Int. J. Nonlinear Anal. Appl. 15 (2024) 11, 271–283
ISSN: 2008-6822 (electronic)
http://dx.doi.org/10.22075/ijnaa.2023.30895.4513

Novel concepts of the Randic index in vague graphs with applications

Alimohammad Fallah Andevari, Yahya Talebi*

Department of Mathematics, Faculty of Mathematical Sciences, University of Mazandaran, Babolsar, Iran

(Communicated by Javad Damirchi)

Abstract

A topological index is a numerical quantity for the structure graph of the molecule and it can be represented through graph theory. Also, its application not only in the field of chemistry can also be applied in areas including computer science, networking, etc. Hence, this paper introduces the Randic index of a vague graph and vague subgraph with their properties. The upper and lower boundaries of the Randic index of vague graphs are studied with some isomorphic properties. Likewise, the Randic index of directed vague graphs is introduced. Finally, an application of the Randic index in construction has been presented.

Keywords: Vague set, vague graphs, Randic index, boundaries, isomorphic
2020 MSC: 05C99, 03E72

1 Introduction

Many real-world situations can accessibly be explained by means of a diagram consisting of a set of points together with lines joining certain pairs of these points. Notice that in such diagrams one is mainly interested in whether or not two given points are joined by a line; the manner in which they are joined is immaterial. A mathematical abstraction of situations of this type gives rise to the concept of a graph. To exemplify the objects and the connection between them, the graph nodes and edges are being employed accordingly. Fuzzy graphs are intended to demonstrate the connection structure among objects so that the concrete object existence (node) and the relationship between two objects (edge) are matters of degree. Many of the issues and phenomena around us are associated with complexities and ambiguities that make it difficult to express certainly. These difficulties were alleviated by the introduction of a fuzzy set by Zadeh [30]. Rosenfeld [24] proposed the idea of the fuzzy graph in 1975. The existence of a single degree for a true membership could not resolve the ambiguity on uncertain issues, so the need for a degree of membership was felt. Afterward, to overcome the existing ambiguities, Gau and Buehrer [8] introduced false membership degrees and defined a vague set as the sum of degrees not greater than 1. Kuffman [9] presented fuzzy graphs based on Zadeh's fuzzy relation [31, 32]. Vague graph notion was introduced by Ramakrishna [15]. Mordeson et al. [10] studied operations on fuzzy graphs. Borzooei et al. [3, 4, 5, 6] investigated new concepts of vague graphs. Akram et al. [1, 2] defined vague hypergraphs, vague cycles, and vague trees with several examples. Samanta et al. [25] studied irregular bipolar fuzzy graphs. Binu et al. [7] introduced connectivity index of a fuzzy graph. Poulik et al. [11, 12, 13, 14] studied indices of graphs under bipolar fuzzy environment. Sebastian et al. [26] presented connectivity parameters

*Corresponding author
Email addresses: alimohammad.fallah@yahoo.com (Alimohammad Fallah Andevari), talebi@uzm.ac.ir (Yahya Talebi)

Received: June 2023 *Accepted:* November 2023

in generalized fuzzy graphs. Rashmanlou et al. [16, 17, 18, 19, 20, 28, 29] investigated new results in vague graphs. Rao et al. [21, 22, 23] studied equitable domination in vague graphs. Zeng et al. [33] given certain properties of single-valued neutrosophic graphs. Shao et al. [27] presented new concepts in intuitionistic fuzzy graphs.

Connectivity index is one of the most important topics that has many applications in computer science, operation research, networks, and medical sciences. So, in this paper, Randic index of vague graph and vague subgraph are introduced with their properties. The upper and lower boundaries of Randic index of vague graphs are studied with some isomorphic properties. Randic index of directed vague graphs are introduced. Finally, an application of Randic index in construction has been presented.

2 Preliminaries

A fuzzy graph is of the form $G = (\sigma, \mu)$, which is a pair of mapping $\sigma : V \to [0, 1]$ and $\mu : V \times V \to [0, 1]$ as is defined as $\mu(m, n) \leq \sigma(m) \wedge \sigma(v)$, $\forall m, n \in V$, and μ is a symmetric fuzzy relation on σ and $\wedge$ denotes minimum.

Definition 2.1. [8] A VS A is a pair (t_A, f_A) on set V where t_A and f_A are taken as real valued functions which can be defined on $V \to [0, 1]$, so that $t_A(m) + f_A(m) \leq 1$, for all $m \in V$.

Definition 2.2. [15] $G = (A, B)$ is called a vague graph on a crisp graph G^*,which $A = (t_A, f_A)$ is a vague set on V and $B = (t_B, f_B)$ is a vague set on $E \subseteq V \times V$ so that $t_B(ab) \leq \min(t_A(a), t_A(b))$ and $f_B(ab) \geq \max(f_A(a), f_A(b))$, $\forall ab \in E$.

Definition 2.3. [28] Let $G = (A, B)$ and $G' = (A', B')$ be two vague graphs of the graphs $G^* = (V, E)$ and $G'^* = (V', E')$, respectively. If there exist a bijective mapping $f : V \to V'$ so that $t_A(m) = t_{A'}(f(m))$, $f_A(m) = f_{A'}(f(m))$, $\forall m \in V$ and $t_B(mn) = t_{B'}(f(m)f(n))$, $f_B(mn) = f_{B'}(f(m)f(n))$, for all $mn \in E$, then f is called an isomorphism.

Definition 2.4. [4] A vague graph $G' = (A', B')$ is said to be a subgraph of a vague graph $G = (A, B)$ if $V' \subseteq V$, $E' \subseteq E$ so that $t_A(m) = t_{A'}(m)$ and $f_A(m) = f_{A'}(m)$, $\forall m \in V'$ and $t_{B'}(mn) = t_B(mn)$, $f_{B'}(mn) = f_B(mn)$, for each edge mn of G'.

Definition 2.5. [5] The open neighborhood degree or degree of a vertex 'm' in a vague graph G is defined as $\deg(m) = (\deg^t(m), \deg^f(m))$, where

$$\deg^t(m) = \sum_{\substack{m \neq n \\ mn \in E}} t_B(mn), \qquad \deg^f(m) = \sum_{\substack{m \neq n \\ mn \in E}} f_B(mn).$$

If $\deg(m) = (d_1, d_2)$, $\forall m \in V$, G is called (d_1, d_2)-regular.

Definition 2.6. [5] The closed neighborhood degree of a node $m \in V$ in a vague graph G is denoted by $\deg[m] = (\deg^t[m], \deg^f[m])$ and is defined as

$$\deg^t[m] = \deg^t(m) + t_A(m), \qquad \deg^f[m] = \deg^f(m) + f_A(m).$$

If $\deg[m] = (f_1, f_2)$, $\forall m \in V$, then G is called (f_1, f_2)-totally regular.

