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# Distribution network design in the supply chain using genetic algorithms

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**Abstract**

This paper is aimed at designing a distribution network of small industries in Arak City. The model presented in this paper will provide optimal rates of order quantity given by a supplier to the producer. NMFC model covers drops in prices and finance costs. Furthermore, the flexible time horizon planning permits the producer to use this model in different time lags like hour, day, and month. The genetic algorithm function has been used in Matlab for achieving the solution space and comparing the output results of the model with two EOQ and JIT models to calculate optimal order quantity. The sensitivity of all parameters has been taken into account to examine its effect on the model which indicates the higher effect of holding and warehousing costs on the total costs.

Keywords: model design, supply chain, distribution network, genetic algorithm, part manufacturing industry
2020 MSC: 68W50

## 1 Introduction

Supply chain management is focused on the exploitation of processes, technologies, and capabilities of suppliers for strengthening competitive advantages [5].

In today's world, we live among a complicated set of supply chains; the chains that move in parallel, some of them intersect each other, and help to fulfill human needs. That's why, if we want to look at the issue more accurately, we'd better use supply chain network instead of supply chains [4]. In fact, distribution refers to allocation of a certain quantity of goods to the consumer that fulfills his needs [9].

Besides product sale and promotion, distributors undertake other activities such as stock management, warehousing affairs, product transportation, and after sale services. Also distributor may be only an intermediate between the producer and the customer so that he never owns the product. This type of distributor undertakes mainly affairs related to the product sale and promotion. In both states, by development of customer expectations and change of available products, distributors follow up customer needs constantly and fulfill them through existing products [7]. When the suppliers have a long distance with the customers, use of a distribution center for transferring a high quantity of products to a place near the final customers creates the advantages of increasing volume in transpirations with long distance. An efficient distribution network must try for achieving different goals of supply chain, from cost reduction to high accountability to the customer needs and reduction of delivery time and so on. The main effective factors are accountability, diversity of products, product availability, and visibility of orders [8].

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Arab et. al. [1] has solved a new mathematical model for scheduling in the distribution networks by optimizing multi-objective mass particles. Tavakoli [13] has evaluated demand risk, dangerous goods transportation, and startup costs in the supply chain. Sharifi [8] has identified and prioritized effective factors on efficiency of product distribution network. Grant & Banomyong [6] carried out a study on the design of distribution chain of products used in Thailand and Japan. Zanjirani [14] has designed a competitive supply chain. Costantino [3] has designed distribution networks by a hierarchical optimization method.

It must be noted that no independent and comprehensive study about the effective factors on the efficiency of product distribution network that determines the relative effect of each factor has been conducted in the country by present. So, deficiency of studies in this regard is quite evident. Therefore, the main question of the present paper is that how is the model of distribution network design in part manufacturing companies of Arak by using optimizer approach and meta-heuristic genetic algorithm.

## 2 Materials and Methods

The present paper is a descriptive survey in terms of research method. It is also an applied research in terms of objective, and finally it is a field research in terms of data collection method.

The statistical universe is comprised of the staff and managers of a part manufacturing company in the automobile industry in Arak industrial complex. The statistical universe includes all managers and experts of the part manufacturing company whose number reaches 56 in the factory and sale centers all over the country. Due to the type of research and limited number of experts, the census method was used for sampling. And 3 senior managers of the company were selected for interview and finalizing the results.

## 3 Findings

### 3.1 Objective Function

The cost of products reduces when the quantity of order increases $E_i = \{1, 2, 3, \cdots, \alpha_0\}$. $Q_i^{e_i-1}$ is the set of product price reduction, $Q_i^{e_i}$ is lower and upper limit of price reduction for product $i$ $(Q_c)$. If $q_i^j$ is between $Q_i^{e_i-1}$ is the cost of $L_i^j \cdot P_i^{e_i}$, the price of product $i$ will be $Q_i^{e_i}$, selling product $i$ at the time point $j$. so we have:

$$\text{if} \quad Q_i^{e_i-1} \leq q_i^j < Q_i^{e_i} \quad \text{then} \quad L_i^j = p_i^{e_i} q_i^j, \ \forall\ i, j \tag{3.1}$$

The producer needs loan to purchase product requirements and the most common type of loan is the payment that each time point has a fixed amount at the time horizon. The rate of interest equals $r$. for example, the rate of annual interest is 6 percent and the interval is one month and the interest rate for one month equals $r = \frac{0.06}{12} = 0.005$.

