Int. J. Nonlinear Anal. Appl. 15 (2024) 8, 273-287 ISSN: 2008-6822 (electronic) http://dx.doi.org/10.22075/ijnaa.2021.23854.2627



# Investigating the impact of an optimal fiscal policy on the Iran economy with externality based on the Ramsey solution

Hossein Marzban\*, Zahra Dehghan Shabani, Ahmad Sadrai Javaheri, Mahsa Mohaghegh Zadeh

Department of Economics, Faculty of Economics, Management and Social Sciences, Eram University Complex, Shiraz University, Shiraz, Iran

(Communicated by Mohammad Bagher Ghaemi)

## Abstract

Based on the literature on pollution tax on capital and labor, we have analyzed the impact of an optimal fiscal policy (tax) on the Iran economy based on the Ramsey solution. To achieve this, we created a dynamic economic-climatic model for the Iranian economy by adding externalities to a simple general equilibrium model without a foreign sector. Thus first we developed a dynamic model in two cases with and without side effects. Findings indicate that in the conditions of no pollution of the production sector, the model reaches equilibrium with zero capital tax. When we take into account the pollution in the model, in order to have a unique optimal point and keep the economy on the saddle path and it converged towards an optimal point, so capital-based pollution tax became essential. In our model, endogenous labor plays an important role as it indicates an increase in household environmental awareness, the supply of working hours, consumption, production, and environmental quality. In the models with exogenous labor, low household awareness of environmental quality leads to more severe degradation of environmental quality. According to our results considering the Ramsey solution on the economy, in a situation where: labor is endogenous, capital-based pollution tax is implemented, with a dynamic general economic-climatic equilibrium model, two environmental parameters are vital to be taken care of, "the weight of environmental quality" and "the polluting technology". In an environment where we use increasingly polluting technology, in the absence of an increasing pollution tax regime, the quality of the environment and society's welfare will diminish subsequently.

Keywords: Dynamic optimal control model, Pollution tax, Small economy, Ramsey solution. 2020 MSC: Primary 91B64; Secondary 91B51, 91B76

# 1 Introduction

Taxes have always been noticed by governments as a reliable source of revenue, but because this important tool of government is disrupting the economy, economists have thought of introducing new tax bases that would minimize inefficiencies in the economic system. Green taxes have such a feature.

Among the types of taxes, pollution tax is cost-based; Therefore, it is very widespread and brings a good income for the government, so it can replace other tax bases. This, on the one hand, reduces the distorting effect of other taxes and, on the other hand, has many advantages in pollution reduction for society.

\*Corresponding author

*Email addresses:* dr.marzban@gmail.com (Hossein Marzban), zahradehghan2003@yahoo.com (Zahra Dehghan Shabani), sadraei@shirazu.ac.ir (Ahmad Sadrai Javaheri), m.mohagheghzadeh@gmail.com (Mahsa Mohaghegh Zadeh)

The purpose of this study is to provide a comprehensive solution to solve the problems of economics at a sustainable level and during this research, it is examined that, is the public desire for society to reduce pollution and receive carbon tax from the government and then spend all of it for society to improve the quality of the climate and maintain its original quality? Can the government maintain the quality of the ecosystem and increase the welfare of individuals, or reduce the welfare of society as a result of reducing the emission of pollutants and returning the utility and welfare of society to its original level?!

The problem of determining the optimal structure of taxes by financing a certain level of expenditure is called the Ramsey problem. In the sample household model, the Ramsey problem is the determination of optimal taxes to maximize the utility of the sample household, that provided the revenues required by the government. Generally, in this method, the utility of the household is dependent on consumption and working hours (leisure) and the household maximizes its utility by setting a budget with a determined pollution tax rate.

The government's problem is to adjust tax rates to maximize the utility of the sample household, given its limited budget and tax revenues to finance expenditures. The government also faces the fact that the consumption of goods and services and the supply of labor must be done in accordance with and limited to the choices of private agents, which indicates the limitation of the options available to the government in choosing tools.

Also, the government is facing limited economic resources. Therefore, in this study, we seek to use the Ramsey problem associated with pollution with the use of a general equilibrium model and study the impact of pollution tax on the economy and welfare.

Given the importance of the effectiveness of government policies on improving the quality of the environment and increasing the welfare of society, the modeling of this paper begins with a closed economy model taking into account environmental pollution. Theoretically in general equilibrium economic models based on Ramsey problem solving, a planner pays attention to the distortion and excess burden of consumption, capital, and labor taxes on the welfare of the society and in addition to controlling the revenue and expenses of the government, such that the balance of budget, constraints are, etc. are available to it.

The main results of this study show the importance of optimal fiscal policies, which undoubtedly the economic stability policy is simultaneously involved with the rules of policy and fiscal policy rules. Therefore, considering the importance of optimal fiscal policies, to provide a model based on the Iranian economy based on solving the Ramsey problem and based on the analysis of welfare by applying additional constraints in the above general equilibrium model, a model is compiled and then to steady analysis and calibrated The model is evaluated in the mentioned economic literature and the model is simulated and the effect of pollution taxation on the economy and welfare of the society is examined.

# 2 Literature Review

Environmental quality and environmental awareness are two important components of the macro dynamic model literature, and the existence of a non-zero capital tax constraint is an integral part of the non-environmental model review literature. This literature suggests that the optimal tax factor may include positive labor and capital income tax rates. In the following, some studies related to the title of the research will be further examined.

Acemoglu, Golosov and Tsyvinski [1] showed that when taxes and supporting public goods are not decided by a utilitarian policymaker, doing policies can't be committed since in the long-term if the policy maker suffers more damage due to pollution than others, wealth tax won't be zero for a long time and will finally increase.

Angelopoulos et al. [2] used a stochastic dynamic general equilibrium model based on their findings with exogenous labor with varying degrees of environmental policy tools in uncertain circumstances, and compared government policy options with the first best alternative. They considered the Ramsey tax as the second best alternative to the pollution reduction policy, and compared the two policies in the face of economic and environmental uncertainty. The results of this study suggest that the welfare effects of higher economic growth depend on the size and degree of effectiveness of the environmental policy, while the performance of the Ramsey tax when there is a shock to the economy is in line with economic growth cycles, and is in the opposite direction of the cycles when the shock enters the environment.

