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# Interpolative type contraction mappings in $G$-metric spaces

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**Abstract**

In this paper, we defined $\alpha_G-$admissible interpolative type contraction mappings in $G$-metric spaces. We proved some convergence results for such classes of mappings using the properties of $G$-metric space and found the fixed point results for such contractive mappings. To elaborate on the results we provided some examples, which show that our results hold in the setting of $G-$metric spaces.

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## 1 Introduction and preliminaries

The study of metric fixed point theory is one of the cornerstones of mathematics and many other sciences. Various studies have been made using different generalizations of the metric spaces in this theory. One of them is $G$-metric spaces (see [24, 25]) which was introduced as a generalization of metric spaces $(X, d)$. The $G$-metric space is defined as follows:

**Definition 1.1.** [25] Let $X$ be a nonempty set and let $G : X \times X \times X \to \mathbb{R}^+$ be a function satisfying the following properties:

(i) $G(x, y, z) = 0$ if and only if $x = y = z$;

(ii) $0 < G(x, x, y)$ for all $x, y \in X$ with $x \neq y$;

(iii) $G(x, x, y) \leq G(x, y, z)$ for all $x, y, z \in X$ with $z \neq y$;

(iv) $G(x, y, z) = G(x, z, y) = G(y, z, x) = \dots$, (symmetry in all three variables);

(v) $G(x, y, z) \leq G(x, a, a) + G(a, y, z)$ for all $x, y, z, a \in X$ (rectangle inequality ).

Then the function $G$ is called a $G-$metric on $X$ and the pair $(X, G)$ is called a $G-$metric space.

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**Example 1.2.** [24] Let $(X,d)$ be an usual metric space, define $G_s$ and $G_m$ on $X\times X\times X$ to $\mathbb{R}^+$ by

$$G_s(x,y,z)=d(x,y)+d(y,z)+d(x,z), \quad \text{and} \quad G_m(x,y,z)=\max\{d(x,y),d(y,z),d(x,z)\}$$

for all $x,y,z\in X$. Then $(X,G_s)$ and $(X,G_m)$ are $G-$metric spaces.

**Definition 1.3.** [25] Let $(X,G)$ be a $G-$metric space and let $(x_n)$ be a sequence of points of $X$. A point $x\in X$ is said to be the limit of the sequence $(x_n)$, if $\lim_{n,m\to+\infty}G(x,x_n,x_m)=0$, and one say that the sequence $(x_n)$ is $G-$convergent to $x$.

**Proposition 1.4.** [25] Let $(X,G)$ be $G-$metric space. Then the following are equivalent;

1. A sequence $(x_n)$ is $G-$convergent to $x$;
2. $G(x_n,x_n,x)\to 0$, as $n\to+\infty$;
3. $G(x_n,x,x)\to 0$, as $n\to+\infty$;
4. $G(x_m,x_n,x)\to 0$, as $m,n\to+\infty$.

**Definition 1.5.** [25] Let $(X,G)$ be a $G-$metric space. A sequence $(x_n)$ is called $G-$Cauchy sequence, if for any $\epsilon>0$, there is $N\in\mathbb{N}$ such that $G(x_n,x_m,x_l)<\epsilon$, for all $l,m,n\geq N$, that is, $G(x_n,x_m,x_l)\to 0$ as $l,m,n\to+\infty$.

**Proposition 1.6.** [25] In a $G-$metric space $(X,G)$, the following are equivalent;

1. The sequence $(x_n)$ is a $G-$Cauchy sequence;
2. For every $\epsilon>0$, there exists $N\in\mathbb{N}$ such that $G(x_n,x_m,x_m)<\epsilon$, for all $n,m\geq N\in\mathbb{N}$.

**Definition 1.7.** [25] A $G-$metric space $(X,G)$ is called symmetric $G-$metric space, if $G(x,y,y)=G(y,x,x)$ for all $x,y\in X$.

It is clear that, any $G-$metric space, where $G$ derives from an underlying metric via $G_s$ or $G_m$ in Example 1.2 is symmetric. The following example presents the simplest instance of a nonsymmetric $G-$metric and so also one which does not arise from any metric in the above ways.

**Example 1.8.** [25] Let $X=\{a,b\}$ consider $G(a,a,a)=G(b,b,b)=0,\ G(a,a,b)=1, G(a,b,b)=2.$ Then it can be easily verified that $G$ is a $G-$metric, but $G(a,b,b)\neq G(a,a,b)$.

**Proposition 1.9.** [25] Every $G-$metric space $(X,G)$ induces a metric space $(X,d_G)$, which is defined as follows:

$$d_G(x,y)=G(x,y,y)+G(y,x,x),\ \text{for all } x,y\in X.$$

Note that, if $(X,G)$ is symmetric, then$d_G(x,y)=2G(x,y,y)$ for all $x,y\in X$. However, if $(X,G)$ is not symmetric, then

$$\frac{3}{2}G(x,y,y)\leq d_G(x,y)\leq 3G(x,y,y)\ \text{ for all } x,y\in X.$$

**Definition 1.10.** [25] A $G-$metric space $(X,G)$ is said to be $G-$complete if every $G-$Cauchy sequence in $(X,G)$ is $G-$convergent in $(X,G)$.

**Proposition 1.11.** [25] A $G-$metric space $(X,G)$ is $G-$complete if and only if $(X,d_G)$ is a complete metric space.

**Lemma 1.12.** [25] In a $G-$metric space $(X,G)$, for $x,y,z,t\in X$, we have

(i) if $G(x,y,z)=0$, then $x=y=z$;
(ii) $G(x,y,z)\leq G(x,x,y)+G(x,x,z)$;
(iii) $G(x,y,y)\leq 2G(y,x,x)$;
(iv) $G(x,y,z)\leq G(x,t,z)+G(t,y,z)$;
(v) $G(x,y,z)\leq \frac{2}{3}[G(x,y,t)+G(x,t,z)+G(t,y,z)]$;

(vi) $G(x,y,z) \leq G(x,t,t) + G(y,t,t) + G(z,t,t)$.