Fixed payment $f_i^j$ at any time interval $j$, to $n$ time points for loan $L_i^j$ with the interest rate $r$ equals:

$$F_i^j = \frac{r(1+r)^{n-j+1}}{(1+r)^{n-j+1} - 1}, \ \forall 1, j \tag{3.2}$$

The producer receives the loan $L_i^j$ at the beginning of time interval and starts to refund a fixed amount at the end of the interval. If $P(q_i^j)$ is the total cost of product $i$ at the interval $j$ that equals total amount of interest refund for product $i$ from time point $j$ to the end of time horizon, then we have:

$$P(q_i^j) = (n - j + 1) \star F_i^j; \ \forall\ i, j \tag{3.3}$$

and to see how equation (3.3) is used, $P(q_i^n)$ is calculated by equation (3.4)

$$\begin{aligned} &Q_i^{e_i-1} \leq q_i^n < Q_i^{e_i} \\ &P(q_i^n) = (n - n + 1)F_i^n = L_i^n \frac{r(1+r)^{n-n+1}}{(1+r)^{n-n+1} - 1} = p_i^{e_i} q_i^n \frac{r(1+r)}{(1+r) - 1}. \end{aligned} \tag{3.4}$$

Equation (3.4) means that if producer orders $q_i^n$, the cost of product $i$ equals $P_i^{e_i}$ and the loan that producer needs will equal $L_i^n = P_i^{e_i} \star q_i^n$. So the total amount of refund including interest from the beginning to the end of time point

$n$ equals $P_i^{e_i} q_i^n \frac{r(1+r)}{(1+r)-1}$ that equals fixed payable cost $F_i^r$. We consider $R_v^j(q_i^j : i \in c_v)$ equaling product transportation cost that includes cost $c_v$. As mentioned earlier, the classification system NMFC for each different weight range $k \in K$ has a certain price; so we will have:

$$\text{if} \quad a_{k-1} \leq \sum_{i \in c_v} q_i^j w_i < a_k, \quad \text{then}$$

$$R_v^j(q_i^j : i \in c_v) = 0.01 \min \left[ v_k^v \sum_{i \in c_v} q_i^j w_i, v_{k+1}^v \alpha_k \right]; \ \forall\, k, v, j \tag{3.5}$$

In equation (3.5), if the weight range equals $K$, the producer carries out a comparison between transportation cost of the existing weight range, $v_k^v \sum_{i \in c_v} q_i^j w_i$ and the next weight range $v_{k+1}^v \alpha_k$ to calculate the minimum transportation cost in $R_v^j(q_i^j : i \in c_v)$. When the product $i$ is ordered, the fixed cost of order is created. The order cost of product $i$ at the interval $j$ equals:

$$O(q_i^j) = o_i(\min[q_i^j, 1]), \ \forall\, i, j \tag{3.6}$$

Product $i$ has a single holding cost $h_i$ at each time interval. Total cost of holding for the ordered quantity of product $i$ between time intervals $j$ and $j+1$ equals equation (3.7).

