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Optimizing redundancy allocation problem with repairable components based on the Monte Carlo simulation

Mohammadreza Shahriari[a,*], Peyman Hajizadeh[a], Arash Zaretalab[b]

[a] *Faculty of Industrial Management, South Tehran Branch, Islamic Azad University, Tehran, Iran*

[b] *Department of Business Management, Shahr-e-Qods Branch, Islamic Azad University, Tehran, Iran*

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Abstract

The optimization of reliability is crucial across various engineering domains. The redundancy allocation problem (RAP) is among the key challenges within reliability. This study introduces an RAP incorporating repairable components and a k-out-of-n sub-systems structure. The objective function aims to maximize system reliability while adhering to cost and weight constraints. The goal is to determine the optimal number of components for each subsystem, including the appropriate allocation of repairmen to each subsystem. Given that this model is classified as an Np-Hard problem, we employed a genetic algorithm (GA) to solve the proposed model. Additionally, response surface methodology (RSM) was utilized to fine-tune the algorithm parameters. To calculate the reliability of each subsystem, as well as the overall system reliability, a Monte Carlo simulation was employed. Lastly, a numerical example was solved to assess the algorithm's performance.

Keywords: reliability, redundancy allocation problem, k-out-of-n sub-systems, common cause failures, genetic algorithm
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1 Introduction

The redundancy allocation problem (RAP) has emerged as a promising framework for enhancing the reliability of practical production and operational systems in various domains, including aircraft engines, water pumping stations, and more. Researchers have recognized the need to make RAP more realistic by incorporating specific assumptions and constraints that align with the characteristics of these systems. These modifications have resulted in the development of different problem categories within RAP, considering factors such as the operational state of components (binary or multi-state), the failure rate pattern of components (constant or time-dependent), and the arrangement of components within the system (active or standby) [8].

To address the inherent complexities of RAP and account for practical considerations, numerous studies have approached the problem by treating system reliability and cost as objective functions. This paradigm shift has transformed RAP into a bi-objective or even multi-objective problem, enabling a more comprehensive exploration of

*Corresponding author
Email addresses: shahriari.mr@gmail.com (Mohammadreza Shahriari), arash_zaretalab@yahoo.com (Arash Zaretalab)

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the trade-offs between system reliability and cost. This approach allows decision-makers to make informed choices that strike an optimal balance between reliability and cost in real-world applications [19, 31].

The primary objective of reliability systems is to identify strategies for increasing the lifespan of components and systems. The redundancy allocation problem (RAP) is a widely recognized model in the field of reliability that aims to enhance system reliability by introducing parallel components within subsystems [25].

Fyffe et al. [9] were among the pioneering researchers to introduce RAP. Their model focused on maximizing system reliability while considering cost and weight constraints, and they employed dynamic programming to solve the problem. Nakagawa and Miyazaki [16] presented a nonlinear reliability optimization model, while Ha. C and Kuo [11] formulated RAP as a non-convex integer programming problem, specifically addressing similar subsystems. Tillman et al. [30] conducted a comprehensive review of 144 studies on reliability problems, examining various redundancy models and proposing a common reliability-redundancy method. In 1992, Chern [3] demonstrated that RAP belongs to the class of Np-Hard problems due to its computational complexity. Consequently, many heuristic and meta-heuristic methods were employed to tackle this problem at that time. Notably, Sharma and Venkateswarn [28], Aggarwal [1], Aggarwal et al. [2], Gopal et al. [10], and Nakagawa and Nakashima [17] proposed heuristic methods that exhibited similarities. Nakagawa and Nakashima [17] further compared the performance of NN, GAGI, and MSV heuristic algorithms for solving the general RAP.

In terms of solution approaches, Ida et al. [13] and Yokota et al. [32] were the first to utilize genetic algorithms (GA) for solving RAP with series-parallel configuration and different failure modes. Coit and Smith [4, 5] presented a mathematical model for RAP that maximized system reliability while considering uncertainty in component reliability, and they utilized GA to solve the problem. Coit, D.W., and Smith [6, 7] made modifications to the objective function of the model and employed GA to solve the revised version.

