Int. J. Nonlinear Anal. Appl. 16 (2025) 2, 209–217
ISSN: 2008-6822 (electronic)
http://dx.doi.org/10.22075/ijnaa.2023.31367.4621

Financial timeseries prediction by a hybrid model of chaos theory, multi-layer perceptron and metaheuristic algorithm

Mostafa Sohouli Vahed[a], Mohammad Ali Aghaei[b,*], Fariborz Avazzadeh Fath[c], Ali Pirzad[d]

[a] *Department of Accounting, Yasuj Branch, Islamic Azad University, Yasuj, Iran*

[b] *Department of Accounting, Tarbiat Modares University, Tehran, Iran*

[c] *Department of Accounting, Gachsaran Branch, Islamic Azad University, Gachsaran, Iran*

[d] *Department of Management, Yasuj Branch, Islamic Azad University, Yasuj, Iran*

(Communicated by Zakieh Avazzadeh)

Abstract

Many researchers proved that hybrid models have better results in comparison with independent models. A combination of different methods could enhance the accuracy of time series prediction. Hence, this research used the hybrid of three methods of chaos theory, multi-layer perceptron and metaheuristic algorithm to increase the power of the model forecasting. Artificial neural networks have properly considered complex nonlinear relations and are good comprehensive approximators. Multi-objective evolutionary algorithms such as multi-objective particle swarm optimization are good at solving multi-objective optimization issues. This algorithm organized the combination of parent and children populations by elitist strategy, decreased the messy comparing factors to improve the solution variety and avoided to use of niche factors. Chaos theory controls the complexities of stochastic systems. So, this research offers Tehran Stock Exchange Index (TSEI) prediction by a hybrid model of chaos theory, multi-layer perceptron and metaheuristic algorithm. The results show that in perceptron-based mode, RMSE measures are gradually increased in all intervals. The continuous decrease of RMSE shows that the perceptron-based model could show consistency with the whole data flow. This matter could offer a better learning and consistency process by perceptron-based models to predict stock prices, as this type of learning could apply more experiences for forecasting future behaviour in order to change the system content.

Keywords: financial timeseries, chaos theory, multi-layer perceptron, metaheuristic algorithm
2020 MSC: 34H10, 62M10

1 Introduction

The stock market is a vital part of the whole economy. Obviously, there is a strong relationship between the stock market and the economy. The more improper the economic situation, the weaker the companies' function and accordingly the weaker the stock market. On the contrary, the companies could show better function in case of a proper economic situation and accordingly, the stock market could expect improved economic conditions.

*Corresponding author
Email addresses: sohoulimostafa@gmail.com (Mostafa Sohouli Vahed), aghaeim@modares.ac.ir (Mohammad Ali Aghaei), fariborz-avazzadeh-fath@iaug.ac.ir, favazzadeh2010@gmail.com (Fariborz Avazzadeh Fath), Alipirzad65@yahoo.com (Ali Pirzad)

Received: May 2023 *Accepted:* July 2023

The challenge of predicting financial time series is building a good prediction model which calculates delicate subtle and clear changes in data. There are many articles about different traditional statistical methods such as Moving Average, Exponential Smoothing, Auto-Regressive Integrated Moving Averages (ARIMA) etc. to predict financial time series [3, 11]. However, these methods are strong in terms of statistics but failed to precisely predict the experimental data. Further, different calculating intelligent techniques such as artificial neural networks, fuzzy systems, swarm intelligence-based motels and others were suggested to predict time series. Most of them intended to be more precise in forecasting. Anyway, they are not perfect [12]. In this research, hybrid prediction models (hybrids) in the smoothing calculation paradigm which combine different prediction models in different arrangements and syntheses gravitated provable superior predictions in comparison with independent models.

Many researchers showed that hybrid models have better results than independent models. Reid [16] and Bates and Granger [2] are the pioneers in different models of hybrid time-series. Bates and Granger [2] showed that a good combination of different forecasting methods could result in better than independent methods. In this matter, Makridakis et al. [10] reported that the combination or generality of different models should be generally better in terms of prediction accuracy. Ginzburg and Horn [9] stated that the combination of different ANNs may enhance the accuracy of time series prediction. Clemen [6] offered a comprehensive review of different hybrid prediction models. A good hybrid prediction model could usually:

1. Improve the prediction function.
2. Remove the defects of independent models.
3. Decrease the model's uncertainties [5].

