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Stability of (1,2)-total pitchfork domination

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Abstract

Let $G = (V, E)$ be a finite, simple, and undirected graph without an isolated vertex. We define a dominating D of $V(G)$ as a total pitchfork dominating set if $1 \leq |N(t) \cap V - D| \leq 2$ for every $t \in D$ such that $G[D]$ has no isolated vertex. In this paper, the effects of adding or removing an edge and removing a vertex from a graph are studied on the order of minimum total pitchfork dominating set $\gamma^t_{pf}(G)$ and the order of minimum inverse total pitchfork dominating set $\gamma^{-t}_{pf}(G)$. Where $\gamma^t_{pf}(G)$ is proved here to be increasing by adding an edge and decreasing by removing an edge, which are impossible cases in the ordinary total domination number.

Keywords: total domination, stability of domination, pitchfork domination
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1 Introduction

Let $G = (V, E)$ be a graph without isolated vertices has vertex set V of order n and edge set E of size m. The degree of a vertex v of a graph G is denoted by $deg(v)$ and defined as the number of edges incident with v where v is pendant vertex when $deg(v) = 1$ and isolated when $deg(v) = 0$. For graph basic concepts and theoretic terminology one can see [21]. The study of domination and its applications has a large area in graph theory. For a detailed survey of domination, we refer to [22, 23, 24]. Ore [27] introduced the expression dominating sets in graphs. Several sorts of dominations are given in [1, 11, 13, 18, 26, 28]. The effects of removing an edge or vertex or adding an edge are studied on more types of graph domination as in [12, 14, 15, 16, 17, 25, 29]. Al-Harere and Abdlhusein introduced pitchfork domination in 2020, where they proved several bounds and properties for this parameter see [1]-[10]. The effects of adding or removing an edge and removing a vertex from the graph on the order of minimum total pitchfork dominating set are studied here. The order of the total pitchfork dominating set is proved here to be increasing by adding an edge and decreasing by removing an edge, which are impossible cases in the ordinary total dominating set. The study of these effects has an important advantage to learning ways of treatments for any added or damaged nods or links of the system or networks to avoid losing some properties of the system and to give the best services with minimum costs.

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2 Changing and Un-Changing $\gamma^t_{pf}(G)$

In this section, we discuss the stability of $\gamma^t_{pf}(G)$ when we deleting a vertex or edge or adding an edge from G. If $G-v$ has a total pitchfork dominating set, then we partition the vertices of G into three sets:

$V^0 = \{v \in V : \gamma^t_{pf}(G-v) = \gamma^t_{pf}(G)\}$, $V^+ = \{v \in V : \gamma^t_{pf}(G-v) > \gamma^t_{pf}(G)\}$ and $V^- = \{v \in V : \gamma^t_{pf}(G-v) < \gamma^t_{pf}(G)\}$.

Similarly, edges set can be partitioned into:

$E^0_* = \{e \in E : \gamma^t_{pf}(G*e) = \gamma^t_{pf}(G)\}$, $E^+_* = \{e \in E : \gamma^t_{pf}(G*e) > \gamma^t_{pf}(G)\}$ and $E^-_* = \{e \in E : \gamma^t_{pf}(G*e) < \gamma^t_{pf}(G)\}$,

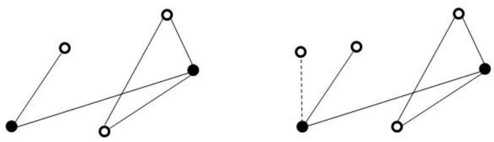
where

$$* = \begin{cases} -, & \text{if } e \in G \\ +, & \text{if } e \in \overline{G}. \end{cases}$$

Theorem 2.1. For any graph G having a $\gamma^t_{pf}-$set. If there is a vertex $v \in V$ such that $G-v$ having a total pitchfork domination, then $V^*_- \neq \phi$, where $* = 0$ or $-$ or $+$.

