Int. J. Nonlinear Anal. Appl. 16 (2025) 2, 365–371
ISSN: 2008-6822 (electronic)
http://dx.doi.org/10.22075/ijnaa.2023.31435.4629

Effective implementation of sine-cosine wavelet in pricing discrete double barrier option

Amirhossein Sobhani[a,*], Mohammad Hossein Beheshti[b]

[a] *Department of Mathematics, Statistics and Computer Science, Semnan University, Semnan, Iran*

[b] *Department of Biostatistics and Epidemiology, Faculty of Medicine, Tehran Medical Sciences, Islamic Azad University, Tehran, Iran*

(Communicated by Zakieh Avazzadeh)

Abstract

In this article, the problem of pricing discrete double barrier options which only monitored at specific times is investigated. According to the Black-Scholes framework, the option price would be obtained from recursively solving the Black-Sholes partial differential equations on the monitoring intervals. In this way, the sine-cosine wavelet approach is applied in approximating the yielded analytical expression. Finally, an operational matrix form is derived which is highly comparable with other methods. According to the method of the present paper, the computational time is nearly fixed against increases in the number of monitoring dates.

Keywords: Barrier Options, Black-Scholes Framework, Sine-Cosine Wavelet
2020 MSC: Primary 91G60; Secondary 65Q20

1 Introduction

Financial derivatives have been developed in the last decades. Options are derivatives which extensively considered by investors in the markets. They are commonly traded in various types, *namely call (put) options and European (American) options*. For example, a European call (put) option is a contract that gives its owner the right to buy (sell) the underlying asset for a predetermined price, namely exercise price, at a specific time in the future, namely expiry time. Path-dependent options, *especially barrier options*, are advanced options that have recently emerged in the financial markets. Barrier options which are usually traded in two general types, *namely single and double*, play an important role in managing risk in the financial markets. In this paper, the problem of pricing a knock-out discrete double barrier option is investigated which would be useless if the price of the underlying asset touches one of the two barriers before the expiry time T. Here, the motion of underlying stock is checked only at specific times, namely *monitoring dates* $0 = t_0 < t_1 < \cdots < t_N = T$. Different approaches have been provided in recent decades for pricing barrier options. Numerical methods based on adaptive mesh models, trinomial trees, and quadrature method are applied in pricing path-dependent options in [1], [8] and [10] receptively. Fusai et. al. obtained an analytical solution for a single barrier option with the aid of z-transform [7]. In [4, 5], the Fourier-cosine expansion method is used for pricing barrier options. Milev and Tagliani presented a numerical method based on the quadrature method for pricing the double barrier option in [9]. A numerical method for pricing barrier options based on projection methods has been

*Corresponding author
Email addresses: a_sobhani@semnan.ac.ir (Amirhossein Sobhani), mh.beheshti@iautmu.ac.ir (Mohammad Hossein Beheshti)

Received: August 2023 *Accepted:* November 2023

presented in [6]. In [11], a numerical method with the aid of Legendre multiwavelet has been proposed. Also, the method of Lagrange interpolation on Jacobi nodes is implemented in [12]. These approaches are working efficiently for pricing discrete barrier options and the method of the present paper performs in a high agreement with them. The main goal of this paper is to present a new effective numerical approach for pricing discrete double barrier options based on the sine-cosine wavelet method that simultaneously provides easy computer implementation with minimum memory requirements and very short computational times. This paper is organized as follows. In Section 2 the mathematical model based on the Black-Scholes framework is presented for pricing discrete double barrier options. An introduction to the sine-cosine wavelet is provided in section 3. Section 4 is devoted to the utilization of the sine-cosine wavelet method in approximating the analytical approach expressed in Section 2. In this section, a convenient operational matrix form has been yielded that significantly controls the computational time even for a large number of monitoring dates. The accuracy and effectiveness of the presented method are investigated in section 5. In this section, it is shown that the obtained results are in good agreement with the quadrature method in [10] as a benchmark. Furthermore, the CPU time of the present method is nearly fixed against increases in the number of monitoring dates.

