\ \forall \ell\in\mathbb{L} \tag{2.15}$$

$$x_{i,t} \le \theta_i^{max} \sum_{k\in\mathbb{K}} q_{i,t,k} \quad \forall i\in\mathbb{I}, \forall t\in\mathbb{T} \tag{2.16}$$

$$x_{i,t} \le m_i x_{i,t-1} + \theta_i^{max} \bar{f}_i\left(2-\eta_{i,t-1,k}-\eta_{i,t,k}\right) \qquad \forall i\in\mathbb{I}, \forall t\in\mathbb{T}: t\ge 2,\ \forall k\in\mathbb{K} \tag{2.17}$$

$$\delta_\ell \in \{0,1\} \qquad \forall \ell\in\mathbb{L}$$

$$\delta'_{i,k} \in \{0,1\} \qquad \forall i\in\mathbb{I}, \forall k\in\mathbb{K}$$

$$\eta_{i,t,k} \in \{0,1\} \qquad \forall i\in\mathbb{I}, \forall t\in\mathbb{T},\ \forall k\in\ \mathbb{K}$$

$$y_i, w_i \ge 0 \quad \forall i\in\mathbb{I}$$

$$u_{t,\ell} \ge 0 \qquad \forall t\in\mathbb{T}, \forall \ell\in\mathbb{L}$$

$$q_{i,t,k} \ge 0, \quad x_{i,t} \ge 0, \qquad \forall i\in\mathbb{I}, \forall t\in\mathbb{T},\ \forall k\in\ \mathbb{K}. \tag{2.18}$$

The objective function (2.1) maximizes the total profit of carbon credits and revenue from EOR operations. Constraint (2.2) ensures that at most one type is selected for the primary pipeline.

Constraint set (2.3) indicates that the secondary pipelines are not established if there is no primary pipeline, and constraint (2.4) ensures that at most one type is selected for the secondary pipeline associated with each reservoir. Constraint sets (2.5) and (2.6) imply that the operating life of reservoir i (in the case of utilization) equals e_i consecutive periods. Constraint sets (2.7) and (2.8) determine the time period in which the activity of reservoir i (in the case of utilization) is started.

Constraint (2.9) insures that for each reservoir i the start time of operation belongs in the range $[a_i, a'_i]$. Constraint set (2.10) determine the range of the amount of CO_2 flow injected into reservoir i during each period of its activity, provided that the reservoir i is utilized and the secondary pipeline of type k is used for its operation. Constraint sets (2.11) and (2.12) ensure that if the secondary pipeline of type k is used for the operation of reservoir i, then for each time period t of utilizing the reservoir i, the variable $q_{i,t,k}$ is equal to w_i, otherwise 0. Constraint set (2.13) indicates that the amount of CO_2 stored in a reservoir during its operational life cannot exceed its capacity. Constraint set (2.14) guarantees that in each period, the total amount of flow transferred through the secondary pipelines is equal to the amount of flow transferred through the primary pipeline; further, the amount of flow on the primary pipeline cannot exceed the maximum flow. Constraint set (2.15) ensures that the amount of flow along the primary pipeline is between specified lower and upper bounds in each period. Constraint sets (2.16) and (2.17) indicate the decrease in oil yield during the oil recovery periods. Constraint set (2.18) express the type of variables.

In the following section, the deterministic model is extended to incorporate uncertainty.

3 Extension of deterministic model to a two-stage stochastic model

In deterministic model, all parameters are assumed to be definite; however, in practice, the initial oil yield of reservoir i (θ_i^{max}) and the rate of decrease in the yield of reservoir i (m_i) are affected by uncertainty. In this section, we assume that these two parameters are indeterminate, and the uncertainty is resolved upon the utilization of reservoir i is started. Let $\mathbb{S}$ (indexed by s) be the set of all possible scenarios, and assume that p_s shows the occurrence probability of scenario s where $\sum_{s \in \mathbb{S}} p_s = 1$. For each reservoir i and under each scenario s, the following uncertain parameters are defined:

$\theta_{i,s}^{max}$ The amount (in $Mbbls$) of oil that can be recovered per megaton of CO_2 injection at the beginning of the utilization of reservoir i under scenario s

$m_{i,s}$ The rate of decrease in the yield of reservoir i from one period to the next one under scenario s (unit-less)

The stochastic model aims to maximize the total expected profit. In the first stage, we decide on establishing the primary and secondary pipelines, and on the start time of the operations for each reservoir (if we decide to utilize that reservoir). As soon as the operations start for a reservoir, the uncertain parameters associated with that reservoir are resolved. Afterward, in the second stage, it is decided that what amount of CO_2 is transferred through the established pipelines. Thus, we encounter with a two-stage SP problem.

Except for $\theta_{i,s}^{max}$ and $m_{i,s}$, other parameters in the stochastic model have the same definition as in the deterministic model. The definition of decision variables δ_ℓ, $\delta'_{i,k}, \eta_{i,t,k}$ and y_i in the stochastic model is the same as in deterministic model; these are first-stage variables of our stochastic model. However, the other variables will be scenario dependent (second-stage variables), as defined below:

Second-stage decision variables

$u_{t,\ell,s}$ The nonnegative continuous variable representing the amount of CO_2 flow on the primary pipeline in period t under scenario s, if the primary pipeline of type ℓ is exploited for the operations, otherwise is 0 (in Mt)

$w_{i,s}$ The nonnegative continuous variable indicates the amount of CO_2 flow transferred through the secondary pipeline associated with reservoir i in each period of its utilization under scenario s (in Mt)

$q_{i,t,k,s}$ The nonnegative continuous variable representing the amount of flow on the secondary pipeline associated with reservoir i in period t under scenario s, which is equal to $w_{i,s}$ if the reservoir i is utilized in period t

under scenario s and the secondary pipeline of type k is used for it, otherwise it is 0 (in Mt)

$x_{i,t,s}$ The nonnegative continuous variable representing the amount of oil recovered by reservoir i in period t under scenario s (in $Mbbls$)

Using these notations, we can formulate the problem in the stochastic case. The two-stage stochastic model consists of objective function (3.1) together with constraints (3.2)–(3.18) stated below:

$$\begin{aligned}\max z = & -\sum_{\ell\in\mathbb{L}} g_\ell \delta_\ell - \sum_{i\in\mathbb{I}}\sum_{k\in\mathbb{K}} g'_{i,k}\delta'_{i,k} \\ & + \sum_{s\in\mathbb{S}} p_s \left(-\sum_{t\in\mathbb{T}} \beta_t \left(\sum_{\ell\in\mathbb{L}} dh_\ell u_{t,\ell,s} + \sum_{k\in\mathbb{K}}\sum_{i\in\mathbb{I}} d'_i h'_{i,k} q_{i,t,k,s}\right)\right. \\ & \left. + \sum_{t\in\mathbb{T}} \beta_t \sum_{i\in\mathbb{I}} v_i x_{i,t,s} + \sum_{t\in\mathbb{T}} \beta_t \sum_{k\in\mathbb{K}}\sum_{i\in\mathbb{I}} b\alpha_i q_{i,t,k,s}\right) \end{aligned} \tag{3.1}$$

Subject to:

$$\sum_{\ell\in\mathbb{L}} \delta_\ell \le 1 \tag{3.2}$$

$$\sum_{k\in\mathbb{K}} \delta'_{i,k} \le \sum_{\ell\in\mathbb{L}} \delta_\ell \qquad \forall i \in \mathbb{I} \tag{3.3}$$

$$\sum_{k\in\mathbb{K}} \eta_{i,t,k} \le 1 \quad \forall i \in \mathbb{I}, t \in \mathbb{T} \tag{3.4}$$

$$\sum_{t\in\mathbb{T}} \eta_{i,t,k} = e_i \delta'_{i,k} \qquad \forall i \in \mathbb{I}\ ,\ k \in \mathbb{K} \tag{3.5}$$