3 Randic index of a vague graph

Definition 3.1. The Randic index of a vague graph $G = (A, B)$ is shown by $RI(G)$ and described as:

$$\begin{aligned} RI(G) =& (RI^t(G), RI^f(G)) \\ =& \Bigg(\sum_{i \neq j m_i m_j \in E} \left(t_A(m_i) t_A(m_j) \deg^t(m_i) \deg^t(m_j) \right)^{-\frac{1}{2}}, \sum_{\substack{i \neq j \\ m_i m_j \in E}} \Big(f_A(m_i) f_A(m_j) \deg^f(m_i) \deg^f(m_j) \Big)^{-\frac{1}{2}} \Bigg), \end{aligned}$$

that $\deg^t(m_i)$ and $\deg^f(m_i)$ are the true and false part of degree of the node m_i, respectively.

Example 3.2. Consider the vague graph G as Figure 1. Here, $\deg(m) = (1, 2.2)$, $\deg(n) = (0.8, 1.2)$, $\deg(z) = (0.6, 1.6)$, $\deg(w) = (0.7, 1.8)$, $\deg(k) = (0.5, 1.3)$.

$$\sum_{\substack{i \neq j \\ v_i v_j \in E}} \left(t_A(v_i) t_A(v_j) \deg^t(v_i) \deg^t(v_j) \right)^{-\frac{1}{2}} = 4.566 + 5.271 + 6.301 + 5.455 + 4.083 + 5.592 + 3.985, 7.220 = 42.473,$$

and

$$\sum_{\substack{i \neq j \\ v_i v_j \in E}} \left(f_A(v_i) f_A(v_j) \deg^f(v_i) \deg^f(v_j) \right)^{-\frac{1}{2}} = 1.163 + 1.538 + 1.521 + 1.150 + 1.207 + 1.235 + 1.123 + 1.575 = 10.512.$$

Therefore, $RI(G) = (RI^t(G), RI^f(G)) = (42.473, 10.512)$.

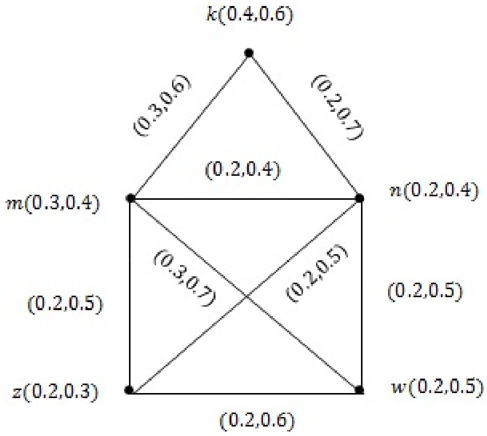

Figure 1: Vague graph G.

Theorem 3.3. Let $G = (A, B)$ be a connected vague graph and $G' = (A', B')$ so that $V' = V - \{r_n\}$, $r_n \in V$ with $|V| = n$. Then, $RI^t(G) \geq RI^t(G')$ and $RI^f(G) \geq RI^f(G')$.

Proof . Since G has n nodes i.e., $V = \{r_1, r_2, \cdots, r_n\}$, $V' = \{r_1, r_2, \cdots, r_{n-1}\}$. Hence, V' must be a subset of V as so G' is a vague subgraph of G. Hence, $t_A(r_i) = t_{A'}(r_i)$, $f_A(r_i) = f_{A'}(r_i)$, $t_B(r_i r_j) = t_{B'}(r_i r_j)$ and $f_B(r_i r_j) = f_{B'}(r_i r_j)$, $\forall r_i \in V'$ and $\forall r_i r_j \in E'$. Now, $\deg^t(r_i) = \sum_{\substack{r_j \\ r_i r_j \in E}} t_B(r_i r_j)$ and $\deg^f(r_i) = \sum_{\substack{r_j \\ r_i r_j \in E}} f_B(r_i r_j)$ i.e., $\deg^t(r_i)$ and $\deg^f(r_i)$ are sum of the true and false membership values of the edges incident in r_i in G, respectively. Then, $\deg^f(r_i) \deg^f(r_j)$ is a positive real number. Similarly we can show that $\deg'^f(r_i) \deg'^f(r_j)$ is a positive real number too. Thus,

$$\left(t_A(r_i) t_A(r_j) \deg^t(r_i) \deg^t(r_j) \right)^{-\frac{1}{2}} \geq 0,$$

$$\left(t_{A'}(r_i) t_{A'}(r_j) \deg'^t(r_i) \deg'^t(r_j) \right)^{-\frac{1}{2}} \geq 0,$$

$$\left(f_A(r_i) f_A(r_j) \deg^f(r_i) \deg^f(r_j) \right)^{-\frac{1}{2}} \geq 0,$$

$$\left(f_{A'}(r_i) f_{A'}(r_j) \deg'^f(r_i) \deg'^f(r_j) \right)^{-\frac{1}{2}} \geq 0.$$

So,

$$\sum_{\substack{1\leq i\neq j\leq n\\ r_ir_j\in E}}\Big(t_A(r_i)t_A(r_j)\deg^t(r_i)\deg^t(r_j)\Big)^{-\frac{1}{2}}\geq\sum_{\substack{1\leq i\neq j\leq n-1\\ r_ir_j\in E'}}\Big(t_{A'}(r_i)t_{A'}(r_j)\deg'^t(r_i)\deg'^t(r_j)\Big)^{-\frac{1}{2}}$$

and

$$\sum_{\substack{1\leq i\neq j\leq n\\ r_ir_j\in E}}\Big(f_A(r_i)f_A(r_j)\deg^f(r_i)\deg^f(r_j)\Big)^{-\frac{1}{2}}\geq\sum_{\substack{1\leq i\neq j\leq n-1\\ r_ir_j\in E'}}\Big(f_{A'}(r_i)f_{A'}(r_j)\deg'^f(r_i)\deg'^f(r_j)\Big)^{-\frac{1}{2}}.$$

Therefore $RI^t(G)\geq RI^t(G')$ and $RI^f(G)\geq RI^f(G')$. $\square$

Example 3.4. Consider the vague graph $G'=(A',B')$ of Figure 2 and vague graph G of Figure 1. Clearly, $t_A(v_i)\geq t_{A'}(v_i)$, $f_A(v_i)\leq f_{A'}(v_i)$, $\forall v_i\in V'$ and $t_B(v_iv_j)\geq t_{B'}(v_iv_j)$, $f_B(v_iv_j)\leq f_{B'}(v_iv_j)$, $\forall v_iv_j\in E'$. Hence, G' is a vague subgraph of the vague graph G.

$$RI^t(G)=6.301+6.301+6.301+6.301+4.761+8.333=38.298,$$

and

$$RI^f(G')=0.598+1.804+1.521+1.408+1.317+1.929=8.577.$$

Therefore, $RI^t(G)\geq RI^t(G')$ and $RI^f(G)\geq RI^f(G')$.