$$H(q_i^j) = h_i l_i^j; \ \forall\, i, j. \tag{3.7}$$

Total holding cost for each product $i \in J$ along the whole time horizon will equal:

$$\begin{aligned}
&\sum_{i \in I} \sum_{j \in J} H(q_i^j) = \sum_{i \in I} h_i l_i^0 + \sum_{j \in J} h_i l_i^j) \\
&= \sum_{i \in I} (h_i l_i^0 + h_i l_i^1 + h_i l_i^2 + \cdots + h_i l_i^n) \\
&= \sum_{i \in I} (h_i l_i^0 + h_i (l_i^0 + q_2^j - d_i^0) + h_i (l_i^1 + q_2^j - d_i^1) + \cdots + h_i (l_i^{n-1} + q_2^j - d_i^{n-1})) \\
&\vdots \\
&= \sum_{i \in I} h_i \left[ (n+1) l_i^0 + \sum_{j \in J} \Big( (n-j+1) q_i^j - (n-j) d_i^j \Big) - n d_i^0 \right] \\
&= \sum_{i \in I} \sum_{j \in J} h_i \left[ (n+1) l_i^0 - (n-j) d_i^j - n d_i^0 + (n-j+1) q_i^j \right]
\end{aligned} \tag{3.8}$$

## 4 Model

Total cost includes cost of purchase, order, transportation, holding, and warehousing. By considering total cost as $C(q_i^j)$ in equations (3.4), (3.5), (3.6), and (3.7), we will have:

$$\begin{aligned}
&C(q_i^j) = \sum_{i \in I} \sum_{j \in J} \Big( P(q_i^j) + o(q_i^j) + H(q_i^j) \Big) + \sum_{j \in J} \sum_{v \in v} R_v^j(q_i^j;\ i \in c_v), \\
&P(q_i^j) = (n-j+1) \frac{r(1+r)^{n-j+1}}{(1+r)^{n-j+1} - 1} p_i^{e_i} q_i^j \quad \text{if} \quad Q_i^{e_i - 1} \leq q_i^j < Q_i^{e_i}, \ \forall\, i, j \\
&O(q_i^j) = o_i(\min[q_i^j, 1]), \ \forall\, i, j \\
&R_v^j(q_i^j : i \in c_v) = 0.01 \min \left[ v_k^v \sum_{i \in c_v} q_i^j w_i, v_{k+1}^v \alpha_k \right] \\
&\text{if} \quad \alpha_{k-1} \leq \sum_{i \in c_v} q_i^j w_i < \alpha_k; \ \forall\, k, v, j
\end{aligned} \tag{4.1}$$

To optimize the price $q_i^j$, we need to minimize total cost $C(q_i^j)$. So the model will be presented as below.

$$\sum_{J=0}^{j} q_i^J \geq -l_i^0 + \sum_{J=0}^{j} d_i^0,\ \forall i, j$$
$$\sum_{J=0}^{j} q_i^J \leq \min\left[\left[\frac{\alpha_i}{w_i}\right], \alpha_i - l_i^0 + \sum_{J=0}^{j-1} d_i^l\right],\ \forall\ i, j \quad \text{and}$$
$$q_i^j \in IN^0;\ \forall\ i, j. \tag{4.2}$$

### 4.1 Solving the model

Genetic algorithm model is used for solving equation 10. GA is started by the primary set of solutions called population. The members of population are called chromosomes that are evaluated based on a pre-defined fitness function which is total cost in our case. Each chromosome includes several genes. Gene indicates quantity of product i at the interval j. for example, if we have 10 products and 12 intervals, we will have 120 genes (volume of order) in each chromosome (figure 1). Chromosomes are created by successful replications that are called generation. A new generation is made by change of chromosomes in the existing population during crossover and mutation (figure 2).

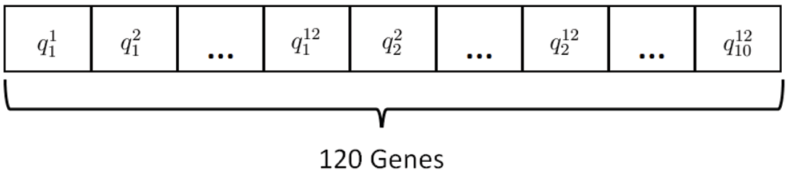


Figure 1: a chromosome with 12o genes

To make the related items in the model, we use the presented method in which generation, crossover, and mutation are selected randomly.

$$[x, fval, \text{exit } flag, output] = ga(fitnessfcn, nvars, A, b, [\ ], [\ ], LB, UB, [\ ], IntCon, Options).$$