Anonymous [3] used the dynamic general equilibrium model to measure the impact of green tax and concluded that green tax does not increase the welfare of society and will not reduce any environmental problems and unemployment.

Barrage [4] investigated the sensitivity of the formula of optimal carbon tax rate calculation discussed by Golosov et al. [15], and then examined how carbon taxation can be part of a fiscal policy. He showed that carbon taxation can be part of a fiscal policy, and this paper is based on existing theories and the optimal carbon tax is calibrated in a stochastic dynamic economic-climate equilibrium model despite disruptive fiscal policy.

Bureau [6] in a study performed for a 14% reduction of carbon dioxide in France, showed that carbon tax should be levied for 31 Euros for each ton of carbon dioxide emission. The results show the different impacts on the welfare of rich and poor households.

Bjertnaes and Faehn [5] in a study performed for Norway, showed that optimal financial policy as taxation on pollution due to energy consumption reduces production, employment, and consumption and if a subsidy is paid to the industry for pollution control, the society total welfare will increase.

Dessus and Bussolo [11] in studies performed for Costa Rica found out that receiving taxes from any pollutant just reduces the same pollutant which indicates that execution of the policy of pollution tax won't generally reduce the pollution.

Dissou and Eyland [12] in a study in Canada demonstrated that if optimal financial policy such as carbon taxation, at a rate of 40 \$ per ton of pollutant, is implemented alongside subsidy payment equivalent to the tax received by society, the competitiveness of the industry decreases. Consequently, taxation will improve and further reduce society's income. Internal gross production decreases by 13% with subsidy payment, and by 17% without it. The society's welfare exhibits a similar trend, with overall welfare decreasing following the carbon tax. However, welfare reduction can be moderated if subsidies are paid to society.

Economides and Philippopoulos [13] found that when governments aim for higher economic growth, the most effective policy is to tax polluting activities and then use the obtained revenues for pollution control measures. Economides and Philippopoulos (2008) reached the same conclusion in their study.

Flores and Graves [14] suggested that the endogeneity of labor supply often leads to a less realistic evaluation of optimally due to the increase in the provision of public goods. If labor supply is assumed to be endogenous and constant, the Le Chatelier-Samuelson principle holds. This issue is directly related to the fact that the cost of public goods increases, leading to a higher final valuation of private goods. Simultaneously, the payment rate for public goods decreases, resulting in a higher final cost of the rest.

Harati et al. [16] (2012) calculated consumption growth rates in the market equilibrium and social planning on the steady-state trajectory in their study. They used the production tax rate as an instrument to match these two rates. Subsequently, they solved the model using experimental methods tailored to the Iranian economy. The results show that the optimal pollution tax rate is approximately 15%.

Hill [17] analyzed the costs of pollution reduction through the use of environmental taxes and the cost of tax exemptions, both with and without employment limitations. It was found that a reduction of carbon dioxide emissions between 5-25% could reduce costs by more than 9% if environmental taxes are used instead of other taxes. Furthermore, the shifting of tax exemptions from certain industries can reduce costs and will also affect the emissions of other gases such as nitrogen dioxide and sulfur dioxide, thereby reducing pollution.

Hwan Bae [18] studied the welfare consequences of green tax modification in small open economies in Pennsylvania. He simulated the possible consequences of the substitution of carbon taxes with common taxes in a computable general equilibrium model. This scholar studied the numerical results on the consumer welfare consequences, supply and demand of factors and goods, import and export demands as three scenarios of factors immobility, factors mobility without environmental damage function and mobility of factors with environmental damage function for local and national carbon tax. According to the results of this study, the total welfare profits of three Pigou effects, reconstruction of tax income, and tax mutual effect are higher than their welfare damages and so environmental taxes increase welfare.

Iosifidi and Jafarey [19] in their study, investigated the role of taxes in a generalized Ramsey model in the United Kingdom. They estimate a dynamic general equilibrium model in which they consider the labor to be endogenous and include pollution as a negative environmental externality. The results of this study are in contrast to the results of Jadd (1985) and Chamley (1986) and indicate that, in the long run, the existence of environmental externalities causes the capital tax rate to be non-zero.

Jones, Manuelli and Rossi [20] studied the dynamism model for wealth and labor taxation and showed that if the rate of imposed tax is not high enough, a low profit will be obtained from human capital and the optimal income tax rate of physical capital will be non-zero.

Liang et al. [23] in a study performed in China showed that optimal carbon taxation which leads to 10-15% carbon dioxide reduction, without payment of subsidy to production results in a reduction of internal gross production, but when the commercial and energy-demanding sections are subject to receiving a subsidy, the production increase will be possible.

Ostadzad and Hadian [27] evaluated the optimal level of pollution tax in the Iranian economy. They utilized a three-agent model, calibrating it using available statistics to estimate the optimal level of pollution tax at 7.8 thousand Rials per tonne of carbon dioxide.

Ostadzad and Hadian [27] in a paper calculated the income tax optimal rate with and without considering environmental considerations using an endogenous growth model for Iran's economy. For this purpose, at first, a triplet generalized growth model including household, firm, and government was extended, and then using Iran's economy parameters, the mentioned model was calibrated and the tax optimal rates were calculated in various scenarios. The results obtained from model estimation indicate that the income tax optimal rate for Iran's economy despite pollution and considering environmental considerations is 22.2% and without considering environmental considerations is 20.5%. Also, changes in economic growth rate, capital final production, governmental oil incomes, and change in pollution-dependent factors impact the income tax optimal rate. The results of the sensitivity analysis of this study for Iran's economy indicate that in case of a reduction of oil income, for remaining in a steady state, the optimal rate of income tax optimal rate should increase. Higher economic growth will raise the optimal tax rate. An increase in societal sensitivity to pollution also necessitates a rise in the optimal income tax rate for optimal social welfare.

Pajouyan and Amin Rashti [29] examined green taxation on pollutant goods using a Rotterdam system model, demonstrating that the implementation of such a tax can reduce demand for these products.

Majdzadeh and Ostadzadeh [24] applied an endogenous growth model and comparative static analysis to examine the policies of implementing energy tax, enhancing pollution prevention technology, and subsidizing green inputs for pollution control in the Iranian economy. Their findings suggest that by adopting appropriate policies, the government can guide economic agents toward the selection of an optimal solution for resource allocation, thereby achieving sustainable economic growth .