TThe ambiguity and nonlinear of a time-series may not be always because of being random, it could be because of deterministic phenomena like chaos which is extremely sensitive to primary conditions. A financial time-series could be affected by economic, social, industrial and geopolitics factors. These time-series are ambiguous, messy and imperfect. Notwithstanding, the prediction of financial time-series has great practical potential in terms of financial achievements. Visually, a chaotic time-series and a non-chaotic time-series are similar, so the chaotic time-series have been traditionally modelled like financial time-series to show inherent randomness. Hence, the prediction of the financial time-series which are complex and important needs more complex and stronger hybrid techniques to be developed.

Poincaré [15], one of the pioneers of chaos theory suggested a new method to model the nonlinear dynamic behaviour of a deterministic complex system by putting a given numerical financial time-series in the corresponding fuzzy space by using parameters such as lag and embedded dimension in which the lag is time lag and the embedded dimension is referred to several required variations for offering nonlinear dynamics of a chaotic system.

Generally, Artificial Neural Networks (ANNs) including Multi-Layer Perceptron (MLP) could thoroughly take complex nonlinear relations. They could be properly generalized and are good comprehensive approximators [19]. However, the convergence is slow in MLP, the local minimum could be affected by the education process and it is hard to measure it. Multi-objective evolutionary algorithms (MOEAs) such as multi-objective particle swarm optimization (MOPSO) and Non-Dominated Sorting Genetic Algorithm II (NSGA-II) are good at solving multi-objective optimization issues [7]. NSGA-II has amended the non-dominated sorting genetic algorithm and decreased the calculation complexity. This algorithm organized the combination of parent and children populations by elitist strategy, decreased the messy comparing factors to improve the solution variety and avoided to use of niche factors.

This research offers a 3-stage hybrid financial time-series prediction model. These models review the chaos in both stages. In the hybrid model (Chaos+MLP+NSGA-II), the chaos in the dataset is first modelled by minimum lag and then by minimum embedded dimension (Stage 1). The resulting multivariate time-series shall be put in MLP in the second stage. The third stage shall review the chaos in residuals. If there is chaos, it is properly modelled and the result in multivariate time-series from residuals shall go to the NSGA-II of the autoregressive model; otherwise, is used polynomial regression to model the residuals. Predicted values in stage 3 and predicted values in stage 2 are algebraically added to reach final improved predictions.

The subject of this research is developing a procedure to assess the stock value in the Tehran Stock Exchange based on predicting effective characteristics by data mining. In this research, a new procedure is offered to develop return rate prediction. In this t-stage model, the criteria which could affect the return rate are determined. In the next step, the return rate is predicted by data mining methods. Eventually, the effective factors on the return rate are classified by using a hybrid model. In other words, this research is looking for a method to improve portfolios for decision-makers and investors.

Despite using this model by many analyzers and users, but there are criticisms which are presented in the theoretical basics part. Notwithstanding these researches, there is no experimental research to review these three models (chaos, multilayer perceptron and metaheuristic algorithms) concurrently to predict the return on stocks. This research considers a new model to predict stock returns. In other words, a 3-stage model using a hybrid model of chaos theory, multilayer perceptron and metaheuristic algorithms is used to predict the return on stocks.

2 Chaos theory

Chaos theory is the area of deterministic dynamics which recommends that apparently, random accidents could result from normal questions because of the complex systems they are involved. Chaos in a time-series could be modelled by building a corresponding fuzzy space by using lag (l) and embedded dimension (m). This process transforms the given nonlinear single-dimension time-series to its equal multidimensional form. This procedure was primarily offered by Packard et al. [14] and its mathematical model was explained by Ticknor [18]. If $Y = \{y_1, y_2, ..., y_k, y_{k+1}, ..., y_N\}$ is a numerical time-series, it could be totally embedded in a m-dimension fuzzy space offered by vector $P_j = \{y_j, y_{j+l}, y_{j+2l}, ..., y_{j+(m-1)l}\}$ where $j = 1, 2, ..., N - (m-1)l/\Delta t$ is named m embedded dimensions ($m \geq \theta$ where θ is catchy dimension), l is time lag and Δt is sampling time.