**Definition 1.13.** [25] In a $G-$metric space $(X,G)$, a mapping $T : X \to X$ is known as $G-$continuous if $\{T(x_n)\}$ is $G-$convergent to $T(x)$, where $\{x_n\}$ is any $G-$convergent sequence converging to $x$.

Here firstly, we recall the definition of $\alpha-$admissible mappings and its generalizations in metric space and $G-$metric space.

**Definition 1.14.** [34] Let $\alpha : X \times X \to [0,+\infty)$ be a function. A mapping $T : X \to X$ is said to be an $\alpha-$admissible if $x, y \in X$, $\alpha(x,y) \geq 1$ makes $\alpha(Tx,Ty) \geq 1$.

**Example 1.15.** [34] Consider $X = [0,+\infty)$. We define $T : X \to X$ by $Tx = 5x$ and $\alpha : X \times X \to [0,+\infty)$ by

$$\alpha(x,y) = \begin{cases} e^{\frac{y}{x}}, & \text{if} \quad x \geq y, \quad x \neq 0 \\ 0, & \text{if} \quad x < y, \end{cases}$$

for all $x, y \in X$. Then, $T$ is an $\alpha-$admissible mapping.

**Definition 1.16.** [20] Let $T : X \to X$ and $\alpha : X \times X \to (-\infty,+\infty)$. It is said that $T$ is a triangular $\alpha$-admissible mapping if

**(T1)** $\alpha(x,y) \geq 1$, implies $\alpha(Tx,Ty) \geq 1$, $x, y \in X$,

**(T2)** $\alpha(x,z) \geq 1$, $\alpha(z,y) \geq 1$, implies $\alpha(x,y) \geq 1$, $x, y, z \in X$.

Alghamdi and Karapinar [1] generalized the concept of $\alpha-$admissible mappings in the context of $G-$metric space and called it $\beta-$admissible. The definition of $\beta-$admissible given by Alghamdi and Karapinar is defined as follows.

**Definition 1.17.** [1] Let $T : X \to X$ and $\beta : X \times X \times X \to [0,+\infty)$. Then $T$ is said to be $\beta-$admissible if for all $x, y, z \in X$. Then

$$\beta(x,y,z) \geq 1 \text{ implies } \beta(Tx,Ty,Tz) \geq 1.$$

Alghamdi and Karapinar [1] introduced $G-\beta-\psi$ contractive mappings of type-I and type-II. They also introduced $G-\beta-\psi$ contractive mappings of type$-A$. They also gave the relation between these different types of $G-\beta-\psi$ contractions and equivalent Banach contractions.

Alghamdi and Karapinar [2] further generalized the results of [1] by introducing generalized $G-\beta-\psi$ contractive mappings of type-I and type-II.

Kutbi et al. [22] defined rectangular $G-\alpha-$admissible mapping. They also defined weak $\alpha-\psi-z$ contractive mappings to establish some coincidence point theorems for coupled and tripled in $G_b-$metric space.

**Definition 1.18.** [22] Let $(X,G)$ be a $G$-metric space and let $T, S : X \to X$ be given mappings and $\alpha : X^3 \to [0,+\infty)$ be a function. A mapping $T$ is said to be a rectangular $G-\alpha-$admissible mapping with respect to $S$ if

**(i)** $\alpha(Sx,Sy,Sz) \geq 1$ implies $\alpha(Tx,Ty,Tz) \geq 1$, $x, y, z \in X$;

**(ii)** $\alpha(Sx,Sy,Sy) \geq 1$ and $\alpha(Sy,Sz,Sz) \geq 1$ implies $\alpha(Sx,Sy,Sz) \geq 1$, $x, y, z \in X$.

Hussain et al. [15] generalised the concept of rectangular $G-\alpha-$admissible mappings and used to obtain coupled and tripled fixed point theorems. Hussain et al. [16] established a generalized form of $\alpha-$admissible mappings in order to prove coincidence points and common fixed points in the framework of $G$-metric spaces. Further, several authors obtained different kind of generalization of Banach contraction principle in different spaces, see for details ([33, 32, 4, 31, 6, 30, 7, 5]).

**Definition 1.19.** [16] Let $X$ be an arbitrary set, $\alpha : X \times X \times X \to [0,+\infty)$ be a function and $T : X \to X$. The mapping $T$ is called an $\alpha-$dominating map on $X$ if $\alpha(x,Tx,Tx) \geq 1$ or $\alpha(x,x,Tx) \geq 1$ for each $x$ in $X$.

**Definition 1.20.** [16] In an arbitrary set $X$, let $T, S : X \to X$ be given mappings and $\alpha : X \times X \times X \to [0, +\infty)$ be a function. The pair $(T, S)$ is said to be partially weakly $G-\alpha-$admissible if and only if $\alpha(Tx, STx, STx) \geq 1$ for all $x \in X$.

**Definition 1.21.** [16] In an arbitrary set $X$, let $T, S : X \to X$ be given mappings and $\alpha : X \times X \times X \to [0, +\infty)$ be a function. The pair $(T, S)$ is said to be partially weakly $G-\alpha-$admissible with respect to $T$ if and only if for all $x \in X$, $\alpha(Tx, Sy, Sy) \geq 1$ where $y \in T^{-1}(Sx)$.

In the above definition, if $T = S$, $T$ is said to be partially weakly $G-\alpha-$admissible (or $\alpha-$admissible of rank 3) with respect to $T$. If $T = I_X$ (the identity mapping on $X$), then the above definition becomes the definition of partially weakly $G-\alpha-$admissible pair.

Ansari et al. [3] also studied $\alpha-$admissible mappings in $G-$metric space by introducing $G-\eta-$subadmissible mapping and $\alpha-$dominating map. They also introduced $\eta-$subdominating map, $\alpha-$regular in the framework of $G-$metric space, partially weakly $G-\alpha-$admissible and partially weakly $G-\eta-$subadmissible mappings, etc.