Proof . Assume that D is a $\gamma^t_{pf}-$set in G, then we show that V^*_- is non empty set as follows:

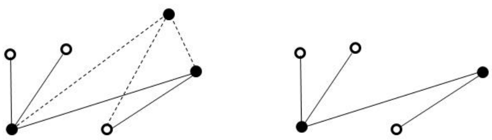
Case 1: $V^0_- \neq \phi$: If $v \in V-D$ is an end vertex dominated by u which is dominates other vertex w, then $\gamma^t_{pf}(G-v) = \gamma^t_{pf}(G)$ and $v \in V^0_-$. Hence, $V^0_- \neq \phi$, see Figure 1.



(a) G (b) $G-v$

Figure 1: $\gamma^t_{pf}(G-v) = \gamma^t_{pf}(G)$

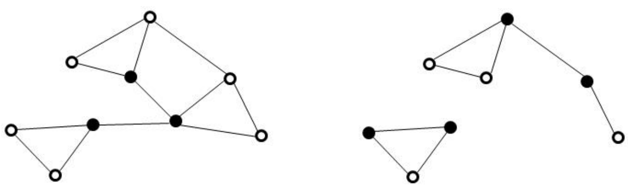
Case 2: $V^-_- \neq \phi$: If $v \in D$ is adjacent with one or more vertices of D which are dominate the same vertex from $V-D$, then $\gamma^t_{pf}(G-v) < \gamma^t_{pf}(G)$ and $v \in V^-_-$. Hence, $V^-_- \neq \phi$. For example see Figure 2.



(a) G (b) $G-v$

Figure 2: $\gamma^t_{pf}(G-v) < \gamma^t_{pf}(G)$

Case 3: $V^+_- \neq \phi$: If G has three joined cycles subgraphs C_3, when we delete the vertex of one cycle which is adjacent with other cycle, then the order of D will increase. Then, $\gamma^t_{pf}(G-v) > \gamma^t_{pf}(G)$ and $v \in V^+_-$. Hence, $V^+_- \neq \phi$. For example see Figure 3. □



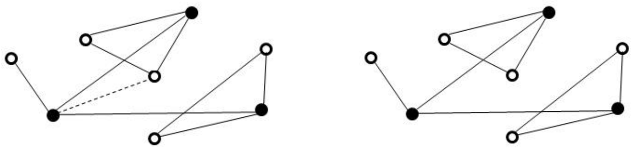
(a) G (b) $G-v$

Figure 3: $\gamma^t_{pf}(G-v) > \gamma^t_{pf}(G)$

Theorem 2.2. For any graph G having a $\gamma^t_{pf}-$set. If $e \in E$ and $G - e$ having a total pitchfork domination, then $E^*_- \neq \phi$, where $* = 0$ or $-$ or $+$.

Proof . Assume that D is a $\gamma^t_{pf}-$set in G, then we show that E^*_- is non empty set as follows:

Case 1: $E^0_- \neq \phi$: If a vertex $v \in D$ is adjacent with some vertices of D, then we can delete an edge $e = v\,u$ for any $u \in D$ to get the result. So if a vertex $w \in V - D$ is dominated by some vertices of D one of them such u dominates two vertices. Then, we can delete an edge $e = u\,w$ to get $\gamma^t_{pf}(G - e) = \gamma^t_{pf}(G)$. Thus, $e \in E^0_-$ and $E^0_- \neq \phi$, see Figure 4.



(a) G (b) $G - e$

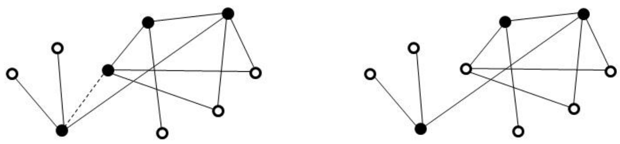
Figure 4: $\gamma^t_{pf}(G - e) = \gamma^t_{pf}(G)$

Case 2: $E^+_- \neq \phi$: If a graph G has two cycles joined by a bridge e incident on two vertices of D, then D in $G - e$ must contain two vertices of every cycle. So $\gamma^t_{pf}(G - e) > \gamma^t_{pf}(G)$. Thus, $e \in E^+_-$ and $E^+_- \neq \phi$, see Figure 5.