$$\sum_{\substack{t'\in\mathbb{T} \\ t'\ge t+2}} \eta_{i,t',k} \le e_i\left(1 - \eta_{i,t,k} + \eta_{i,t+1,k}\right) \quad \forall i \in \mathbb{I}\ , k \in \mathbb{K}, \forall t \in \mathbb{T} : t \le T-2 \tag{3.6}$$

$$\eta_{i,1,k} \le y_i \le 1 + T\left(1 - \eta_{i,1,k}\right) \qquad \forall i \in \mathbb{I}\ , \forall k \in \mathbb{K} \tag{3.7}$$

$$-t\left(\eta_{i,t-1,k} - \eta_{i,t,k}\right) \le y_i \le t + T\left(1 + \eta_{i,t-1,k} - \eta_{i,t,k}\right) \qquad \forall i \in \mathbb{I}\ , \forall k \in \mathbb{K}, t \in \mathbb{T} : t \ge 2 \tag{3.8}$$

$$a_i \sum_{k\in\mathbb{K}} \delta'_{i,k} \le y_i \le a'_i \sum_{k\in\mathbb{K}} \delta'_{i,k} \qquad \forall i \in \mathbb{I} \tag{3.9}$$

$$\sum_{k\in\mathbb{K}} \max(\underline{f}_i, \underline{w}_k)\delta'_{i,k} \le w_{i,s} \le \sum_{k\in\mathbb{K}} \min(\bar{f}_i, \overline{w}_k)\delta'_{i,k} \quad \forall i \in \mathbb{I}\ , \forall s \in \mathbb{S} \tag{3.10}$$

$$w_{i,s} - \bar{f}_i\left(1 - \eta_{i,t,k}\right) \le q_{i,t,k,s} \le w_{i,s} \quad \forall i \in \mathbb{I}, \forall t \in \mathbb{T}\ , \forall k \in \mathbb{K}, \forall s \in \mathbb{S} \tag{3.11}$$

$$q_{i,t,k,s} \le \bar{f}_i \eta_{i,t,k} \quad \forall i \in \mathbb{I}, \forall t \in \mathbb{T}\ , \forall k \in \mathbb{K}, \forall s \in \mathbb{S} \tag{3.12}$$

$$\sum_{t\in\mathbb{T}} \alpha_i q_{i,t,k,s} \le c_i \qquad \forall i \in \mathbb{I}\ , \forall k \in \mathbb{K}, \forall s \in \mathbb{S} \tag{3.13}$$

$$\sum_{i\in\mathbb{I}}\sum_{k\in\mathbb{K}} q_{i,t,k,s} = \sum_{\ell\in\mathbb{L}} u_{t,\ell,s} \le \bar{F}_t \qquad \forall t \in \mathbb{T}, \forall s \in \mathbb{S} \tag{3.14}$$

$$\underline{u}_\ell \delta_\ell \le u_{t,\ell,s} \le \overline{u}_\ell \delta_\ell \qquad \forall t \in \mathbb{T},\ \forall \ell \in \mathbb{L}, \forall s \in \mathbb{S} \tag{3.15}$$

$$x_{i,t,s} \le \theta^{max}_{i,s} q_{i,t,k,s} \qquad \forall i \in \mathbb{I}, \forall t \in \mathbb{T}, \forall k \in \mathbb{K}, \forall s \in \mathbb{S} \tag{3.16}$$

$$x_{i,t,s} \le m_{i,s} x_{i,t-1,s} + \theta^{max}_{i,s} \bar{f}_i\left(2 - \eta_{i,t-1,k} - \eta_{i,t,k}\right) \qquad \forall i \in \mathbb{I}, t \in \mathbb{T} : t \ge 2\ , k \in \mathbb{K}, \forall s \in \mathbb{S} \tag{3.17}$$