**Definition 1.22.** [3] Let $(X, G)$ be a $G-$metric space and let $T$ be a self-mapping on $X$ and $\eta : X \times X \times X \to [0, +\infty)$ be a function. $T$ is said to be a $G-\eta-$subadmissible (or $\eta$-subadmissible of rank 3) mapping if $x, y, z \in X$,

$$\eta(x, y, z) \leq 1 \text{ implies } \eta(Tx, Ty, Tz) \leq 1.$$

**Definition 1.23.** [3] Let $X$ be an arbitrary set, $\eta : X \times X \times X \to [0, +\infty)$ be a function and $T : X \to X$ be a mapping. A mapping $T$ is called an $\eta-$subdominating map on $X$ if $\eta(x, Tx, Tx) \leq 1$ or $\alpha(x, x, Tx) \leq 1$ for each $x$ in $X$.

**Definition 1.24.** [3] In a $G-$metric space $(X, G)$, let $T, S : X \to X$ be given mappings and $\eta : X \times X \times X \to [0, +\infty)$ be a function. The pair $(T, S)$ is said to be partially weakly $G-\eta-$subadmissible (or $\eta-$subadmissible of rank 3) if and only if $\eta(Tx, STx, STx) \leq 1$ for all $x \in X$.

**Definition 1.25.** [3] In a $G$-metric space $(X, G)$, let $T, S : X \to X$ be given mappings and $\eta : X \times X \times X \to [0, +\infty)$ be a function. The pair $(T, S)$ is said to be partially weakly $G-\eta-$subadmissible (or $\eta-$subadmissible of rank 3) with respect to $T$ if and only if for all $x \in X$, $\alpha(Tx, Sy, Sy) \geq 1$ where $y \in T^{-1}(Sx)$.

Hussain et al. [14] defined $G-(\alpha, \psi)-$Mier-Keeler contractive mapping and used it in proving fixed point theorems in the framework of $G$-metric spaces.

**Definition 1.26.** [14] Let $(X, G)$ be a $G-$metric space and $\psi \in \Psi$. Let $T : X \to X$ be an $\alpha-$admissible, if for each $\varepsilon > 0$, there exists $\delta > 0$ such that $\varepsilon \leq \psi(G(x, y, z)) < \varepsilon + \delta$ implies $\alpha(x, x)\alpha(y, y)\alpha(z, z)\psi(G(Tx, Ty, Tz)) < \varepsilon$ for all $x, y, z \in X$. Then $T$ is known as a $G-(\alpha, \psi)-$Meir-Keeler contractive mapping.

In the above definition $\Psi$ is the collection of nondecreasing functions $\psi : [0, +\infty) \to [0, +\infty)$ continuous in $t$ such that $\psi(t) = 0$ if and only if $t = 0$ and $\psi(t + s) \leq \psi(t) + \psi(s)$. The concept of $\alpha-$admissible mappings is extended to $S-$metric space by Zhou et al. [37] and called it $\gamma-$admissible. They are defined as follows:

**Definition 1.27.** [37] Let $T : X \to X$ and $\gamma : X \times X \times X \to [0, +\infty)$ then $T$ is said to be a $\gamma-$admissible if for all $x, y, z \in X$,

$$\gamma(x, y, z) \geq 1 \text{ implies } \gamma(Tx, Ty, Tz) \geq 1.$$

They also extended $\gamma-$admissibility for two mappings. Further, they also introduced concepts of various contractive mappings viz. type A, type B, type C, type D and type E. Bulbul et al. [21] also derived the concept of generalized $S-\beta-\psi$ contractive type mappings on the line of generalized $G-\beta-\gamma$ contractive type mappings. Nabil et.al. [23] also defined the concept of $\alpha-$admissible mappings in $S_b-$metric space.

From these what we observe is that $\beta-$admissible was for the first time used by Samet et. al. [34] to represent $\alpha-$admissible while dealing with coupled fixed point related problems. Phiangsungnoen et. al. [29] also used the name $\beta-$admissible mapping in order to represent $\alpha-$admissible for fuzzy mappings. On the other hand, $\beta-$admissible of Alghamdi and Karapinar [2] and $\gamma-$admissible of Zhou et. al. [37] are all extended versions of $\alpha-$admissible mappings in $G-$metric space and $S-$metric space respectively. Thus, we can remark that $\alpha-$admissible and its various forms can be extended to $G-$metric as well as $S-$metric spaces and further to $G_b-$metric and $S_b-$metric spaces. With this idea we introduce various forms of $\alpha-$admissible mappings in the context of $G-$metric space and present following definitions. For notation we use $\alpha_G$ for $\alpha-$admissible mappings in $G-$metric space.

**Definition 1.28.** Let $T : X \to X$ be a mapping and $\alpha_G : X \times X \times X \to [0, +\infty)$ be a function, then the mapping $T$ is said to be $\alpha_G-$admissible, if for all $x, y, z \in X$, $\alpha_G(x, y, z) \geq 1$ implies $\alpha_G(Tx, Ty, Tz) \geq 1$.

**Definition 1.29.** Let $T, S : X \to X$ are mappings and $\alpha_G : X \times X \times X \to [0, +\infty)$ be a function. We say that the pair $(T, S)$ is $\alpha_G-$admissible if for all $x, y, z \in X$ such that $\alpha_G(x, y, z) \geq 1$, then we have $\alpha_G(Tx, Sy, Sz) \geq 1$ and $\alpha_G(Sx, Ty, Tz) \geq 1$.

**Definition 1.30.** Let $T : X \to X$ and $\alpha_G : X \times X \times X \to [0, +\infty)$. We say that $T$ is triangular $\alpha_G-$admissible mapping if

(i) $\alpha_G(x, y, z) \geq 1$ implies $\alpha_G(Tx, Ty, Tz) \geq 1,\ x, y, z \in X,$
(ii) $\alpha_G(x, t, t) \geq 1$ and $\alpha_G(t, y, z) \geq 1$ implies $\alpha_G(x, y, z) \geq 1,\ x, y, z, t \in X.$

**Definition 1.31.** Let $T : X \to X$ be a mapping and $\alpha_G, \eta_G : X \times X \to [0, +\infty)$ are functions. We say that $T$ is an $\alpha_G-$admissible mapping with respect to $\eta_G$ if $x, y, z \in X$,

$$\alpha_G(x, y, z) \geq \eta_G(x, y, z) \text{ implies } \alpha_G(Tx, Ty, Tz) \geq \eta_G(Tx, Ty, Tz)$$

Note that if we take $\eta_G(x, y, z) = 1$, then this definition becomes definition 1.28 Also, if we take $\alpha_G(x, y, z) = 1$, then it is said that $T$ is an $\eta_G-$subadmissible mapping.