This study introduces a RAP with a series-parallel configuration and k-out-of-n sub-systems. The components are repairable, and each repairman is responsible for repairing components within a specific subsystem. The objective function aims to maximize system reliability, and the system variables include the number and type of components within each subsystem, as well as the allocation of repairmen. The paper is divided into five sections, with the second part focusing on problem definition, the third part addressing the methodologies employed for solving the problem, a numerical example presented in the fourth part, and the final section covering the conclusion and potential avenues for further research [20, 22].

2 Problem definition

In this paper, we present a novel and comprehensive approach to address the single-objective redundancy allocation problem (RAP) with a focus on maximizing system reliability while considering cost and weight constraints. The RAP model is particularly relevant in the context of complex systems with serially connected subsystems, where each subsystem comprises parallel components operating under a k-out-of-n structure.

The k-out-of-n system is widely acknowledged and frequently employed to evaluate the reliability of systems equipped with multiple components. Essentially, a k-out-of-n system implies that the entire system functions successfully as long as a minimum of k out of the n components are operational. This framework not only encompasses both parallel and series systems but also allows for the accommodation of diverse component configurations and assumptions regarding component lifetimes within its structure.

In our proposed model, we make the assumption that the components are repairable, and for efficient maintenance, each repairman is exclusively assigned to work on the components within an allocated subsystem. By doing so, we can determine the optimal number and type of components for each subsystem and also identify the most appropriate allocation of repairmen to achieve an optimal balance between reliability and maintenance efficiency.

However, the complexity involved in calculating the total repair time of components in each subsystem and the overall system reliability makes analytical approaches impractical. To overcome this challenge, we adopt a Monte Carlo simulation method for their estimation, which allows us to generate numerous random samples and approximate the system's reliability and maintenance performance effectively. This approach has been successfully applied in various engineering and reliability studies [27, 29, 31].

To provide a sound foundation for our model, we outline several key assumptions that underpin its design and implementation. Firstly, we assume that the system follows a series-parallel configuration, which is a widely adopted arrangement in real-world engineering systems due to its balance between reliability and cost. Secondly, components are assumed to exhibit a constant failure rate (CFR) to simplify the reliability calculations and ensure consistency across the subsystems. Thirdly, the number of subsystems is considered deterministic, offering a realistic representation of fixed and predefined system structures.

Furthermore, our model accommodates the presence of different types of components, each having unique characteristics in terms of cost and operation rate. This flexibility allows us to incorporate a wide range of component options, enhancing the applicability of our approach to diverse engineering scenarios.

The consideration of repairable components is crucial as it reflects real-world situations, where malfunctioning components can be repaired and brought back into operation. This aspect adds an extra layer of complexity to the optimization process, making it essential to find the right balance between component redundancy and repair resources allocation.

Lastly, our model allows each subsystem to be allocated different types of components, providing an opportunity to tailor the system design to meet specific reliability and cost requirements for individual subsystems.

By integrating these assumptions and leveraging the Monte Carlo simulation approach, our proposed model offers valuable insights into the optimization of both reliability and cost aspects of the system. The results obtained from this study will aid decision-makers and engineers in making informed choices for system design, contributing to enhanced reliability and cost-effectiveness in a wide range of engineering applications. Moreover, our approach is not limited to a specific industry but can be adapted and applied to various sectors, including aerospace, manufacturing, transportation, and energy systems.

Several key assumptions underpinning our model include:

- The system follows a series-parallel configuration.
- Components exhibit a constant failure rate (CFR).
- The number of subsystems is deterministic.
- Different types of components are available for each subsystem.
- Components vary in terms of cost and operation rate.
- The components are repairable.
- Each subsystem can be allocated different types of components.

  By incorporating these assumptions and leveraging the Monte Carlo simulation approach, our model offers insights into optimizing the reliability and cost aspects of the system.

In this section, the notations used in this paper are presented as follows. Then the presented mathematical model is analyzed. The notations used in this paper are as follows:

$R(t)$: System reliability at time t,

s : Number of subsystems,

i : Subsystems index, $i = 1, 2, ..., s$,

k_i : Minimum number of components in subsystem i,

v_i : Maximum number of components in subsystem i,

j : Component type index, $j = 1, 2, 3, 4$,

n_{ij} : Number of component type j that allocated to subsystem i,

w_{ij} : Weight of component type j that allocated to subsystem i,

W : Upper bound of system weight,

c_{ij1} : Price of component type j that allocated to subsystem i,

c_{i2} : Price of recruitment each repairman for subsystem i,

c_{i3} : The time dependent repair cost of components that allocated to subsystem i,

C : Upper bound of system cost,

m_i : Number of repairman recruitment for repairing components of subsystem i,

t_i : Total repairing time of component in subsystem i,

t : Mission time of system.

