Int. J. Nonlinear Anal. Appl. 15 (2024) 10, 33–50
ISSN: 2008-6822 (electronic)
http://dx.doi.org/10.22075/ijnaa.2023.30986.4533

Designing and explaining a portfolio optimization model using the cuckoo optimization algorithm

Zeinab Kazemi[a], Gholamreza Zomorodian[a,*], Mirfeiz Fallah Shams Layalestani[a], Farhad Hanifi[b]

[a]Department of Financial Engineering, Central Tehran Branch, Islamic Azad University, Tehran, Iran

[b]Department of Business Management, Central Tehran Branch, Islamic Azad University, Tehran, Iran

(Communicated by Sirous Moradi)

Abstract

Choosing the right portfolio is a crucial goal for investors. Two key factors to consider when selecting a portfolio are return and portfolio risk. This problem can be written as a mathematical programming equation and can be solved quickly. For instance, some investors choose stocks based on past performance, while others consider factors like liquidity when making their selections. Additionally, fundamental and technical analysis are often used in stock and portfolio selection. However, the strategies and methods used to select stocks and portfolios can vary depending on the current market conditions and the investor's level of knowledge. This article focuses on designing and explaining a portfolio optimization model using the cuckoo optimization algorithm. Results show that the best value of the utility function increases as the number of iterations increases. The growth value of the best value of the utility function is higher in the initial iterations and gradually decreases until it reaches zero, indicating that the algorithm has converged to the optimal solution. This research fills a gap in the study of investment portfolio optimization using nested optimization models. Additionally, the study finds that shares selected from industries with better performance make up a higher proportion of the portfolio.

Keywords: portfolio optimisation, cuckoo optimisation algorithm, stock selection, investors
2020 MSC: 37N40

1 Introduction

One of the important topics discussed in the stock market that investors, whether natural or legal persons, should consider is choosing the optimal investment portfolio [25]. In this regard, investors seek to choose the best investment portfolio according to the amount of risk and its potential return. The traditional investment approach is to strive for the highest return while minimizing risk. In other words, investors view return as a favorable factor and the variance of returns (risk) as an unfavorable element [17].

According to financial theories, investors have preferences that make them not risk-averse but loss-averse and, therefore, willing to bear high risk. Also, individuals may make decisions under the influence of society or other individuals, contrary to traditional theories [10]. However, investment risk is one of the most important issues for investors. The results of many traditional studies show a positive relationship between risk and return [7, 8].

*Corresponding author
Email addresses: Zeinabkazemi1987@gmail.com (Zeinab Kazemi), gh.zomorodian@gmail.com (Gholamreza Zomorodian), fallahshams@gmail.com (Mirfeiz Fallah Shams Layalestani), hanifi_farhad@yahoo.com (Farhad Hanifi)

Received: February 2023 *Accepted:* June 2023

Therefore, one of the challenges in forming an asset portfolio is determining the optimal ratio or weight of assets in the investment portfolio to reduce risk. In portfolio optimization, the main issue is the optimal selection of assets and securities that can be prepared with a certain amount of capital. Although minimizing risk and maximizing investment returns seems simple, several methods have been used to form an optimal portfolio [9]. By accepting the traditional investment theory and the basic assumption of investors' risk aversion, the challenge of forming an optimal stock portfolio can be solved.

In 1952, Markowitz [12] solved this challenge by pointing out that by forming a portfolio at a certain level of risk, one can achieve more returns or, on the contrary, bear less risk at a certain level of return [7, 8]. Markowitz [13] expressed the modern portfolio theory (MPT) as a mathematical formula. He designed and solved a constrained optimization problem to obtain the optimal weight of investments in the portfolio (which meets the condition of maximum return at a certain level of risk or minimum risk at a certain level of return for the investment portfolio desired by the investor) by which the optimal weight vector of investments in the portfolio can be obtained [22]. In fact, Markowitz [13] determined the optimal allocation of an investor's wealth to various investments that he wishes to hold by maximizing the return or minimizing the portfolio's risk at a certain level of return. Markowitz's most important idea is to use the standard deviation of investment portfolio returns as a measure of risk. The Markowitz model considers the same term as standard portfolio optimization [19].

However, during the portfolio optimization process, we also face some realistic limitations, such as stock size, number of stocks, transaction cost, and portfolio size. This model is based on assumptions that are rarely true in practice. When the aforementioned real restrictions (ceiling and floor restrictions, restrictions on investment weight and types of financial risks, etc.) are added to the portfolio optimization, the problem quickly becomes a highly complex set, resulting in an optimal problem. We will face the creation of a comprehensive portfolio, and here Markowitz's solution and contract methods, such as quadratic programming, will no longer be applicable [15]. In such a situation, metaheuristic optimization methods are usually used to interact with the extensive portfolio optimization problem.

In this research, a multi-objective operational research model has been used to obtain the weight of investment shares, maximize investment returns, and minimize other factors such as systemic and non-systematic risks. This model is an extended model of Markowitz's mean-variance model to which some limitations have been added, the most important of which are [16]: 1- Limiting the number of portfolio shares and 2- Incorporating ceiling and floor restrictions.

Ceiling and floor restrictions are what make this research different from the past and have an innovative aspect. These limitations are as follows:

1. Limitation on the number of shares in the portfolio: The number of assets in a portfolio is often either fixed or capped at a given value. Through the introduction of a binary variable Z_i (which means that if asset i is present in the portfolio, it is equal to one, and otherwise, it is zero), this limitation can be expressed in the form of the following relationship (w_i: shows the share of each weight gives) [21]:

$$\sum_{i=1}^{n} Z_i \leq K \qquad \text{where} \quad Z = \begin{cases} 1, & \text{if } w_i > 0 \\ 0, & \text{if } w_i = 0 \end{cases}$$

   This restriction is imposed to facilitate portfolio management and reduce portfolio management costs. However, the above unequal form is quite common. Also, a lower limit can be introduced in the form of the following relationship:

$$K_{\min} \leq \sum_{i=1}^{n} Z_i \leq K_{\max}$$

   At the same time, this limitation can also be expressed in the form of equality:

$$\sum_{i=1}^{n} Z_i = k$$

   It should be mentioned that the limit of the number of stocks in the portfolio is sometimes considered an objective function.

2. Ceiling and floor restrictions:
   Applying ceiling and floor restrictions, additionally considering a minimum and maximum ratio (ε_i and γ_i, respectively) for the weight of each asset is allowed to be kept in the portfolio, such that $W_i \geq 0, \varepsilon_i \leq W_i \leq \gamma_i$,

where $(i = 1, ..., k)$. In other words, the share of the portfolio for a particular asset changes in a given interval [24].

$$\varepsilon_i \leq W_i \leq \gamma_i, \quad (i = 1, ..., k)$$

Ceiling limits (upper limit limits) are introduced to prevent exceeding a specific asset ratio, and floor limits (lower limit) are used to avoid the cost of managing very low asset ratios [18].

This research uses more effective and efficient metaheuristic optimization algorithms to solve the problem of comprehensive portfolio optimization. Moreover, by comparing the answers, the error probability can be almost zero. During this research, metaheuristic optimization methods are well-designed and researched, and then they are used to optimize the portfolio despite some fundamental limitations in the market. Finally, the developed algorithms are implemented to solve the extended portfolio optimization problem.

Therefore, what distinguishes the current research from other similar research and is considered as the innovation of the upcoming research:

- Solving the problem of comprehensive portfolio optimization using more effective and efficient metaheuristic algorithms, which use the latest and most efficient metaheuristic optimization algorithms to calculate the optimal weight of baskets, which can reduce the probability of error to almost zero by comparing the answers.
- In this research, more effective and efficient metaheuristic optimization algorithms are used to solve the problem of comprehensive portfolio optimization. By comparing the answers, the error probability can be almost zero. During this research, metaheuristic optimization methods are well-designed and researched, and then they are used to optimize the portfolio despite some fundamental limitations in the market. The developed algorithms are all implemented to solve the extended portfolio optimization problem.