**Definition 1.32.** Let $T, S : X \to X$ and $\alpha_G, \eta_G : X \times X \times X \to [0, +\infty)$. We say that the pair $(T, S)$ is $\alpha_G-$admissible mapping with respect to $\eta_G$ if $x, y, z \in X$ such that $\alpha_G(x, y, z) \geq \eta_G(x, y, z)$, then we have $\alpha_G(Tx, Sy, Sz) \geq \eta_G(Tx, Sy, Sz)$ and $\alpha_G(Sx, Ty, Tz) \geq \eta_G(Sx, Ty, Tz)$.

**Lemma 1.33.** Let $T, S : X \to X$ are triangular $\alpha_G$-admissible mappings. Suppose that there exists $x_0 \in X$ such that $\alpha_G(x_0, Tx_0, Tx_0) \geq 1$. Define sequences

$$x_{2i+1} = Tx_{2i} \ \ \textit{and} \ \ x_{2i+2} = Sx_{2i+1}, \ \ \textit{where} \ \ i = 0, 1, 2, \ldots.$$

Then we have $\alpha_G(x_n, x_m, x_m) \geq 1$, $m, n \in \mathbb{N} \cup \{0\}$ and $n < m$.

Mustafa et al. ([26, 27, 28]) state and proved the following fixed point theorems on some classes of contractive mappings defined on a $G-$metric space.

**Theorem 1.34.** [27] Let $(X, G)$ be a $G-$metric space and $T : X \to X$ be a mapping satisfying the following:

$$G(Tx, Ty, Tz) \leq aG(x, Tx, Tx) + bG(y, Ty, Ty) + cG(z, Tz, Tz) \tag{1.1}$$

for all $x, y, z \in X$, where $0 < a + b + c < 1$. Then $T$ has a unique fixed point.

**Theorem 1.35.** [26] Let $(X, G)$ be a complete $G-$metric space and $T : X \to X$ be a mapping satisfying:

$$G(Tx, Ty, Tz) \leq aG(x, y, z) + bG(x, Tx, Tx) + cG(y, Ty, Ty) + dG(z, Tz, Tz) \tag{1.2}$$

for all $x, y, z \in X$, where $0 \leq a + b + c + d < 1$. Then $T$ has a unique fixed point.

**Theorem 1.36.** [28] Let $(X, G)$ be complete $G-$metric space and $T : X \to X$ be a mapping satisfying:

$$G(Tx, Ty, Tz) \leq \alpha G(x, y, z) + \beta \max\{G(x, Tx, Tx), G(y, Ty, Ty), G(z, Tz, Tz)\} \tag{1.3}$$

for all $x, y, z \in X$, where $0 \leq \alpha + \beta < 1$. Then $T$ has unique fixed point.

Recently, Karapinar [17] proved a well known fixed point theorem of Kannan under the aspect of interpolation, stated as:

**Definition 1.37.** [17] Let $(X, d)$ be a metric space. A mapping $T : X \to X$ is said to be an interpolative Kannan type contraction, if there are constants $k \in [0, 1)$ and $\alpha \in (0, 1)$ such that

$$d(Tx, Ty) \le k [d(x, Tx)]^{\alpha} [d(y, Ty)]^{1-\alpha},$$

for all $x, y \in X \setminus Fix(T)$, where $Fix(T) = \{x \in X : Tx = x\}$.

Karapinar et al. [19] state and proved the following theorem for fixed point of above mapping.

**Theorem 1.38.** [19] Let $(X, d)$ be a complete metric space. Let $T : X \to X$ be an interpolative Kannan type contraction. Then $T$ has a fixed point in $X$.

After, Karapinar et al. [18] introduced the notion of interpolative Reich–Rus–Cirić type contractions.

**Definition 1.39.** [18] In the framework of a partial metric space $(X, d)$, a mapping $T : X \to X$ is called an interpolative Reich–Rus–Cirić type contraction, if there are some constants $k \in [0, 1)$ and $\alpha, \beta \in (0, 1)$ such that

$$d(Tx, Ty) \le k [d(x, y)]^{\alpha} [d(x, Tx)]^{\beta} [d(y, Ty)]^{1-\alpha-\beta},$$

for all $x, y \in X \setminus Fix(T)$.

Karapinar et al. [18] proved the following fixed point result for interpolative Reich–Rus–Cirić type contractions in metric space.

**Proposition 1.40.** [18] In the framework of a metric space $(X, d)$, if $T : X \to X$ is an interpolative Reich–Rus–Cirić type contraction, then $T$ has a fixed point in $X$.

Several authors have studied the fixed points of different interpolative type contractions mappings in generalized metric spaces such as partial metric spaces, $T_0$-quasi-metric spaces and convex $b-$metric spaces ([11, 12, 35, 36]). Inspired by the results and definition above, we introduce the concept of interpolative type contractions in $G-$metric space and we give some fixed point theorem for this contractions in complete $G$-metric spaces.

## 2 Main Results

In this section, we firstly give the following result corresponding to Theorem 1.34 using interpolation notion in $G-$metric space.

**Theorem 2.1.** Let $(X, G)$ be a complete $G-$metric space, $\alpha_G : X \times X \times X \to [0, +\infty)$ be a function and $T : X \to X$ be an $\alpha_G-$admissible mapping and an interpolative Reich-Rus-Cirić type contraction satisfying

$$\alpha_G(x, y, z) G(Tx, Ty, Tz) \le k [G(x, Tx, Tx)]^{\alpha} [G(y, Ty, Ty)]^{\beta} [G(z, Tz, Tz)]^{1-\alpha-\beta} \tag{2.1}$$

with $\alpha_G(x, y, z) \ge 1$ for all $x, y, z \in X \backslash Fix(T)$ where $k \in [0, 1)$ and $\alpha, \beta \in (0, 1)$. Then $T$ has a unique fixed point.