Moghimi et al. [25] used the 2001 input-output table and a computable general equilibrium model to assess the welfare and environmental effects of two policies: green taxation and fuel subsidy reduction. They used the MSP technique and GAMS software to examine welfare changes (both with and without considering environmental effects), changes in energy demand, and changes in the shares of carbon dioxide, nitrate, and carbohydrate pollutants under five tax scenarios. Their results show that levying a fuel tax reduces both intermediate and consumer demand for fossil fuels. In all scenarios, considering the positive effect of pollution reduction, welfare changes are positive, and its rate increases with the tax rate. Under both policies, the highest welfare growth rate considering environmental effects is at a 10% tax rate (the third scenario).

Wissema and Dellink [32] conducted a study in Ireland and found that a tax rate of 10-15 Euros per tonne of carbon dioxide leads to a 25% reduction in pollution. The welfare reduction due to this tax was estimated to be one percent .

The remainder of this paper is organized as follows: Section 2 reviews experiments from various countries, Section 4 presents the model, Section 5 provides a solution for decentralized competitive equilibrium, and Section 6 checks the stability of the model and compares it with a similar model where labor is endogenous. In addition, it compares the results with those of Judd and Chamley, offers numerical examples, and specifies the dynamic responses to parametric shocks for some parameters of interest in the model. Section 7 presents the conclusion.

# **3** Theoretical Base

### 3.1 Environmental destructive effects of air pollution

Air pollution has significant effects on health, the environment, and the economy. According to the World Health Organization, about 7 million people die each year from air pollution in the world, half of which is due to external pollution. Air pollution is currently the biggest threat to environmental health and causes and exacerbates many diseases such as asthma and cancer. According to the International Energy Agency, if energy policies do not change, by 2040 the number of premature deaths due to external pollution will reach 4.5 million.

#### 3.2 Economic destructive effects of air pollution

Apart from environmental damage, air pollution also imposes many economic costs on countries. Air pollution has its effects on the economy in several ways; Increasing the cost of human life, reducing the ability of people to function, increasing the important effects on food, destroying historical and cultural monuments, reducing the ability of ecosystems to meet the needs of communities and increasing the cost of restoration. Recent studies show that the health costs of using fossil fuels are 600 percent higher than the taxes levied on them; This means that for every dollar of subsidy given to the fossil energy sector, six dollars of health costs increase.

#### 3.3 The adverse effects of climate change on Iran

Greenhouse gases are naturally present in the Earth's atmosphere, but human activities and the pollution caused by these activities abnormally increase the amount of these gases. As a result, the heat from the sun's rays is trapped in the Earth's atmosphere and raises the Earth's temperature. Climate change is a phenomenon that has adverse effects on human life and the planet. According to research and evaluations conducted in the Climate Change Enabling Plan under the United Nations Framework Convention on Climate Change and using scenarios proposed by the IPCC, if the concentration of carbon dioxide doubles by 2100, the average temperature in Iran will increase by 1.5 to 4.5 degrees Celsius, which will cause significant changes in water resources, energy demand, agricultural production, and coastal areas. Changing temperature patterns, declining water resources, rising sea levels, coastal degradation, loss of agricultural and food crops, deforestation, drought intensification and threat to human health are some of the direct detrimental effects of climate change. One of the indirect effects of climate change is the economic damage caused by the countermeasures of developed countries.

# 4 Description of Economy

In this section, we explain and place our basic framework. Some statements are based on the fact that in the preferences of individuals, labor decisions are endogenous and their weight in the utility function is proportional to the weight of consumption and the quality of the environment. Then we explain the firm's decisions, laws of natural resources, and resources restriction are explained and a model is created with the government budget restriction.

#### 4.1 Households

We have assumed that the population size is constant and equal to one. The representative household with infinite life maximizes the intertemporal utility:

$$\sum_{t=0}^{\infty} \beta^t U(c_t, l_t, Q_t), \tag{1}$$

where c is private consumption, l is leisure, Q is environmental quality, and  $\beta \in (0,1)$  is the discount factor. Our utility function, without loss of generality, is as follows:

$$U(c_t, l_t, Q_t) = \frac{(c_t^{\mu_1} l_t^{\mu_2} Q_t^{1-\mu_1-\mu_2})^{1-\sigma}}{1-\sigma},$$
(2)

where  $(1 - \mu_1 - \mu_2)$ ,  $\mu_1$ ,  $\mu_2 \in (0, 1)$  are preference parameters indicating the weights assigned to consumption, leisure, and environmental quality. Note that  $\sigma \ge 0$  represents the degree of risk aversion. Each household has a unit of time, which can be allocated to leisure,  $l_t$ , or labor,  $n_t$ , such that  $n_t + l_t = 1$ . Each household can save in the form of capital,  $K_t$ , which earns a return,  $V_t$ .

Households supply labor inelastically and receive labor income. They also receive profits,  $\pi_t$ . Each household must pay a portion of its income to the government as linear taxes. The tax rates on capital and labor are denoted by  $\tau_t^k$ and  $\tau_t^l$  respectively. The budget constraint of the household is as follows:

$$k_{t+1} - (1 - \delta^k)k_t + c_t = y_t = (1 - \tau_t^l)w_t n_t + (1 - \tau_t^k)r_t k_t + \pi_t,$$
(3)

where  $k_{t+1}$  is the capital stock at the end of the period,  $k_t$  is the capital stock at the beginning of the period, and  $\delta^k \in (0, 1)$  is the rate of capital depreciation.