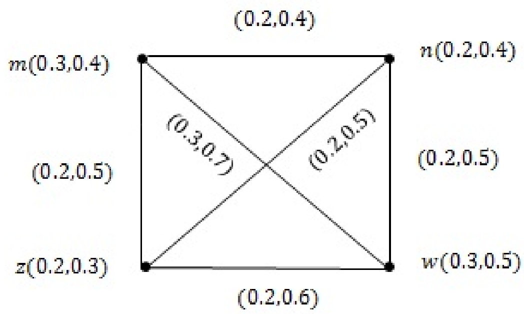

Figure 2: Vague subgraph G' of the vague graph G of Fig. 1.

Theorem 3.5. Let $G=(A,B)$ be a vague graph of the graph $G^*=(V,E)$. Then,

$$RI^t(G)\geq\frac{m}{n-1}\quad and\quad RI^f(G)\geq\frac{m}{n-1},$$

where, $n=|V|$ and $m=|E|$.

Proof . Since G is a vague graph, $0\leq t_A(m_i)\leq 1$, $0\leq t_A(m_j)\leq 1$, $0\leq f_A(m_i)\leq 1$, $0\leq f_A(m_j)\leq 1$, $\forall m_i,m_j\in V$. So,

$$t_A(m_i)t_A(m_j)\leq 1,\quad f_A(m_i)f_A(m_j)\leq 1.\qquad (i)$$

Again, since $|V|=n$, every vertex is connected with at most $(n-1)$ vertices. But, $\deg^t(m_i)=\sum_{m_im_j\in E}^{m_j}t_B(m_im_j)$, $\deg^f(m_i)=\sum_{m_im_j\in E}^{m_j}f_B(m_im_j)$, so $\deg^t(m_i)\leq n-1$ and $\deg^f(m_i)\leq n-1$, $\forall m_i\in V$. This implies that

$$\deg^t(m_i)\deg^t(m_j)\leq(n-1)^2\quad\text{and}\quad\deg^f(m_i)\deg^f(m_j)\leq(n-1)^2.\qquad (ii)$$

So, from (i) and (ii), we have:

$$t_A(m_i)t_A(m_j)\deg^t(m_i)\deg^t(m_j) \le (n-1)^2 \quad and \quad f_A(m_i)f_A(m_j)\deg^f(m_i)\deg^f(m_j) \le (n-1)^2.$$

Hence,

$$\Big(t_A(m_i)t_A(m_j)\deg^t(m_i)\deg^t(m_j)\Big)^{-\frac{1}{2}} \ge \frac{1}{n-1}$$

and

$$\Big(f_A(m_i)f_A(m_j)\deg^f(m_i)\deg^f(m_j)\Big)^{-\frac{1}{2}} \ge \frac{1}{n-1}.$$

Therefore,

$$\sum_{\substack{i\neq j\\ m_im_j\in E}} \Big(t_A(m_i)t_A(m_j)\deg^t(m_i)\deg^t(m_j)\Big)^{-\frac{1}{2}} \ge \frac{m}{n-1}$$

and

$$\sum_{\substack{i\neq j\\ m_im_j\in E}} \Big(f_A(m_i)f_A(m_j)\deg^f(m_i)\deg^f(m_j)\Big)^{-\frac{1}{2}} \ge \frac{m}{n-1}.$$

Thus, $RI^t(G) \ge \frac{m}{n-1}$ and $RI^f(G) \ge \frac{m}{n-1}$. □

Example 3.6. Consider the vague graph G of Example 3.1. Here, $n = |V| = 5$, $m = |E| = 8$, $RI^t(G) = 42.473$, $RI^f(G) = 10.512$, and $\frac{m}{n-1} = \frac{8}{5-1} = 2$. Therefore, $RI^t(G) > \frac{m}{n-1}$ and $RI^f(G) > \frac{m}{n-1}$.

Theorem 3.7. Let $G = (A, B)$ be a strong vague graph so that A is a constant function and $|V| = n$. Then, $RI^t(G) \le \frac{n}{2v_1^2}$ and $RI^f(G) \le \frac{n}{2v_2^2}$, where, $(v_1, v_2) = (t_A(m_i), f_A(m_i))$, $m_i \in V$.

Proof . Since a strong vague graph is always a vague subgraph of the corresponding complete vague graph, using Theorem 3.5, it can easily prove that $RI^t(G) \le \frac{n}{2v_1^2}$ and $RI^f(G) \le \frac{n}{2v_2^2}$. □

Theorem 3.8. Let G be a complete vague graph so that A is a constant function. Then $RI(G) = \left(\frac{n}{2v_1^2}, \frac{n}{2v_2^2}\right)$, where $n = |V|$ and $(v_1, v_2) = (t_A(m_i), f_A(m_i))$, $m_i \in V$.

Proof . Since A is a constant and $t_A(m_i) = v_1$ and $f_A(m_i) = v_2$, $m_i \in V$, $t_A(m_i) = v_1$, $f_A(m_i) = v_2$, $\forall m_i \in V$. Since G is complete vague graph, $t_B(m_im_j) = \min\{t_A(m_i), t_A(m_j)\} = v_1$ and $f_B(m_im_j) = \max\{f_A(m_i), f_A(m_j)\} = v_2$, $\forall m_im_j \in E$.

Again, G is complete and $|V| = n$, so there are $\frac{n(n-1)}{2}$ pair of vertices and hence $\frac{n(n-1)}{2}$ edges and also every vertex in G is adjacent to $(n-1)$ number of vertices. Then,

$$\deg^t(m_i) = \sum_{\substack{m_j\\ m_im_j\in E}} t_B(m_im_j) = v_1 \cdot v_1 \cdots v_1(n-1\ times) = (n-1)v_1,$$

and

$$\deg^f(m_i) = \sum_{\substack{m_j\\ m_im_j\in E}} f_B(m_im_j) = v_2 \cdot v_2 \cdots v_2(n-1\ times) = (n-1)v_2.$$

Therefore,

$$\begin{aligned} RI^t(G) &= \sum_{\substack{i\neq j\\ m_im_j\in E}} \Big(t_A(m_i)t_A(m_j)\deg^t(m_i)\deg^t(m_j)\Big)^{-\frac{1}{2}} \\ &= \frac{n(n-1)}{2}(v_1v_1(n-1)v_1(n-1)v_1)^{-\frac{1}{2}} = \frac{n}{2v_1^2}, \end{aligned}$$

and

$$RI^f(G) = \sum_{\substack{i\neq j \\ m_im_j\in E}} \left(f_A(m_i)f_A(m_j)\deg^f(m_i)\deg^f(m_j)\right)^{-\frac{1}{2}}$$
$$= \frac{n(n-1)}{2}(v_2v_2(n-1)v_2(n-1)v_2)^{-\frac{1}{2}} = \frac{n}{2v_2^2}.$$

Hence, $RI(G) = (RI^t(G), RI^f(G)) = (\frac{n}{2v_1^2}, \frac{n}{2v_2^2})$. □

Proposition 3.9. If two vague graphs G and G' are isomorphic to each other, then, $RI(G) = RI(G')$.