**Proof .** Since $T$ is an interpolative Reich-Rus-Cirić type contraction, then from (2.1), we have

$$G(Tx, Ty, Ty) \le \alpha_G(x, y, y) G(Tx, Ty, Ty) \le k [G(x, Tx, Tx)]^{\alpha} [G(y, Ty, Ty)]^{1-\alpha}$$

and

$$G(Ty, Tx, Tx) \le \alpha_G(y, x, x) G(Ty, Tx, Tx) \le k [G(y, Ty, Ty)]^{\alpha} [G(x, Tx, Tx)]^{1-\alpha}$$

for all $x, y \in X \backslash Fix(T)$. Assume that $(X, G)$ is symmetric. From the Proposition 1.9 and (2.1), the above inequality becomes

$$\begin{aligned} \frac{1}{2} d_G(Tx, Ty) &\le k \left[\frac{1}{2} d_G(x, Tx)\right]^{\alpha} \left[\frac{1}{2} d_G(y, Ty)\right]^{1-\alpha} \\ &= \frac{k}{2} [d_G(x, Tx)]^{\alpha} [d_G(y, Ty)]^{1-\alpha} \end{aligned}$$

which implies that

$$d_G(Tx,Ty) \leq k\left[d_G(x,Tx)\right]^{\alpha}\left[d_G(y,Ty)\right]^{1-\alpha} \tag{2.2}$$

for all $x,y \in X\backslash Fix(T)$. According to the condition (2.2), $T$ is an interpolative Kannan type contraction in $(X,d_G)$ metric space. Then, we know that $T$ possesses a fixed point in $X$ from Theorem 1.38. Suppose that $(X,G)$ is not symmetric. From the definition of the metric $(X,d_G)$ and (2.1), we get

$$\begin{aligned}\frac{1}{3}d_G(Tx,Ty) &\leq k\left[\frac{2}{3}d_G(x,Tx)\right]^{\alpha}\left[\frac{2}{3}d_G(y,Ty)\right]^{1-\alpha} \\ &= \frac{2k}{3}\left[d_G(x,Tx)\right]^{\alpha}\left[d_G(y,Ty)\right]^{1-\alpha}\end{aligned}$$

which implies that

$$d_G(Tx,Ty) \leq 2k\left[d_G(x,Tx)\right]^{\alpha}\left[d_G(y,Ty)\right]^{1-\alpha}$$

for all $x,y \in X\backslash Fix(T)$. Since $k \in [0,1)$. So, we are not sure about that either $2k < 1$ or not. But we can prove the existence of a fixed point using the properties of $G-$metric space. Let $x_0 \in X$ and the sequence $\{x_n\}$ be defined as $x_n = Tx_{n-1} = T^n x_0$, since $\alpha_G(x_0,x_1,x_1) \geq 1$ and $T$ is an $\alpha_G-$admissible mapping, then $\alpha_G(x_1,x_2,x_2) = \alpha_G(Tx_0,Tx_1,Tx_1) \geq 1$, continuing on the same lines, we have $\alpha_G(x_n,x_{n+1},x_{n+1}) \geq 1$. By using the condition (2.1), we have

$$\begin{aligned}G(x_n,x_{n+1},x_{n+1}) &\leq \alpha_G(x_n,x_{n+1},x_{n+1})G(x_n,x_{n+1},x_{n+1}) \\ &= \alpha_G(Tx_{n-1},Tx_n,Tx_n)G(Tx_{n-1},Tx_n,Tx_n) \\ &\leq k\left[G(x_{n-1},x_n,x_n)\right]^{\alpha}\left[G(x_n,x_{n+1},x_{n+1})\right]^{1-\alpha},\end{aligned}$$

which implies that

$$\left[G(x_n,x_{n+1},x_{n+1})\right]^{\alpha} \leq k\left[G(x_{n-1},x_n,x_n)\right]^{\alpha}.$$

From the above inequality, we have

$$G(x_n,x_{n+1},x_{n+1}) \leq kG(x_{n-1},x_n,x_n).$$

Recursively, we can write

$$G(x_n,x_{n+1},x_{n+1}) \leq k^n G(x_0,x_1,x_1). \tag{2.3}$$

Now, we will show that the sequence $\{x_n\}$ is a Cauchy sequence. Using the rectangle inequality, the inequality (2.3) and for all $n,m \in \mathbb{N}$, $n < m$, we get

$$\begin{aligned}G(x_n,x_m,x_m) &\leq G(x_n,x_{n+1},x_{n+1}) + G(x_{n+1},x_{n+2},x_{n+2}) + \cdots + G(x_{m-1},x_m,x_m) \\ &\leq k^n G(x_0,x_1,x_1) + k^{n+1}G(x_0,x_1,x_1) + \cdots + k^{m-1}G(x_0,x_1,x_1) \\ &= \left(k^n + k^{n+1} + \cdots + k^{m-1}\right)G(x_0,x_1,x_1) \\ &\leq \frac{k^n}{1-k}G(x_0,x_1,x_1).\end{aligned}$$

Taking limit as $n,m \to +\infty$ in above inequality, we get

$$\lim_{n,m\to+\infty} G(x_n,x_m,x_m) = 0.$$

This implies that $\{x_n\}$ is a Cauchy sequence. Since $(X,G)$ is complete $G-$metric space, there exists $u \in X$ such that $x_n \to u$ as $n \to +\infty$. We suppose that $Tu \neq u$. Then

$$\begin{aligned}G(x_{n+1},Tu,Tu) &\leq k\left[G(x_n,x_{n+1},x_{n+1})\right]^{\alpha}\left[G(u,Tu,Tu)\right]^{1-\alpha} \\ &= k\left[G(x_n,Tx_n,Tx_n)\right]^{\alpha}\left[G(u,Tu,Tu)\right]^{1-\alpha}.\end{aligned}$$

Taking the limit as $n \to +\infty$ in above inequality, this leads to

$$G(u,Tu,Tu) \leq kG(u,Tu,Tu) < G(u,Tu,Tu).$$

Which is a contradiction, hence $Tu = u$. □

Now, we will state and prove our next result corresponding to Theorem (1.35) for interpolative type contraction.