According to the explanations mentioned in this article, we seek to design and explain the portfolio optimization model by using the cuckoo optimization algorithm, and finally, we seek to answer the questions of the optimal portfolio with return goals, financial risks (risks; What is the market, liquidity and systematic) with metaheuristic algorithms in listed companies? Furthermore, can the portfolio formed using metaheuristic optimization algorithms guide the market to achieve maximum profit? Finally, is there a significant difference between the models proposed in portfolio optimization?

2 Literature review

In recent years, studies have been conducted to optimize stock portfolios, with researchers and investors investing in various algorithms to improve portfolio performance. Bačević et al. [2] investigated the relationship between operating capital flow and the market value of listed companies using seasonal stochastic algorithms. Their results showed that capital operations are significantly related to market value management and have different effects on companies in different situations. This helps create a benchmark for the market or industry and provides a scientific basis and decision support for target companies as they implement their investments. Additionally, Feshari and Nazari [6] used a multi-objective evaluation approach in their research titled "Prediction of the mean-variance model for the selection of mandatory portfolio assets" to achieve improved results compared to the traditional average variance criteria and the standard Markowitz mean-variance model.

They used an artificial neural network model to predict future capital returns and then performed the optimization process using multi-objective evaluation algorithms. The research results showed that the Pareto solutions approach, which includes maintaining sufficient diversity, resulted in better outcomes compared to the Markowitz model.

In summary, there has been a growing interest in optimizing stock portfolios through various algorithms and studies. Bačević et al. [2] and Feshari and Nazari [6] are examples of research that have contributed to this field by investigating the relationship between operating capital flow and market value, and using a multi-objective evaluation approach to improve portfolio performance. These studies provide valuable insights and decision-making support for investors and researchers looking to optimize their stock portfolios.

Also, Wang et al. investigated multivariate dependence risk and portfolio optimization: an application to a mineral stock portfolio. They proposed an integrated framework for modeling and evaluating relatively large dependent matrices using minimum risk of optimal securities and considering five risk indicators within the global financial crisis. The methodology was applied to two portfolios of the mining sector with 20 assets (iron ore-nickel and gold) from the stock exchange. Results showed that the proposed method is a powerful tool for modeling the change of dependence risk under three different periodic scenarios in addition to optimizing portfolios with complex dependence patterns, and on average, the portfolio optimization results converged in some stocks [23].

Additionally, Babazadeh and Esfahanipour focused on average deviation and risk in their study, conducting a robust analysis for adverse (downside) risk in portfolio management in stock markets with price volatility. They investigated the average deviation and adverse risk frameworks concerning portfolio management, using the highly volatile Karachi Stock Exchange in Pakistan as an example. They proposed and addressed factors that affect portfolio optimization, such as the right size of the portfolio, the process of sorting the portfolio, the butterfly effect in choosing suitable algorithms, and the endogeneity problem. The results showed that the adverse risk framework performed better than the mean-variance Markowitz framework, and that if the asset returns have high skewness, then the amount of difference will be significant. They recommended using adverse risk instead of variance as a measure of risk in investment decisions [1].

Markowitz [13] defines risk measures using variance or standard deviation. Mazraeh et al. (2022) further investigated this concept in their study, "Optimization of the portfolio (stock portfolio) for total sequence assets: limit risk index versus Markowitz." Using the returns of 500 S&P stocks from 2001 to 2011, they investigated portfolio optimization strategies based on the marginal risk index (ERI). In this method, the multivariate limit value theory minimizes the possibility of significant portfolio losses. With data on more than 400 stocks, their study appears to be the first to apply the marginal value technique to large-scale portfolio management. The primary purpose of this research was to investigate the practical application potential of ERI. The performance of this strategy was measured against the minimum portfolio variance and the equal weight portfolio. These basic strategies are essential criteria for large-scale applications. Their study included annual portfolio profit, maximum drawdown, transaction costs, portfolio density, and asset dispersion in the portfolio. They also investigated the effect of an alternative estimator of the sequence index. The results showed that for wide-sequence assets, the ERI strategy performed much better than the minimum variance and equal-weight portfolios [14].

Additionally, Erwin and Engelbrecht [5] in their research titled "Reminder Algorithm for Optimizing Principled Mandatory Capital with Transaction Costs", achieved the following results: They proposed the reminder approach, which is a combination of the algorithm and programming, to investigate the problem of selecting optimal capital with linear transaction costs. In this method, in addition to specifying assets, including the capital, it also includes the business operations performed at the time of capital balancing.

Moreover, finally, Chen et al. [3] in their research titled "Application of Multi-Objective Genetic Algorithms for Public Project Portfolio Selection", have reached the following results: They considered the possible rapid increase of portfolios and the preference relationship of asymmetry in the decision-maker to implement portfolio time and capital calculations. High complexity in actual conditions requires evaluation algorithms, but in providing objectives, evaluation algorithms are inefficient, so to overcome the problem, an extensive multi-objective genetic algorithm was used. In this decision-making situation, it is assumed that the decision-maker can evaluate the parameters for structuring the superior relationship. On the other hand, in this research, a method was assumed in which each heterogeneous group obtains its best portfolio. Then, these individual solutions are aggregated into a group with the best acceptable portfolio, in which the group's satisfaction or dissatisfaction is maximized.

3 Research method

3.1 Modeling and solving the model using the cuckoo optimization algorithm

In this section, a stock portfolio with low risk and high return was selected by presenting a proposed model based on the cuckoo optimization algorithm.

3.1.1 Modeling

In this part, we design a model that includes the initial model of the cuckoo optimization algorithm along with various changes and limitations. The proposed model is an extension of the cuckoo optimization algorithm.

- Proposed model

  The Cuckoo Optimization Algorithm (COA) is presented by Rajabioun [20] to provide an optimal solution to the problems. The model used in this article is developed and presented as follows:

$$\max \lambda \left(\sum_{i=1}^{N} w_i \mu_i\right) - (1-\lambda) \left(\sum_{i=1}^{N}\sum_{j=1}^{N} w_i w_j cov(\mu_i, \mu_j)\right)$$

where $w_i = \frac{x_i c_i z_i}{\sum_{j=1}^{N} x_i c_i z_i}$ $\&\lambda \in [0,1]$

st:

$$\sum_{i=1}^{N} z_i = M \leq N; \quad N \in N, \quad \forall i = 1, ..., N \quad z_i \in \{0,1\} \tag{3.1}$$

$$\sum_{i=1}^{N} x_i c_i z_i \leq B \tag{3.2}$$

$$0 \leq B_{L_i} \leq x_i c_i \leq B_{H_i} \leq B \tag{3.3}$$

$$\sum_{i_s} w_{i_s} \geq \sum_{i_s} w_{i_{s'}}; \quad \forall y_s, y_{s'} \neq 0; \quad s, s' \in \{1, 2, ..., \mathbb{S}\}, \quad s < s' \tag{3.4}$$

where $y_s = \begin{cases} 1, & \text{if } \sum_{i_s} z_i > 0 \\ 0, & \text{if } \sum_{i_s} z_i = 0 \end{cases}$; $i_{s,s'} \in \{1, 2, ..., N\{\}\}$

- Symbolisation

  N: the number of shares available, M: the portfolio size,λ is a weighting parameter that determines how much the investor values the portfolio's performance. Obviously, $(1-\lambda)$, it determines how much the investor values the variance of the portfolio.