Proof . Let $G = (A, B)$ and $G' = (A', B')$ be two isomorphic vague graphs. Then, there is a bijection $g : G \to G'$ so that $t_A(m_i) = t_{A'}(g(m_i)) = t_A(m'_i)$, $f_A(m_i) = f_{A'}(g(m_i)) = f_A(m'_i)$, $\forall m_i \in V$ and $t_B(m_im_j) = t_{B'}(g(m_i)g(m_j)) = t_B(m'_im'_j)$, $f_B(m_im_j) = f_{B'}(g(m_i)g(m_j)) = f_B(m'_im'_j)$, $\forall m_im_j \in E$. So, for every $m_i \in V$ there exists a vertex $m'_i \in V'$ so that $\deg(m_i) = \deg(m'_i)$. Thus, we have the proof. □

Example 3.10. Consider the vague graphs G and G' of Figure 3. Here, $t_{A'}(g(m_i)) = t_A(n_i)$, $f_{A'}(g(m_i)) = f_A(n_i)$, $t_B(m_im_j) = t_{B'}(g(m_i)g(m_j)) = t_B(n_in_j)$, $f_B(m_im_j) = f_{B'}(g(m_i)g(m_j)) = f_B(n_in_j)$, $\forall 1 \le i, j \le 3$.

Here, $RI^t(G) = 35.64 + 35.46 + 50 = 120.92 = RI^t(G')$ and $RI^f(G) = 6.80 + 5.55 + 6.80 = 19.15 = RI^f(G')$. Thus, $RI(G) = RI(G')$.

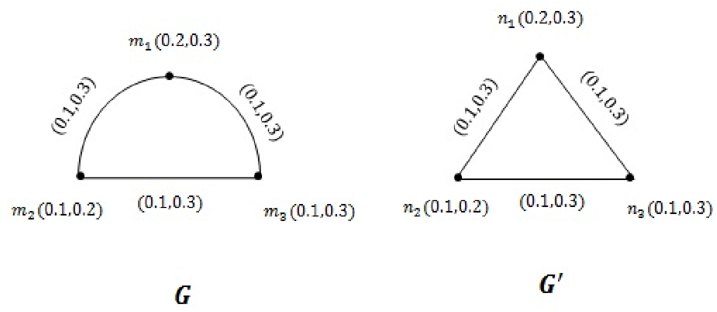

Figure 3: Two isomorphic vague graphs G and G'.

Theorem 3.11. Let $G = (A, B)$ be a vague graph so that $\delta_1 = \min\{\deg^t(m_i)\}$, $\delta_2 = \min\{\deg^f(m_i)\}$, $\Delta_1 = \max\{\deg^t(m_i)\}$ and $\Delta_2 = \max\{\deg^f(m_i)\}$, $\forall m_i \in V$. Then, $k_1\Delta_1^{-1} \le RI^t(G) \le k_1\delta_1^{-1}$ and $k_2\Delta_2^{-1} \le RI^f(G) \le k_2\delta_2^{-1}$, where

$$k_1 = \sum_{m_im_j\in E} \frac{1}{\sqrt{t_A(m_i)t_A(m_j)}}, \qquad k_2 = \sum_{m_im_j\in E} \frac{1}{\sqrt{f_A(m_i)f_A(m_j)}}.$$

Proof . Since G is a vague graph and $m_i \in V$, $\deg^t(m_i) = \sum_{\substack{m_j \\ m_im_j\in E}} t_B(m_im_j)$ and $\deg^f(m_i) = \sum_{\substack{m_j \\ m_im_j\in E}} f_B(m_im_j)$. Again, $\delta_1 = \min\{\deg^t(m_i)\}$, $\delta_2 = \min\{\deg^f(m_i)\}$, $\Delta_1 = \max\{\deg^t(m_i)\}$ and $\Delta_2 = \max\{\deg^f(m_i)\}$, $\forall m_i \in V$, so we have $\delta_1 \le \deg^t(m_i) \le \Delta_1$ and $\delta_2 \le \deg^f(m_i) \le \Delta_2$, $\forall m_i \in V$. Then, $\delta_1^2 \le \deg^t(m_i)\deg^t(m_j) \le \Delta_1^2$ and $\delta_2^2 \le \deg^f(m_i)\deg^f(m_j) \le \Delta_2^2$, $\forall m_im_j \in V$. Thus,

$$t_A(m_i)t_A(m_j)\delta_1^2 \le t_A(m_i)t_A(m_j)\deg^t(m_i)\deg^t(m_j) \le t_A(m_i)t_A(m_j)\Delta_1^2$$

and

$$f_A(m_i)f_A(m_j)\delta_2^2 \le f_A(m_i)f_A(m_j)\deg^f(m_i)\deg^f(m_j) \le f_A(m_i)f_A(m_j)\Delta_2^2,$$

for all $m_i, m_j \in V$. So,

$$\sum_{\substack{i\neq j\\ m_im_j\in E}} \Big(t_A(m_i)t_A(m_j)\Delta_1^2\Big)^{-\frac{1}{2}} \leq \sum_{\substack{i\neq j\\ m_im_j\in E}} \Big(t_A(m_i)t_A(m_j)\deg^t(m_i)\deg^t(m_j)\Big)^{-\frac{1}{2}}$$
$$\leq \sum_{\substack{i\neq j\\ m_im_j\in E}} \Big(t_A(m_i)t_A(m_j)\delta_1^2\Big)^{-\frac{1}{2}}$$

and

$$\sum_{\substack{i\neq j\\ m_im_j\in E}} \Big(f_A(m_i)f_A(m_j)\Delta_2^2\Big)^{-\frac{1}{2}} \leq \sum_{\substack{i\neq j\\ m_im_j\in E}} \Big(f_A(m_i)f_A(m_j)\deg^f(m_i)\deg^f(m_j)\Big)^{-\frac{1}{2}}$$
$$\leq \sum_{\substack{i\neq j\\ m_im_j\in E}} \Big(f_A(m_i)f_A(m_j)\delta_2^2\Big)^{-\frac{1}{2}}.$$

Hence,

$$\Delta_1^{-1}\sum_{m_im_j\in E}\frac{1}{\sqrt{t_A(m_i)t_A(m_j)}} \leq RI^t(G) \leq \delta_1^{-1}\sum_{m_im_j\in E}\frac{1}{\sqrt{t_A(m_i)t_A(m_j)}}$$

and

$$\Delta_2^{-1}\sum_{m_im_j\in E}\frac{1}{\sqrt{f_A(m_i)f_A(m_j)}} \leq RI^f(G) \leq \delta_2^{-1}\sum_{m_im_j\in E}\frac{1}{\sqrt{f_A(m_i)f_A(m_j)}}.$$