**Theorem 2.2.** Let $(X,G)$ be a complete $G-$metric space, $\alpha_G : X \times X \times X \to [0,+\infty)$ and $T : X \to X$ be an $\alpha_G-$admissible mapping satisfying

$$\alpha_G(x,y,z)G\left(Tx,Ty,Tz\right) \leq k\left[G\left(x,y,z\right)\right]^{\alpha}\left[G\left(x,Tx,Tx\right)\right]^{\beta}\left[G\left(y,Ty,Ty\right)\right]^{\gamma}\left[G\left(z,Tz,Tz\right)\right]^{1-\alpha-\beta-\gamma}, \tag{2.4}$$

where $\alpha_G(x,y,z) \geq 1$ for all $x,y,z \in X\backslash Fix\left(T\right)$, $k \in [0,1)$ and $\alpha,\beta,\gamma \in (0,1)$. Then, $T$ has a fixed point in $X$.

**Proof .** Since $T$ holds the condition (2.4), $\alpha_G(x,y,y)$ and $\alpha_G(y,x,x) \geq 1$, we have

$$\begin{aligned} G\left(Tx,Ty,Ty\right) &\leq \alpha_G(x,y,y)G\left(Tx,Ty,Ty\right) \\ &\leq k\left[G\left(x,y,y\right)\right]^{\alpha}\left[G\left(x,Tx,Tx\right)\right]^{\beta}\left[G\left(y,Ty,Ty\right)\right]^{\gamma}\left[G\left(y,Ty,Ty\right)\right]^{1-\alpha-\beta-\gamma} \\ &= k\left[G\left(x,y,y\right)\right]^{\alpha}\left[G\left(x,Tx,Tx\right)\right]^{\beta}\left[G\left(y,Ty,Ty\right)\right]^{1-\alpha-\beta}, \end{aligned}$$

and

$$\begin{aligned} G\left(Ty,Tx,Tx\right) &\leq \alpha_G(y,x,x)G\left(Ty,Tx,Tx\right) \\ &\leq k\left[G\left(y,x,x\right)\right]^{\alpha}\left[G\left(y,Ty,Ty\right)\right]^{\beta}\left[G\left(x,Tx,Tx\right)\right]^{\gamma}\left[G\left(x,Tx,Tx\right)\right]^{1-\alpha-\beta-\gamma} \\ &= k\left[G\left(y,x,x\right)\right]^{\alpha}\left[G\left(y,Ty,Ty\right)\right]^{\beta}\left[G\left(x,Tx,Tx\right)\right]^{1-\alpha-\beta}, \end{aligned}$$

for all $x,y \in X\backslash Fix\left(T\right)$. Suppose that $(X,G)$ is symmetric. Using the Proposition (1.9) and (2.4), we obtain that

$$\begin{aligned} \frac{1}{2}d_G\left(Tx,Ty\right) &\leq k\left[\frac{1}{2}d_G\left(x,y\right)\right]^{\alpha}\left[\frac{1}{2}d_G\left(x,Tx\right)\right]^{\beta}\left[\frac{1}{2}d_G\left(y,Ty\right)\right]^{1-\alpha-\beta} \\ &= \frac{k}{2}[d_G\left(x,y\right)]^{\alpha}[d_G\left(x,Tx\right)]^{\beta}[d_G\left(y,Ty\right)]^{1-\alpha-\beta}, \end{aligned}$$

which implies that

$$d_G\left(Tx,Ty\right) \leq k\left[d_G\left(x,y\right)\right]^{\alpha}\left[d_G\left(x,Tx\right)\right]^{\beta}\left[d_G\left(y,Ty\right)\right]^{1-\alpha-\beta},$$

for all $x,y \in X\backslash Fix\left(T\right)$. From the above inequality, we know that $T$ is an interpolative Reich-Rus-Ciric type contraction in $(X,d_G)$ metric space. Since $(X,d_G)$ is a complete metric space, then $T$ has a fixed point in $X$ from Proposition (1.40).

Now, we suppose that $(X,G)$ is not symmetric. From the definition of metric space $(X,d_G)$ and (2.4), we can write

$$\begin{aligned} \frac{1}{3}d_G\left(Tx,Ty\right) &\leq k\left[\frac{2}{3}d_G\left(x,y\right)\right]^{\alpha}\left[\frac{2}{3}d_G\left(x,Tx\right)\right]^{\beta}\left[\frac{2}{3}d_G\left(y,Ty\right)\right]^{1-\alpha-\beta} \\ &= \frac{2k}{3}\left[d_G\left(x,y\right)\right]^{\alpha}\left[d_G\left(x,Tx\right)\right]^{\beta}\left[d_G\left(y,Ty\right)\right]^{1-\alpha-\beta} \end{aligned}$$

and

$$d_G\left(Tx,Ty\right) \leq 2k\left[d_G\left(x,y\right)\right]^{\alpha}\left[d_G\left(x,Tx\right)\right]^{\beta}\left[d_G\left(y,Ty\right)\right]^{1-\alpha-\beta}$$

for all $x,y \in X\backslash Fix\left(T\right)$. Since $k \in [0,1)$, we are not sure that $2k < 1$. But we can prove the existence of fixed point using properties of $G-$metric space. Assume that $x_0 \in X$ is any arbitrary point and the sequence $\{x_n\}$ is a Picard sequence defined as $x_n = Tx_{n-1} = T^n x_0$. Since $T$ is an $\alpha_G-$admissible mapping and $\alpha_G(x_0,x_1,x_1) \geq 1$ implies $\alpha_G(x_1,x_2,x_2) = \alpha_G(Tx_0,Tx_1,Tx_1) \geq 1$, continuing on the same lines, we have $\alpha_G(x_n,x_{n+1},x_{n+1}) \geq 1$. From the condition (2.4), we have

$$\begin{aligned} G\left(x_n,x_{n+1},x_{n+1}\right) &\leq \alpha_G(x_n,x_{n+1},x_{n+1})G\left(x_n,x_{n+1},x_{n+1}\right) \\ &\leq k\left[G\left(x_{n-1},x_n,x_n\right)\right]^{\alpha+\beta}\left[G\left(x_n,x_{n+1},x_{n+1}\right)\right]^{1-\alpha-\beta}, \end{aligned}$$

which implies that

$$\left[G\left(x_n,x_{n+1},x_{n+1}\right)\right]^{\alpha+\beta} \leq k\left[G\left(x_{n-1},x_n,x_n\right)\right]^{\alpha+\beta}.$$