$$\begin{aligned}
&\delta_\ell \in \{0,1\} \quad \forall \ell \in \mathbb{L} \\
&\delta'_{i,k} \in \{0,1\} \quad \forall i \in \mathbb{I}, \forall k \in \mathbb{K} \\
&\eta_{i,t,k} \in \{0,1\} \quad \forall i \in \mathbb{I}, \forall t \in \mathbb{T},\ \forall k \in\ \mathbb{K} \\
&y_i \geq 0 \quad \forall i \in \mathbb{I} \\
&u_{t,\ell,s} \geq 0 \quad \forall t \in \mathbb{T},\ \forall \ell \in \mathbb{L}, \forall s \in \mathbb{S} \\
&w_{i,s} \geq 0 \quad \forall i \in \mathbb{I}, \forall s \in \mathbb{S} \\
&q_{i,t,k,s} \geq 0, \quad x_{i,t,s} \geq 0 \quad \forall i \in \mathbb{I}, \forall t \in \mathbb{T},\ \forall k \in\ \mathbb{K}, \forall s \in \mathbb{S}
\end{aligned} \tag{3.18}$$

The objective function (3.1) maximizes the total expected profit. Constraint set (3.2)–(3.9) have the same description as in constraint set (2.2)–(2.9) in the deterministic model. Moreover, constraint set (3.10)–(3.17) have the same descriptions as in constraint set (2.10)–(2.17), with the difference that they are specified for each scenario. Constraint set (3.18) express the type of decision variables.

4 Computational experiments

In this section, we evaluate the proposed models on two case-studies taken from Abdoli, et al., [1] and compare them with the models presented in Abdoli, et al., [1]. All experiments are performed on a laptop with a Core-i5 CPU and 8GB of RAM. The models are implemented in the GAMS software, and the CPLEX solver embedded in GAMS is used in its default settings to solve them.

4.1 4.1 Case-study 1

Consider a CCS-EOR system with three oil reservoirs, and assume that the secondary pipelines are directly connected to the CO_2 source (i.e., there is no primary pipeline and the branching point is CO_2 source itself). Suppose that there is only one type of secondary pipe (i.e., $\mathbb{K} = \{1\}$), and consider a 20-year time horizon. The value of parameters in the deterministic case is as follows: $d = 0$, $\bar{F}_t = 22$ (for each $t \in \mathbb{T}$), $\rho = 8$, $b = 23$, $\underline{w}_1 = 2$, $\overline{w}_1 = 15$. In addition, the value of parameters associated with the reservoirs is summarized in Table1.

Table 1: Reservoirs characteristics in case-study 1

Parameters associated with reservoir i	Reservoirs		
	$i = 1$	$i = 2$	$i = 3$
d_i	150	200	130
a_i	1	1	1
a'_i	5	10	5
c_i	100	150	100
$\underline{f_i}$	2	2	2
$\bar{f_i}$	15	15	15
e_i	15	10	15
α_i	0.95	0.85	0.35
v_i	100	70	90
θ_i^{max}	2.5	3.0	3.5
m_i	0.95	0.90	0.75
$g'_{i,1}$	100	100	100
$h'_{i,1}$	0.43	0.43	0.43

The deterministic model associated with case-study 1 has 634 constraints and 147 and 64 continuous and binary variables, respectively. It is solved to optimality via solver CPLEX in 0.60 seconds. A comparison of our deterministic model and that of Abdoli, et al., [1], in terms of model size and solving time, for case-study 1, is prepared in Table 2.

Table 2: Our deterministic model vs the corresponding model of Abdoli, et al., [1] for case-study 1

	Our model	Model of [1]
Number of continuous variables	147	147
Number of binary variables	64	73
Number of constraints	634	642
Solving time	0.60	0.57

Now, we consider the stochastic version of case-study 1. Following Abdoli, et al., [1] suppose that for the vector (θ_1^{max}, m_1), three values $(2.50, 0.95)$, $(1.75, 0.92)$ and $(3.25, 0.98)$, for the vector (θ_2^{max}, m_2), two values $(2.00, 0.85)$ and $(4.00, 0.95)$, and for the vector $(\theta_3^{\max}, m_3)$, two values $(2.25, 0.65)$ and $(4.75, 0.85)$, each one with equal probabilities, are possible and assume that these parameters are independent. Thus, the scenario set will be $\mathbb{S} = \{1, 2, \ldots, 12\}$ containing 12 scenarios each one with occurrences probability of 1/12.