2 The Pricing Model

In this section, the mathematical model based on the Black-Scholes framework is presented for pricing knock-out discrete double barrier options, i.e. a call option that becomes useless if the stock price hits lower barrier L or upper barrier U at the specific monitoring dates $0 = t_0 < t_1 < \cdots < t_N = T$. If the barriers are not touched by the underlying asset price in the monitoring dates, the pay-off at the maturity time T is $max(S_T - E, 0)$, where E is the exercise price.

In this manner, assume that the price of the underlying asset, especially a stock, follows the geometric Brownian motion:

$$dS_t = rS_t + \sigma S_t dB_t.$$

where r, σ, and S_t are the risk-free rate, the volatility, and the stock price at time t respectively.

According to the well-known Black-Scholes framework, the price of knock-out discretely monitored double barrier call option as a function of stock price at time to maturity $t \in (t_d, t_{d+1})$, namely $\mathcal{P}(S, t, d)$, is obtained from forward solving the following partial differential equations [3]:

$$-\frac{\partial \mathcal{P}}{\partial t} + rS\frac{\partial \mathcal{P}}{\partial S} + \frac{1}{2}\sigma^2 S^2 \frac{\partial^2 \mathcal{P}}{\partial S^2} - r\mathcal{P} = 0, \tag{2.1}$$

with the following initial conditions:

$$\mathcal{P}(S, t_0, 0) = (S - E)\,\mathbf{1}_{(\max(E,L) \le S \le U)};\ d = 0$$
$$\mathcal{P}(S, t_d, d) = \mathcal{P}(S, t_d, d-1)\,\mathbf{1}_{(L \le S \le U)};\ d = 1, 2, ..., N-1\ .$$

where $\mathcal{P}(S, t_d, d-1) := \lim_{t \to t_d} \mathcal{P}(S, t, d-1)$.

In the following two steps, the PDEs in (2.1) would be transformed into the heat equations which have analytical solution. As a first step, by implementing the change of variable $z = \ln\left(\frac{S}{L}\right)$ and denoting $C(z, t, d) := \mathcal{P}(S, t, d)$, the PDEs in (2.1) is converted to:

$$-C_t + \mu C_z + \frac{\sigma^2}{2} C_{zz} = rC \tag{2.2}$$
$$C(z, t_0, 0) = L(e^z - e^{E^*})\mathbf{1}_{(\delta \le z \le \theta)};\ d = 0$$
$$C(z, t_d, d) = C(z, t_d, d-1)\,\mathbf{1}_{(0 \le z \le \theta)}\ ;\ d = 1, 2, ..., N-1$$

where $E^* = \ln\left(\frac{E}{L}\right)$; $\mu = r - \frac{\sigma^2}{2}$; $\theta = \ln\left(\frac{U}{L}\right)$ and $\delta = \max\{E^*, 0\}$. As a second step, the transformations $C(z, t, d) := e^{\alpha z + \beta t} h(z, t, d)$ are made where

$$\alpha = -\frac{\mu}{\sigma^2};\ c^2 = \frac{\sigma^2}{2};\ \beta = \alpha\mu + \alpha^2 \frac{\sigma^2}{2} - r$$

then the PDEs in (2.2) is reduced to the following heat equations:

$$-h_t + c^2 h_{zz} = 0$$
$$h(z, t_0, 0) = Le^{-\alpha z}\left(e^z - e^{E^*}\right)\mathbf{1}_{(\delta \le z \le \theta)};\ d = 0$$
$$h(z, t_d, d) = h(z, t_d, d-1)\,\mathbf{1}_{(0 \le \theta \le z)};\ d = 1, ..., N-1.$$