Given these considerations, the household's problem is given by:

$$\max_{\{c_t, l_t, Q_t\}_{t=0}^{\infty}} \sum_{t=0}^{\infty} \beta^t \left( \frac{(c_t^{\mu_1} (1-n_t)^{\mu_2} Q_t^{1-\mu_1-\mu_2})^{1-\sigma}}{1-\sigma} \right)$$
  
s.t.  $k_{t+1} - (1-\delta^k) k_t + c_t = (1-\tau_t^l) w_t n_t + (1-\tau_t^k) r_t k_t + \pi_t,$ 

where  $w_t$ ,  $r_t$ , and the policy are given. The first-order conditions of this problem are as follows:

$$U_{c_t} = \lambda_t,\tag{4}$$

Marzban, Dehghan Shabani, Sadrai Javaheri, Mohaghegh Zadeh

$$\frac{c_t}{1-n_t} = \frac{\mu_1}{\mu_2} (1-\tau_t^l) w_t, \tag{5}$$

$$U_{c_t} = \beta U_{c_{t+1}} [(1 - \tau_{t+1}^k) r_{t+1} + 1 - \delta^k].$$
(6)

The last equation is the Euler equation for capital. This equation tells us that along the optimal path, the marginal utility at each time point equals the opportunity cost.

# 4.2 Firms

$$y_t = Ak_t^{\alpha} n_t^{1-\alpha} = f(k_t, n_t), \tag{7}$$

where  $\alpha \in (0, 1)$  is the product elasticity relative to the private capital and  $1 - \alpha \in (0, 1)$  is the private labor elasticity. A is the factor of total productivity or index of production technology that is assumed to be constant.

In each period, the sample firm receives  $n_t$ ,  $r_t$  as given and uses labor services and household capital. The problem of the firm is as follows:

$$\max_{\{c_t, l_t, k_{t+1}\}_{t=0}^{\infty}} \pi_t = y_t - w_t n_t - r_t k_t.$$
(8)

By solving first-order conditions of this problem, we have:

$$r_t = \frac{ay_t}{k_t},\tag{9}$$

$$w_t = \frac{(1-\alpha)y_t}{n_t},\tag{10}$$

where  $\pi = 0$ .

### 4.3 Rule of natural resources motion

$$Q_{t+1} = (1 - \delta^q)\tilde{Q}_t + \delta^q Q_t - p_t + \nu g_t, \tag{11}$$

where  $\widetilde{Q} \geq 0$  is environmental quality without pollution,  $p_t$  is current pollution flow,  $\delta^q \in (0, 1)$  is a degree of environmental stability,  $g_t$  is general payment for activities control and  $\nu \geq 0$  indicates the public expenditure for pollution control and transfers to natural resources units. The pollution flow generated by the production is as follows:

$$p_t = \varphi A k_t^{\alpha} n_t^{1-\alpha}, \tag{12}$$

where  $\varphi$  is the pollution technology indicator, which shows the pollution reduction rate imposed by each product unit.

#### 4.4 Government budget constraint

The government obtained revenues from taxes on labor and capital and on the expenditure side, the government pays for its pollution control policy,  $g_t$ . Assuming a balanced of budget, we have:

$$g_t = Ak_t^{\alpha} n_t^{1-\alpha} [\alpha \tau_t^k + (1-\alpha)\tau_t^l].$$
(13)

#### 4.5 Resource constraints (technology)

The product can be used by the household, increase capital, or be consumed by the government. Therefore, the resource constraint will be as follows:

$$c_t + g_t + k_{t+1} = y_t + (1 - \delta^q)k_t.$$
(14)

## **5** Decentralized Competition Equilibrium

In this section, we will solve the problem described in Section 4 for a decentralized competitive equilibrium to which the following assumptions are true. In this equilibrium, (i) the household maximizes welfare, (ii) the firm maximizes benefits, (iii) all constraints are imposed, and (iv) all markets are clear.

The DCE model of the economy described above is given by the following equations:

$$c_t/(1-n_t) = \frac{\mu_1}{\mu_2}(1-\tau_t^l)w_t,$$
(15)

$$U_{c_t} = \beta U_{c_{t+1}}[(1 - \tau_{t+1}^k)r_{t+1} + 1 - \delta^k],$$
(16)

$$Q_{t+1} = (1 - \delta^q)\overline{Q}_t + \delta^q Q_t - \phi A k_t^\alpha n_t^{1-\alpha} + \nu g_t, \qquad (17)$$

$$g_t = Ak_t^{\alpha} n_t^{1-\alpha} [\alpha \tau_t^k + (1-\alpha)\tau_t^l], \qquad (18)$$

$$c_t + k_{t+1} = Ak_t^{\alpha} n_t^{1-\alpha} - g_t + (1 - \delta^q) k_t.$$
(19)

DCE model is based on primary provisions for the variables of  $K_0$  and  $Q_0$  with first-order conditions of the sample firm problem, with endogenous variables A and  $\phi$  with the given policy  $(\tau^k, \tau^l)$  which has resulted in  $r_t = a_t n_t$  and  $w_t = (1-a)A_t^a D_t^{-a}$ . Then, we have a system containing five equations as  $\{c_t, n_t, k_{t+1}, Q_{t+1}, g_t\}_{t=0}^{\infty}$ . If we simply discard the subscript t, we can also get a long-term DCE.

#### 5.1 Steady state

To find the steady state, we solve the above system for g, k, Q, n, c so that it represents the state of the variable in the steady state of each variable. So we will have:

(1 - a) (1 -

11. ( .

$$c^* = \frac{\mu_1}{\mu_2} (1 - \tau^l) (1 - \alpha) A X^{a/(1-a)} (1 - X^{1/(1-a)} k^*), \tag{20}$$

$$n^* = X^{1/(1-a)}k^*, (21)$$

$$Q^* = \overline{Q} - k^* A X / ((1 - \delta^q)), \tag{22}$$

$$g^* = AXk^*[\alpha\tau^k + (1-\alpha)\tau^l], \qquad (23)$$

Where:

$$k^* = \frac{\frac{\mu_1}{\mu_2}(1-\alpha)AX^{a/(1-a)}(1-\tau^l)}{\delta^k - \left[\frac{\mu_1}{\mu_2}(1-\alpha) + \alpha(1-\tau^k) + (1-\alpha)(1-\tau^l)\right]AX}$$
(24)

$$X = \frac{(1 - \beta + \beta \delta^k)}{a\beta A(1 - \tau^k)}.$$
(25)

Steady state variables indicate that increase of capital tax  $\tau^k$  has led to reduction of  $k^*, c^*$ , but  $q^*, Q^*, n^*$  increase. The agents replace consumption with labor and the increased tax helps the government to have more payment for pollution control policy which improves the quality of the environment. Similar results are obtained for  $\tau^{l}$  increases, with this difference that the labor rate decreased in equilibrium decreased. In addition, a change in the environmental parameter  $\nu$  and q only affects the quality of the environment. In particular, a change in pollution technology or an increase in government payments to control pollution improves the quality of the environment. An increase in proportion  $\mu_1/\mu_2$  (consumption weight) or  $\beta$  increase in, will result in an increase in consumption, labor, and capital in a steady state. If the government payment for pollution control is reduced, environmental quality decreases. Then, we will have similar results with consumption weight increase (consumption weight or reduction of environmental quality weight reduction) or we may say that the agent should be very cautious and pay more attention to the future.