  μ_i: yield of the i-th stock, $cov(\mu_i, \mu_j)$: covariance between the yield of the i-th stock and the yield of the j-th stock, x_i: the number of the i-th stock in the portfolio, c_i: the price of the i-th stock, w_i: the ratio of the i-th stock in the portfolio (the ratio of the budget allocated to the i-th stock from the total available budget), z_i: zero and one variable that indicates the presence or absence of the i-th share in the portfolio, B: the total available budget, B_{L_i}: the minimum budget that can be invested on the i-th share, B_{H_i}: the maximum budget that can be invested on the i-th share, i_s: The index of the share that belongs to the industry of s, S: the total number of industries, y_s: a variable of zero and one that indicates the presence or absence of a share of the s-th industry in the basket.

- Description of the model

- The goal of the model is to maximise the return of the portfolio $\left(\sum_{i=1}^{N} w_i \mu_i\right)$ and also to minimise its variance $\left(\sum_{i=1}^{N}\sum_{j=1}^{N} w_i w_j cov(\mu_i, \mu_j)\right)$ (it should be noted that the standard deviation of the return (square root of the variance) is considered as the risk of the portfolio) [17]. These two goals are combined by the weighting parameter λ and have become $\max \lambda \left(\sum_{i=1}^{N} w_i \mu_i\right) - (1-\lambda) \left(\sum_{i=1}^{N}\sum_{j=1}^{N} w_i w_j cov(\mu_i, \mu_j)\right)$ in the form of a goal [3].

- Constraint 1 specifies the size of the basket (the number of stocks in the basket).

- Constraint 2 considers the total available budget. However, the maximum value of $\sum_{j=1}^{N} x_i c_i z_i$ should be equal to the investor's budget.

- Constraint 3 places upper and lower limits on the capital allocated to each set of available shares. If there is no such limit, the capital allocated to a share may be a large percentage of the available budget, which is undesirable.

- Constraint 4 states that in M shares selected from among N available shares, more capital should be allocated to the shares belonging to the industry with better performance. It is assumed that industry 1 has the best performance and industry S has the weakest performance, and the industries are sorted in descending order in terms of performance. Suppose that there are shares from two industries, "s" and "s'", which s has a better performance than $s'(s < s')$ in the basket (i.e. $y_s, y_{s'} \neq 0$), in this case, the relation $\sum_{i_s} w_{i_s} \geq \sum_{i_{s'}} w_{i_{s'}}$ states that the total proportion of shares belonging to the industry is greater than the proportion of shares belonging to the industry s'.

3.1.2 Choosing the optimal basket using the cuckoo optimization algorithm

In this part, the presented model is solved using the cuckoo optimization algorithm, and the optimal basket is determined. The cuckoo optimization algorithm is based on Darwin's evolutionary theory, and the solution to the problem is gradually improved. A chromosome represents each potential solution. The algorithm starts with a set of solutions called the initial population. Then, the solutions from one population are used to produce the following population, and the new population is expected to be better than the previous population. Selection from the current population to create the next population is based on their fitness. Naturally, more appropriate solutions have more chances to reproduce. This process continues until the end condition is established (such as the number of populations or the improvement rate of the solution) [4]. The general procedure of the proposed cuckoo optimization algorithm is as follows:

1. Construction of the initial population (parents) of size n_{pop}
2. Calculation of the value of the utility function for each parent using the following relationship [11]:

$$\lambda\left(\sum_{i=1}^{N} w_i\mu_i\right) - (1-\lambda)\left(\sum_{i=1}^{N}\sum_{j=1}^{N} w_i w_j cov(\mu_i,\mu_j)\right) - \left(1 - \frac{\sum_{j=1}^{N} x_i c_i z_i}{B}\right)$$

3. Selecting from parents using the roulette wheel mechanism and pairing them two by two to create children using the combination operator.
4. Selecting parents randomly in order to create children using the mutation operator.
5. Calculating the value of the utility function for each child.
6. Sorting the chromosomes in the population (parents and children) in descending order of the values of the utility function from the highest value to the lowest value and then choosing the first n_{pop} chromosome from the sorted list as the parent of the following population.
7. Repeating steps 3 to 6 until the stop criterion is established.

3.2 Coding

One of the important factors in the design of the cuckoo optimisation algorithm is the design of the coding system to determine the way of displaying the problem's solution and the mutual effect of this coding on the combination and mutation operators. In the genetic algorithm, each potential answer is represented by a chromosome.

In this research, each potential answer is represented by a two-part chromosome placed next to each other. The first part of the chromosome (the integer part) takes integer values, and the second part (the binary part) takes zero and one values. It should be noted that the length of each of these sections (the number of genes in each) is equal to the number of available stocks (N). The length of the chromosome is equal to twice the number of stocks available. The i-th gene in each part of the chromosome represents the i-th contribution $(i = 1, 2, ..., N)$.

If the i-th gene $(i = 1, 2, ..., N)$ in the binary section takes a value of zero, it means that the i-th share is not in the basket (i.e. $z_i = 0$), and if it takes a value of one, it means that the i-th share is in the basket (i.e. $z_i = 1$). The value of the i-th gene $(i = 1, 2, ..., N)$ in the correct chromosome indicates how many i-th shares should be purchased. By multiplying the genes of the first part of the chromosome in the second part (multiplying the i-th gene value of the first part by the i-th gene value of the second part), a chromosome is obtained whose i-th gene represents the number of the i-th share in the basket (i.e. x_i).

3.3 Primary population

The initial population is the initial answer to the problem. The cuckoo optimisation algorithm starts searching and exploring the answer space from these answers. The way to generate the initial population in different meta-heuristic methods differs according to the problem. In some problems, the initial population is generated completely randomly. Additionally, according to the characteristics of the problem, innovative methods are used while maintaining the property of randomness to generate the initial population. It is also possible to randomly generate a percentage of the initial population and the rest using heuristic methods.

In this research, the initial population is generated completely randomly. It should be noted that the chromosome is acceptable to apply within the model's limits. Otherwise, a new chromosome will be generated randomly. It should be noted that the population size is 20 $(n_{pop} = 20)$.

3.4 Assessment

The value of the desirability function is calculated using the following relationship to determine each chromosome's degree of desirability (the degree of fitness or the degree of competence). The higher the value of the chromosomal utility function, the better that chromosome (potential answer).

$$\lambda\left(\sum_{i=1}^{N} w_i\mu_i\right) - (1-\lambda)\left(\sum_{i=1}^{N}\sum_{j=1}^{N} w_i w_j cov(\mu_i,\mu_j)\right) - \left(1 - \frac{\sum_{j=1}^{N} x_i c_i z_i}{B}\right)$$

The utility function is the same as the objective function of the $\lambda\left(\sum_{i=1}^{N} w_i\mu_i\right) - (1-\lambda)\left(\sum_{i=1}^{N}\sum_{j=1}^{N} w_i w_j cov(\mu_i,\mu_j)\right)$ model, from which the value $\left(1 - \frac{\sum_{j=1}^{N} x_i c_i z_i}{B}\right)$ has been subtracted. The term $\left(1 - \frac{\sum_{j=1}^{N} x_i c_i z_i}{B}\right)$ is considered a penalty for constraint 2 (the budget constraint). The penalty amount will be zero if the entire available funds are spent on the stock purchase (i.e. X). Naturally, the smaller the amount of the total available budget is spent on buying shares, the greater the penalty will be; as a result, the value of the utility function will decrease. In fact, with the maximisation of the utility function, in addition to the maximisation of the objective function (maximisation of the return and minimisation of the variance of the portfolio), constraint 2 (budget constraint) tends to be equality.