2.1 Seida method

This method estimates Lyapunov exponent λ by equation (2.1):

$$\lambda = \lim_{t\to\infty} \frac{1}{t} \ln \frac{|\Delta g(y_0, t)|}{|y_0|} \tag{2.1}$$

where, y_0 is a point in state space in which a circuit is created by an equational system. $\lambda \geq 0$ means there is chaos, otherwise there is no chaos in timeseries.

2.2 Akaike information criterion (AIC)

Akaike Information Criterion [1] is a method of choosing proper lag in modeling autoregression aspect of data of an autoregressive timeseries. This criterion defines the j lab which minimizes $\frac{\log(SSR(j))}{N} + (j+1)\frac{c(N)}{N}$ in which $SSR(j)$ is the set of quadratic residues for vector autoregression (VAR) with j lags and N is number of observations and $c(N) = 2$.

2.3 Cao method

Cao [4] suggested a method to determining the embedding dimension for a given set. If $Y = \{y_1, y_2, ..., y_k, y_{k+1}, ..., y_N\}$ is a timeseries, we could reproduce it as time lag vectors in fuzzy space in equation (2.2):

$$Y_i^m = \{y_j, y_{j+l}, y_{j+2l}, ..., y_{j+(m-1)l}\}; \quad i = 1, 2, ..., n - (m-1)l \tag{2.2}$$

where, Y_i^m is the ith reproduced vector. The nearest neighborhood corresponding with Y_i^m which is shown by ψY_i^m is as equation (2.3).

$$\psi Y_i^m = \{\psi y_j, \psi y_{j+l}, \psi y_{j+2l}, ..., \psi y_{j+(m-1)l}\}; \quad i = 1, 2, ..., n - (m-1)l \tag{2.3}$$

Equation (2.4) is defined similar to FNN method:

$$\phi_2(i, m) = \frac{\|y_i^{m+1} - \psi y_i^m\|}{\|y_i^m - \psi y_i^m\|} \tag{2.4}$$

where, $\|\cdot\|$ is the maximum norm which is resulted to Euclidean distance, y_i^{m+1} is the ith reproduced vector and ψY_i^m is the nearest neighborhood in embedded dimension of $m+1$. To avoid FNN issues, the mean average of all $\phi_2(i, m)$ is defined as equation (2.5):

$$E(m) = \frac{1}{n - ml} \sum_{i=1}^{n-ml} \phi_2(i, m) \tag{2.5}$$

$E(m)$ is just a function of 1 and m. to review its change from m to $m+1$, the equation (2.6) is defined:

$$E_1(m) = \frac{E(m+1)}{E(m)} \tag{2.6}$$

where, $E_1(m)$ in $m \geq m_0$ turns the change for m_0, so m_0 is the minimum embedded dimensions we need.

2.4 Multilayer perceptron (MLP)

Multilayer Perceptron is by far the most popular feedforward network architecture. It is very strong and varied and has many applications in different fields. Typically, it consists of an input layer, a hidden layer and an output layer. It is trained by well-known backpropagation algorithm in which the weights of layers are iterated updated. When it is trained, the network can be used to predict new and unseen data. It is one of the varied networks which solves both classification and regression issues.

2.5 Multi-objective evolutionary algorithms (MOEAs)

Multi-objective evolutionary algorithms (MOEAs) are applied to multi-objective optimization issues. MOEAs maintain non-dominance solutions an algorithmically goes to Pareto front, keep the variety of Pareto optimization front solutions and offer a few Pareto solution to decision makers [7].

Deb et al. [8], Coelo Coello et al. [7] and Mukhopadhyay et al. [13] have suggested good reviews of MOEAs. Two multi-objective optimization algorithms of this research are as follows:

1. Multi-objective Particle Swarm Optimization (MOPSO) by using numerical optimization
2. Non-Dominated Sorting Genetic Algorithm II (NSGA-II)

NSGA-II [8] is the amended NSGA [17] which uses a better sorting algorithm, puts in the elitism and no need to niche or any other parameter. In NSGA-II, the population randomly commences based on decision variances. Then, the population is sorted based on the non-dominance of solutions which results in different hierarchical fronts. When the non-dominance sorting is completed, the crowding distance is determined for each solution depending on its distance from other individuals of the populations in the target space. Each one of them is similarly selected based on their non-dominance rank and crowding distance. Then, the re-combination and mutation factors for selective parental solutions are applied to build a new generation. This process continues to reach a final criterion. In each next generation t, the children population Q_t and the current generation population P_t are combined and the solution for further generation is selected. The elitism is applied as all former and current superior solutions are finished in the next generation. Then, the population is sorted based on non-dominancy. Hence, the new generation is formed without any front unless the population size becomes bigger than the current population size. If adding all solutions in front F_i results in population P_{t+1} which is bigger than N, then solutions in front F_i are descending and sorted based on their crowding distance up to reach the population size of N. This process is repeated to form the next generations. This process is shown in Fig 1.