#### 5.2 Linearization

With the substitution of equation 18 in other equations of DCE, our model will turn to the following equations:

$$c_t + k_{t+1} = Ak_t^{\alpha} n_t^{1-\alpha} - Ak_t^{\alpha} n_t^{1-\alpha} [\alpha \tau_t^k + (1-\alpha)\tau_t^l] + (1-\delta^q)k_t.$$
(26)

$$c_t/(1-n_t) = \frac{\mu_1}{\mu_2} (1-\tau_t^l)(1-\alpha) A k_t^{\alpha} n_t^{-\alpha},$$
(27)

$$U_{c_t} = \beta U_{c_{t+1}} [(1 - \tau_{t+1}^k) \alpha A k_{t+1}^{(\alpha - 1)} n_{t+1}^{1 - \alpha} + 1 - \delta^k],$$
(28)

$$Q_{t+1} = (1 - \delta^q)\overline{Q} + \delta^q Q_t - \phi A k_t^{\alpha} n_t^{1-\alpha} + \nu A k_t^{\alpha} n_t^{1-\alpha} [\alpha \tau_t^k + (1 - \alpha) \tau_t^l].$$
<sup>(29)</sup>

We have linearized equations 26-29 using the Taylor series around the steady state. We have supposed that  $\tau^k, \tau^l$  are endogenous and the values are got from the Ramsey optimization problem. After the substitution of the remained parameters with the values used in the literature of Angelopoulos, Economides, and Philippopoulos (2010) it was found that our model is stable.

#### 5.3 Impulse-Response and Revealed Facts

To see how labor endogeneity affects equilibrium results, we compared our DVE model with the DCE model in which the labor variable was exogenous. Our findings show that in both of these economies, the dynamic responses to sustained uncertainty are an increase in some interested parameters in the models.

For this comparison, we equated the amount of output in both models. In these models, capital is a state variable. Therefore, we set its initial value for the assumed values in the model with endogenous labor at t = 0. Initially, all variables are at the level of their steady state. Figure 5.3 shows how the economy responds with endogenous labor to a one percent increase in environmental quality weight. We have seen that when both  $\mu_1, \mu_2$  are reduced, as discussed



Figure 1: DCE Economy response to the one percent increase in the weight of Env. Quality in the model with endogenous labor

above, there is a substitution channel for investment and consumption, and other channels for labor and consumption, and the quality of the environment is definitely will increase steadily.

Households internalize the fact that the product is harmful to the environment and thus their production and consumption are reduced. In this way, taxes paid to the government are reduced and fewer funds are needed for control policies. Through this mechanism, welfare will definitely be developed and improved. And interestingly, in the model where labor is endogenous, people's preferences for a better environment are positively related to variables such as pollution emissions, which characterize the quality of the environment. These findings are real and consistent with the revealed facts supposed in this section.

Figure 2 indicates how an economy with an exogenous labor force responds to a one percent increase in environmental quality. The current results are very different; Productivity and capital increase, consumption decreases, and even if government payments for pollution control policies increase, pollution will increase more than before. This will lead to a decline in environmental quality. Agents only can substitute the present and future consumption time with each other. But obviously, this is not enough to improve the quality of the environment. The result is an increase in



Figure 2: DCE Economy response to the one percent increase in the weight of Env. Quality in a model with exogenous labor

welfare, despite the fact that the quality of the environment has declined. This is in stark contrast to the findings of a positive correlation between environmental quality and welfare shown in Figure 5.3.

Figure 3 indicates how the economy with endogenous labor responds to a one percent increase in the pollution parameter. The increase in the pollution parameter means that agents use more pollution technology in the production of their products, so they pay less attention to the quality of the environment. We observed that all variables, except consumption, are reduced. The current consumer will replace labor and investment, and the quality of the environment will certainly decline. The household does not internalize the fact that the product is harmful to the environment and therefore welfare is reduced.



Figure 3: DCE Economy response to the one percent increase in the pollution parameter in the model with endogenous labor

Finally, Figure 4 indicates the response of the economy with the exogenous labor to one percent increase in the pollution parameter  $\phi$ . Again, our findings are very different from the state where the economy's labor is endogenous, observed in Figure 3. Future consumption is again is substituted by present consumption and therefore present consumption increases. They produce more of a polluting product and consume more. In this way, they pay more taxes to the government, which is used in pollution control policies.

We can see that in both cases, the change in the quality of the environment affects well-being, and this effect is



Figure 4: DCE Economy response to the one percent increase in the pollution parameter in a model with exogenous labor

greater when the labor is endogenous than when it is exogenous. Thus Le Chatelier's-Samuelson principle is confirmed, and the exogenous labor will lead to a less evaluation of increase in the quality of the environment, which is a public good.

The importance of endogenous labor is further supported by the revealed facts related to environmental awareness and the quality of the environment. We measured the quality of the environment by focusing on  $CO_2$ . This measurement is common in the experimental literature for several reasons. First, air pollution is one of the most important indicators of environmental quality and  $CO_2$  is one of the criteria for air pollution. Second,  $CO_2$  is one of the main air pollutants and has a significant impact on human health, ecosystems, and the economy.

Finally, Bernayer and Kobe (2009) showed that most types of air pollution (like  $SO_2$ ,  $CO_2$ ,  $N_2O$ , NOx) behave very similarly in different countries, and so  $CO_2$  includes general and public procedures in universal air pollution.

# 6 Ramsey problem with environmental externality

In this section, our DCE framework is replaced with the Ramsey tax framework. Ramsey tax creates the discussions of linearization against wealth tax in the long term and in macroeconomic dynamic models.