Our stochastic model associated with case-study 1 has 4880 constraints and 1720 and 64 continuous and binary variables, respectively. It is solved to optimality via solver CPLEX in 3.5 seconds. A comparison of our stochastic model and that of Abdoli, et al., [1], in terms of model size and solving time, for case-study 1, is showed in Table 3.

Table 3: Our stochastic model vs the corresponding model of Abdoli, et al., [1] for case-study 1

	Our model	Model of Abdoli, et al., [1]
Number of continuous variables	1720	1753
Number of binary variables	64	876
Number of constraints	4880	38713
Solving time	3.5	1879

4.2 Case-study 2

Case-study 2 is an instance involving 6 oil reservoirs and a primary pipeline of length 150 km. Suppose that there is only one type of primary and secondary pipes (i.e., $\mathbb{L} = \{1\}$ and $\mathbb{K} = \{1\}$), and consider a 30-year planning horizon. The value of parameters in the deterministic case is as follows: $d = 150$, $\bar{F}_t = 15$ (for each $t \leq 10$) and $\bar{F}_t = 25$ (for each $11 \leq t \leq 30$), $\rho = 10$, $b = 15$, $\underline{u}_1 = 0$, $\bar{u}_1 = 25$, $\underline{w}_1 = 1$, $\overline{w}_1 = 10$, $g_1 = 95$, $h_1 = 0.25$. In addition, the value of parameters associated with reservoirs is summarized in Table 4.

Table 4: Reservoirs characteristics in case-study 2

The parameters associated with reservoir i	Reservoirs					
	$i = 1$	$i = 2$	$i = 3$	$i = 4$	$i = 5$	$i = 6$
d_i	50	30	70	100	45	60
c_i	100	150	75	200	200	150
a_i	6	1	6	1	1	1
a'_i	10	15	15	15	15	15
$\underline{f_i}$	1	1	1	1	1	1
$\bar{f}_i$	5	7	8	10	9	7
e_i	20	15	15	10	10	15
α_i	0.50	0.90	0.95	0.85	0.80	0.75
v_i	80	80	80	80	80	80
θ_i^{max}	2.63	0.98	0.79	4.50	3.00	1.40
m_i	0.90	080	0.95	0.97	0.93	0.92
$g'_{i,1}$	95	95	95	95	95	95
$h'_{i,1}$	0.25	0.25	0.25	0.25	0.25	0.25

The deterministic model associated with the case-study 2 has 1766 constraints and 403 and 187 continuous and binary variables, respectively. It is solved to optimality via solver CPLEX in 4.29 seconds. A comparision of our deterministic model and that of Abdoli, et al., [1], in terms of model size and solving time, for case-study 2, is prepared in Table 5 .

Table 5: Our deterministic model vs the corresponding model of [1] for case-study 2

	Our model	Model of Abdoli, et al., [1]
Number of continuous variables	403	403
Number of binary variables	187	217
Number of constraints	1766	1787

Now, following Abdoli, et al., [1], suppose that in case-study 2, for parameters (θ_1^{max}, m_1), two values $(5.35, 0.82)$ and $(3.75, 0.94)$, for parameters (θ_2^{max}, m_2), two values $(0.75, 0.75)$ and $(2.45, 0.96)$, for parameters (θ_3^{max}, m_3), two values $(0.45, 0.91)$ and $(1.35, 0.97)$, and for parameters (θ_4^{max}, m_4), two values $(2.50, 0.84)$ and $(6.25, 0.98)$, each one with equal probabilities, are possible. Also, suppose that the vector of parameters (θ_5^{max}, m_5) accepts only one value $(3.00, 0.93)$, and the vector of parameters (θ_6^{max}, m_6) receives only one value $(1.40, 0.92)$. Thus, the scenario set will be $\mathbb{S} = \{1, 2, \ldots, 16\}$ containing 16 scenarios each one with occurance probability of$1/16$. The stochastic model associated with case-study 2 has 17246 constraints and 6343 and 187 continuous and binary variables, respectively. It is solved to optimality via solver CPLEX in 302 seconds. A comparision of our stochastic model and that of Abdoli, et al., [1], in terms of model size and solving time, for case-study 2, is prepared in Table 6.