Therefore, $k_1\Delta_1^{-1} \leq RI^t(G) \leq k_1\delta_1^{-1}$ and $k_2\Delta_2^{-1} \leq RI^f(G) \leq k_2\delta_2^{-1}$. □

Example 3.12. Consider the VG G of Example 3.10. Here, $V = \{m_1, m_2, m_3\}$, $E = \{m_1m_2, m_1m_3, m_2m_3\}$, $(\deg^t(m_1), \deg^f(m_1)) = (0.2, 0.6) = (\deg^t(m_2), \deg^f(m_2)) = (\deg^t(m_3), \deg^f(m_3))$. So, $\delta_1 = 0.2 = \Delta_1$ and $\delta_2 = 0.6 = \Delta_2$. $RI^t(G) = 120.92$ and $RI^f(G) = 19.15$. Now,

$$k_1 = \sum_{m_im_j\in E}\frac{1}{\sqrt{t_A(m_i)t_A(m_j)}} = \frac{1}{\sqrt{0.2\times 0.1}} + \frac{1}{\sqrt{0.2\times 0.1}} + \frac{1}{\sqrt{0.1\times 0.1}}$$
$$= 7.072 + 7.072 + 10 = 24.185,$$

and

$$k_2 = \sum_{m_im_j\in E}\frac{1}{\sqrt{f_A(m_i)f_A(m_j)}} = \frac{1}{\sqrt{0.2\times 0.3}} + \frac{1}{\sqrt{0.3\times 0.3}} + \frac{1}{\sqrt{0.2\times 0.3}}$$
$$= 4.083 + 3.333 + 4.083 = 11.499.$$

So, $k_1\delta_1^{-1} = \frac{24.185}{0.2} = 120.92 = k_1\Delta_1^{-1}$ and $k_2\delta_2^{-1} = \frac{11.499}{0.6} = 19.15 = k_2\Delta_2^{-1}$. Therefore, $k_1\Delta_1^{-1} = RI^t(G) = k_1\delta_1^{-1}$ and $k_2\Delta_2^{-1} = 19.15 = RI^f(G)$.

Theorem 3.13. Let $G = (A, B)$ be a vague graph so that $|V| = n$. Then,

$$RI^t(G) \geq \frac{1}{n-1}\sum_{m_im_j\in E}\frac{1}{t_A(m_i)t_A(m_j)}$$

and

$$RI^f(G) \geq \frac{1}{n-1}\sum_{m_im_j\in E}\frac{1}{f_A(m_i)f_A(m_j)}.$$

Proof . Since $|V| = n$, let $V = \{m_1, m_2, \cdots, m_n\}$. Then, each vertex of G is connected with at most $n-1$ other vertices. Therefore, $\sum_{\substack{j \\ m_im_j \in E}} t_B(m_im_j) \leq (n-1)t_A(m_i)$ and $\sum_{\substack{j \\ m_im_j \in E}} f_B(m_im_j) \leq (n-1)f_A(m_i)$. So,

$$\deg^t(m_i) \leq (n-1)t_A(m_i) \quad \text{and} \quad \deg^f(m_i) \leq (n-1)f_A(m_i),$$

for all $m_i \in V$. Hence,

$$t_A(m_i)t_A(m_j)\deg^t(m_i)\deg^t(m_j) \leq t_A(m_i)t_A(m_j)(n-1)t_A(m_i)(n-1)t_A(m_j)$$

and

$$f_A(m_i)f_A(m_j)\deg^f(m_i)\deg^f(m_j) \leq f_A(m_i)f_A(m_j)(n-1)f_A(m_i)(n-1)f_A(m_j),$$

for all $m_im_j \in E$. So,

$$\sum_{\substack{i \neq j \\ m_im_j \in E}} \Big(t_A(m_i)t_A(m_j)\deg^t(m_i)\deg^t(m_j)\Big)^{-\frac{1}{2}} \geq \frac{1}{(n-1)} \sum_{m_im_j \in E} \frac{1}{t_A(m_i)t_A(m_j)}$$

and

$$\sum_{\substack{i \neq j \\ m_im_j \in E}} \Big(f_A(m_i)f_A(m_j)\deg^f(m_i)\deg^f(m_j)\Big)^{-\frac{1}{2}} \geq \frac{1}{(n-1)} \sum_{m_im_j \in E} \frac{1}{f_A(m_i)f_A(m_j)}.$$

Thus, $RI^t(G) \geq \frac{1}{(n-1)} \sum_{m_im_j \in E} \frac{1}{t_A(m_i)t_A(m_j)}$ and $RI^f(G) \geq \frac{1}{(n-1)} \sum_{m_im_j \in E} \frac{1}{f_A(m_i)f_A(m_j)}$. □

Definition 3.14. The degree of a vertex m_i in a vague digraph $\overrightarrow{G} = (A, \overrightarrow{B})$ is $\deg(m_i) = (\deg^t(m_i), \deg^t f(m_i))$, where

$$\deg^t(m_i) = \sum_{\substack{j \\ i \neq j}} \big(t_B(\overrightarrow{m_im_j}) + t_B(\overrightarrow{m_jm_i})\big), \qquad \deg^f(m_i) = \sum_{\substack{j \\ i \neq j}} \big(f_B(\overrightarrow{m_im_j}) + f_B(\overrightarrow{m_jm_i})\big).$$

Example 3.15. Consider the vague digraph $\overrightarrow{G}$ of Figure 4. Here, $(t_A(m_i), f_A(m_i)) = (0.2, 0.4)$, $i = 1, 2, 3, 4$. $\deg^t(m_1) = 0.6 = \deg^t(m_2) = \deg^t(m_3) = \deg^t(m_4)$, $\deg^f(m_1) = 2.2 = \deg^f(m_2) = \deg^f(m_3) = \deg^f(m_4)$. So, $\deg(m_1) = (0.6, 2.2) = \deg(m_2) = \deg(m_3) = \deg(m_4)$.

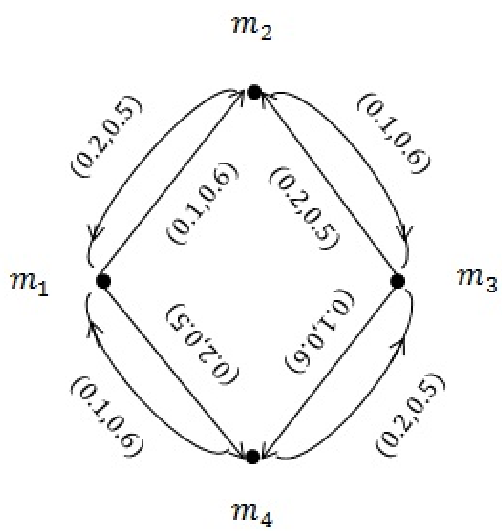

Figure 4: A vague digraph $\overrightarrow{G}$ with membership values of vertices are as $(0.2, 0.4)$.