(a) G (b) $G - e$

Figure 5: $\gamma^t_{pf}(G - e) > \gamma^t_{pf}(G)$

Case 3: $E^-_- \neq \phi$: Suppose that there is a vertex $u \in D$ dominates exactly two end vertices and adjacent with more than one vertex of D. Let $v \in D$ is adjacent with u where every vertex which is dominated by v is also dominated by other vertex of D. So v is adjacent with a vertex of D dominates only one vertex. Let $e = u\,v$, then in $G - e$ we can put $v \in V - D$ to be $D - v$ is a $\gamma^t_{pf}-$set of $G - e$. Hence, $\gamma^t_{pf}(G - e) < \gamma^t_{pf}(G)$ and $E^-_- \neq \phi$, see Figure 6. □



(a) G (b) $G - e$

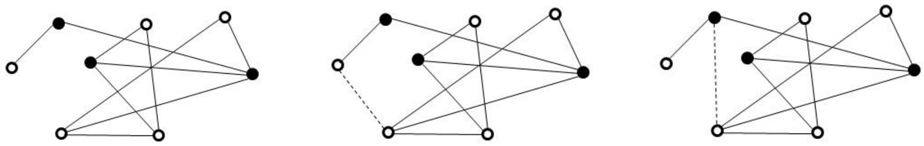
Figure 6: $\gamma^t_{pf}(G - e) < \gamma^t_{pf}(G)$

According to case 3 of the above theorem, we proved that the order of a total pitchfork dominating set can be decreasing by removing an edge. This case is impossible in the ordinary total dominating set [19].

Theorem 2.3. For any graph G having a $\gamma^t_{pf}-$set. If $e \notin E$ and $G + e$ having a total pitchfork domination, then $E^*_+ \neq \phi$, where $* = 0$ or $-$ or $+$.

Proof . Assume that D is a $\gamma^t_{pf}-$set in G, then we show that E^*_+ is non empty set as follows:

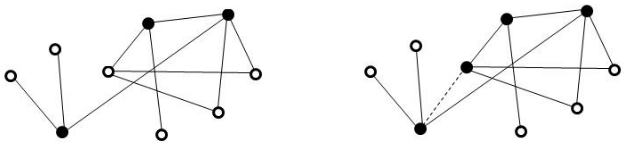
Case 1: $E^0_+ \neq \phi$: If a vertex $v \in D$ is dominating only one vertex, then adding an edge between v and any other vertex of $V - D$ will not affect on the order of $\gamma^t_{pf}-$set. Also, when we add an edge between any two vertices of $V - D$ or between any two vertices of D will not affect on the order of $\gamma^t_{pf}-$set. Hence, $\gamma^t_{pf}(G + e) = \gamma^t_{pf}(G)$ and $E^0_+ \neq \phi$. For example see Figure 7.



(a) G (b) $G+e$ (c) $G+e$

Figure 7: $\gamma^t_{pf}(G+e)=\gamma^t_{pf}(G)$

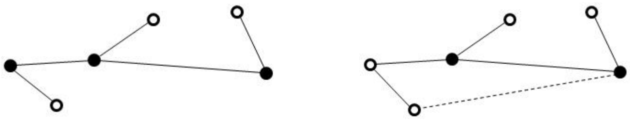
Case 2: $E^+_+\neq\phi$: If every vertex $v\in D$ dominates exactly two vertices, then adding an edge between v and any other vertex of $V-D$ which isn't dominated by it will increase the order of $\gamma^t_{pf}-$set. Hence, $\gamma^t_{pf}(G+e)>\gamma^t_{pf}(G)$ and $E^+_+\neq\phi$. For example see Figure 8.



(a) G (b) $G+e$

Figure 8: $\gamma^t_{pf}(G+e)>\gamma^t_{pf}(G)$

Case 3: $E^-_+\neq\phi$: If $v\in D$ dominates only one end vertex w and adjacent with a vertex from D which also dominates only one vertex. When we add e between w and any vertex of D dominates one vertex and non-adjacent with v, then $D-\{v\}$ is a γ^t_{pf} of $G+e$. Hence, $\gamma^t_{pf}(G+e)<\gamma^t_{pf}(G)$ and $E^-_+\neq\phi$. For example see Figure 9. □



(a) G (b) $G+e$

Figure 9: $\gamma^t_{pf}(G+e)<\gamma^t_{pf}(G)$

According to case 2 of the above theorem, we proved the order of a total pitchfork dominating set can be increasing by adding an edge. This case is impossible in the ordinary total dominating set [20].