Combination operator:

In the recombination process, a pair of chromosomes is selected from the parents and combined to create a new pair of chromosomes.

In this research, a pair of chromosomes is first selected for the combination process using the roulette wheel mechanism. In this selection mechanism, the higher the fitness of a chromosome, the more likely it is to be selected. Then, the mathematical combination operator creates a new pair of chromosomes. It should be noted that the new chromosomes must comply with the model's limitations. Otherwise, the unacceptable chromosome is removed, and the combination operator is applied again.

Jump operator:

Leaping is critical to the success of the Cuckoo optimisation algorithm, as it avoids getting stuck in local optima and determines the direction of the search. In other words, the mutation causes more points of the solution space to be searched and the possibility of obtaining better solutions increases.

In this research, a chromosome is randomly selected from the parents. Then, the uniform mutation operator is used on the correct part of the chromosome, and the swap mutation operator is used on the binary part to create a new chromosome. It should be noted that the new chromosome must apply within the model's limits. Otherwise, that chromosome is not accepted, so it is removed, and the mutation operator has applied again.

Set parameters:

To determine the type of operators of the proposed algorithm (combination and mutation operators), the size of the population (n_{pop}), the number of generations to stop the algorithm and other influential parameters, the proposed algorithm has been adjusted, that is, the influential parameters have been tested with different values, and finally the best values have been reported.

Stop criterion:

The stop criterion for running the algorithm in the proposed algorithm is the number of iterations (number of population or generation produced). In this research, the number of generations to stop is 1000 generations.

4 Findings

4.1 The results of the cuckoo optimisation algorithm on the proposed model

As mentioned earlier, the statistical population of the research is related to 69 companies from the five mentioned industries from 2011 to 2019. In order to display the results quickly, we code each industry and each company and use the relevant code to display the results.

It is assumed that the importance of return and variance of the portfolio is the same for the investor (i.e. $\lambda = 0.5$). Also, the available budget is equal to 100 million Rials (i.e. $B = 100000000$), and at least 99.8% of the available budget must be spent in the proposed basket. Therefore, the maximum budget that can be invested in the i-th share is 25% of the total available budget (i.e. $B_{H_i} = 0.25B$).

4.1.1 The test of the normality of the distribution of variables

The Kolmogorov-Smirnov test was used to check the normality of the distribution of the variables. The output table of the K-S test in SPSS software for this variable is described in table 1. According to the above table and Kolmogorov Smirnov's Z statistic, since the significance level for all variables is greater than 0.05, the H0 hypothesis is confirmed, so with 95% confidence, it can be said that the mentioned variables in the above models have a normal distribution.

Table 1: Test of normality of distribution of variables

Variable	Significance level	Z Kolmogorov Smirnov	Result
Stock returns	0.4696	0.6697	The distribution is normal
Stock risk	0.0688	0.4528	The distribution is normal
Stock liquidity	0.1225	0.3458	The distribution is normal
Return on assets	0.1498	0.2918	The distribution is normal
Earning per share	0.1194	0.0895	The distribution is normal
Price to earnings per share	0.2591	0.9437	The distribution is normal

Table 2: The results of the cuckoo optimisation algorithm for a portfolio of 5 stocks in 10 executions

Execution number	The value of the utility function	Portfolio return	Portfolio variance	Percentage of budget spent
1	0.188	0.378	0.00059	99.922
2	0.169	0.345	0.00663	99.995
3	0.161	0.325	0.00321	99.996
4	0.167	0.342	0.00543	99.955
5	0.178	0.358	0.00157	99.989
6	0.189	0.378	0.00121	99.933
7	0.209	0.422	0.00279	99.933
8	0.185	0.373	0.00216	99.985
9	0.176	0.358	0.00437	99.961
10	0.186	0.375	0.00182	99.985
Average	0.181	0.365	0.00298	99.971
Standard deviation	0.013	0.025	0.00184	0.025

Table 3: The results of the cuckoo optimisation algorithm for a portfolio of 7 stocks in 10 executions

Execution number	The value of the utility function	Portfolio return	Portfolio variance	Percentage of budget spent
1	0.197	0.397	0.00098	99.918
2	0.205	0.412	0.00099	99.994
3	0.205	0.413	0.00189	99.986
4	0.167	0.336	0.00053	99.981
5	0.181	0.363	0.00074	99.969
6	0.192	0.387	0.00163	99.984
7	0.175	0.355	0.00382	99.958
8	0.171	0.344	0.00068	99.974
9	0.178	0.361	0.00343	99.962
10	0.187	0.377	0.0022	99.971
Average	0.186	0.375	0.00169	99.97
Standard deviation	0.012	0.026	0.001	0.02

The results of the proposed cuckoo optimisation algorithm for the five stock portfolio in 10 times of execution are given in table 2, and the information about the best five stock portfolio resulting from these ten times of execution is shown in table 9. As can be seen in table 9, in a basket of 5, if 2275 shares of 2, 1727 shares of 8, 769 shares of 14, 1696 shares of 40 and 811 shares of 43 are purchased, the return of the portfolio is maximised, and the variance of the portfolio is minimised. In this case, the optimal portfolio will be 42.26%, and its optimal risk (standard deviation) will be equal to 0.28%. On the other hand, the budget constraint is also met (we assumed that at least 99.8% of the total available budget should be spent in the proposed portfolio). Also, as can be seen, the better the industrial performance (the lower it ranks or the smaller the corresponding code), the shares selected from that industry have a higher proportion of the portfolio. Industry 1, 59.96%, Industry 3, 23.31% and Industry 4, 16.72% of the portfolio.

Table 4: The results of the cuckoo optimisation algorithm for a portfolio of 10 stocks in 10 executions

Execution number	**The value of the utility function**	**Portfolio return**	**Portfolio variance**	**Percentage of budget spent**
1	0.204	0.413	0.0017	99.833
2	0.172	0.356	0.0108	99.991
3	0.179	0.36	0.0012	99.988
4	0.203	0.408	0.0004	99.97
5	0.201	0.405	0.0013	99.991
6	0.192	0.385	0.0008	99.999
7	0.188	0.381	0.0025	99.949
8	0.185	0.371	0.0007	99.989
9	0.198	0.398	0.0005	99.984
10	0.201	0.405	0.0009	99.972
Average	0.192	0.388	0.0021	99.967
Standard deviation	0.01	0.019	0.0029	0.0465

Table 5: Cuckoo optimisation algorithm results for 12 stock portfolio in 10 executions

Execution number	**The value of the utility function**	**Portfolio return**	**Portfolio variance**	**Percentage of budget spent**
1	0.177	0.356	0.0019	99.991
2	0.185	0.373	0.0019	99.986
3	0.201	0.406	0.0031	99.959
4	0.199	0.401	0.0018	99.962
5	0.184	0.37	0.0012	99.977
6	0.189	0.386	0.006	99.974
7	0.182	0.365	0.0007	99.998
8	0.195	0.391	0.0011	99.988
9	0.179	0.362	0.0034	99.991
10	0.2	0.403	0.0007	99.894
Average	0.189	0.381	0.0022	99.972
Standard deviation	0.0085	0.0175	0.0015	0.0287

Table 6: Cuckoo optimisation algorithm results for 15 stock portfolios in 10 executions

Execution number	**The value of the utility function**	**Portfolio return**	**Portfolio variance**	**Percentage of budget spent**
1	0.19	0.383	0.0021	99.997
2	0.187	0.376	0.0007	99.99
3	0.182	0.366	0.0019	99.993
4	0.206	0.414	0.0009	99.995
5	0.178	0.363	0.0063	99.988
6	0.2	0.404	0.002	99.994
7	0.203	0.409	0.0011	99.987
8	0.196	0.395	0.0011	99.998
9	0.193	0.387	0.0002	99.981
10	0.199	0.399	0.0005	99.999
Average	0.194	0.389	0.0017	99.992
Standard deviation	0.0087	0.0165	0.0016	0.0054