Definition 3.16. The Randic index of a vague digraph $\overrightarrow{G} = (A, \overrightarrow{B})$ is denoted by $RI(\overrightarrow{G})$ and is defined as:

$$\begin{aligned} RI(\overrightarrow{G}) =& (RI^t(\overrightarrow{G}), RI^f(\overrightarrow{G})) \\ =& \Bigg(\sum_{\substack{i\neq j \\ m_l m_j \in E}} \big(t_A(m_i)t_A(m_j)\deg^t(m_i)\deg^t(m_j)\big)^{-\frac{1}{2}}, \sum_{\substack{i\neq j \\ m_l m_j \in E}} \big(f_A(m_i)f_A(m_j)\deg^f(m_i)\deg^f(m_j)\big)^{-\frac{1}{2}} \Bigg). \end{aligned}$$

Example 3.17. Consider the vague digraph $\overrightarrow{G}$ of Example 3.15. We have:

$$RI^t(\overrightarrow{G}) = \sum_{\substack{i\neq j \\ m_l m_j \in E}} \big(t_A(m_i)t_A(m_j)\deg^t(m_i)\deg^t(m_j)\big)^{-\frac{1}{2}} = 4 \cdot (0.2 \times 0.2 \times 0.6 \times 0.6)^{-\frac{1}{2}} = 3.333$$

and

$$RI^f(\overrightarrow{G}) = \sum_{\substack{i\neq j \\ m_l m_j \in E}} \big(f_A(m_i)f_A(m_j)\deg^f(m_i)\deg^f(m_j)\big)^{-\frac{1}{2}} = 4 \cdot (0.4 \times 0.4 \times 2.2 \times 2.2)^{-\frac{1}{2}} = 4.54.$$

Hence, $RI(\overrightarrow{G}) = (RI^t(\overrightarrow{G}), RI^f(\overrightarrow{G})) = (33.33, 4.54)$.

Theorem 3.18. Let $\overrightarrow{G} = (A, \overrightarrow{B})$ be a vague digraph with $|V| = m$ so that $A = (t_A, f_A)$ is constant. If $\deg(m_i) = (r_1, r_2)$, $\forall m_i \in V$, then $RI(\overrightarrow{G}) = m(\frac{1}{v_1 r_1}, \frac{1}{v_2 r_2})$, where $(v_1, v_2) = (t_A(m_i), f_A(m_i))$, $\forall m_i \in V$.

Proof . Consider the vague digraph $\overrightarrow{G} = (A, \overrightarrow{B})$ so that $A = (t_A, f_A)$ is constant, $deg(m_i) = (r_1, r_2)$, and $(v_1, v_2) = (t_A(m_i), f_A(m_i))$, $\forall m_i \in V$.

$$RI^t(\overrightarrow{G}) = \sum_{\substack{i\neq j \\ m_i m_j \in E}} \big(t_A(m_i)t_A(m_j)\deg^t(m_i)\deg^t(m_j)\big)^{-\frac{1}{2}} = m(v_1 v_1 r_1 r_1)^{-\frac{1}{2}} = \frac{m}{v_1 r_1},$$

and

$$RI^f(\overrightarrow{G}) = \sum_{\substack{i\neq j \\ m_i m_j \in E}} \big(f_A(m_i)f_A(m_j)\deg^f(m_i)\deg^f(m_j)\big)^{-\frac{1}{2}} = m(v_2 v_2 r_2 r_2)^{-\frac{1}{2}} = \frac{m}{v_2 r_2}.$$

Thus, $RI(\overrightarrow{G}) = (RI^t(\overrightarrow{G}), RI^f(\overrightarrow{G})) = m(\frac{1}{v_1 r_1}, \frac{1}{v_2 r_2})$. □

Example 3.19. Consider the vague digraph $\overrightarrow{G}$ of Example 3.17. Here, $|V| = m = 4$, degree of each $vertex = (r_1, r_2) = (0.6, 2.2)$, membership value of each $vertex = (v_1, v_2) = (0.2, 0.4)$ and $RI(\overrightarrow{G}) = (33.33, 4.54)$. Now, $\frac{m}{v_1 r_1} = \frac{4}{0.2 \times 0.6} = 33.33$ and $\frac{m}{v_2 r_2} = \frac{4}{2.2 \times 0.4} = 4.54$. Hence, Theorem 3.18 is verified.

Theorem 3.20. Let $G = (A, B)$ be a regular and totally regular vague graph so that there is an edge between every pair of vertices and $|V| = n$. Then, $RI(G) = \frac{n(n-1)}{2}(\frac{1}{v_1 r_1}, \frac{1}{v_2 r_2})$, where $r_1 = \deg^t(m_i)$, $r_2 = \deg^f(m_i)$, $v_1 = t_A(m_i)$, $v_2 = f_A(m_i)$, $\forall m_i \in V$.

Proof . Since G is regular, $\deg^t(m_i) = r_1$, $\deg^f(m_i) = r_2$, $\forall m_i \in V$. Again, G is totally regular vague graph, so $\deg^t[m_i] = d_1$ (say), $\deg^f[m_i] = d_2$ (say), $\forall m_i \in V$. We know that, $\deg^t[m_i] = \deg^t(m_i) + t_A(m_i)$ and $\deg^f[m_i] = \deg^f(m_i) + f_A(m_i)$, $\forall m_i \in V$.

Thus, $v_1 = t_A(m_i) = \deg^t[m_i] - \deg^t(m_i) = d_1 - r_1$ and $v_2 = f_A(m_i) = \deg^f[m_i] - \deg^f(m_i) = d_2 - r_2$, $\forall m_i \in V$. Hence, A is constant function and $v_1 = t_A(m_i)$, $v_2 = f_A(m_i)$, $\forall m_i \in V$. Since $|V| = n$, and there is an edge between every pair of vertices in G, so there are $\frac{n(n-1)}{2}$ number of edges in G. Now,

$$RI^t(G) = \sum_{\substack{i \neq j \\ m_i m_j \in E}} \big(t_A(m_i) t_A(m_j) \deg^t(m_i) \deg^t(m_j)\big)^{-\frac{1}{2}} = \frac{n(n-1)}{2}(v_1 v_1 r_1 r_1)^{-\frac{1}{2}}$$
$$= \frac{n(n-1)}{2v_1 r_1},$$

and

$$RI^f(G) = \sum_{\substack{i \neq j \\ m_i m_j \in E}} \big(f_A(m_i) f_A(m_j) \deg^f(m_i) \deg^f(m_j)\big)^{-\frac{1}{2}} = \frac{n(n-1)}{2}(v_2 v_2 r_2 r_2)^{-\frac{1}{2}}$$
$$= \frac{n(n-1)}{2v_2 r_2}.$$

Therefore, $RI(G) = (RI^t(G), RI^f(G)) = \frac{n(n-1)}{2}(\frac{1}{v_1 r_1}, \frac{1}{v_2 r_2})$. □

4 Application of randic index in construction

Definitely, sending loved ones to the nursing home will bbe a difficult decision for everyone, and because there is emotional dependence between parents and children, this issue prevents the right decision.