3 Changing and Un-Changing $\gamma^{-t}_{pf}(G)$

In this section, we discuss the stability of $\gamma^{-t}_{pf}(G)$ when G is modified by removing a vertex or edge and when adding an edge. If $G-v$ has an inverse total pitchfork dominating set, then we partition the vertices of G into three sets:

$$\ddot{V}^0=\{v\in V:\gamma^{-t}_{pf}(G-v)=\gamma^{-t}_{pf}(G)\},\ \ddot{V}^+=\{v\in V:\gamma^{-t}_{pf}(G-v)>\gamma^{-t}_{pf}(G)\},\text{ and }\ddot{V}^-=\{v\in V:\gamma^{-t}_{pf}(G-v)<\gamma^{-t}_{pf}(G)\}.$$

Similarly, edges set can be partitioned into:

$$\ddot{E}^0_*=\{e\in E:\gamma^{-t}_{pf}(G*e)=\gamma^{-t}_{pf}(G)\},\ \ddot{E}^+_*=\{e\in E:\gamma^{-t}_{pf}(G*e)>\gamma^{-t}_{pf}(G)\},\text{ and }\ddot{E}^-_*=\{e\in E:\gamma^{-t}_{pf}(G*e)<\gamma^{-t}_{pf}(G)\},$$

where

$$*=\begin{cases}-, & \text{if } e\in G\\ +, & \text{if } e\in\overline{G}.\end{cases}$$

The vertices of a $\gamma^{-t}_{pf}(G)$ are referred by red color, while vertices of a $\gamma^t_{pf}(G)$ are referred by black color.

Theorem 3.1. For any graph G having a γ_{pf}^{-t}−set. If $e \in E$ and $G - e$ having an inverse total pitchfork domination, then $\ddot{E}_{-}^{*} \neq \phi$, where $* = 0$ or $+$.

Proof . Assume that D^{-1} is a γ_{pf}^{-t}−set in G, then we show that $\ddot{E}_{-}^{*}$ is non empty set as follows:

Case 1: $\ddot{E}_{-}^{0} \neq \phi$: If a vertex $v \in V - D^{-1}$ is dominated by two or more vertices of D^{-1}, then we can delete $e = v\,w$ such that $w \in D^{-1}$ and dominated exactly two vertices. Hence, $\gamma_{pf}^{-t}(G-e) = \gamma_{pf}^{-t}(G)$. Thus, $e \in \ddot{E}_{-}^{0}$ and $\ddot{E}_{-}^{0} \neq \phi$. For example see Figure 10.

(a) G (b) $G - e$

Figure 10: $\gamma_{pf}^{-t}(G-e) = \gamma_{pf}^{-t}(G)$

Case 2: $\ddot{E}_{-}^{+} \neq \phi$: If a vertices $v,\, w \in D^{-1}$ are adjacent only together in $G[D^{-1}]$, when we delete $e = v\,w$, then we must increase the order of D^{-1} to be v and w not isolated in $G[D^{-1}]$. Hence, $\gamma_{pf}^{-t}(G-e) > \gamma_{pf}^{-t}(G)$. Thus, $e \in \ddot{E}_{-}^{+}$ and $\ddot{E}_{-}^{+} \neq \phi$. For example see Figure 11. □

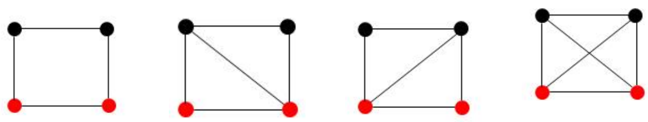
(a) G (b) $G - e$

Figure 11: $\gamma_{pf}^{-t}(G-e) > \gamma_{pf}^{-t}(G)$

Theorem 3.2. For any graph G having a γ_{pf}^{-t}−set. If $e \notin E$ and $G + e$ having an inverse total pitchfork domination, then $\ddot{E}_{-}^{*} \neq \phi$, where $* = 0$ or $+$ or $-$.