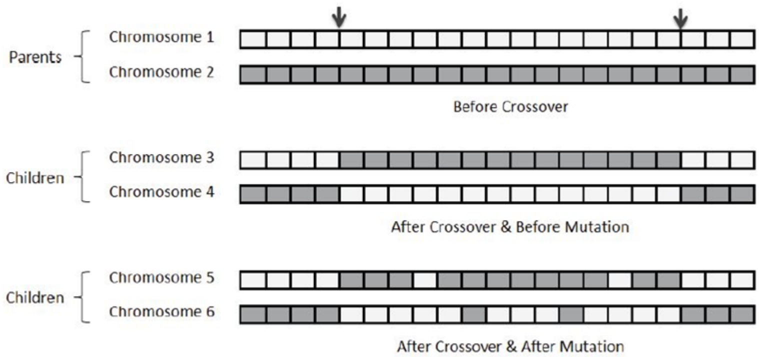


Figure 2: crossover and mutation

We first work with the input parameters. The number of accurate variable is nvars. A is the non linear matrix of unequal limitations and b is vector of non linear limitations in the form of $A_x \leq b$.

$$-\sum_{J=0}^{j} q_i^J \leq -\sum_{J=0}^{j} d_i^J + l_i^0,\ \forall i, j$$
$$\sum_{J=0}^{j} q_i^J \leq \min\left[\left[\frac{\alpha_i}{w_i}\right], \alpha_i - l_i^0 + \sum_{J=0}^{j-1} d_i^l\right],\ \forall\ i, j \tag{4.3}$$

The symbol "[]" represents replacement for a linear or non linear matrix of equal limitations, linear or non linear vector of equal limitations and functions of non linear limitations.

LB and UP are vectors of lower and upper limits. Options are the final input whose structure is as following.

$$\text{option} = \text{gaoptionset}('\text{Generations}', \text{value1}, '\text{PopulationSize}', \text{value2}, '\text{EliteCount}', \text{value3})$$

Population size of value 1 determines that how many chromosomes exist in each generation. With a large population size, the genetic algorithm searches the solution space more carefully for increasing minimum probability to a relative minimum. EliteCount is the number of chromosomes that remain with no change and go to the next generation. For an accurate problem, the minimum amount of EliteCount equals:

$$value3 = 0.05 \star \min(\max(10 \star nvar, 40)100)$$

One of the output parameters of GA is $x$ which is the best point that GA determines during generation production and $fval$ is the best fitness function found for point $x$. another parameter is $exitfval$. GA uses penalty function instead of fitness function to stop criterion. The last output parameter provides some information about the algorithm function.

## 5 Numerical study parameters

This part of the model will show a numerical study about 10 products and 12 time points and the lag between the points equals 1 month. Table 1 presents our objective in this case study $d_i^j$. The tables show products 1 to 9 that have a fixed demand every month. Items No. 2 and 73 have a fixed demand in a certain lag. And items No. 4, 5, and 8 have a certain model for their demand volume and items 6 and 7 do not have a certain model for their demand.

Table 1: the rate of demand for the studied industrial parts

| | | $j \in J$ | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 |
| $i \in I$ | 1 | 43 | 43 | 43 | 43 | 43 | 43 | 43 | 43 | 43 | 43 | 43 | 43 |
| | 2 | 145 | 0 | 145 | 0 | 145 | 0 | 145 | 0 | 145 | 0 | 115 | 0 |
| | 3 | 117 | 0 | 0 | 117 | 0 | 0 | 117 | 0 | 0 | 117 | 0 | 0 |
| | 4 | 17 | 20 | 17 | 20 | 17 | 20 | 17 | 20 | 17 | 20 | 17 | 20 |
| | 5 | 322 | 0 | 334 | 0 | 284 | 0 | 290 | 0 | 274 | 0 | 287 | 0 |
| | 6 | 38 | 19 | 56 | 41 | 51 | 51 | 37 | 34 | 34 | 41 | 52 | 39 |
| | 7 | 0 | 0 | 0 | 0 | 0 | 364 | 0 | 0 | 0 | 0 | 0 | 364 |
| | 8 | 500 | 300 | 83 | 500 | 300 | 83 | 500 | 300 | 83 | 500 | 300 | 83 |
| | 9 | 101 | 101 | 101 | 101 | 101 | 101 | 101 | 101 | 101 | 101 | 101 | 101 |
| | 10 | 126 | 124 | 128 | 0 | 105 | 119 | 122 | 128 | 0 | 121 | 122 | 120 |