The above heat equations have analytical solution as below, see [13]:

$$h(z,t,d) = \begin{cases} L\int_{\delta}^{\theta} k\left(z-\xi,t\right)e^{-\alpha\xi}\left(e^{\xi}-e^{E^*}\right)d\xi\,;\ d=0 \\ \int_{0}^{\theta} k\left(z-\xi,t-t_d\right)h\left(\xi,t_d,d-1\right)d\xi\,;\ d=1,2,...,N-1 \end{cases}$$

where

$$k(z,t) = \frac{1}{\sqrt{4\pi c^2 t}}e^{-\frac{z^2}{4c^2 t}}. \tag{2.3}$$

In the case of discrete barriers, the movement of the stock price is solely checked at dates equally spaced namely daily, weekly, or monthly. Therefore the following expression for N monitoring dates is assumed:

$$t_d = d\tau \quad \text{where} \quad \tau = \frac{T}{N}.$$

Hence by focusing on monitoring dates t_d, the following recursive expressions for $f_d\left(z\right) := h(z,t_{d+1},d)$, will be obtained:

$$f_d(z) = \int_0^{\theta} k(z-\xi,\tau)f_{d-1}\left(\xi\right)d\xi;\ d=1,2,3,...,N-1 \tag{2.4}$$

where $f_0\left(\xi\right) = Le^{-\alpha z}\left(e^z - e^{E^*}\right)\mathbf{1}_{(\delta\leq\xi\leq\theta)}$. According to the results have been obtained above, the price of the Knock-out Discrete Double Barrier Option on a stock of price S at time to maturity T would be evaluated as:

$$\mathcal{P}\left(S,T,N-1\right) = e^{\alpha z+\beta T}f_{N-1}\left(z\right) \tag{2.5}$$

where $z = \log\left(\frac{S}{L}\right)$ and f_{N-1} is obtained from (2.4).

3 sine-cosine Wavelet

In this section, a brief introduction to sine-cosine wavelet, or CASW for short, is presented. Let $m \in \mathbb{Z}$ and $CASW_m(t) = \cos(2m\pi t) + \sin\left(2m\pi t\right)$, then CASW $\psi_{n,m}(t)$ for any non-negative integer k are defined on $[0,1)$ as follows:

$$\psi_{n,m}(t) = \begin{cases} 2^{\frac{k}{2}}CASW_m\left(2^k t - n\right) & \frac{n}{2^k} \leq t < \frac{n+1}{2^k} \\ 0 & o.w \end{cases}$$

where $n = 1,2,...,2^k-1$. Hence the functions $\{\psi_{n,m}(t); n=1,...,\infty, m\in\mathbb{Z}\}$ constitute an orthonormal basis functions for $L^2[0,1]$, see for more details [14]. Therefore any function $f(t) \in L^2[0,1]$ could be expanded as:

$$f(t) = \sum_{n=1}^{\infty}\sum_{m\in\mathbb{Z}} a_{m,n}\psi_{n,m}(t)$$

where $a_{m,n} = \langle f,\psi_{n,m}\rangle = \int\limits_0^1 f(t)\psi_{n,m}(t)dt$. Also, consider the functions $\tilde{\psi}_{n,m}(t) = \sqrt{\theta}^{-1}\psi_{n,m}(t/\theta)$. Then $\{\tilde{\psi}_{n,m}(t); n = 1,...,\infty, m\in\mathbb{Z}\}$ is an orthonormal basis for $L^2[0,\theta]$ and any function $f(t) \in L^2[0,\theta]$ could be expanded as:

$$f(t) = \sum_{n=1}^{\infty}\sum_{m\in\mathbb{Z}} a_{m,n}\tilde{\psi}_{n,m}(t)$$

where $a_{m,n} = \left\langle f,\tilde{\psi}_{n,m}\right\rangle = \int\limits_0^1 f(t)\tilde{\psi}_{n,m}(t)dt$. Now, let the operator $P_{J,M}$ is defined for tow integers J and M as follows:

$$P_{J,M}\left(f\right) = \sum_{n=1}^{2^J}\sum_{m=-M}^{M} a_{m,n}\tilde{\psi}_{n,m}(t) = \vec{a}'_{M,J}\Psi_{J,M} \qquad \forall f\in L^2[0,\theta] \tag{3.1}$$