You may face a strong reaction from those around you regarding this decision. Probably those with the sentence "they worked so hard for you". They may try to blame you or their wrong judgment may put a lot of pressure on your spirit, but ultimately it is you who will make the final decision about them. Maybe your situation is not such that you can be with your elderly loved one all the time and the decision you have made is only for his good. In any case, being in an environment where he is constantly under care is much better than being alone. Contrary to what everyone thinks, the nursing home is not an inappropriate place for the elderly. It is a central nursing home with a clean and calm environment where nurses and doctors take care of the elderly 24 hours a day and provide the best services to the elderly patients in case of any serious problems with medical emergencies. Among the advantages of nursind home, we can mention the security of elderly people in the nursing home, suitable environmental conditions and space, helping the elderly in doing personal things, and health and treatment services.

Therefore, in this section, by using the concept of Randic index, we try to determine the most suitable places to build a nursing home. Therefore, we consider six cities in Iran (Mazandaran province) named Amol, Ramsar, Chalus, Fereidoonkenar, Nowshahr, and Tonekabon. The corresponding symbols of each city in the graph are Am, Ra, Ch, Fe, No, and To respectively.

It should be noted that in this vague graph, the nodes represent the cities and the edges represent the quality of the roads as well as the access to intercity transportation. The node $Ch(0.4, 0.1)$ shows that the city of Chalus

Table 1: Weight of nodes in G

G	Am	Ch	No	Fe	To	Ra
(t_A, f_A)	$(0.6, 0.2)$	$(0.4, 0.1)$	$(0.4, 0.2)$	$(0.2, 0.3)$	$(0.5, 0.1)$	$(0.3, 0.4)$

Table 2: Weight of edges in G

G	Am-Ch	Am-Fe	Fe-To	Ch-To	Ch-No	To-No	To-Ra
(t_B, f_B)	$(0.4, 0.5)$	$(0.2, 0.4)$	$(0.2, 0.5)$	$(0.3, 0.2)$	$(0.3, 0.3)$	$(0.2, 0.4)$	$(0.3, 0.4)$

Table 3: The population of cities

G	Amol	Nowshahr	Chalus	Tonekabon	Ramsar	Fereidoonkenar
Population	401639	138913	1165546	166132	74179	67000

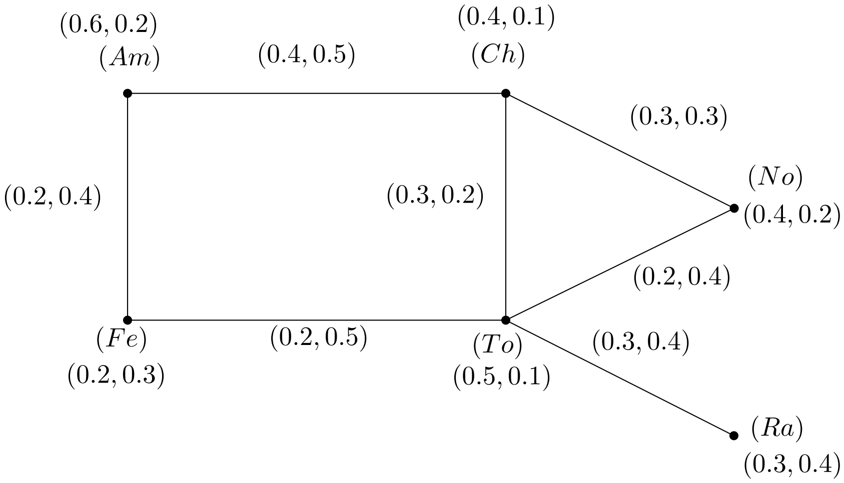

Figure 5: Vague graph G

Table 4: Distance between cities

cities	Am-Fe	Am-Ch	Am-Ra
Distance	36Km	102Km	183Km
cities	Am-No	Am-To	Fe-Ch
Distance	95Km	159Km	108Km
cities	Fe-To	Fe-Ra	Fe-No
Distance	162Km	186Km	98Km
cities	Ch-To	Ch-Ra	Ch-No
Distance	56Km	80Km	13Km
cities	To-Ra	To-No	No-Ra
Distance	25Km	66Km	89Km

has 40% of the necessary equipment to buid a nursing home and the false membership part of this node shows that only 10% of the population of this city are not agree with the construction of a nursing home in the city. The To-No edge shows that this route has about 20% of the vehicles needed to transport elderly, and in terms of standard traffic conditions, it has 40% of the necessary quality for the transportation of vehicles.

$$\deg(Am) = (1, 0.7), \quad \deg(Ch) = (1, 1), \quad \deg(Fe) = (0.4, 0.9), \quad \deg(To) = (1, 1.5),$$
$$\deg(No) = (0.5, 0.7), \quad \deg(Ra) = (0.3, 0.4),$$
$$\sum_{\substack{i \neq j \\ v_i v_j \in E}} \left(t_A(v_i) t_A(v_j) \deg^t(v_i) \deg^t(v_j) \right)^{-\frac{1}{2}} = 2.04 + 4.56 + 5 + 2.23 + 3.54 + 3.16 + 4.71 = 25.24$$
$$\sum_{\substack{i \neq j \\ v_i v_j \in E}} \left(f_A(v_i) f_A(v_j) \deg^f(v_i) \deg^f(v_j) \right)^{-\frac{1}{2}} = 8.47 + 5.20 + 5 + 8.19 + 8.47 + 6.94 + 6.49 = 48.76.$$

So, $RI(G) = (RI^t(G), RI^f(G)) = (25.24, 48.76)$.

$$RI^t(G - Am) = 18.64, RI^t(G - Ch) = 17.43,$$
$$RI^t(G - Fe) = 15.68, RI^t(G - To) = 10.14,$$
$$RI^t(G - No) = 18.54, RI^t(G - Ra) = 20.53,$$

$$RI^f(G-Am)=35.09, RI^f(G-Ch)=23.63,$$
$$RI^f(G-Fe)=38.56, RI^f(G-To)=22.14,$$
$$RI^f(G-No)=33.35, RI^f(G-Ra)=42.27.$$

As we can see in the above calculation, if we remove the node To, then the node Ra is autoatically removed, so, Ramsar is not suitable for building a nursing house. In addition, in terms of the necessary facilities for construction, as well as the population, it is at the lowest possible level abviously, for node Fe, firstly it has a low population, and secondly, it dose not has suitable conditions in terms of the necessary facilities for building a nursing house, and thirdly, by removing it, the false membership value of the Randic index will not decrease significantly. Hence, we conclude that Fereidoonkenar is not a suitable place to build a nursing home. But Tonekabon city has the necessary tools for construction, and by removing it, the amount of true membership value of Randic index is significantly reduced. Likewise, most cities are connected to it by a short distance, which can make it easier to transport the elderly to this city. Therefore, it can be the best place to build a nursing home.