Table 2: studied parameters

| $i$ | $l_i^0$ | $s_i$ | $\omega_i$ | $S_i$ | $w_i$ | $\alpha_i$ | $W_i$ | $v$ |
|---|---|---|---|---|---|---|---|---|
| 1 | 45 | 5.12 | 163.84 | 409.60 | 13107.20 | 80 | 32.0 | 3 |
| 2 | 140 | 10.00 | 228.80 | 10000.0 | 254196.80 | 1000 | 22.88 | 4 |
| 3 | 100 | 18.29 | 457.33 | 6402.67 | 182933.33 | 350 | 25.00 | 4 |
| 4 | 17 | 5.53 | 66.91 | 552.96 | 6021.73 | 90 | 12.10 | 7 |
| 5 | 300 | 8.23 | 275.66 | 9874.29 | 358354.28 | 1200 | 33.50 | 3 |
| 6 | 40 | 9.60 | 119.04 | 2400.0 | 28569.60 | 240 | 12.40 | 7 |
| 7 | 0 | 4.09 | 116.16 | 4014.08 | 104546.30 | 900 | 28.36 | 4 |
| 8 | 100 | 4.32 | 150.55 | 17280.0 | 752760.0 | 4000 | 34.85 | 3 |
| 9 | 0 | 5.83 | 76.39 | 1283.04 | 19099.8 | 220 | 13.10 | 7 |
| 10 | 128 | 9.41 | 316.24 | 17882.35 | 632470.58 | 1900 | 33.60 | 3 |

Table 2 shows all the research parameters. The first column is the index column and the next column includes the level of initial inventory $l_i^0$. Third and fourth columns show size $S_i$ and weight $w_i$ of products. The maximum volume of warehouse for product $i$ is $s_i$ in the fifth column and the maximum weight $w_i$ is in the sixth column. The maximum number of product i that can be stored in the factory warehouse is $\alpha_i$ in the seventh column. Eighth and ninth columns include weight per cubic foot $w_i$ and a set of classifications $v = \{3, 4, 5\}$.

Table 3 shows price costs. For example, $\forall\ i \in I$, $\forall\ e_i \in E_i$ and $P_i^{e_i}$. The table shows that for the product 5, the price cost equals $P_5^{2_5} = 9.0$ per unit. If the number of purchased units is between $Q_5^{t_5} = 51$ and $Q_5^{2_5} = 5$, the purchase cost with $r = 0.005$ will equal:

$$\text{if} \quad Q_s^{t_s} \leq q_s^J < Q_s^{2_s}$$

$$P(q_s^J) = (13 - J)\frac{0.005(1.005)^{13-j}}{(1.005)^{13-j} - 1} p_s^{2_s} q_s^J;\ \forall\ J$$

Table 3: the relation between price cost per unit and lag $|Q_i^{e_i - 1} - Q_i^{e_i}|$