where

$$\vec{a}_{M,J} = \left[a_{-M,1},\, a_{-M,2},...,a_{-M,2^J}, a_{-M+1,1},\, a_{-M+1,2},...,a_{-M+1,2^J},...,a_{M,1},\, a_{M,2},...,a_{M,2^J}\right] \tag{3.2}$$

$$\Psi_{J,M} = \left[\tilde{\psi}_{-M,1},\, \tilde{\psi}_{-M,2},...,\tilde{\psi}_{-M,2^J}, \tilde{\psi}_{-M+1,1},\, \tilde{\psi}_{-M+1,2},...,\tilde{\psi}_{-M+1,2^J},...,\tilde{\psi}_{M,1},\, \tilde{\psi}_{M,2},...,\tilde{\psi}_{M,2^J}\right] \tag{3.3}$$

Because the operator $P_{J,M}$ is defined as a linear combinantion of the functions $\tilde{\psi}_{n,m}(t)$, then it is clear that $P_{J,M}$ is an orthogonal projection operator from $L^2[0,\theta]$ to $X_{J,M}$, i.e:

$$P_{J,M} : L^2[0,\theta] \to X_{J,M}$$

where $X_{J,M}$ is defined as $X_{J,M} := span\left\{\tilde{\psi}_{n,m}(t); n = 1, ..., 2^J, m = -M, ..., M\right\}$. It is important to note that the $P_{J,M}(f)$ would be used as an approximation to the function $f(t) \in L^2[0,\theta]$.

4 Pricing by Sine-Cosine Wavelet

As mentioned in section 2, the pricing problem would be done through a formula in (2.5) and a recursive expression in (2.4). Here the implementation of CASW to obtain a numerical approach for pricing the option according to (2.4) and (3.1) is provided. In this manner, consider the compact operator $\mathcal{K} : L^2([0,\theta]) \to L^2([0,\theta])$ as follows:

$$\mathcal{K}(f)(z) := \int_0^\theta \kappa(z-\xi,\tau) f(\xi) d\xi. \tag{4.1}$$

where κ is defined in (2.3). With attention to the definition of operator $\mathcal{K}$, recursive expression in (2.4) can be rewritten as below:

$$f_d = \mathcal{K} f_{d-1} \quad d = 1, 2, 3, ..., N-1 \tag{4.2}$$

Now, let $\tilde{f}_{d,J} = P_{J,M}\mathcal{K}\left(\tilde{f}_{d-1,J}\right) = (P_{J,M}\mathcal{K})^d (f_0)$, $d = 1, 2, 3, ..., N-1$ where $(P_{J,M}\mathcal{K})(f) := P_{J,M}(\mathcal{K}(f))$. Since $\tilde{f}_{d,J} \in X_{J,M}$, it could be written as follows:

$$\tilde{f}_{d,J} = \sum_{i=0}^{(2M+1)2^J} a_{d,i}\overline{\psi}_i(z) = \Psi'_{J,M}(x) F_d,$$

where $F_d = [a_{d,1}, a_{d,2}, \cdots, a_{d,(2M+1)2^J}]'$ and $\overline{\psi}_i(x)$ is the ith element of $\Psi_{J,M}$. By the recursive expression in (4.2), the following expression for $\tilde{f}_{d,J}$ would be obtained:

$$\tilde{f}_{d,J} = (P_J\mathcal{K})^{d-1}\left(\tilde{f}_{1,J}\right). \tag{4.3}$$

Because X_J is a finite-dimensional linear space, so the linear operator $P_J\mathcal{K}$ on X_J is corresponded to matrix K of size $(2M+1)2^J \times (2M+1)2^J$ whose elements are as below:

$$k_{ij} = \int_0^\theta \int_0^\theta \overline{\psi}_i(\eta)\overline{\psi}_j(\xi)\kappa(\eta-\xi,\tau) d\xi d\eta \ .$$