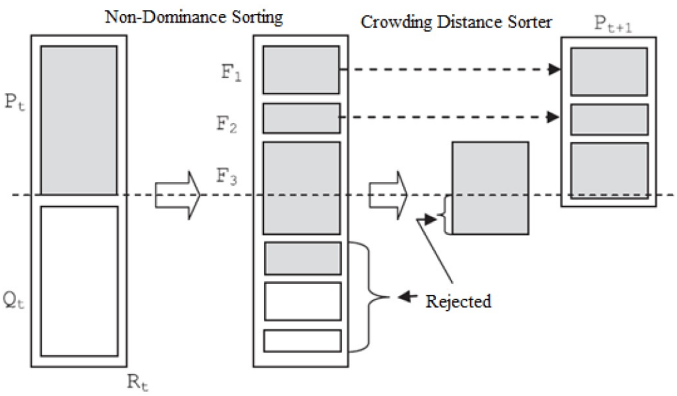

Figure 1: Non-Dominated Sorting Genetic Algorithm II (NSGA-II) [8]

3 Recommended hybrid model

3.1 Multi-objective problem formulation

The prediction of financial timeseries is formulated as a multi-objective optimization problem as follows:

$$\text{Minimizing } O_1 = \text{Mean Square Error (MSE)} \tag{3.1}$$

$$\text{Maximizing } O_1 = \text{Directed Statistics (Dstat)} \quad (3.2)$$

where, MSE and Dstate are defied in equations (3.1) and (3.2), respectively. In predicting financial timeseries, minimizing the prediction mean error is so important. Furthermore, the prediction of financial timeseries path is also important. So, we choose two above-mentioned goals. NSGA-II solves this multi-objective optimization problem directly. However, it should be changed to a numerical optimization issue with equal wights in accordance to MSE and Dstate by using MOSPO as follows:

Minimizing $O_1 \_\_\_ O_2$ where O_1 and O_2 are defined as above.

3.2 Explanation of 3-state hybrid model

Table 1 shows the applied notations in the hybrid model. The detailed explanation of 3-stage prediction model is as follows:

If $Y = \{y_1, y_2, ..., y_k, y_{k+1}, ..., y_N\}$ is a financial timeseries from N data point in $t = 1, 2, ..., k, k+1, N$, respectively, the 3-stage prediction which is shown in equation 2 is continued as follows:

Stage 1: Reproduction of Fuzzy Space

1. Check Y if there is chaos. If yes, reproduce fuzzy space from Y by minimum lag (l_1) and minimum embedded dimension (m_1).
2. Categorize the reproduced Y to $Y_{Train} = \{y_t; \quad t = l_1m_1 + 1, l_1m_1 + 2, ..., k\}$ and $Y_{Train} = \{y_t; \quad t = l_1m_1 + 1, l_1m_1 + 2, ..., k\}$.

Table 1: Applied notations in 3-stage hybrid model and their interpretation

Notation	Interpretation
l_1, m_1	Values of applied lag and embedded dimensions in stage 1
l_2, m_2	Values of applied lag and embedded dimensions in stage 2
e_t	Error in t time
$\widehat{e_t}$	Predicted error in t time
$a_0, a_1, a_2, ...$	Confidences which should be determined
$g1(.)$	A nonlinear function to predict by using MLP
$g2(.)$	A linear function to predict by using polynomial regression
y_t	Data actual point in t time
$\widehat{y_t}$	Date predicted point in t time after stage 1
$\widehat{\widehat{y_t}}$	Date predicted point in t time after stage 2

1. Training phase
   (a) Stage 2: Using MLP
      i. Train MLP by training set of Y_{Train}.
      ii. Early predict by equation (3.3), corresponding errors and following equation (3.4):

$$\begin{aligned} &\widehat{y_t} = g_1(y_{t-l_1}, y_{t-2l_1}, ..., y_{t-m_1l_1}) \\ &t = l_1m_1 + 1, l_1m_1 + 2, ..., k \end{aligned} \quad (3.3)$$

$$\begin{aligned} &e_t = y_t - \widehat{y_t} \\ &t = l_1m_1 + 1, l_1m_1 + 2, ..., k \end{aligned} \quad (3.4)$$