| Item $i \in I$ | Price Cost $e_i \in E_i$ | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
| 1 | 1-21 | 21-51 | 51-121 | 121-351 | 351-∞ | | | | |
| | 12.5 | 11.25 | 10.50 | 10.25 | 10.06 | | | | |
| 2 | 1-51 | 51-101 | 101-136 | 136-501 | 501-∞ | | | | |
| | 18.75 | 17.75 | 16.25 | 15.75 | 15.00 | | | | |
| 3 | 1-∞ | | | | | | | | |
| | 19.25 | | | | | | | | |
| 4 | 1-101 | 101-∞ | | | | | | | |
| | 8.75 | 8.00 | | | | | | | |
| 5 | 1-51 | 51-151 | 151-251 | 251-401 | 401-801 | 801-∞ | | | |
| | 9.25 | 9.00 | 8.75 | 8.13 | 7.50 | 7.16 | | | |
| 6 | 1-16 | 16-22 | 22-61 | 61-251 | 251-∞ | | | | |
| | 13.75 | 13.50 | 13.25 | 12.50 | 12.25 | | | | |
| 7 | 1-101 | 101-201 | 201-301 | 301-401 | 401-501 | 501-601 | 601-701 | 701-801 | 801-∞ |
| | 10.0 | 9.50 | 9.25 | 9.00 | 8.75 | 8.50 | 8.00 | 7.75 | 7.50 |
| 8 | 1-501 | 501-2001 | 2001-∞ | | | | | | |
| | 6.00 | 5.25 | 5.00 | | | | | | |
| 9 | 1-101 | 101-201 | 201-∞ | | | | | | |
| | 7.75 | 7.50 | 7.25 | | | | | | |
| 10 | 1-121 | 121-501 | 501-1201 | 1201-∞ | | | | | |
| | 11.00 | 10.75 | 10.50 | 10.00 | | | | | |

Table 4 shows the transportation cost $v_k^v$. To understand how to use table 4, we consider $j = 4$ and $v = 3$, we use last column of table 2. Also table 2 provides values of $w_i$ that $i \in C_3$. If $\sum_{i \in C_3} q_i^4 w_i = 1800\ lb$, then we will have:

$$\sum_{i \in C_3} q_i^4 w_i = 63.84 q_1^4 + 275.66 q_5^4 + 150.55 q_8^4 + 326.24 q_{10}^4 = 1800$$

Final cost of transportation will be calculated as below.

$$1000 \leq \sum_{i \in C_3} q_i^4 w_i < 2000,$$

$$R_3^4(q_i^4;\ i \in C_3) = 0.01 \min[2.35 \star 1800, 2.07 \star 2000] = 44.40$$

A lower level of $42.40 = 2.07 \star 2000$ is exerted and will be paid to the factory owner for $k = 4$. If $\sum_{i \in C_3} q_i^4 w_i$ equals 1500, then the final cost of transportation will be as following.

If

$$1000 \leq \sum_{i \in C_3} q_i^4 w_i < 2000.$$

Then

$$R_3^4(q_i^4;\ i \in C_3) = 0.01 \min[2.35 \star 1500, 2.07 \star 2000] = 32.25$$

A lower rate of $2.35 \star 1500 = 35.25$ has been implemented and will be paid to the factor owner for $k = 3$.

Tables 5 and 6 show fixed costs of order $Q_i$ and holding $h_i$ per month.

Table 4: cost of parts transportation per 100 pounds of body weight of goods

| | | $k \in K$ | | | | | | |
|---|---|---|---|---|---|---|---|---|
| | | 1 | 2 | 3 | 4 | 5 | 6 | 7 |
| | | $a_{k-1} - a_k$ | | | | | | |
| | | 0-500 | 500-1000 | 1000-2000 | 2000-5000 | 5000-10000 | 10000-20000 | 20000-200000 |
| $v \in \mathcal{V}$ | 3 | 2.90 | 2.57 | 2.35 | 2.07 | 1.57 | 1.38 | 0.78 |
| | 4 | 3.25 | 2.67 | 2.55 | 2.46 | 1.88 | 1.64 | 0.99 |
| | 7 | 4.00 | 3.70 | 3.20 | 2.50 | 2.16 | 2.03 | 1.09 |

Table 5: costs of order of industrial parts

| i | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
|---|---|---|---|---|---|---|---|---|---|---|
| $o_i$ | 69.13 | 170.37 | 110.36 | 19.35 | 201.05 | 76.63 | 82.83 | 216.39 | 104.17 | 156.41 |

Table 6: costs of holding in the industrial parts warehouse

| $i$ | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
|---|---|---|---|---|---|---|---|---|---|---|
| $h_i$ | 5.17 | 7.53 | 4.59 | 1.65 | 4.31 | 5.96 | 4.37 | 2.48 | 3.98 | 4.94 |