Table 7: Cuckoo optimisation algorithm results for 17 stock portfolios in 10 executions

Execution number	**The value of the utility function**	**Portfolio return**	**Portfolio variance**	**Percentage of budget spent**
1	0.192	0.384	0.0001	99.999
2	0.198	0.398	0.0003	99.984
3	0.201	0.403	0.0003	99.995
4	0.186	0.375	0.002	99.981
5	0.186	0.373	0.0008	99.984
6	0.189	0.378	0.0003	99.976
7	0.202	0.404	0.0005	99.995
8	0.184	0.370	0.0009	99.995
9	0.203	0.408	0.0006	99.995
10	0.207	0.416	0.0013	99.99
Average	0.195	0.391	0.0007	99.99
Standard deviation	0.0079	0.0158	0.0005	0.0073

Table 8: Cuckoo optimisation algorithm results for 20 stocks portfolio in 10 executions

Execution number	**The value of the utility function**	**Portfolio return**	**Portfolio variance**	**Percentage of budget spent**
1	0.191	0.385	0.0024	99.997
2	0.196	0.394	0.0006	99.993
3	0.195	0.392	0.0011	99.982
4	0.186	0.373	0.0006	99.999
5	0.196	0.394	0.0006	99.989
6	0.19	0.382	0.0001	99.989
7	0.195	0.393	0.0019	99.992
8	0.185	0.371	0.0009	99.996
9	0.199	0.4	0.0001	99.996
10	0.192	0.386	0.0012	99.984
Average	0.193	0.387	0.001	99.992
Standard deviation	0.0045	0.0089	0.0006	0.0052

Table 9: Information about the best basket of 5 items obtained from 10 times execution

	Share code	**Industry code**	**Number of shares**	**Share ratio in the basket**	**Industry ratio in basket**
	2	1	2275	0.216	
	8	1	1727	0.171	0.599
A portfolio of five	14	1	769	0.212	
	40	3	1696	0.233	0.233
	43	4	811	0.167	0.167
The value of the utility function	0.2092				
Portfolio return	0.4225				
Portfolio variance	0.0027				
Percentage of budget spent	99.9339				

Table 10: Information about the best basket of 7 items obtained from 10 times execution

	Share code	**Industry code**	**Number of shares**	**Share ratio in the basket**	**Industry ratio in basket**
	1	1	1133	0.179	
	7	1	395	0.078	
	8	1	1904	0.188	0.76
A portfolio of seven	14	1	325	0.089	
	15	1	1565	0.224	
	43	4	317	0.065	0.239
	60	4	778	0.173	
The value of the utility function	0.2055				
Portfolio return	0.4132				
Portfolio variance	0.0018				
Percentage of budget spent	99.9867				

The results of the proposed cuckoo optimisation algorithm for the 7-stock portfolio in 10 executions are given in Table 3, and the information about the best 7-stock portfolio from these ten executions is shown in Table 4. As can be seen in table 10, in a basket of 7, if 1, 7, 8, 14, 15, 43 and 60 shares are purchased in the number mentioned in the table, the yield of the basket is maximised, and the variance of the basket is minimised (return The optimal portfolio, in this case, will be equal to 41.33% and its optimal risk (standard deviation) will be equal to 4.35%. On the other hand, the budget constraint is also met (we assumed that at least 99.8% of the total available budget should be spent in the proposed portfolio). Also, as can be seen, the better the industrial performance (the lower rank or the smaller the corresponding code), the shares selected from that industry have a higher proportion of the portfolio. Industry 1, 76.08% and Industry 4, 23.92% of the portfolio.

The results of the proposed cuckoo optimisation algorithm for the 10-stock portfolio in 10 times of execution are given in Table 4, and the information about the best 10-stock portfolio resulting from these ten times of execution is shown in Table 11. As can be seen in the table 11, in a basket of 10, if the number of shares 6, 8, 18, 24, 30, 38, 43, 61, 66 and 69 are purchased in the number mentioned in the table, the efficiency of the basket is maximised, and The variance of the portfolio is minimised (the optimal return of the portfolio, in this case, will be equal to 41.40% and its optimal risk (standard deviation) will be equal to 4.20%). On the other hand, the budget constraint is also met (we assumed that at least 99.8% of the total available budget should be spent in the proposed portfolio). Also, as can be seen, the better the industrial performance (the lower rank or the smaller the corresponding code), the shares selected

Table 11: Information about the best basket of 10 obtained from 10 executions

	Share code	Industry code	Number of shares	Share ratio in the basket	Industry ratio in basket
A portfolio of ten	6	1	4709	0.1658	0.264
	8	1	990	0.0981	
	18	2	4	0.0009	0.21
	24	2	1938	0.2098	
	30	3	231	0.0441	0.191
	38	3	772	0.1471	
	43	4	504	0.104	0.167
	61	4	429	0.0638	
	66	5	717	0.122	0.165
	69	5	294	0.0439	
The value of the utility function	0.2044				
Portfolio return	0.4139				
Portfolio variance	0.0017				
Percentage of budget spent	99.8338				

from that industry have a higher proportion of the portfolio.

In this basket, the question may arise that in the results, why it is suggested that share 18 be bought in the number of 4 shares, if it is suggested to buy another share like share 6, in the number of 4709, and the reason for these differences in what The reason is that in this numerical example, there is no limit for the minimum budget that can be invested on the i-th share (i.e. $B_{L_i} = 0$). In other words, the i-th share can be purchased in any quantity. If the investor does not want a significant difference in the number of shares purchased, it is better to consider a limit for the minimum budget that can be invested on the i-th share.

Table 12: Information about the best basket of 12 items obtained from 10 times execution

	Share code	Industry code	Number of shares	Share ratio in the basket	Industry ratio in basket
A portfolio of twelve	28	3	360	0.0744	0.5179
	39	3	543	0.1866	
	40	3	1513	0.2079	
	41	3	183	0.0489	
	43	4	40	0.0082	0.2566
	50	4	632	0.0699	
	54	4	300	0.0758	
	60	4	81	0.0181	
	61	4	569	0.0845	
	66	5	541	0.0919	0.2254
	68	5	719	0.0056	
	69	5	519	0.0774	
The value of the utility function	0.2013				
Portfolio return	0.4066				
Portfolio variance	0.0031				
Percentage of budget spent	99.9596				

The results of the proposed cuckoo optimisation algorithm for the 12 stock portfolio in 10 times of execution are given in table 5, and the information about the best 12 stock portfolio resulting from these ten times of execution is shown in table 12. As can be seen in table 12, in a basket of 12, if the number of shares mentioned in the table is purchased, the yield of the basket is maximised, and the variance of the basket is minimised (the optimal yield of the basket, in this case, will be equal to 40.66% and its optimal risk (standard deviation) will be equal to 5.59%. On the other hand, the budget constraint is also met (we assumed that at least 99.8% of the total available budget should be spent in the proposed portfolio). Also, as can be seen, the better the industrial performance (the lower rank or the smaller the corresponding code), the shares selected from that industry have a higher proportion of the portfolio.