The mathematical model under the above assumptions is as follows:

$$\max R(t) = \prod_{i=1}^{s} R_i(t) \tag{2.1}$$

S.t:

$$k_i \leq \sum_{i=1}^{k} n_{ij} \leq v_i \tag{2.2}$$

$$\sum_{i=1}^{s} \sum_{j=1}^{4} w_{ij}.n_{ij} \leq W \tag{2.3}$$

$$\sum_{i=1}^{s} \sum_{j=1}^{4} c_{ij1}.n_{ij} + \sum_{i=1}^{s} (c_{i2}.m_i + c_{i3}.t_i) \leq C \tag{2.4}$$

3 Solution methodology

The genetic algorithm (GA), originally introduced by John Holland [12], is a population-based evolutionary algorithm. It begins with an initial population and iteratively applies various operators to improve the fitness function of individuals within the population. GA has found wide-ranging applications in discrete optimization problems, including production scheduling, reliability, and maintenance. Several studies have highlighted the superiority of GA in solving complex redundancy allocation problems when compared to other meta-heuristic algorithms [15, 18, 21, 23, 26]. Thus, in this paper, we have adapted a GA to address the proposed mathematical model.

For determining the total repair time of components in each subsystem and calculating the reliability of both individual subsystems and the overall system, we employed a simulation method. Additionally, we utilized a general GA to identify the optimal system parameters.

Genetic algorithm (GA) is considered one of the most effective algorithms for solving the redundancy allocation problem (RAP). It was originally introduced by Holland in 1992 [12]. Figure 1 presents the pseudocode of the GA algorithm, showcasing its step-by-step execution.

```
Parameter setting (Population size, number of iterations, Crossover Rate, Mutation Rate)
For i = 1 to number of Population size do
        ith Population generated Randomly
        ith Population evaluated by the fitness function
End For
For it = 1 to number of iterations do
        Number of Childs = Population size * Crossover Rate
        For i = 1 to number of Childs do
                Two parents are selected by the roulette wheel selection
                ith Child is generated by the performing uniform crossover operator on the selected parents
                ith Child evaluated by the fitness function
        End For
        number of mutated solutions = Pop Size * Mutation Rate
        For i = 1 to number of mutated solutions do
                A parent is selected by the roulette wheel selection
                ith mutated solution is generated by performing uniform Mutation operator on the selected
                parent
                ith mutated solution evaluated by the fitness function
        End For
        Merging Population
        Updating Best solution
End For
```

Figure 1: Pseudocode of GA

The chromosome used for illustrating the solution of the presented model is presented in Figure 2.

	Sub-system 1	2	$\cdots$	s
Component type 1	n_{11}	n_{21}	$\cdots$	n_{s1}
Component type 2	n_{12}	n_{22}	$\cdots$	n_{s2}
Component type 3	n_{13}	n_{23}	$\cdots$	n_{s3}
Component type 4	n_{14}	n_{24}	$\cdots$	n_{s4}
Repairman	m_1	m_2	$\cdots$	m_s

Figure 2: Chromosome of the problem

In our implementation, we incorporated crossover and mutation operators to generate new generations, while also applying elitism to preserve good solutions in each generation.

The crossover operator involves selecting two parents from the current population. We then generate a random integer number between 1 and s, where s represents the total number of components. Using this crossover point, we swap the first part of the first parent with the second part of the second parent, and vice versa. This process is illustrated in Figure 3.

Please note that Figure 3 is not visible in the provided text, but it likely provides a visual representation of the crossover operation described above.