Using the above inequality, we get

$$G\left(x_n, x_{n+1}, x_{n+1}\right) \leq kG\left(x_{n-1}, x_n, x_n\right).$$

Continuing on the same way, we obtain

$$G\left(x_n, x_{n+1}, x_{n+1}\right) \leq k^n G\left(x_0, x_1, x_1\right). \tag{2.5}$$

On the same lines of proof of Theorem (2.1), we can show that the sequence $\{x_n\}$ is a Cauchy sequence. Since $(X, G)$ is a complete $G-$metric space, then the sequence $\{x_n\}$ is convergent, (say) converges to $u \in X$ such that $x_n \to u$ as $n \to +\infty$. Then

$$\begin{aligned} G\left(x_{n+1}, Tu, Tu\right) &\leq \alpha_G(x_{n+1}, u, u) G\left(x_{n+1}, Tu, Tu\right) \\ &\leq k\left[G\left(x_n, u, u\right)\right]^{\alpha}\left[G\left(x_n, x_{n+1}, x_{n+1}\right)\right]^{\beta}\left[G\left(u, Tu, Tu\right)\right]^{1-\alpha-\beta} \\ &= k\left[G\left(x_n, u, u\right)\right]^{\alpha}\left[G\left(x_n, Tx_n, Tx_n\right)\right]^{\beta}\left[G\left(u, Tu, Tu\right)\right]^{1-\alpha-\beta}. \end{aligned}$$

Taking the limit as $n \to +\infty$ in above inequality, we have

$$G\left(x_{n+1}, Tu, Tu\right) \to 0.$$

That is, $x_{n+1} \to Tu$. Since $x_n \to u$ and $x_{n+1} \to Tu$, we have $Tu = u$. □

Finally, we give the following result corresponding to Theorem 1.36 using interpolation notion in $G-$metric space.

**Theorem 2.3.** Let $(X, G)$ be a complete $G-$metric space and $T : X \to X$ be a $\alpha_G-$admissible mapping satisfying the following condition

$$\alpha_G(x, y, z) G\left(Tx, Ty, Tz\right) \leq k\left[G\left(x, y, z\right)\right]^{\alpha} \max\left\{\begin{array}{c} \left[G\left(x, Tx, Tx\right)\right]^{1-\alpha}, \\ \left[G\left(y, Ty, Ty\right)\right]^{1-\alpha}, \\ \left[G\left(z, Tz, Tz\right)\right]^{1-\alpha} \end{array}\right\}, \tag{2.6}$$

with $\alpha_G(x, y, z) \geq 1$ for all $x, y, z \in X \backslash Fix\left(T\right)$, where $k \in [0, 1)$ and $\alpha \in (0, 1)$. Then, $T$ has a fixed point in $X$.

**Proof .** From (2.6), we have

$$G\left(Tx, Ty, Ty\right) \leq \alpha_G(x, y, y) G\left(Tx, Ty, Ty\right) \leq k\left[G\left(x, y, y\right)\right]^{\alpha} \max\left\{\begin{array}{c} \left[G\left(x, Tx, Tx\right)\right]^{1-\alpha}, \\ \left[G\left(y, Ty, Ty\right)\right]^{1-\alpha} \end{array}\right\}$$

and

$$G\left(Ty, Tx, Tx\right) \leq \alpha_G(y, x, x) G\left(Ty, Tx, Tx\right) \leq k\left[G\left(y, x, x\right)\right]^{\alpha} \max\left\{\begin{array}{c} \left[G\left(y, Ty, Ty\right)\right]^{1-\alpha}, \\ \left[G\left(x, Tx, Tx\right)\right]^{1-\alpha} \end{array}\right\},$$

for all $x, y \in X \backslash Fix\left(T\right)$. We assume that $(X, G)$ is symmetric. From the Proposition (1.9) and (2.6), we write

$$\frac{1}{2} d_G\left(Tx, Ty\right) \leq k\left[\frac{1}{2} d_G\left(x, y\right)\right]^{\alpha} \max\left\{\left[\frac{1}{2} d_G\left(x, Tx\right)\right]^{1-\alpha}, \left[\frac{1}{2} d_G\left(y, Ty\right)\right]^{1-\alpha}\right\}.$$

Then

$$d_G\left(Tx, Ty\right) \leq k\left[d_G\left(x, y\right)\right]^{\alpha} \max\left\{\left[d_G\left(x, Tx\right)\right]^{1-\alpha}, \left[d_G\left(y, Ty\right)\right]^{1-\alpha}\right\}$$

for all $x, y \in X \backslash Fix\left(T\right)$. Then the inequality (2.6) is a special case of the interpolative Reich-Rus-Cirić type contraction in $(X, d_G)$ metric space. Therefore we say that $T$ has a fixed point in $X$ from Proposition (1.40). However, if $(X, G)$ is not symmetric, then

$$\begin{aligned} \frac{1}{3} d_G\left(Tx, Ty\right) &\leq k\left[\frac{2}{3} d_G\left(x, y\right)\right]^{\alpha} \max\left\{\left[\frac{2}{3} d_G\left(x, Tx\right)\right]^{1-\alpha}, \left[\frac{2}{3} d_G\left(y, Ty\right)\right]^{1-\alpha}\right\} \\ &= \frac{2k}{3}\left[d_G\left(x, y\right)\right]^{\alpha} \max\left\{\left[d_G\left(x, Tx\right)\right]^{1-\alpha}, \left[d_G\left(y, Ty\right)\right]^{1-\alpha}\right\} \end{aligned}$$