Table 6: Our stochastic model vs the corresponding model of Abdoli, et al., [1] for case-study 2

	Our model	Model of Abdoli, et al., [1]
Number of continuous variables	6343	6433
Number of binary variables	187	3472
Number of constraints	17246	190697
Solving time	302	24 hours

5 Conclusions

In this paper, the joint CCS-EOR planning in a multi-reservoir EOR system was addressed. First, an equivalent reformulation of the deterministic model of Abdoli, et al., [1] was developed, that in which the planning time horizon is no longer divided into equally length sections. Then, the uncertainty of the oil yield of reservoirs and its depletion factor was incorporated to extend the deterministic model to a two-stage SP model. Due to the exogenous nature of uncertainty in our stochastic model and absence of NA constraints, it is expected that our model be of less time complexity, confirmed with computational experiments over two case-studies.

For the future work, the proposed stochastic model can be extended by incorporating the uncertainty of the maximum amount of CO_2 that the common source can supply (i.e., the parameter $\bar{F_t}$). Additionally, the modification of the proposed model to deal more than one CO_2 common source can also be considered for future work. Moreover, the duration of the operation of each reservoir can be considered as a decision variable instead of a fixed parameter, leads to a more practical model. From the resolution aspect, presenting an appropriate heuristic or exact method to solve the proposed stochastic model would be a valuable direction for future research.

Data availability statement

All data, models, and codes that support the findings of this study are available from the corresponding author upon reasonable request.

References

[1] B. Abdoli, F. Hooshmand, and S. Mir Hassani, *A novel stochastic programming model under endogenous uncertainty for the CCS-EOR planning problem,* Appl. Energy **338** (2023), 1–23.

[2] E. Adu, Y. Jhang, and D. Liu, *Current situation of carbon dioxide capture, storage, and enhanced oil recovery in the oil and gas industry,* Canad. J. Chem. Engin. **97** (2019), no. 5, 1048–1076.

[3] W. Ampomah, R. Balch, M. Cather, R. Will, D. Gunda, Z. Dai, and M. Soltanian, *Optimum design of CO_2 storage and oil recovery under geological uncertainty,* Appl. Energy **195** (2017), 80–92.

[4] A. Elkamel, H. Hashim, P.L. Douglas, and E. Croiset, *Optimization of energy usage for fleet-wide power generating system under carbon mitigation options,* AIChE J. **55** (2009), no. 12, 3168–3190.

[5] A. Ettehad, *Storage compliance in coupled CO_2-EOR and storage,* Greenhouse Gases: Sci. Technol. **4** (2014), no. 1, 66–80.

[6] V. Goel and I.E. Grossmann, *A class of stochastic programs with decision dependent uncertainty,* Math. Program. Ser. B **108** (2006), no. 2-3, 355-–394.

[7] J. Guo, C. Huang, J. Wang, and X. Meng, *Integrated operation for the planning of CO_2 capture path in CCS–EOR project,* J. Petrol. Sci. Engin. **186** (2020), 106720.

[8] Y.J. He, Y. Zhang, Z.F. Ma, N.V. Sahinidis, R.R. Tan, and D.C.Y. Foo, *Optimal source–sink matching in carbon capture and storage systems under uncertainty,* Ind. Engin. Chem. Res. **53** (2014), no. 2, 778–785.

[9] H. Herzog, *Dioxide Capture and Storage*, USA: Helm Hepburn, Oxford University Press, 2009.

[10] F. Hooshmand and S.A. MirHassani, *Efficient constraint reduction in multistage stochastic programming problems with endogenous uncertainty,* Optim. Meth. Software **31** (2016a), no. 2, 359–376.

[11] H.R. Jahangiri and D. Zhang, *Ensemble based co-optimization of carbon dioxide sequestration and enhanced oil recovery,* Int. J. Greenhouse Gas Control **8** (2012), 22–33.