Integer Random Variable Between 1 and 14=8

Parent 1 (Sub-sysytem)

Component type	1	2	3	4	5	6	7	8	9	10	11	12	13	14
1	2	2	1	0	0	0	2	1	2	0	0	1	1	0
2	0	2	1	2	2	0	2	1	2	2	0	0	0	4
3	1	0	1	1	0	1	1	2	0	2	2	0	0	0
4	0	0	0	1	2	1	0	0	0	2	2	3	1	0
m	1	1	0	1	1	0	2	1	1	2	1	1	0	1

Parent 2 (Sub-sysytem)

Component type	1	2	3	4	5	6	7	8	9	10	11	12	13	14
1	1	2	0	2	1	0	2	4	0	1	2	2	0	2
2	1	1	2	2	1	2	0	2	0	1	4	0	1	2
3	0	1	2	0	0	2	0	0	0	2	0	2	2	1
4	0	2	0	2	2	0	2	0	4	0	0	0	1	0
m	0	1	1	2	1	1	1	2	1	1	2	1	1	1

Offspring 1 (Sub-sysytem)

Component type	1	2	3	4	5	6	7	8	9	10	11	12	13	14
1	2	2	1	0	0	0	2	1	0	1	2	2	0	2
2	0	2	1	2	2	0	2	1	0	1	4	0	1	2
3	1	0	1	1	0	1	1	2	0	2	0	2	2	1
4	0	0	0	1	2	1	0	0	4	0	0	0	1	0
m	1	1	0	1	1	0	2	1	1	1	2	1	1	1

Offspring 2 (Sub-sysytem)

Component type	1	2	3	4	5	6	7	8	9	10	11	12	13	14
1	1	2	0	2	1	0	2	4	2	0	0	1	1	0
2	1	1	2	2	1	2	0	2	2	2	0	0	0	4
3	0	1	2	0	0	2	0	0	0	2	2	0	0	0
4	0	2	0	2	2	0	2	0	0	2	2	3	1	0
m	0	1	1	2	1	1	1	2	1	2	1	1	0	1

Figure 3: Crossover operator

For mutation operator, we select one parent and then generate an integer number between 1 and s. Then, we change the first part of the parent with the second part as shown in Figure 4.

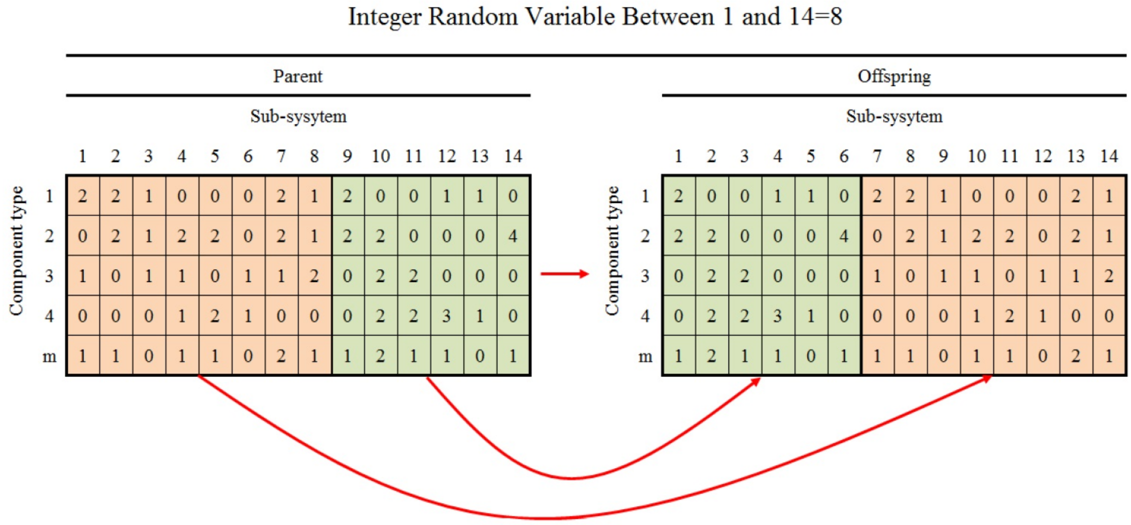

Figure 4: Mutation operator

For evaluation of each chromosome, we consider a fitness function, which considers the reliability of each chromosome. After generating initial chromosomes, crossover, and mutation operator, the new chromosomes may be infeasible. For avoiding infeasible selection of chromosomes, we consider a penalty function for fitness function. The penalty function presented in Equations (3.1) to (3.3) are as follows:

$$\text{Fitness Function} = \frac{R(t)}{PF_1 \times PF_2} \tag{3.1}$$

$$PF_1 = \max\left\{\frac{\sum_{i=1}^{s}\sum_{j=1}^{4} w_{ij} n_{ij}}{W}, 1\right\} \tag{3.2}$$

$$PF_1 = \max\left\{\frac{\sum_{i=1}^{s}\sum_{j=1}^{4} c_{ij1}.n_{ij} + \sum_{i=1}^{s}(c_{i2}.m_i + c_{i3}.t_i)}{C}, 1\right\} \tag{3.3}$$