Proof . Assume that D^{-1} is a γ_{pf}^{-t}−set in G, then we show that $\ddot{E}_{-}^{*}$ is non empty set as follows:

Case 1: $\ddot{E}_{-}^{0} \neq \phi$: If a vertex $v \in V - D^{-1}$ is dominated by only one vertex of D^{-1}, then we can add $e = v\,w$ for any $w \in D^{-1}$ dominates exactly one vertex. Hence, $\gamma_{pf}^{-t}(G+e) = \gamma_{pf}^{-t}(G)$. Thus, $e \in \ddot{E}_{-}^{0}$ and $\ddot{E}_{-}^{0} \neq \phi$. For example see Figure 12.



(a) G (b) $G + e$ (c) $G + e$ (d) $G + e_1 + e_2$

Figure 12: $\gamma_{pf}^{-t}(G+e) = \gamma_{pf}^{-t}(G)$

Case 2: $\ddot{E}_{-}^{+} \neq \phi$: For any vertex $v \in D^{-1}$ dominates exactly two vertex. If we add an $e = v\,u$ for any $u \in V - D^{-1}$, then v dominates three vertices, that is contradiction. So $u \in D^{-1}$. Hence, $\gamma_{pf}^{-t}(G+e) > \gamma_{pf}^{-t}(G)$. Thus, $e \in \ddot{E}_{-}^{+}$ and $\ddot{E}_{-}^{+} \neq \phi$. For example see Figure 13.

Case 3: $\ddot{E}_{-}^{-} \neq \phi$: For any two adjacent vertices $u,\, v \in D^{-1}$ and dominate the same one vertex and $deg(u) = deg(v) = 1$ in $G[D^{-1}]$. If we add an $e = v\,w$ for any $w \in D^{-1}$, then $u \in V - D^{-1}$. Hence, $\gamma_{pf}^{-t}(G+e) < \gamma_{pf}^{-t}(G)$. Thus, $e \in \ddot{E}_{-}^{-}$ and $\ddot{E}_{-}^{-} \neq \phi$. For example see Figure 14. □

(a) G (b) $G+e$

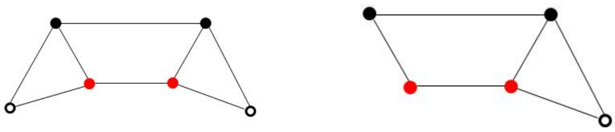
Figure 13: $\gamma_{pf}^{-t}(G+e) > \gamma_{pf}^{-t}(G)$

(a) G (b) $G+e$

Figure 14: $\gamma_{pf}^{-t}(G+e) < \gamma_{pf}^{-t}(G)$

Proposition 3.3. For any graph G having a $\gamma_{pf}^{-t}-$set. If $v \in V$ and $G-v$ having an inverse total pitchfork domination, then $\ddot{V}_{-}^{0} \neq \phi$.

Proof . Assume that D^{-1} is a $\gamma_{pf}^{-t}-$set in G. Let $v \in V$ such that $v \notin D$ and $v \notin D^{-1}$, if every vertex from D or D^{-1} which dominates v is also dominates other vertex. Then, $\gamma_{pf}^{-t}(G-v) = \gamma_{pf}^{-t}(G)$ and $v \in \ddot{V}_{-}^{0}$. Hence, $\ddot{V}_{-}^{0} \neq \phi$. For example see Figure 15. □



(a) G (b) $G-v$

Figure 15: $\gamma_{pf}^{-t}(G-v) = \gamma_{pf}^{-t}(G)$

Remark 3.4. For any graph G with a total pitchfork dominating set. If $G-v$ have an end vertex, then $G-v$ has no inverse total pitchfork dominating set.

Remark 3.5. For any graph G of order $n \leq 4$ with a total pitchfork dominating set. Then, $G-v$ has no inverse total pitchfork dominating set.

Remark 3.6. For any graph G with a total pitchfork dominating set. If v is a support vertex in G, then $G-v$ has no total pitchfork dominating set.