[12] J. Jiang, Z. Rui, R. Hazlett, and J. Lu, *An integrated technical-economic model for evaluating CO_2 enhanced oil recovery development,* Appl. Energy **247** (2019), 190–211.

[13] T. Jonsbraten, *Optimization models for petroleum field exploitation*, PhD thesis, Norwegian School of Economics and Business Administration, 1998.

[14] F. Kamali and Y. Cinar, *Co-optimizing enhanced oil recovery and CO_2 storage by simultaneous water and CO_2 injection,* Energy Explor. Exploit. **32** (2014), 281–300.

[15] S. Kashkooli, A. Gandomkar, M. Riazi, and M. Tavallali, *Coupled optimization of carbon dioxide sequestration and CO_2 enhanced oil recovery,* J. Petrol. Sci. Engin. **208** (2022), 109257.

[16] R.S. Middleton, *A new optimization approach to energy network modeling: anthropogenic CO_2 capture coupled with enhanced oil recovery,* Int. J. Energy Res. **37** (2013), no. 14, 1794–1810.

[17] R.S. Middleton and J.M. Bielicki, *A scalable infrastructure model for carbon capture and storage: SimCCS,* Energy Policy **37** (2009), no. 3, 1052–1060.

[18] R. S. Middleton, M.J. Kuby, and J.M. Bielicki, *Generating candidate networks for optimization: The CO_2 capture and storage optimization problem,* Comput. Envir. Urban Syst. **36** (2012), no. 1, 18–29.

[19] M.A. Safarzadeh and S.M. Motahhari, *Co-optimization of carbon dioxide storage and enhanced oil recovery in oil reservoirs using a multi-objective genetic algorithm (NSGA-II),* Petrol. Sci. **11** (2014), 460—468.

[20] R.R. Tan, K.B. Aviso, S. Bandyopadhyay, and D.K.S. Ng, *Continuous-time optimization model for source–sink matching in carbon capture and storage systems,* Ind. Engin. Chem. Res. **51** (2012), no. 30, 10015–10020.

[21] R.R. Tan, K.B. Aviso, S. Bandyopadhyay, and D.K.S. Ng, *Optimal source–sink matching in carbon capture and storage systems with time, injection rate, and capacity constraints,* Envir. Progress Sustain. Energy **32** (2013), no. 2, 411–416.

[22] J.F.D. Tapia, J.Y. Lee, R.E.H. Ooi, D.C.Y. Foo, and R.R. Tan, *Optimal CO_2 allocation and scheduling in enhanced oil recovery (EOR) operations,* Appl. Energy **184** (2016), 337–345.

[23] J. Tapia, J. Lee, R. Ooi, D. Foo, and R. Tan, *Planning and scheduling of CO_2 capture, utilization and storage (CCUS) operations as a strip packing problem,* Process Safety Envir. Protect. **104** (2016a), 358–372.

[24] J.F.D. Tapia and R.R. Tan, *Fuzzy optimization of multi-period carbon capture and storage systems with parametric uncertainties,* Process Safety Envir. Protect. **92** (2014), no. 6, 545–554.

[25] X. Wang, K. Veld, P. Marcy, S. Huzurbazar, and V. Alvarado, *Economic co-optimization of oil recovery and CO_2*

sequestration, Appl. Energy **222** (2018), 132–147.

[26] J. You, W. Ampomah, and Q. Sun, *Co-optimizing water-alternating-carbon dioxide injection projects using a machine learning assisted computational framework,* Appl. Energy **279** (2020b), 115695.

[27] J. You, W. Ampomah, Q. Sun, E. J. Kutsienyo, R.S. Balch, Z. Dai, M. Cather, and X. Zhang, *Machine learning based co-optimization of carbon dioxide sequestration and oil recovery in CO_2-EOR project,* J. Cleaner Product. **260** (2020a), 120866.

[28] S. Zhang, L. Liu, L. Zhang, Y. Zhuang, and J. Du, *An optimization model for carbon capture utilization and storage supply chain: A case study in Northeastern China,* Appl. Energy **231** (2018), 194–206.