5 Conclusion

Vague graphs have more precision, flexibility and compatibility, as compared to the fuzzy graphs. Today, vague graphs play an important role in social networks and allow users to find the most effective person in a group or organization. Connectivity index has many applications in psycholigy, medical science, social groups, and computer networks. Therefore, in this paper, Randic index of vague graph and vague subgraph are presented with their properties. The upper and lower boundaries of Randic index of vague graphs are studied with some isomorphic properties. Also, Randic index of directed vague graphs are introduced. In our future work, we will introduce vague incidence graphs and study some connectivity indices on it.

References

[1] M. Akram, A. Farooq, A.B. Saeid, and K.P. Shum, *Certain types of vague cycles and vague trees*, J. Intell. Fuzzy Syst. **28** (2015), no. 2, 621–631

[2] M. Akram, N. Gani, and A.B. Saeid, *Vague hypergraphs*, J. Intell. Fuzzy Syst. **26** (2014), 647–653.

[3] R.A. Borzooei and H. Rashmanlou, *Ring sum in product intuitionistic fuzzy graphs*, J. Adv. Res. Pure Math. **7** (2015), no. 1, 16–31.

[4] R.A. Borzooei and H. Rashmanlou, *Domination in vague graphs and its applications*, J. Intell. Fuzzy Syst. **29** (2015), 1933–1940.

[5] R.A. Borzooei and H. Rashmanlou, *Degree of vertices in vague graphs*, J. Appl. Math. Inf. **33** (2015), no. 5, 545–557.

[6] R.A. Borzooei, H. Rashmanlou, S. Samanta, and M. Pal, *Regularity of vague graphs*, J. Intell. Fuzzy Syst. 30 (2016), 3681–3689.

[7] M. Binu, S. Mathew, and J.N. Mordeson, *Connectivity index of a fuzzy graph and its application to human trafficking*, Fuzzy Sets Syst. **360** (2019), 117–136.

[8] W.L. Gau and D.J. Buehrer, *Vague sets*, IEEE Trans. Syst. Man Cybernet. **23** (1993), no. 2, 610–614.

[9] A. Kauffman, *Introduction à la Théorie des Sous-Ensembles Flous à l'usage des ingénieurs*, Vol. 1, Eléments Théoriques de Base, Masson, Paris, 1973.

[10] J.N. Mordeson and P. Chang-Shyh, *Operations on fuzzy graphs*, Inf. Sci. **79** (1994), no. 3-4, 159–170.

[11] S. Poulik and G. Ghorai, *Certain indices of graphs under bipolar fuzzy environment with applications*, Soft Comput. **24** (2020), 5119–5131.

[12] S. Poulik and G. Ghorai, *Determination of journeys order based on graph's Wiener absolute index with bipolar fuzzy information*, Inf. Sci. **545** (2021), 608–619.

[13] S. Poulik and G. Ghorai, *Detour g-interior nodes and detour g-boundary nodes in bipolar fuzzy graph with applications*, Hacettepe J. Math. Stat. **49** (2020), no. 1, 106–119.

[14] S. Poulik and G. Ghorai, *Applications of graphs complete degree with bipolar fuzzy information*, Complex Intell. Syst. **8** (2022) 1115–1127.

[15] N. Ramakrishna, *Vague graphs*, Int. J. Cogn. Comput. **7** (2009), 51–58.

[16] H. Rashmanlou and R.A. Borzooei, *Vague graphs with application*, J. Intell. Fuzzy Syst. **30** (2016), 3291–3299.

[17] H. Rashmanlou, S. Samanta, M. Pal, and R.A. Borzooei, *A study on bipolar fuzzy graphs*, J. Intell. Fuzzy Syst. **28** (2015), 571–580.

[18] H. Rashmanlou and R.A. Borzooei, *Product vague graphs and its applications*, J. Intell. Fuzzy Syst. **30** (2016), 371–382.

[19] H. Rashmanlou, S. Samanta, M. Pal, and R.A. Borzooei, *Intuitionistic fuzzy graphs with categorical properties*, Fuzzy Inf. Engin. **7** (2015), no. 3, 317–334.

[20] H. Rashmanlou, S. Samanta, M. Pal, and R.A Borzooei, *Bipolar fuzzy graphs with Categorical properties*, Int. J. Comput. Intell. Syst. **8** (2015), no. 5, 808–818.

[21] Y. Rao, S. Kosari, Z. Shao, X. Qiang, M. Akhoundi, and X. Zhang, *Equitable domination in vague graphs with application in medical sciences*, Front. Phys. **9** (2021), 635642.

[22] Y. Rao, S. Kosari, and Z. Shao, *Certain properties of vague graphs with a novel application*, Mathematics **8** (2020), no. 10, 1647.

[23] Y. Rao, S. Kosari, Z. Shao, R. Cai, and L. Xinyue, *A Study on Domination in vague incidence graph and its application in medical sciences*, Symmetry **12** (2020), no. 11, 1885.

[24] A. Rosenfeld, *Fuzzy graphs, fuzzy sets and their applications*, Zadeh, L.A., Fu, K.S., Shimura, M., Eds.; Academic Press: New York, NY, USA, 1975, pp. 77–95.

[25] S. Samanta and M. Pal, *Irregular bipolar fuzzy graphs*, Int. J. Appl. Fuzzy Sets **2** (2012), 91–102.

[26] A. Sebastian, J.N. Mordeson and S. Mathew, *Generalized fuzzy graph connectivity parameters with application to human trafficking*, Mathematics **8** (2020), no. 3, 424.

[27] Z. Shao, S. Kosari, H. Rashmanlou, and M. Shoaib, *New concepts in intuitionistic fuzzy graph with application in water supplier systems*. Mathematics **8** (2020), no. 8, 1241.

[28] A.A. Talebi, H. Rashmanlou, and N. Mehdipoor, *Isomorphism on vague graphs*, Ann. Fuzzy Math. Inf. **6** (2013), no. 3, 575–588.

[29] Y. Talebi and H. Rashmanlou, *Application of dominating sets in vague graphs*, Appl. Math. E-Notes **17** (2017), 251–267.

[30] L.A. Zadeh, *Fuzzy sets*, Inf. Control **8** (1965), 338–353.

[31] L.A. Zadeh, *Similarity relations and fuzzy ordering*, Inf. Sci. **3** (1971), 177–200.

[32] L.A. Zadeh, *Is there a need for fuzzy logic?*, Inf. Sci. **178** (2008), 2751–2779.

[33] S. Zeng, M. Shoaib, H. Ali, F. Smarandache, H. Rashmanlou, and F. Mofidnakhaei, *Certain properties of single-valued neutrosophic graph with application in food and agriculture organization*, Int. J. Comput. Intell. Syst. **14** (2021), no. 1, 1516–1540.