Here we show how the results change when we discuss a negative environmental externality. Following Chamley (1986), we replace them with net f prices so  $(\tilde{w}_t, \tilde{r}_t)$  that:

$$\tilde{r}_t = (1 - \tau_t^k) r_t \tag{6.1}$$

$$\tilde{\omega}_t = (1 - \tau_t^l)\omega_t \tag{6.2}$$

In this regard, for instruments of  $w_t, n_t, \tau_t^l, \tau_t^k$  are reduced to two cases. Then, DCE will be as follows:

$$\frac{c_t}{(1-n_t)} = \frac{\mu_1}{\mu_2} \tilde{\omega}_t \tag{6.3}$$

$$U_{c_t} = \beta U_{c_{t+1}} (\tilde{r}_{t+1} + 1 - \delta^k) \tag{6.4}$$

$$Q_{t+1} = (1 - \delta^q)\bar{Q} + \delta^q Q_t - \phi A k_t^\alpha n_t^{1-\alpha} + \nu g_t$$

$$(6.5)$$

$$g_t = Ak_t^{\alpha} n_t^{1-\alpha} - \tilde{\omega}_t n_t - \tilde{r}_t k_t \tag{6.6}$$

$$c_t + k_{t+1} - (1 - \delta^q)k_t + g_t = Ak_t^{\alpha} n_t^{1-\alpha}$$
(6.7)

The first-order conditions of the above problem are as follows:

$$U_{c_{t}} = \frac{1}{1 - n_{t}} \lambda_{t} + \chi_{t} - \frac{\partial U_{c_{t}}}{\partial c_{t}} \left[ \psi_{t-1}(\tilde{r}_{t+1} + 1 - \delta) - \psi_{t} \right]$$

$$U_{n_{t}} = \frac{c_{t}}{\langle t - \chi_{t} \rangle^{2}} \lambda_{t} - (1 - \alpha) A k_{t}^{\alpha} n_{t}^{1 - \alpha} (\xi_{t} - \zeta_{t} \phi + \chi_{t}) +$$
(6.8)

$$\gamma_{n_{t}} = \frac{1}{(1 - n_{t})^{2}} \lambda_{t} - (1 - \alpha) A \kappa_{t}^{\alpha} n_{t}^{-\alpha} (\xi_{t} - \zeta_{t} \phi + \chi_{t}) + \delta (U_{ct} / \mu_{t})$$

$$\xi_{t}\tilde{\omega}_{t} + \frac{\langle \psi_{t+1}, \psi_{t-1}\rangle}{\partial n_{t}} [\psi_{t} - \psi_{t-1}(\tilde{r}_{t+1} + 1 - \delta)]$$
(6.9)

$$\chi_t = \beta \left[ \chi_{t+1}(f_k + 1 - \delta^{\kappa}) + \xi_{t+1}(f_k - \dot{r}_{t+1}) - \zeta_{t+1}\phi f_k \right]$$
(6.10)

$$U_{Q_t}\left[\psi_t(\tilde{r}_{t+1}+1-\delta^k)-\psi_{t+1}\right] = \frac{\zeta_t}{\beta} - U_{Q_{t+1}} - \zeta_{t+1}\delta^q$$
(6.11)

$$\xi_t k_t = \psi_{t-1} U_{c_t} \tag{6.12}$$

$$\frac{\lambda_t \mu_1}{\mu_2} = \xi_t n_t \tag{6.13}$$

$$\nu\zeta_t = \xi_t + \chi_t \tag{6.14}$$

$$\frac{\mu_1 \tilde{\omega}_t}{1} = \frac{c_t}{1} \tag{6.15}$$

$$\mu_2 = \mu_t = n_t = 0$$

$$U_{c_k} = \beta U_{c_{k+1}} (\tilde{r}_{t+1} + 1 - \delta^k)$$
(6.16)

$$Q_{t+1} = (1 - \delta^q)\bar{Q} + \delta^q Q_t - \phi A k_t^\alpha n_t^{1-\alpha} + \nu g_t \tag{6.17}$$

$$Ak_t^{\alpha} n_t^{1-\alpha} - \tilde{\omega}_t n_t - \tilde{r}_t k_t = g_t \tag{6.18}$$

$$c_t + k_{t+1} = Ak_t^{\alpha} n_t^{1-\alpha} + (1-\delta^q)k_t - g_t$$
(6.19)

Equation (6.10) tells us that the final increase of the capital instrument in time t, increases the accessible goods rate in t + 1 by  $(f_k + 1 - \delta)$  with final social value  $X_{t+1}$ . Moreover, the tax revenue increased by  $(f_k - \tilde{r}_{t+1})$  enables the government to reduce their debts on other taxes with similar amounts. This increase has a final social value equal to  $\epsilon_{t+1}$  which has been defined as the additional load imposed on the society due to tax disorder.

 $\beta$  is the discount in t + 1 and  $X_t$  is the final social value of the intermediate goods in time t. Then,  $\epsilon_t, X_t$  on all t rates are positive. Finally, it is observed that the increase in capital instrument of environmental quality by  $\phi_k^f$  is more similar to final social value t + 1.

By discarding the time subscript, the long-term provision is obtained. For simplifying the first order conditions, place  $\alpha = 1, U(C_t, L_t, Q)$  in the production function which is then limited to:

$$U(c_t, l_t, Q_t) = \mu_1 \ln(c_t) + \mu_2 \ln(l_t) + (1 - \mu_1 - \mu_2) \ln(Q_t)$$
(6.20)

As done for the DEC equations system, the system of equations 37-48 is linearized around the steady state and the Taylor series is used. Similar values for parameters are used. It was found out the model is stable and again we have a unique equilibrium where the economy gets convergent towards this equilibrium.