**The results of numerical study in part manufacturing**

Table 7 shows a set of orders in the form of $q_i^j$, $\forall i, j$. The output results have been obtained after 200 runs during 13 hours with the generation number 3000 and population equaling 5000 in the genetic algorithm. In each run, Matlab software will give us the final cost with a set of different order quantities. Then it will compare them and will provide the most optimal value. Figure 3 will show all the results of 200 runs of genetic algorithm in Matlab software and the most optimal solution has been obtained in the $83^{rd}$ run that gives the minimum final price (140185.12 $).

Table 7: the most optimal value of order quantity

| | | $j \in J$ | | | | | | | | | | | | $\sum_{j=1}^{12} q_i^j$ | $\sum_{j=0}^{12} d_i^j - l_i^0$ | Total Cost |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | | | |
| $j \in J$ | 1 | 41 | 43 | 43 | 80 | 6 | 43 | 43 | 43 | 80 | 43 | 6 | 0 | 471 | 471 | |
| | 2 | 5 | 725 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 730 | 730 | |
| | 3 | 17 | 350 | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 368 | 368 | |
| | 4 | 20 | 90 | 0 | 37 | 0 | 37 | 0 | 21 | 0 | 0 | 0 | 0 | 205 | 205 | |
| | 5 | 22 | 1200 | 0 | 269 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1491 | 1491 | |
| | 6 | 17 | 240 | 0 | 97 | 0 | 51 | 48 | 0 | 0 | 0 | 0 | 0 | 453 | 453 | |
| | 7 | 0 | 728 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 728 | 728 | |
| | 8 | 700 | 2732 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 3432 | 3432 | |
| | 9 | 202 | 194 | 26 | 202 | 0 | 101 | 101 | 202 | 0 | 184 | 0 | 0 | 1212 | 1212 | |
| | 10 | 122 | 965 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1087 | 1087 | |
| | | | | | | | | | | | | | | | | 140185.12 |

Inequality of columns $\sum_{j=1}^{12} q_i^j$ and $\sum_{j=0}^{12} d_i^j - l_i^0$ in table 1-7 shows that all goals have been obtained in the time horizon plan. The table shows that any product has its own ordering model. For example, for product No. 7, the order model is the total demand of annual consumption in the second month and for product No. 1, the order is the variable number of that product per month. Tables 8 and 9 show the level of inventory and cost of holding in the warehouse. The maximum cost of holding relates to the product No. 8 and equals 5353.83 Dollars because its annual demand occurs in the first two month of the year. Although it has the highest cost of holding, it is replicated 2 times per year instead of 12 times.

Tables 10, 11 and 12 show respectively the optimal costs of ordering, purchase, and transportation for all the related products in one year.