Consequently, the following matrix operator form would be written for (4.3):

$$\tilde{f}_{d,J} = \Psi'_{J,M} K^{d-1} F_1. \tag{4.4}$$

where the elements of the vector $F_1 = [a_{1,1}, a_{1,2}, \cdots, a_{1,(2M+1)2^J}]$ are calculated as below:

$$a_{1,i} = \int_0^\theta \int_\delta^\theta \overline{\psi}_i(\eta)\kappa(\eta-\xi,\tau) f_0(\xi) d\xi d\eta \ , \qquad i = 1, 2, ..., (2M+1)2^J.$$

Therefore, the approximated price of the knock-out discrete double barrier option on a stock of price S at the time to maturity T can be evaluated as follows:

$$\mathcal{P}(S, T, N-1) \simeq e^{\alpha z + \beta T} \tilde{f}_{N-1,J}(z) \tag{4.5}$$

where $z = \log\left(\frac{S}{L}\right)$ and $\tilde{f}_{N-1,J}$ from (4.4). Notice that the matrix form of relation (4.4) implies that the computational time of the presented algorithm is almost fixed and does not depend on the number of monitoring dates. In the following, it is proven that the approximated price is convergent in $L^2[0,\theta]$:

Firstly, from the fact that the continuous projection operators $P_{J,M}$ converge pointwise to identity operator I, then the operator $(P_{J,M}\mathcal{K})$ is also a compact and converges in operator's norm to $\mathcal{K}$, i.e:

$$\lim_{J\to\infty} \|(P_{J,M}\mathcal{K}) - \mathcal{K}\| = 0. \tag{4.6}$$

see for more details [2].

Secondly, according to some properties of the *operator's norm* in $L^2[0,\theta]$, especially triangular inequality, the following expressions are obtained:

$$\begin{aligned} \|f_d - \tilde{f}_{d,J}\| &= \|\mathcal{K}f_{d-1} - (P_{J,M}\mathcal{K})\tilde{f}_{d-1,J}\| = \|\mathcal{K}f_{d-1} - \mathcal{K}\tilde{f}_{d-1,J} + \mathcal{K}\tilde{f}_{d-1,J} - (P_{J,M}\mathcal{K})\tilde{f}_{d-1,J}\| \\ &\le \|\mathcal{K}f_{d-1} - \mathcal{K}\tilde{f}_{d-1,J}\| + \|\mathcal{K}\tilde{f}_{d-1,J} - (P_{J,M}\mathcal{K})\tilde{f}_{d-1,J}\| \\ &\le \|\mathcal{K}\|\|f_{d-1} - \tilde{f}_{d-1,J}\| + \|\mathcal{K} - P_{J,M}\mathcal{K}\|\|\tilde{f}_{d-1,J}\|. \end{aligned} \tag{4.7}$$

Hence by using the expression in (4.6) and the induction on $d = 1, 2, ..., N-1$, the following relation is obtained:

$$\lim_{J\to\infty} \|f_d - \tilde{f}_{d,J}\| = 0. \tag{4.8}$$

Therefore the approximating approach in (4.5) is convergent. Furthermore, by assuming $\mathcal{N} = (2M+1)2^J$, the complexity of our algorithm becomes $\mathcal{O}(\mathcal{N}^2)$ that dose not depend on number of monitoring dates.

5 Numerical Result

In this section, the price of the knock-out discrete double barrier option is evaluated according to (4.5) with $(2M+1)2^J$ CASW basis functions for diffident values of time to maturity T, risk-free rate r, volatility σ, exercise price E, stock price S, lower and upper barriers L, U. In this manner, two examples are provided in which the accuracy and efficiency of the present paper are investigated. In examples (1 2), quadrature method $QUAD - K200$ in [10] is considered as a benchmark. In example (1), it is shown that the CPU time of the present method is nearly fixed against increases in the number of monitoring dates. In example (2), the method of CASW is also compared to the common trinomial method and adaptive mesh model(AMM-8) in [8] and [1] respectively. Therefore, tables (1) and (2) show the accuracy and effectiveness of the presented paper in comparison with the mentioned methods. The source code of this method was written in MATLAB 2015 on a 3.2 GHz Intel Core i5 PC with 8 GB RAM.