#### 6.1 Judd - Chamley model for Ramsey problem

Equation (6.10) reduces in the long-term to:

$$\beta[(r - \tilde{r}_t)\xi + (r + 1 - \delta)\chi - r\phi\zeta] = \chi.$$
(6.21)

Equation (6.16) in the long-term yields  $(1-\delta) = \frac{1}{\beta} - \tilde{r}$ . Substituting this result into equation (6.20) and rearranging, we get:

$$(r - \tilde{r}_t)(\chi + \xi) - r\phi\zeta = 0. \tag{6.22}$$

Now, two cases are studied, where  $\phi = 0$  and  $\phi \neq 0$ . In the case where the environmental externality is zero, equation (6.21) simplifies to:

$$^{k}(\chi + \xi) = 0. \tag{6.23}$$

The final social value of X goods is definitely positive, and the final social value of the governmental taxes is non-negative. Therefore, r should be equal to  $\tilde{r}$  such that  $\tau^k$  is equal to zero. This is consistent with the results of Judd (1985) and Chamley (1986). This result can be observed through a simple numerical example. In Table 1,

au

Р.	Descrip.	Val.	Р.	Descrip.	Value
$\begin{array}{c} \alpha \\ \delta^k \\ \sigma \\ \beta \\ \mu_1 \\ \nu \end{array}$	Capital share in production function Capital depreciation rate Curvature parameter in utility function Time discount factor Consumption weight in utility function Transfer Payments to nature units	0.22 0.1 2 0.97 0.2 5	$\begin{array}{c} \mu_2 \\ \tilde{Q} \\ \delta^q \\ A \\ \phi \end{array}$	Leisure weight in utility function Environmental Quality without pollution Persistence of Environmental Quality Long-run total factor productivity Pollution technology in long-run	$0.6 \\ 1 \\ 0.9 \\ 1 \\ 0$

Table 1: Parameter value in numerical example

the parameter values are presented, and the results are presented in the first section of Table 2. The values used for parameters in most dynamic general equilibrium models are similar (Angelopoulos, Economides, and Philippopoulos, 2010, and others). Our findings showed that  $\tau^k = 0$ , and with a welfare discount for t = 100, we have:

$$U^*(c,n,Q) = \frac{(1-\beta^t)}{(1-\beta)}U(c,n,Q) = \frac{(1-\beta^{100})}{(1-\beta)}[(c_t)^{\mu_1}(l_t)^{\mu_2}(Q_t)^{1-\mu_1-2}]^{\frac{1-\sigma}{1-\sigma}} = -37.28275513$$

Table 2: The long-run Value

Var.	Ι	II	Var.	Ι	II
g	0.099	0.116	c	0.156	0.135
$\lambda$	0.575	0.681	n	0.217	0.217
$\psi$	0.101	0.084	Q	5.964	5.168
ς	0.295	0.361	k	0.861	0.737
ε	0.885	1.046	$\tau^k$	0.000	0.1
$\chi$	0.589	0.757	$\tau^l$	0.434	0.483

In the case where  $\phi \neq 0$ , equation (50) is definitely similar to the results of Judd and Chamley. With the replacement of  $\tilde{r}$  with  $\delta(1-\tau^k)$  and using equation (43), we will have:

$$\tau^k = \frac{\phi}{\nu}.\tag{53}$$

It should be noticed that if  $\tau^k < 1$ , then  $\frac{\phi}{\nu} < 1$  or  $\phi < \nu$ . In the second column of Table 2, the results obtained from a numerical example where  $\phi$  is positive and equal to 0.5 are presented. The value of the parameters is similar to the previous ones. It is obvious that  $\tau^k$  is positive and welfare decline when t = 100 is as follows:

$$U^{*}(c,n,Q) = \frac{(1-\beta^{t})}{(1-\beta)}U(c,n,Q) = \frac{(1-\beta^{100})}{(1-\beta)} \left[ (c_{t})^{\mu_{1}} (l_{t})^{\mu_{2}} (Q_{t})^{1-\mu_{1}-2} \right]^{1-\sigma} / (1-\sigma) = -39.49321353$$
(6.24)

Again, environmental externality worsens the environmental quality. Taxes are increased which leads to a lower optimality level to the state when the environmental externality is equal to zero. Then, in our model with environmental externality, the results of the second order condition of Judd and Chamley were obtained, where wealth tax is always positive and the only reaction changes in environmental parameters.

## 6.2 Impulse and Response function for Ramsey economy

# 7 Dynamic Reaction of Ramsey Economy

In this section, the dynamic reaction of the Ramsey economy to the effect of an increase in the uncertainty parameter on some of the considered parameters in our model is observed. Specifically, the effects of a one percent increase in the environmental quality weight and a one percent increase in the pollution parameter  $\phi$  are studied.

Figure 5 shows how the economy reacts to a one percent increase in the environmental quality weight. It was observed that in the long-term, the product, labor, and capital are decreased. Here, the second channel of the trial is the fact that labor is endogenous. Environmental quality to the primary equilibrium is at a significantly higher level. When the household becomes informed that the product is harmful to the environment, they reduce their production and consumption. In the long term, the labor tax increases, leading to an increase in the amount of money used by the government for pollution control policy.

Wealth tax in the long-term doesn't change, as its value is predominantly determined by environmental parameters. Finally, welfare is at a higher level than at the primary equilibrium. These findings are consistent with the facts revealed in sections 3-3.



Figure 5: Ramsey economy response to the one percent increase in the Env. Quality weight

Similarly, a one percent increase in the pollution parameter  $\phi$  has a negative effect on environmental quality, as shown in Figure 6. An increase in the pollution parameter is represented in a way that the agent uses more polluting technology for product production, thereby reducing the weight of control on environmental quality. The household compensates for this by increasing working hours for current consumption, although the reduction in consumption is minimal, as the agent prefers to consume more today rather than in the future. Through this mechanism, capital decreases, and as a result, the wealth tax and working hours increase in the long term.

The agent produces fewer products, and although the amount of taxes paid to the government increases, leading to an increase in the funds used by the government for pollution control policy, the increase in pollution production is prevalent, causing a decline in environmental quality and a reduction in welfare.

# 8 Conclusion

This paper is formed based on wealth tax literature and working hours and studies a dynamic general equilibrium model (DSGE) with externality. Our model without environmental externality gives zero wealth tax results which are consistent with the primary model of Chamley (1985) and Judd (1986), though the inclusion of environmental externality changes these results and results in positive and fixed wealth tax which is merely a reaction to change of environmental parameters.



Figure 6: Ramsey economy response to the one percent increase in the Pollution parameter

One significant factor in the framework of our model is the endogeneity of families' working hours decisions. This assumption is consistent with environmental awareness increase, unemployment, and product decisions with environmental quality. The supply of endogenous labor creates partial changes in the results. Especially in the model without endogenous labor, it is predicted that in fact, reduction of environmental quality weight by the household leads to environmental decline.