   (b) Stage 3: By using MOPSO/NSGA-II based on autoregression model
      i. Review the error set of all above-gained et whether chaos is existed or not. Rebuild the set by l_2 and m_2 and get $\widehat{e_t}$ by MOPSO/NSGA-II based on autoregression model of equation (3.5) if there is chaos; if not, use polynomial regression (PR) g2(x) at the function form of $Y_{out} = \beta_0 + \beta_1 x + \beta_2 x^2 + ... + \beta_n x^n$ to reach $\widehat{e_t}$, like equation (3.6).

$$\begin{aligned} &\widehat{e_t} = a_0 + a_1 e_{t-l_2} + a_2 e_{t-2l_2} + ... + a_{m2} e_{t-m_2l_2} \\ &t = l_1m_1, l_2m_2 + 1, l_1m_1 + l_2m_2 + 2, ..., k \end{aligned} \quad (3.5)$$

$$\begin{aligned} &\widehat{e_t} = g_2(e_t) \\ &t = l_1m_1 + 1, l_1m_1 + 2, ..., k \end{aligned} \quad (3.6)$$

ii. For chaos, get the predictions of final training set by equation (3.7) after stage 2 and for lack of chaos, use equation (3.8):

$$\begin{aligned}&\widehat{\widehat{y_t}} = \widehat{y_t} + \widehat{e_t}\\&t = l_1 m_1, l_2 m_2 + 1, l_1 m_1 + l_2 m_2 + 2, ..., k\end{aligned} \tag{3.7}$$

$$\begin{aligned}&\widehat{\widehat{y_t}} = \widehat{y_t} + \widehat{e_t}\\&t = l_1 m_1 + 1, l_1 m_1 + 2, ..., k\end{aligned} \tag{3.8}$$

2. Trial phase: In this phase, predict the trial set after two stages by putting observations of $t = k+1, k+2, ..., N$ respectively in equations (3.3), (3.4), (3.5), (3.6), (3.7) and (3.8).

3.3 Recommended hybrid model (Chaos+MLP+NSGA-II)

The 3rd recommended hybrid mode consists of 3 stages. In this combination, chaos theory is used to build fuzzy space from financial timeseries of stage 1, MLP is entered in stage 2 and NSGA-II/PR is involved in stage 3 (chaos could be exist or not on prorate basis). It is similar to hybrid model 2 except that it is used NSGA-II to reach optimized values of $a_0, a_1, a_2, ..., a_{m2}$ in equation (3.5).

The algorithm of achieving optimized $a_0, a_1, a_2, ..., a_{m2}$ is continued by NSGA-II as follows:

1. Build a random population from n particles. Each particle is vector of length $m_2 + 1$.
2. Review fitness function i.e. $O_1 = MSE,\ O_2 = Dstat$ of each particle in population by primary predictions and errors.
3. Classify population through following steps:
   (a) Rate the population by non-dominated sorting algorithm.
   (b) Calculate crowding distance.
4. Create a new population by repeating following steps:
   (a) Choose two parents from the population based on crowding choosing factor.
   (b) Replace parents by using accumulation probability to form children.
   (c) Change the new child by mutation probability.
   (d) Combine parents and children.
   (e) Select n number of the best particles for next generation and abandon the remaining.
   (f) Repeat steps 2 to 4 for new population from former generation up to complete all generations.
   (g) After completion of all generations, take the values of best particle fitness means minimum MSE and maximum Dstat. The coordinates of best particle are optimized coefficient $a_0, a_1, a_2, ..., a_{m2}$.

4 Model assessment

The function of applied algorithms in simulation and long-term prediction of stock index based on the hybrid model in every produced dataset was experimentally analyzed. The function of models was estimated by preliminary error and then to assess the function it was used Root Mean Square Error of Approximation RMSE. RMSE values were used as an analysis criterion to compare and rank the prediction function of the experiments of this research.