4 Monte Carlo simulation

Monte Carlo simulation is a powerful mathematical technique employed to assess the likelihood of various outcomes for uncertain events through repeated random sampling. As explained in the book "The Monte Carlo Simulation Method for System Reliability and Risk Analysis" [2], this method enables realistic analysis of complex systems by relaxing many of the restrictive assumptions about system behavior.

In the context of reliability analysis, Monte Carlo simulation proves valuable by constructing a model that captures a range of potential results based on probability distributions assigned to variables with inherent uncertainty [24]. Through the simulation, the analysis recalculates results multiple times, utilizing different sets of random numbers within the defined minimum and maximum value ranges. This iterative process generates a substantial number of probable outcomes, each associated with a corresponding probability of occurrence [14].

Monte Carlo simulation serves as a versatile tool for system reliability analysis, offering a comprehensive understanding of the system's performance and allowing for informed decision-making in the presence of uncertainty. By incorporating the effects of randomness and variability, Monte Carlo simulation contributes to a more robust evaluation of complex systems.

For calculating fitness function of each created chromosomes, we used a simulation method. The flowchart of the event is presented in Figures 5 and 6.

5 Parameter tuning

In this research the parameters of the GA are tuned using the response surface methodology (RSM) technique. the parameters of GA are the population size ($nPop$), crossover operator probability (P_c), mutation operator probability

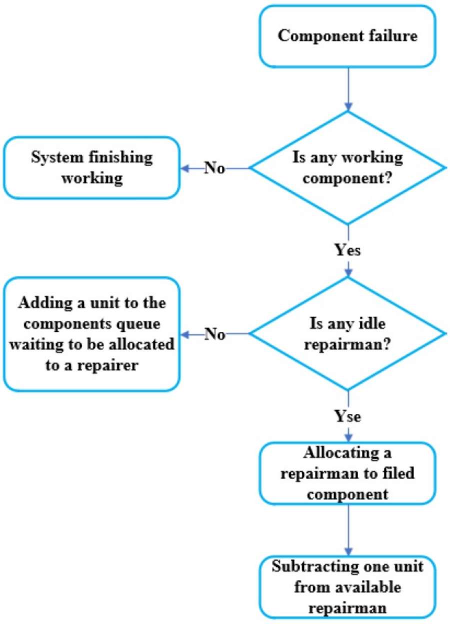

Figure 5: Monte Carlo simulation flowchart of Component failure

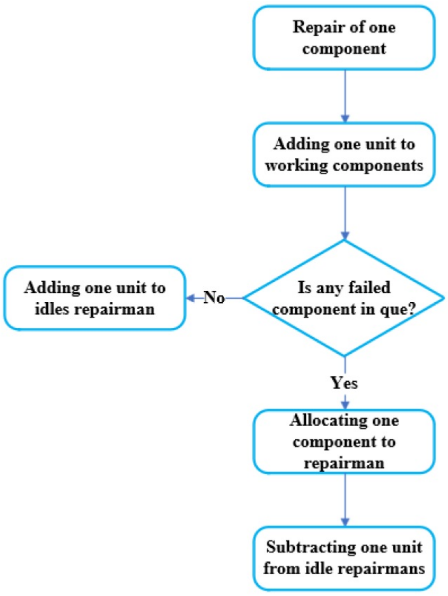

Figure 6: Monte Carlo simulation flowchart of Component repair

(P_m), and algorithm iteration ($MaxIt$) We used Box-Benking's method to tune the algorithm's parameters. The boundaries and optimum values of the algorithm parameters are presented in Table 1.

Table 1: The boundaries and optimum values of algorithm parameters

Parameters	**Optimal value**	**Upper value**	**Lower value**
$nPop$	50	100	100
P_c	0.4	0.7	0.63
P_m	0.1	0.3	0.1
$MaxIt$	100	200	200

6 Numerical example

For illustrating the performance of the presented algorithm, we solve a numerical example. This example has been already solved by Coit [4]. The parameters of examples are presented in Table 2. The other parameters are $(n_{\max} = 6)$, $W = 220$, $C = 500$, and $t = 100$. The values of c_{i2} and c_{i3} are calculated in Equations (6.1) and (6.2).