Using this simple indicator of environmental quality and awareness of it, it is shown that the findings are in contrast to the revealed facts, and in a similar model where the labor was considered endogenous, it is predicted that environmental quality improvement following a positive shock in the household considered weight for environmental quality, has results coincident to the revealed facts.

A study of the uncertainty effect, incomplete competition, and or different production technology is recommended for the extension of this model. As this paper has covered the required backgrounds for the discussion, we suggest these ideas for future studies.

## References

- D. Acemoglu, M. Golosov, M., and A. Tsyvinski, *Political economy of Ramsey taxation*, J. Public Econ. 95 (2011), 467–475.
- [2] K. Angelopoulos, G. Economides, and A. Philippopoulos, First-and Second-Best Allocations under Economic and Environmental Uncertainty, Working Papers 2010 35, Business School- Economics, University of Glasgow, 35 (2010).
- [3] M. Anony, Developing Green Taxation-Summary of a Government Assignment Report 5390, Econ. Soc. Commission Pacific Envir. Sustain. Dev. Division, 2004, pp. 1–23.
- [4] L. Barrage, Sensitivity analysis for Golosov, Hassler, Krusell, and Tsyvinski: Optimal taxes on fossil fuel in general equilibrium, Econometrica 82 (2014), 41–88.
- [5] G.H. Bjertnaes and T. Faehn, Energy taxation in a small, open economy: Social efficiency gains versus industrial concerns, Energy Econ. 30 (2017), no. 4, 2050–2071.
- [6] B. Bureau, Distributional effects of a carbon tax on car fuels in France, Energy Econ. 33 (2011), 121–130.
- [7] A. Bovenberg and S. Smulders, Environmental quality and pollution-augmenting technological change in a twosector endogenous growth model, J. Public Econ. 57 (1995), 369–391.
- [8] C. Chamley, Optimal taxation of capital income in general equilibrium with infinite lives, Econometrica 54 (1986), 607–622.
- [9] I.H. Correia, Should capital income be taxed in the steady state?, J. Public Econ. 60 (1996), 147–151.

- [10] P.A. De Hek, On taxation in a two-sector endogenous growth model with endogenous labor supply, J. Econ. Dyn. Control 30 (2006), 655–685.
- [11] S. Dessus and M. Bussolo, Is there a trade-off between trade liberalization and pollution abatement?, J. Policy Model. 20 (1998), 11–31.
- [12] Y. Dissou and T. Eyland, Carbon control policies, competitiveness, and border tax adjustments, Energy Econ. 33 (2011), 556–564.
- [13] G. Economides and A. Philippopoulos, Growth enhancing policy is the means to sustain the environment, Rev. Econ. Dyn. 11 (2008), 207–219.
- [14] N.E. Flores and P.E. Graves, Optimal public goods provision: Implications of endogenizing the labor/leisure choice, Land Econ. 84 (2008), 701–707.
- [15] M. Golosov, J. Hassler, P. Krusell, and A. Tsyvinski, Optimal taxes on fossil fuel in general equilibrium, Econometrica 82 (2014), 41–88.
- [16] J. Harati, K. Eslamloueyan, and M.A. Ghetmiri, The optimal environmental tax in a generalized growth model with clean technology diffusion and environment quality: The Case of Iran, J. Econ. Model. Res. 3 (2012), no. 7, 97–126.
- [17] M. Hill, Green Tax Reform in Sweden: The Second Dividend and the Cost of Tax Exemptions, Economics Department, Stockholm School of Economics and the Beijer Institute, Stockholm, Sweden, 1999.
- [18] S. Hwan Bae, *The Welfare Consequences of Green Tax Reform in Small Open Economies*, Department of Agricultural Economics and Rural Sociology the Pennsylvania State University, 2005.
- [19] M. Iosifidi and S. Jafarey, The Role of Taxation in a Ramsey Model with Environmental Externality, Department of Economics, City University, London, 2012.
- [20] L.E. Jones, R.E. Manuelli, and P.E. Rossi, On the optimal taxation of capital income, J. Econ. Theory 73 (1997), 93–117.
- [21] K.L. Judd, Redistributive taxation in a simple perfect foresight model, J. Public Econ. 28 (1985), 59–83.
- [22] F.E. Kydland and E.C. Prescott, Time to build and aggregate fluctuations, Econometrica 50 (1982), 1345–1370.
- [23] Q. Liang and X.F. Qian, Externality and agglomeration: A literature review, J. World Econ. 2 (2007), 84–96.
- [24] Sh. Majdzadeh Tabatabaei and A.H. Ostadzad, Study of pollution abatement policy by using comparative static analysis and endogenous growth model: Case study of Iran economy, Econ. Model. 9 (2015), 85–105.
- [25] M. Moghimi, N. Shahnooshi, Sh. Danesh, B. Akbari Moghadam, and M. Daneshvar, The survey of welfare and environmental effects on the green tax & decline subsidy on fuels in Iran by using a computable general equilibrium model, Quart. J. Agricul. Econ. Dev. 19 (2011), no. 3, 79–108.
- [26] C.I. Plosser, Understanding real business Cycles, J. Econ. Perspect. 3 (1989), 51–77.
- [27] A.H. Ostadzad and E. Hadiyan, Estimating the optimal pollution tax for Iranian economy, Quart. J. Econ. Growth Dev. Res. 3 (2013), 57–74.
- [28] A.H. Ostadzad and E. Hadiyan, Optimal income tax rate with and without environmental considerations, J. Appl. Econ. Stud. Iran 4 (2015), 1–25.
- [29] J. Pajouyan and N. Amin Rashti, Green taxes with emphasis on gasoline consumption, Econ. Res. J. 7 (2007), 1.
- [30] J.E. Stiglitz, Pareto efficient and optimal taxation and the new welfare economics, Handbook of Public Economics, 2 (1987), chapter 15, pp. 991–1042.
- [31] S.J. Turnovsky, Fiscal policy, elastic labor supply, and endogenous growth, J. Monetary Econ. 45 (2000), 185–210.
- [32] W. Wissema and R. Dellink, AGE analysis of the impact of a carbon energy tax on the Irish economy, Ecolog. Econ. 61 (2007), 671–683.