More than statistical analysis of errors, image analysis of the evolution of prediction models of this research was used as “Prediction Diagrams”. Based on the theoretical base of statistics and probabilities, if the distribution is fixed, the error will be decreased but it will be increased by changing the distribution. In the regression field, statistical squares average is a standard statistic which is used to determine the error. So, changes in RMSE could provide a reasonable perception of continuous data distribution changes and also content changes. RMSE variance is a statistical test which is measured in this research to measure RMSE values’ sequence variance during the increasing learning process. This test includes measuring the RMSE difference absolute percentage between two successive predictions. As per two predicted values of y_t and y_{t+1} and their corresponding preliminary errors $RMSE_t$ and $RMSE_{t+1}$, RMSE variance was calculated:

$$RMSE_{Disp} = \left|(RMSE_{(t)} - RMSE_{(t-1)}/RMSE_{(t-1)}\right| \tag{4.1}$$

$RMSE_{Disp}$ values for all data in each one of three algorithms were calculated. Then, $RMSE_{Disp}$ standard deviation for each interval and each algorithm were calculated. The content probable changes are determined in those intervals that $RMSE_{Disp}$ standard deviation is more than 0.25. This deviation error threshold was determined based on required accuracy in feasibility studies for economic investments which is arranged from +25% to -25%.

The review of the results in variance analysis shows that three drifts are identified more than great values at the beginning of time-series which is natural in preliminary error. The first one is identified by three algorithms. The second is identified by the perceptron algorithm and the last one in the year is identified by perceptron algorithms and AM rules:

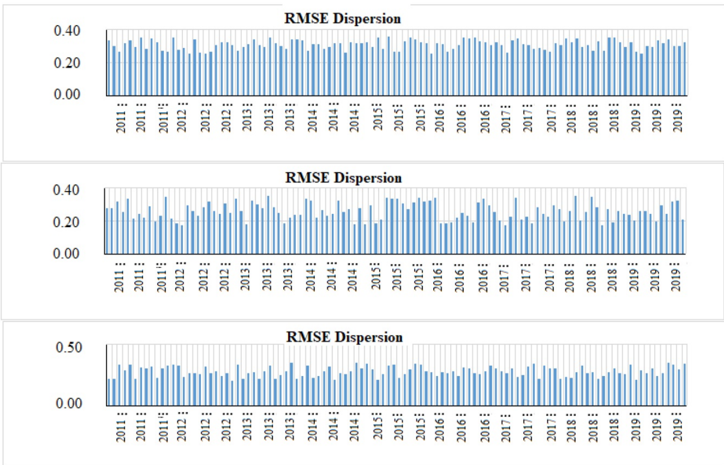

Figure 2: Error Analysis Diagram in Algorithms: A) Perceptron; B) FITMDD and C) AM Rules

In this part, function of the three algorithms in each dataset is measured.

Table 2: RMSE comparison for different algorithms; time intervals (* Best algorithm function, ** Second best algorithm function)

Dataset Interval	Full Stream		
	Perceptron	**FIMTDD**	**AM Rules**
DB01_(t+2)_FIL	4.49 (0.51)	5.35 (0.58)	3.74 (0.29)
DB01_(t+2)_WRP	0.85 (0.03)**	1.46 (0.12)	0.87 (0.04)
DB01_(t+2)_PCA	1.75 (0.10)	2.89 (0.17)	2.00 (0.13)
DB01_(t+2)_CTC	1.37 (0.07)	1.08 (0.02)	0.83 (0.01)
DB03_(t+2)_FIL	1.65 (0.06)	2.35 (0.09)	4.97 (0.53)
DB03_(t+2)_WRP	0.89 (0.02)	0.72 (0.03)*	1.06 (0.03)
DB03_(t+2)_PCA	2.14 (0.30)	1.40 (0.02)	1.42 (0.05)
DB03_(t+2)_CTC	1.44 (0.10)	1.03 (0.06)**	0.87 (0.01)
DB01_(Nt+2)_FIL	2.04 (0.21)	1.47 (0.03)	1.37 (0.10)
DB01_(Nt+2)_WRP	0.72 (0.02)*	1.09 (0.01)	0.74 (0.03)*
DB01_(Nt+2)_PCA	1.33 (0.03)	1.40 (0.01)	1.88 (0.04)
DB01_(Nt+2)_CTC	1.15 (0.05)	1.53 (0.03)	1.17 (0.02)
DB03_(Nt+2)_FIL	0.97 (0.05)	1.27 (0.04)	0.77 (0.03)**
DB03_(Nt+2)_WRP	1.03 (0.04)	1.93 (0.18)	1.22 (0.04)
DB03_(Nt+2)_PCA	1.08 (0.04)	1.40 (0.02)	1.32 (0.00)
DB03_(Nt+2)_CTC	1.35 (0.07)	1.39 (0.05)	1.29 (0.01)

The simulation pattern based on AM rules which use no lag structure or time interval through variable refinery method showed the best function and RMSE=0.37. The FITMDD-based pattern which use lag structures and time interval and refined variables through classification methods showed the second best function in which RMSE=0.38.