The results of the proposed cuckoo optimisation algorithm for a portfolio of 15 stocks in 10 runs are given in Table 6, and the information about the best portfolio of 15 stocks from these ten runs is shown in Table 13. As can be seen in table 13, in a basket of 15, if the specified number of shares mentioned in the table is purchased, the yield of the basket is maximised, and the variance of the basket is minimised (the optimal yield of the basket, in this case, will be equal to 41.45% and Its optimal risk (standard deviation) will be equal to 3.04%. On the other hand, the budget constraint is also met (we assumed that at least 99.8% of the total available budget should be spent in the proposed portfolio). Also, as can be seen, the better the industrial performance (the lower rank or the smaller the corresponding code), the shares selected from that industry have a higher proportion of the portfolio.

Table 13: Information about the best basket of 15 items obtained from 10 executions

	Share code	**Industry code**	**Number of shares**	**Share ratio in the basket**	**Industry ratio in basket**
A portfolio of fifteen	1	1	662	0.1049	0.5908
	2	1	84	0.0079	
	7	1	811	0.1609	
	8	1	629	0.0622	
	9	1	128	0.0362	
	14	1	791	0.2183	
	21	2	60	0.015	0.2096
	24	2	1800	0.1946	
	31	3	35	0.0068	0.1302
	41	3	462	0.1234	
	43	4	24	0.0049	0.0456
	51	4	62	0.0101	
	52	4	72	0.0132	
	57	4	173	0.0173	
	63	5	211	0.0235	0.023
The value of the utility function	0.2067				
Portfolio return	0.4144				
Portfolio variance	0.0009				
Percentage of budget spent	99.995				

Table 14: Information about the best basket of 17 items obtained from 10 executions

	Share code	**Industry code**	**Number of shares**	**Share ratio in the basket**	**Industry ratio in basket**
A portfolio of seventeen	1	1	894	0.1417	0.3919
	3	1	228	0.0124	
	6	1	510	0.0179	
	7	1	768	0.1524	
	11	1	32	0.0097	
	15	1	401	0.0575	
	28	3	73	0.15	0.3536
	29	3	117	0.05	
	33	3	90	0.0095	
	38	3	291	0.0553	
	41	3	838	0.2236	
	42	4	153	0.0501	0.1305
	60	4	360	0.0804	
	63	5	474	0.0528	0.1238
	66	5	371	0.063	
	67	5	5	0.0005	
	68	5	95	0.0074	
The value of the utility function	0.2073				
Portfolio return	0.4161				
Portfolio variance	0.0013				
Percentage of budget spent	99.9908				

The results of the proposed cuckoo optimisation algorithm for a portfolio of 17 stocks in 10 times of execution are given in table 7, and the information about the best portfolio of 17 stocks obtained from these ten times of execution is shown in table 14. As can be seen in table 14, in a basket of 17, if the number of shares mentioned in the table is purchased, the yield of the basket is maximised, and the variance of the basket is minimised (the optimal yield of the basket, in this case, will be equal to 41.62% and Its optimal risk (standard deviation) will be equal to 3.65%. On the other hand, the budget constraint is also met (we assumed that at least 99.8% of the total available budget should be spent in the proposed portfolio). Also, as can be seen, the better the industrial performance (the lower rank or the smaller the corresponding code), the shares selected from that industry have a higher proportion of the portfolio.

The results of the proposed cuckoo optimisation algorithm for the 20 stocks portfolio in 10 execution times are given in table 8, and the information about the best 20 stock portfolio resulting from these ten execution times is shown in table 15. As can be seen in the table 15, in a basket of 20, if the number of shares mentioned in the table is purchased, the yield of the basket is maximised, and the variance of the basket is minimised (the optimal yield of the basket, in this case, will be equal to 40.02%, and its optimal risk (standard deviation) will be equal to 1.39%. On the other hand, the budget constraint is also met (we assumed that at least 99.8% of the total available budget should be

Table 15: Information about the best basket of 20 obtained from 10 times execution

	Share code	Industry code	Number of shares	Share ratio in the basket	Industry ratio in basket
A portfolio of twenty	6	1	2913	0.1024	0.4289
	7	1	710	0.1409	
	8	1	629	0.0622	
	14	1	322	0.0888	
	15	1	240	0.0344	
	17	2	284	0.0543	0.1758
	19	2	145	0.0699	
	21	2	11	0.0027	
	24	2	451	0.0487	
	30	3	11	0.0021	0.1475
	34	3	13	0.0012	
	38	3	758	0.1441	
	43	4	424	0.0873	0.1343
	47	4	134	0.0161	
	52	4	125	0.0229	
	57	4	47	0.0047	
	59	4	10	0.0031	
	64	5	100	0.0439	0.1132
	65	5	3	0.0005	
	66	5	405	0.0688	
The value of the utility function	0.1999				
Portfolio return	0.4001				
Portfolio variance	0.0001				
Percentage of budget spent	99.9967				

spent in the proposed portfolio). Also, as can be seen, the better the industrial performance (the lower rank or the smaller the corresponding code), the shares selected from that industry have a higher proportion of the portfolio.

The graph of the convergence process of the proposed cuckoo optimisation algorithm in different generations for different basket sizes (5, 7, 10, 12, 15, 17 and 20) is shown in figures 1 to 8. In this diagram, the horizontal axis represents the number of iterations (the number of populations or generations produced), and the vertical axis represents the best utility function value in each iteration (population or generation). As can be seen, the utility function's best value increases with the number of iterations. Therefore, the growth value of the best value of the utility function is higher in the initial iterations and gradually decreases until it reaches zero; that is, the algorithm converges to the optimal solution.

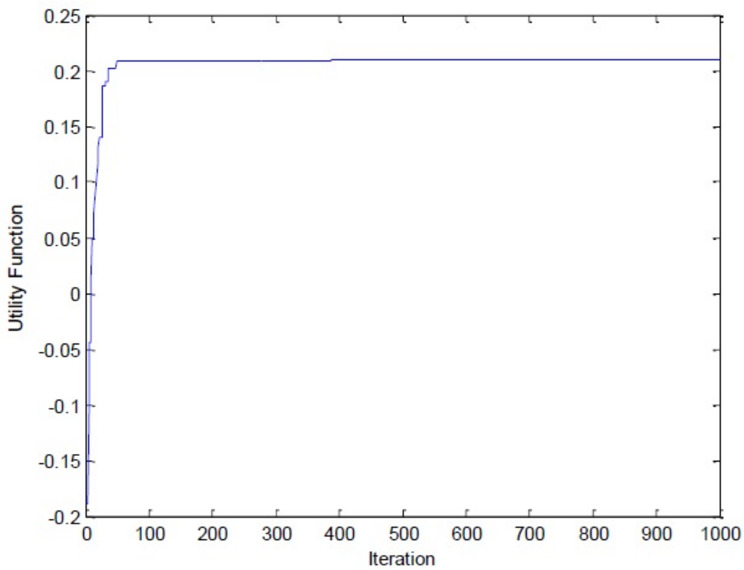

Figure 1: Convergence trend diagram of the proposed cuckoo optimisation algorithm for the best basket of 5

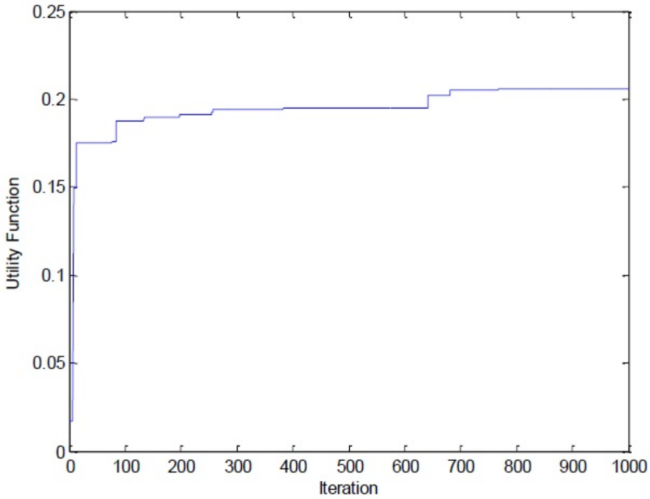

Figure 2: Convergence trend diagram of the proposed cuckoo optimisation algorithm for the best basket of 7