Proposition 3.7. For any graph G with a unique total pitchfork dominating set D. If $D^{-1} = V - D$, then $G-v$ has no inverse total pitchfork dominating set for any $v \in V$.

Proof . Let $v \in V$, then if $G-v$ has no total pitchfork domination, then it has no inverse total pitchfork domination. Suppose that $G-v$ has a total pitchfork domination and let D be a $\gamma_{pf}^{t}-$set of $G-v$. Then, $\gamma_{pf}^{t}(G-v) > \frac{n-1}{2}$, so $G-v$ has no inverse total pitchfork dominating set. □

Proposition 3.8. For any cycle graph C_n ;$(n > 3)$ having an inverse total pitchfork domination. Then, $G-v$ and $G-e$ has no inverse total pitchfork dominating set for any $v \in V$ and $e \in E$.

Proof . Since $C_n - v$ and $C_n - e$ are a path graph which have a pendent vertex and no inverse total pitchfork dominating set (see [8]). □

Theorem 3.9. For any wheel graph $G = W_n$ $(n \geq 3)$ having an inverse total pitchfork domination. Then, $G-v$ has an inverse total pitchfork domination and $\gamma_{pf}^{-t}(G-v) = \gamma_{pf}^{-t}(G)$ if and only if $v \in K_1$ and $n \equiv 0\,(mod\ 4)$.

Proof . Let $v \in K_1$ and $n \equiv 0\,(mod\ 4)$, since $W_n = C_n + K_1$, then $G - v = C_n$. Since C_n having an inverse total pitchfork domination if and only if $n \equiv 0\,(mod\ 4)$ according to [8]. Hence, $G - v$ has an inverse total pitchfork domination and $D^{-1} = V - D$. Therefore, $\gamma_{pf}^{-t}(G - v) = \gamma_{pf}^{-t}(G)$.

Suppose that $G - v$ has an inverse total pitchfork domination such that $\gamma_{pf}^{-t}(G - v) = \gamma_{pf}^{-t}(G)$ for any $v \in C_n$. Let D^{-1} be a $\gamma_{pf}^{-t}(G - v)$. Since W_n has an inverse total pitchfork domination if $n \equiv 0\,(mod\ 4)$ or $n = 3$. Then, there are two cases:

Case1: If $n = 3$, then there is a vertex in D does not dominate by D^{-1}, which is a contradiction.

Case2: If $n \equiv 0\,(mod\ 4)$, then since the vertex of K_1 does not belong to D^{-1} because it is adjacent with more than two vertices of D. Then, $\gamma_{pf}^{t}(G) = \gamma_{pf}^{t}(G - v)$. Therefore, $|D^{-1}| < |D|$, so there is a vertex in D does not dominate by D^{-1} since it was dominated by only v. Hence, we get a contradiction. Thus, $G - v$ has no inverse total pitchfork domination. □

Proposition 3.10. For any complete graph $G = K_n$;$(n \geq 3)$ having an inverse total pitchfork domination. Then, $G - v$ has no inverse total pitchfork dominating set for any $v \in V$.

Proof . Since K_n having an inverse total pitchfork domination if and only if $n = 4$ and $D^{-1} = V - D$ (see[8]). Then, $G - v$ has three vertices and has no γ_{pf}^{-t}−set. □

References

[1] M.A. Abdlhusein, *Doubly connected bi-domination in graphs*, Discrete Math. Algorithms Appl. **13** (2021), no. 2, 2150009.

[2] M.A. Abdlhusein, *Stability of inverse pitchfork domination*, Int. J. Nonlinear Anal. Appl. **12** (2021), no. 1, 1009–1016.

[3] M.A. Abdlhusein, *Applying the (1,2)-pitchfork domination and its inverse on some special graphs*, Bol. Soc. Paran. Mat. **41** (2023), 1–8.

[4] M.A. Abdlhusein and M.N. Al-Harere, *Pitchfork domination and it's inverse for corona and join operations in graphs*, Proc. Int. Math. Sci. **1** (2019), no. 2, 51–55.

[5] M.A. Abdlhusein and M.N. Al-Harere, *Pitchfork domination and its inverse for complement graphs*, Proc. IAM **9** (2020), no. 1, 13–17.