Example 1. In this example, the price of the knock-out discrete double barrier option for different values of stock price $S = 95,\ 100,\ 105\ ,107,\ 110$ and different Numbers of monitoring date $N = 5,\ 25$ is evaluated. The parameters are considered as $r = 0.05$, $\sigma = 0.25$, $E = 100$, $T = 0.5$, $L = 95$ and $U = 110$. In table (1), The numerical results of the presented paper are compared with the quadrature method $QUAD - K200$ in [10] for various numbers of monitoring dates. Table (1) shows the accuracy of the presented method. Furthermore, it is shown that the CPU time of the method is almost constant and does not depend on the number of monitoring dates.

N	S	CASW ($M=1, J=6$)	$\|Error\|$	CASW ($M=1, J=7$)	$\|Error\|$	Benchmark
	95	0.176305	1.800e-03	0.175404	9.0600e-04	0.174498
	100	0.232373	1.3500e-04	0.232526	1.8000e-05	0.232508
5	105	0.225881	1.8000e-04	0.226143	8.2000e-05	0.226061
	107	0.20748	7.5700e-04	0.206866	1.4300e-04	0.206723
	110	0.16912	1.700e-03	0.168259	8.6600e-04	0.167393
CPU time		0.21 s		0.81 s		
	95	0.020119	5.9100e-04	0.019824	2.9600e-04	0.019528
	100	0.04288	7.7000e-05	0.04296	3.0000e-06	0.042957
25	105	0.040819	9.7000e-05	0.040947	3.1000e-05	0.040916
	107	0.033305	2.9900e-04	0.033061	5.5000e-05	0.033006
	110	0.01925	5.6200e-04	0.018971	2.8300e-04	0.018688
CPU time		0.21 s		0.81 s		

Table 1: Double barrier option pricing with parameters: $T = 0.5$, $r = 0.05$, $\sigma = 0.25$, $E = 100$, $L = 95$ and $U = 110$.

Example 2. In this example, the price of the knock-out discrete double barrier option on the stock of price $S = 100$ for different values of lower barrier $L = 80,\ 90,\ 95\ ,99,\ 99.9$ is evaluated. The other parameters are considered as $r = 0.05$, $\sigma = 0.25$, $E = 100$, $T = 0.5$, and $U = 120$. In table (2), The numerical results of the presented paper are compared with the trinomial method and adaptive mesh model(AMM-8) in [8] and [1] for various numbers of monitoring date $N = 5,\ 25,\ 125$. Table (2) shows the accuracy and effectiveness of the presented method.

N	L	CASW ($M = 1, J = 10$)	Trinomial	AMM-8	Benchmark
5	80	2.4499	2.4439	2.4499	2.4499
	90	2.2028	2.2717	2.2027	2.2028
	95	1.6831	1.6926	1.6830	1.6831
	99	1.0811	0.3153	1.0811	1.0811
	99.9	0.9432	-	0.9433	-
25	80	1.9420	1.9490	1.4419	1.9420
	90	1.5354	1.5630	1.5353	1.5354
	95	0.8668	0.8823	0.8668	0.8668
	99	0.2931	0.3153	0.2932	0.2931
	99.9	0.2023	-	0.2024	0.2023
125	80	1.6808	1.7477	1.6807	1.6808
	90	1.2028	1.2370	1.2028	1.2029
	95	0.5531	0.5699	0.5531	0.5532
	99	0.1042	0.1201	0.1043	0.1042
	99.9	0.0513	-	0.0513	0.0513

Table 2: Double barrier option pricing with parameters: $T = 0.5$, $r = 0.05$, $\sigma = 0.25$, $E = 100$, $S = 100$ and $U = 120$.

6 Conclusion and remarks

In this paper, an orthogonal projection method based on sine-cosine wavelets basis functions for pricing discrete double barrier options is implemented. In this way, a matrix form (4.4) for approximating the problem is obtained. Finally, the accuracy and effectiveness of this method are investigated in two examples.

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