5 Conclusion

The perceptron-based pattern which uses lag structures or time intervals through components of chaos theory is in the third place with RMSE=0.43. However, the values of RMSE of the second best function based on AM rules and FITMDD algorithms are 0.52 and 0.54, respectively which are much bigger than the second model based on perceptron with RMSE=0.45 which is similar to the best model based on perceptron multilayer algorithm.

The analysis confirms the former RMSE for perceptron-based and FITMDD models. However, it is are partial differences in the function of AMR rules-based models. MAE values show that AMR rules-based models show the

Table 3: RMSE comparison for variables refinery method; time intervals assessment (* Best algorithm function, ** Second best algorithm function)

Dataset	RMSE					
	Full Stream			**Drift_01**		
	Perceptron	**FIMTDD**	**AM Rules**	**Perceptron**	**FIMTDD**	**AM Rules**
DB01_(t+2)_FIL	4.49 (0.51)	5.35 (0.58)	3.74 (0.29)	0.95 (0.06)	1.79 (0.11)	1.07 (0.05)
DB01_(t+2)_WRP	0.85 (0.03)**	1.46 (0.12)	0.87 (0.04)	0.84 (0.03)	1.28 (0.09)	0.89 (0.05)
DB01_(t+2)_PCA	1.75 (0.10)	2.89 (0.17)	2.00 (0.13)	1.24 (0.02)	2.83 (0.21)	1.97 (0.15)
DB01_(t+2)_CTC	1.37 (0.07)	1.08 (0.02)	0.83 (0.01)	0.65 (0.01)	0.83 (0.02)**	0.72 (0.01)
DB03_(t+2)_FIL	1.65 (0.06)	2.35 (0.09)	4.97 (0.53)	0.74 (0.02)	1.25 (0.02)	0.95 (0.01)
DB03_(t+2)_WRP	0.89 (0.02)	0.72 (0.03)*	1.06 (0.03)	0.76 (0.02)	0.66 (0.03)*	0.93 (0.03)
DB03_(t+2)_PCA	2.14 (0.30)	1.40 (0.02)	1.42 (0.05)	1.36 (0.06)	1.30 (0.02)	1.39 (0.05)
DB03_(t+2)_CTC	1.44 (0.10)	1.03 (0.06)**	0.87 (0.01)	0.79 (0.01)	0.92 (0.05)	0.83 (0.00)
DB01_(Nt+2)_FIL	2.04 (0.21)	1.47 (0.03)	1.37 (0.10)	1.70 (0.14)	1.49 (0.05)	1.31 (0.12)
DB01_(Nt+2)_WRP	0.72 (0.02)*	1.09 (0.01)	0.74 (0.03)*	0.67 (0.02)	0.96 (0.00)	0.73 (0.03)**
DB01_(Nt+2)_PCA	1.33 (0.03)	1.40 (0.01)	1.88 (0.04)	0.93 (0.02)	1.34 (0.02)	1.87 (0.05)
DB01_(Nt+2)_CTC	1.15 (0.05)	1.53 (0.03)	1.17 (0.02)	0.58 (0.01)*	1.35 (0.04)	0.84 (0.01)
DB03_(Nt+2)_FIL	0.97 (0.05)	1.27 (0.04)	0.77 (0.03)**	0.86 (0.05)	1.20 (0.04)	0.68 (0.02)*
DB03_(Nt+2)_WRP	1.03 (0.04)	1.93 (0.18)	1.22 (0.04)	0.76 (0.03)	1.91 (0.22)	1.17 (0.05)
DB03_(Nt+2)_PCA	1.08 (0.04)	1.40 (0.02)	1.32 (0.00)	0.71 (0.02)	1.30 (0.02)	1.28 (0.00)
DB03_(Nt+2)_CTC	1.35 (0.07)	1.39 (0.05)	1.29 (0.01)	0.62 (0.00)**	1.22 (0.07)	1.17 (0.01)

best function in selecting explanatory variables through the classification method.