Table 2: The parameters of examples

	Choice 1 (j=1)			Choice 2 (j=2)			Choice 3 (j=3)			Choice 4 (j=4)		
i	λ_{i1}	c_{i1}	w_{i1}	λ_{i2}	c_{i2}	w_{i2}	λ_{i3}	c_{i3}	w_{i3}	λ_{i4}	c_{i4}	w_{i4}
1	0.005320	1	3	0.000726	1	4	0.00499	2	2	0.008180	2	5
2	0.008180	2	8	0.000619	1	10	0.00431	1	9	–	–	–
3	0.013300	2	7	0.011000	3	5	0.01240	1	6	0.004660	4	4
4	0.007410	3	5	0.012400	4	6	0.00683	5	4	–	–	–
5	0.006190	2	4	0.004310	2	3	0.00818	3	5	–	–	–
6	0.004360	3	5	0.005670	3	4	0.00268	2	5	0.000408	2	4
7	0.010500	4	7	0.004660	4	8	0.00394	5	9	–	–	–
8	0.015000	3	4	0.001050	5	7	0.01050	6	6	–	–	–
9	0.002680	2	8	0.000101	3	9	0.000408	4	7	0.000943	3	8
10	0.014100	4	6	0.006830	4	5	0.001050	5	6	–	–	–
11	0.003940	3	5	0.003550	4	6	0.003140	5	6	–	–	–
12	0.002360	2	4	0.007690	3	5	0.013300	4	6	0.011000	5	7
13	0.002150	2	5	0.004360	3	5	0.006650	2	6	–	–	–
14	0.011000	4	6	0.008340	4	7	0.003550	5	6	0.004360	6	9

$$c_{i2} = 3.\sum_{j=1}^{4} c_{ij1}; \quad i = 1, 3, 6, 9, 12, 14$$

$$c_{i2} = 4.\sum_{j=1}^{3} c_{ij1}; \quad i = 2, 4, 5, 7, 8, 10, 11, 13 \tag{6.1}$$

$$c_{i3} = \frac{c_{i2}}{10}; \quad i = 1, 2, ..., 14 \tag{6.2}$$

We solved the presented example with GA using simulation methods and the optimal solution for this example with 20 times running the algorithm is presented in Figure 7. The total cost and weight of the problem are $C = 499.9375$ and $W = 219$, and the optimal system reliability is $R(100) = 0.9963$.

Sub-sysytem

Component type	1	2	3	4	5	6	7	8	9	10	11	12	13	14
1	0	2	1	1	0	0	3	1	2	0	0	1	0	2
2	0	2	1	2	2	0	0	0	1	0	2	1	1	0
3	1	0	0	0	0	0	0	2	0	2	0	0	2	0
4	2	0	0	0	0	2	0	0	0	0	0	1	0	0
m	1	1	0	1	1	1	1	1	1	2	1	1	2	1

Figure 7: The optimal chromosome of problem

7 Conclusion and further studies

In conclusion, reliability optimization is a crucial concern in various industries, motivating the exploration of more realistic conditions for problem formulation. This paper presented a redundancy allocation problem (RAP) with a series-parallel configuration and k-out-of-n subsystems. The model incorporated repairable components, where each repairman was assigned to specific subsystem components. The objective function aimed to maximize system reliability, while the system variables included the number and type of components in each subsystem and the allocation of repairmen. A simulation method was employed to calculate system reliability, and a genetic algorithm (GA) was utilized to solve the numerical example.

The results demonstrated that the reliability of the system with repairable components surpassed that of the system with non-repairable components, confirming our expectations.

For future studies, it would be valuable to explore different assumptions and scenarios. For instance, incorporating more complex configurations for subsystems could offer additional insights. Moreover, comparing the results obtained from different meta-heuristic algorithms with the outcomes of the presented GA algorithm would contribute to a comprehensive evaluation of their performance.

Overall, by further investigating and refining the RAP model under different conditions, researchers can continue to advance reliability optimization and its practical applications in various industries.

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