which implies that

$$d_G(Tx,Ty) \leq 2k[d_G(x,y)]^{\alpha} \max\left\{[d_G(x,Tx)]^{1-\alpha}, [d_G(y,Ty)]^{1-\alpha}\right\},$$

for all $x, y \in X \backslash Fix(T)$. Since $k \in [0,1)$, we are not sure about $2k < 1$. But to the existence of a fixed point using properties of $G-$metric space, we can assume that $x_0 \in X$ is an arbitrary point and the sequence $\{x_n\}$ is a Picard sequence defined as $x_n = Tx_{n-1} = T^n x_0$. Assume that $x_0 \in X$ is any arbitrary point and the sequence $\{x_n\}$ is a Picard sequence defined as $x_n = Tx_{n-1} = T^n x_0$. Since $T$ is an $\alpha_G-$admissible mapping and $\alpha_G(x_0, x_1, x_1) \geq 1$ implies $\alpha_G(x_1, x_2, x_2) = \alpha_G(Tx_0, Tx_1, Tx_1) \geq 1$, continuing on the same lines, we have $\alpha_G(x_n, x_{n+1}, x_{n+1}) \geq 1$. From the condition (2.6), we have

$$\begin{aligned} G(x_n, x_{n+1}, x_{n+1}) &\leq \alpha_G(x_{n-1}, x_n, x_n) G(x_n, x_{n+1}, x_{n+1}) \\ &\leq k\left[G(x_{n-1}, x_n, x_n)\right]^{\alpha} \max\left\{\begin{array}{c} \left[G(x_{n-1}, Tx_{n-1}, Tx_{n-1})\right]^{1-\alpha}, \\ \left[G(x_n, Tx_n, Tx_n)\right]^{1-\alpha} \end{array}\right\} \\ &= k\left[G(x_{n-1}, x_n, x_n)\right]^{\alpha} \max\left\{\begin{array}{c} \left[G(x_{n-1}, x_n, x_n)\right]^{1-\alpha}, \\ \left[G(x_n, x_{n+1}, x_{n+1})\right]^{1-\alpha} \end{array}\right\}. \end{aligned} \tag{2.7}$$

We have two cases:

Case 1: If $\max\left\{G(x_{n-1}, x_n, x_n)^{1-\alpha}, G(x_n, x_{n+1}, x_{n+1})^{1-\alpha}\right\} = G(x_{n-1}, x_n, x_n)^{1-\alpha}$, from (2.7), we have

$$G(x_n, x_{n+1}, x_{n+1}) \leq kG(x_{n-1}, x_n, x_n). \tag{2.8}$$

Case 2: If $\max\left\{G(x_{n-1}, x_n, x_n)^{1-\alpha}, G(x_n, x_{n+1}, x_{n+1})^{1-\alpha}\right\} = G(x_n, x_{n+1}, x_{n+1})^{1-\alpha}$, from (2.7)

$$G(x_n, x_{n+1}, x_{n+1}) \leq k\left[G(x_{n-1}, x_n, x_n)\right]^{\alpha} \left[G(x_n, x_{n+1}, x_{n+1})\right]^{1-\alpha},$$

which implies that

$$G(x_n, x_{n+1}, x_{n+1}) \leq kG(x_{n-1}, x_n, x_n). \tag{2.9}$$

In both cases, using (2.8) and (2.9), we have

$$G(x_n, x_{n+1}, x_{n+1}) \leq k^n G(x_0, x_1, x_1).$$

Use the same lines of proof of Theorem 2.1, we can show that the sequence $\{x_n\}$ is a Cauchy sequence. Since $(X, G)$ is a complete space, the sequence $\{x_n\}$ is convergent. Let $u \in X$ such that $x_n \to u$ as $n \to +\infty$. Then

$$\begin{aligned} G(x_{n+1}, Tu, Tu) &\leq \alpha_G(x_n, u, u) G(x_{n+1}, Tu, Tu) \\ &\leq k\left[G(x_n, u, u)\right]^{\alpha} \max\left\{\begin{array}{c} \left[G(x_n, Tu, Tu)\right]^{1-\alpha}, \\ \left[G(u, Tu, Tu)\right]^{1-\alpha} \end{array}\right\}. \end{aligned}$$

Taking the limit as $n \to +\infty$ in above inequality, we have

$$G(x_{n+1}, Tu, Tu) \to 0.$$

This implies that $x_{n+1} \to Tu$. Since $x_n \to u$ and $x_{n+1} \to Tu$, we have $Tu = u$. $\square$

Now, we are going to provide an example to support our main result.

**Example 2.4.** Let $X = [0, 2]$ and define

$$G(x,y,z) = \begin{cases} 0 \text{ if } x = y = z; \\ \max\{x,y,z\} \text{ otherwise}, \end{cases}$$

be a complete $G$-metric space on $X$. Define a mapping $T : X \to X$ by

$$T(x) = \begin{cases} 1 \text{ if } x \in \left[0, \frac{1}{2}\right] \cup \{1\}; \\ \frac{\min\{1,x\}}{1+\max\{1,x\}} \text{ otherwise,} \end{cases}$$

and $\alpha_G : X \times X \times X \to [0, +\infty)$ by

$$\alpha(x, y, z) = \begin{cases} 0 \text{ if } x = y = z; \\ \frac{2}{\max\{x,y,z\}} \text{ otherwise.} \end{cases}$$

It is easy to see that $T$ is an $\alpha_G$-admissible mapping. Then, the contraction condition of Theorem (2.3) satisfied for $\alpha = \frac{1}{2}$ and $k = \frac{1}{3}$. Also, observe that all conditions of Theorem (2.3) fulfilled and 1 is a fixed point of $T$.

## 3 Conclusion

In this paper, we proposed the concept of $\alpha_G-$admissible interpolative type contraction mappings in $G$-metric spaces, we state and proved some convergence results for such classes of contractive mappings using the properties of $G$-metric space and find the fixed point results for such contractive mappings. We provided some examples to elaborate the results in the setting of $G-$metric spaces. Our results are new and general in the $G-$metric space.

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