In the FMITDD-based model, the RMSE value decreased a bit through the two first drifts (Drift_01) and (Drift_02) and was extremely dropped through the third drift (Drift_03) (almost 50%). In the AM rules-based model, the RMSE value showed a more continuous decrease in first three intervals. However, the same extreme decrease was seen in last drift (Drift_03). The sudden decrease of RMSE in FMITDD-based models and AM rules showed that their consistency trend is slower and primarily happened at the last drift.

About perceptron-based model, the RMSE values were gradually decreased in all intervals. The continuous decrease of RMSE showed that the perceptron-based model could be compatible in all data stream. This matter may recommend better learning and consistency process than perceptron-based model to predict stock price as this kind of learning arrangement could benefit more experiments for predicting future behavior at this moment in order to change the system content.

References

[1] H. Akaike, *A new look at the statistical model identification*, IEEE Trans. Automatic Control **19** (1974), no. 6, 716–723.

[2] J.M. Bates and C.W.J. Granger, *The combination of forecasts*, Oper. Res. Q. **20** (1969).

[3] G. Box and G. Jenkins, *Time Series Analysis: Forecasting and Control*, Holden-Day, San Francisco, 1976.

[4] L. Cao, *Practical method for determining the minimum embedding dimension of a scalar time series*, Phys. D: Nonlinear Phen. **110** (1997), no. 1-2, 43–50.

[5] C. Chatfield, *Model uncertainty and forecast accuracy*, J. Forecast. **15** (1996), 495–508.

[6] R.T. Clemen, *Combining forecasts: A review and annotated bibliography*, Int. J. Forecast. **5** (1989), 559–583.

[7] A. Coello Coello, G.B. Lamont, and D.A. Van Veldhuisen, *Evolutionary Algorithms for Solving Multi-Objective Problems*, (2nd ed.), Springer series of Genetic Algorithms and Evolutionary Computation, 2007.

[8] K. Deb, A. Pratap, S. Agarwal and T.A.M.T. Meyarivan, *A fast and elitist multiobjective genetic algorithm: NSGA-II*, IEEE Trans. Evol. Comput. **6** (2002), no. 2, 182–197.

[9] I. Ginzburg and D. Horn, *Combined neural networks for time series analysis*, Adv. Neural Inf. Process. Syst. **6** (1993).

[10] S. Makridakis, A. Andersen, R. Carbone, R. Fildes, M. Hibon, R. Lewandowski, J. Newton, E. Parzen, and R. Winkler, *The accuracy of extrapolation (time series) methods: Results of a forecasting competition*, J. Forecast. **1** (1982), 111–153.

[11] S. Makridakis, S. Wheelwright, and R. Hyndman, *Forecasting: Methods and Applications*, edition 3 ed., New York: John Wiley & Sons, 1998.

[12] T. Mitsa, *Temporal Data Mining*, Chapman & Hall/ CRC Data mining and knowledge discovery series, CRC Press, US, 2010.

[13] A. Mukhopadhyay, U. Maulik, S. Bandyopadhyay, and C.A. Coello Coello, *A survey of multiobjective evolutionary algorithms for data mining: Part I*, IEEE Trans. Evolut. Comput. **18** (2014), 4–19.

[14] N.H. Packard, J.P. Crutchfield, J.D. Farmer, and R.S. Shaw, *Geometry from a time series*, Phys. Rev. Lett. **45** (1980), no. 9, 712.

[15] H. Poincaré, *Sur le problème des trois corps et les équations de la dynamique*, Acta Math. **13** (1890), no. 1, A3–A270.

[16] D.J. Reid, *Combining three estimates of gross domestic product*, Economica **35** (1968).

[17] N. Srinivas and K. Deb, *Muiltiobjective optimization using nondominated sorting in genetic algorithms*, Evolut. Comput. **2** (1994), 221–248.

[18] J.L. Ticknor, *A Bayesian regularized artificial neural network for stock market forecasting*, Expert Syst. Appl. **40** (2013), no. 14, 5501–5506.

[19] G. Zhang, B. Patuwo, and M.Y. Hu, *Forecasting with artificial neural networks:: The state of the art*, Int. J. Forecast. **14** (1998).