[6] M.N. Al-Harere and M.A. Abdlhusein, *Pitchfork domination in graphs*, Discrete Math. Algorithms Appl. **12** (2020), no. 2, 2050025.

[7] M.A. Abdlhusein and M.N. Al-Harere, *New parameter of inverse domination in graphs*, Indian J. Pure Appl. Math. **52** (2021), no. 1, 281–288.

[8] M.A. Abdlhusein and M.N. Al-Harere, *Total pitchfork domination and its inverse in graphs*, Discrete Math. Algorithms Appl. **13** (2021), no. 4, 2150038.

[9] M.A. Abdlhusein and M.N. Al-Harere, *Doubly connected pitchfork domination and its inverse in graphs*, TWMS J. App. Eng. Math. **12** (2022), no. 1, 82.

[10] M.A. Abdlhusein and M.N. Al-Harere, *Some modified types of pitchfork domination and its inverse*, Bol. Soc. Paran. Mat. **40** (2022), 1–9.

[11] M.N. Al-Harere and A.T. Breesam, *Variant types of domination in spinner graph*, Al-Nahrain J. **2** (2019), 127–133.

[12] M.N. Al-Harere and P.A. Khuda Bakhash, *Changes of tadpole domination number upon changing of graphs*, Sci. Int. **31** (2019), no. 2, 197–199.

[13] M.N. Al-Harere and P.A. Khuda Bakhash, *Tadpole domination in duplicated graphs*, Discrete Math. Algorithms Appl. **13** (2021), no. 2, 2150003.

[14] M. Amraee, N.J. Rad, and M. Maghasedi, *Roman domination stability in graphs*, Math. Rep. **21** (2019), no. 71, 193–204.

[15] B.A. Atakul, *Stability and domination exponentially in some graphs*, AIMS Math. **5** (2020), no. 5, 5063–5075.

[16] K. Attalah and M. Chellali, *2-Domination dot-stable and dot-critical graphs*, Asian-Eur. J. Math. **21** (2021), no. 5, 2150010.

[17] S. Balamurugan, *Changing and unchanging isolate domination: edge removal*, Discrete Math. Algorithms Appl. **9** (2017), no. 1, 1750003.

[18] A. Das, R.C. Laskar, and N.J. Rad, *On α-domination in graphs*, Graphs Combin. **34** (2018), no. 1, 193–205.

[19] W.J. Desormeaux, T.W. Haynes, and M.A. Henning, *Total domination critical and stable graphs upon edge removal*, Discrete Appl. Math. **158** (2010), 1587–1592.

[20] W.J. Desormeaux, T.W. Haynes, and M.A. Henning, *Total domination stable graphs upon edge addition*, Discrete Math. **310** (2010), 3446–3454.

[21] F. Harary, *Graph Theory*, Addison-Wesley, Reading Mass, 1969.

[22] T.W. Haynes, S.T. Hedetniemi, and P.J. Slater, *Fundamentals of Domination in Graphs*, Marcel Dekker Inc., New York, 1998.

[23] T.W. Haynes, S.T. Hedetniemi, and P.J. Slater, *Domination in Graphs: Advanced Topics*, Marcel Dekker Inc., 1998.

[24] S.T. Hedetniemi and R. Laskar, *Topics in domination in graphs*, Discrete Math. **86** (1990), no. 1-3, 3–9.

[25] M.A. Henning and M. Krzywkowski, *Total domination stability in graphs*, Discrete Appl. Math. **236** (2018), no. 19, 246–255.

[26] A.A. Omran and T.A. Ibrahim, *Fuzzy co-even domination of strong fuzzy graphs*, Int. J. Nonlinear Anal. Appl. **12** (2021), no. 1, 727–734.

[27] O. Ore, *Theory of Graphs*, American Mathematical Society, Providence, R.I., 1962.

[28] S.J. Radhi, M.A. Abdlhusein, and A.E. Hashoosh, *The arrow domination in graphs*, Int. J. Nonlinear Anal. Appl. **12** (2021), no. 1, 473–480.

[29] V. Samodivkin, *A note on Roman domination: Changing and unchanging*, Austr. J. Combin. **71** (2018), no. 2, 303–11.