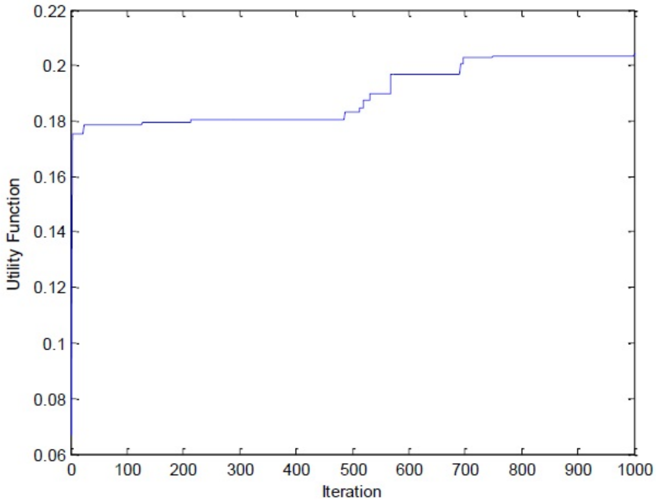

Figure 3: Convergence trend diagram of the proposed cuckoo optimisation algorithm for the best basket of 10

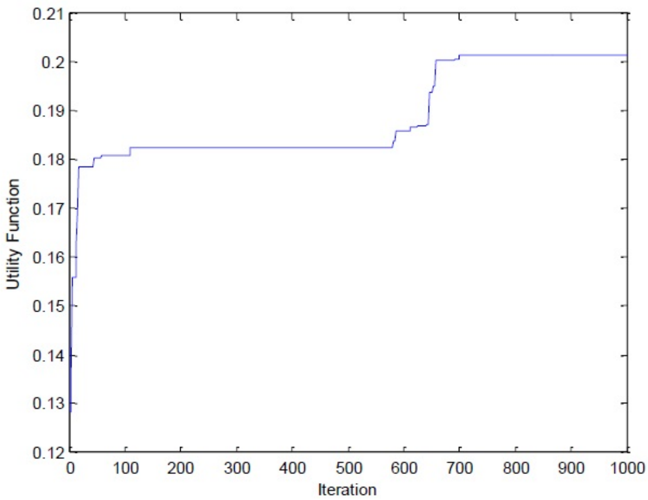

Figure 4: Convergence trend diagram of the proposed cuckoo optimisation algorithm for the best basket of 12

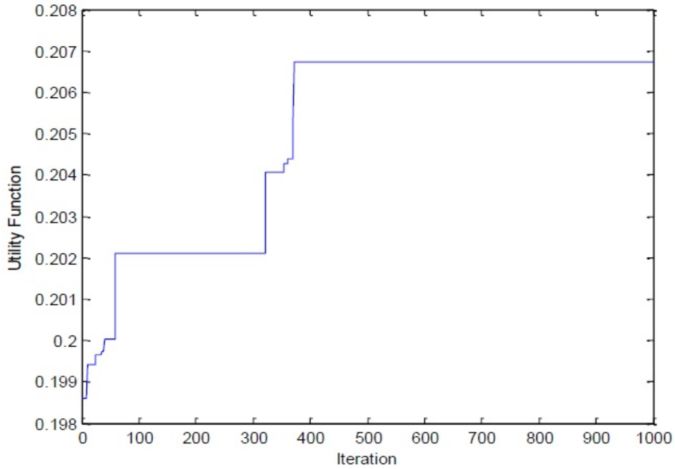

Figure 5: The convergence trend diagram of the proposed cuckoo optimisation algorithm for the best basket of 15

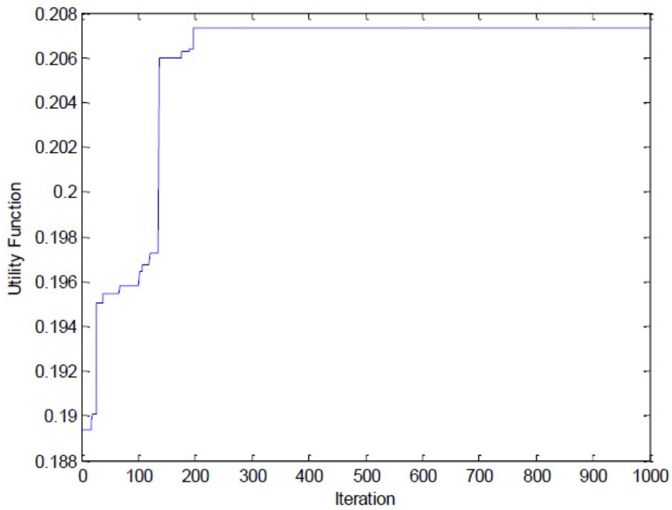

Figure 6: Convergence trend diagram of the proposed cuckoo optimisation algorithm for the best basket of 17

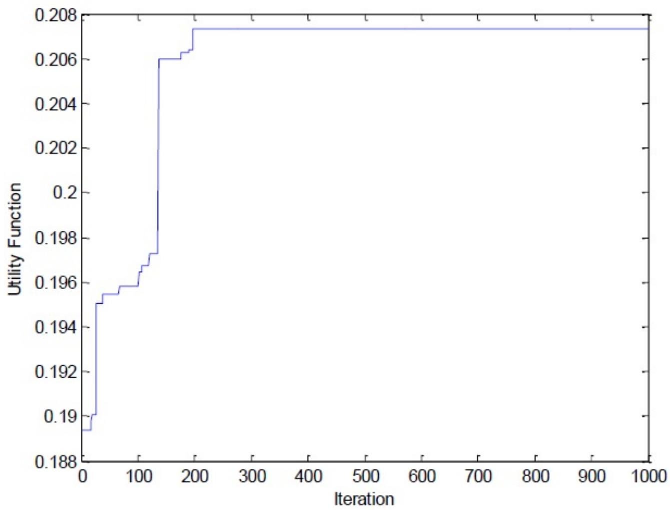

Figure 7: Convergence trend diagram of the proposed cuckoo optimisation algorithm for the best basket of 17

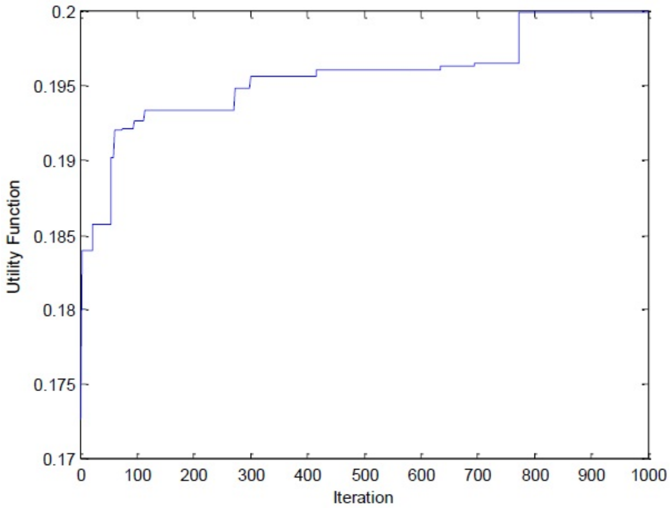

Figure 8: Convergence trend diagram of the proposed cuckoo optimisation algorithm for the best basket of 20

5 Discussion and conclusion

Due to the possible rapid increase of portfolios, the asymmetry preference relationship in the decision-maker should be computable to implement the time-investigation of portfolio and capital. High complexity in actual conditions requires evaluation algorithms, but in providing objectives, evaluation algorithms are inefficient, so to overcome this problem, cuckoo optimization has been used. In this decision-making situation, it is assumed that the decision-maker can evaluate the parameters for structuring the superior relationship. On the other hand, in this research, a method is assumed in which each of the heterogeneous groups obtains its best portfolio, and then these individual solutions are aggregated in a group with the best acceptable portfolio, in which the group's satisfaction or dissatisfaction is maximized.

The simultaneous development and improvement of the cuckoo optimization algorithm and the creation of complex formulations in the financial and economic fields have led to a mutual interest for both research communities. Therefore, the classification chosen for this research distinguishes between the portfolio optimization problem and the use cases in this field.

Besides, in this research, in order to achieve results in the field of how to report the rate of return for the higher effectiveness and efficiency of the monthly return compared to the annual return, two different portfolios are formed using each algorithm and with the help of annual and monthly inputs. In the following, to measure and compare the performance of three groups of algorithms, experts, and novices in the stock market, the collection of their selected portfolios has been done through a questionnaire. All research portfolios, i.e. eight selected portfolios of algorithms, forty selected portfolios of brokers as samples of experts and selected portfolios of individual investors present in the stock market as samples of newbies, in actual market conditions over six months, which is called the test period, were learned and applied.

In the proposed model, due to the permission of borrowing and short selling, we can expect to achieve a portfolio return higher than the highest return on available assets.

The yield and risk of the formed portfolios were determined based on the weights provided by each model. In the Iranian market, the actual returns from both methods are not significantly different. This is because the real risk of optimized portfolios with the stable method is lower than that of optimized portfolios with the classical method.

The findings of the research fill the study gap in the optimization of the investment portfolio, and also, according to the results of the present research, suggestions for using these results are presented as follows:

1. The attention of law-making institutions, including the stock exchange organization, the Tehran Stock Exchange Company, and brokerages and investment companies, to the effectiveness of the censoring model method based on the cuckoo optimization algorithm in optimizing the portfolios.
2. According to the findings of this research, suggesting to capital market activists, decision-makers, financial analysts, and potential and actual investors of the stock exchange to analyze investment plans in financial assets and securities using censorship models based on special attention paid to the cuckoo optimization algorithm

in the optimization of portfolios, as mentioned in this research. Because the use of this method leads to the selection of the optimal investment portfolio with minimum risk and maximum efficiency, while also doubling the transparency of the decision-making environment and the results.

References

[1] H. Babazadeh and A. Esfahanipour, *A novel multi period mean-VaR portfolio optimisation model considering practical constraints and transaction cost*, J. Comput. Appl. Math. **361** (2019), 313–342.

[2] A. Bačević, N. Vilimonović, I. Dabić, J. Petrović, D. Damnjanović and D. Džamić, *Variable neighborhood search heuristic for nonconvex portfolio optimisation*, Eng. Econ. **64** (2019), no. 3, 254–274.

[3] Y. Chen, A. Zhou, and S. Das, *Utilising dependence among variables in evolutionary algorithms for mixed-integer programming: A case study on multi-objective constrained portfolio optimisation*, Swarm Evolut. Comput. **66** (2021), 100928.

[4] A.M. Daryani, M.M. Omran and A. Makui, *A novel heuristic, based on a new robustness concept, for multi-objective project portfolio optimisation*, Comput. Ind. Eng. **139** (2020), 106187.

[5] K. Erwin and A. Engelbrecht, *Improved set-based particle swarm optimisation for portfolio optimisation*, IEEE Symp. Ser. Comput. Intell. (SSCI), IEEE, 2020, pp. 1573–1580.

[6] M. Feshari and R. Nazari, *Portfolio optimisation in selected Tehran stock exchange companies (symbiotic organisms search and memetic algorithms)*, Region. Sci. Inquiry **10** (2018), no. 1, 149–160.

[7] C. John, *Complete neighbourhood search heuristic algorithm for portfolio optimization*, Tanzania J. Eng. Technol. **40** (2021), no. 2, 97–104.

[8] C. John, *High speed hill climbing algorithm for portfolio optimization*, Tanzania J. Sci. **47** (2021), no. 3, 1236–1242.

[9] C.B. Kalayci, O. Ertenlice and M.A. Akbay, *A comprehensive review of deterministic models and applications for mean-variance portfolio optimisation*, Expert Syst. Appl. **125** (2019), 345–368.

[10] A.T. Khan, X. Cao, I. Brajevic, P.S. Stanimirovic, V.N. Katsikis and S. Li, *Non-linear activated beetle antennae search: A novel technique for non-convex tax-aware portfolio optimisation problem*, Expert Syst. Appl. **197** (2022), 116631.

[11] R. Kizys, A.A. Juan, B. Sawik and L. Calvet, *A biased-randomised iterated local search algorithm for rich portfolio optimisation*, Appl. Sci. 9 (2019), no. 17, 3509.

[12] H. Markowitz, *The utility of wealth*, J. Political Econ. **60** (1952), no. 2, 151–158.

[13] H.M. Markowitz, *Foundations of portfolio theory*, J. Finance **46** (1991), no. 2, 469–477.

[14] N.B. Mazraeh, A. Daneshvar and F.R. Roodposhti, *Stock portfolio optimization using a combined approach of multi objective grey wolf optimizer and machine learning preselection methods*, Comput. Intell. Neurosci. **2022** (2022).

[15] J.C. Mba, E. Pindza and U. Koumba, *A differential evolution copula-based approach for a multi-period cryptocurrency portfolio optimisation*, Financ. Markets Portfolio Manag. **32** (2018), no. 4, 399–418.

[16] B.A. Mercangoz, *Particle swarm algorithm: An application on portfolio optimisation*, Metaheuristic approaches to portfolio optimisation, IGI Global, 2019, pp. 27–59.

[17] D.A. Milhomem and M.J.P. Dantas, *Analysis of new approaches used in portfolio optimisation: a systematic literature review*, Production **30** (2020).

[18] N.S.M. Mussafi and Z. Ismail, *A review on Shariah stock portfolio optimization*, DLSU Bus. Econ. Rev. **31** (2021), no. 1, 142–157.

[19] M.J. Naik and A.L. Albuquerque, *Hybrid optimisation search-based ensemble model for portfolio optimisation and return prediction in business investment*, Progress Artif. Intell. **11** (2022), no. 4, 315–331.

[20] R. Rajabioun, *Cuckoo optimisation algorithm*, Appl. Soft Comput. **11** (2011), no. 8, 5508–5518.

[21] D. Venturelli and A. Kondratyev, *Reverse quantum annealing approach to portfolio optimisation problems*, Quantum Machine Intell. **1** (2019), no. 1, 17–30.

[22] J. Wang, *A novel firefly algorithm for portfolio optimisation problem*, IAENG Int. J. Appl. Math. **49** (2019), no. 1, 1–6.

[23] Z. Wang, X. Zhang, Z. Zhang and D. Sheng, *Credit portfolio optimisation: A multi-objective genetic algorithm approach*, Borsa Istanbul Rev. **22** (2022), no. 1, 69–76.

[24] I. Yaman and T.E. Dalkılıç, *A hybrid approach to cardinality constraint portfolio selection problem based on nonlinear neural network and genetic algorithm*, Expert Syst. Appl. **169** (2021), 114517.

[25] M. Zanjirdar, *Overview of portfolio optimisation models*, Adv. Math. finance Appl. **5** (2020), no. 4, 419–435.