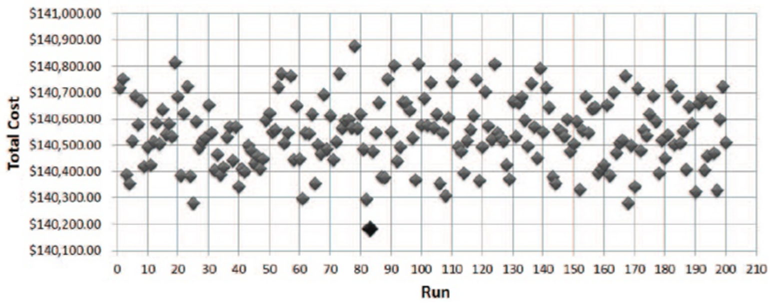


Figure 3: comparing solutions in different runs of software

Table 8: the level of optimal inventory $l_i^j$ for the industrial parts

| $i \in \mathcal{I}$ | $j \in \mathcal{J}$: 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 1 | 43 | 43 | 43 | 43 | 80 | 43 | 43 | 43 | 43 | 80 | 80 | 43 | 0 |
| 2 | 140 | 0 | 725 | 580 | 580 | 435 | 435 | 290 | 290 | 145 | 145 | 0 | 0 |
| 3 | 100 | 0 | 350 | 350 | 234 | 234 | 234 | 117 | 117 | 117 | 0 | 0 | 0 |
| 4 | 17 | 20 | 90 | 73 | 90 | 73 | 90 | 73 | 74 | 57 | 37 | 20 | 0 |
| 5 | 300 | 0 | 1200 | 866 | 1135 | 851 | 851 | 561 | 561 | 287 | 287 | 0 | 0 |
| 6 | 40 | 19 | 240 | 184 | 240 | 189 | 189 | 200 | 166 | 132 | 91 | 39 | 0 |
| 7 | 0 | 0 | 728 | 728 | 728 | 728 | 364 | 364 | 364 | 364 | 364 | 364 | 0 |
| 8 | 100 | 300 | 2732 | 2649 | 2149 | 1849 | 1766 | 1266 | 966 | 883 | 383 | 83 | 0 |
| 9 | 0 | 101 | 194 | 119 | 220 | 119 | 119 | 119 | 220 | 119 | 202 | 101 | 0 |
| 10 | 128 | 124 | 965 | 837 | 837 | 732 | 613 | 491 | 363 | 363 | 242 | 120 | 0 |

Table 9: cost of holding in the warehouse $H(q_i^j)$ for the parts

| $i \in \mathcal{I}$ | $j \in \mathcal{J}$: 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | Holding Cost |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 1 | 33.24 | 31.76 | 31.76 | 31.76 | 59.09 | 31.76 | 31.76 | 31.76 | 31.76 | 59.09 | 59.09 | 31.76 | 0 | 464.62 |
| 2 | 150.67 | 0 | 780.25 | 624.40 | 624.20 | 468.15 | 468.15 | 312.10 | 312.10 | 156.05 | 156.65 | 0 | 0 | 4051.89 |
| 3 | 65.52 | 0 | 229.31 | 229.31 | 153.31 | 153.31 | 153.31 | 76.65 | 76.65 | 76.65 | 0 | 0 | 0 | 1214.02 |
| 4 | 4.02 | 4.72 | 21.26 | 17.25 | 21.26 | 17.25 | 21.26 | 17.25 | 17.48 | 13.47 | 8.74 | 4.72 | 0 | 168.68 |
| 5 | 184.48 | 0 | 737.93 | 532.54 | 697.96 | 523.32 | 523.32 | 344.98 | 344.98 | 176.49 | 176.49 | 0 | 0 | 4242.49 |
| 6 | 33.95 | 16.13 | 203.70 | 156.17 | 203.70 | 160.42 | 160.42 | 169.75 | 140.89 | 112.04 | 77.24 | 33.10 | 0 | 1467.51 |
| 7 | 0 | 0 | 454.49 | 454.49 | 454.49 | 454.49 | 227.25 | 227.25 | 227.25 | 227.25 | 227.25 | 227.25 | 0 | 3181.44 |
| 8 | 35.39 | 106.18 | 966.99 | 937.61 | 760.64 | 654.45 | 625.07 | 448.10 | 341.91 | 312.54 | 135.56 | 29.38 | 0 | 5353.83 |
| 9 | 0 | 57.37 | 110.20 | 67.59 | 124.96 | 67.59 | 67.59 | 67.59 | 124.96 | 67.59 | 114.74 | 57.37 | 0 | 927.57 |
| 10 | 90.34 | 87.51 | 681.04 | 590.71 | 590.71 | 516.60 | 432.62 | 346.52 | 256.19 | 256.19 | 170.79 | 84.69 | 0 | 4103.90 |
| | | | | | | | | | | | | | | 25175.96 |

Cost of transportation has been calculated based on three classes $v = \{3, 4, 7\}NMFC$ among 18 types. From the last column of table 1, products No. 1, 5, 8 and 10 from class No. 3, products 2, 3 and 7 from class No. 4, and products 4, 6 and 9 from class No. 7 have been calculated. The final transportation cost equals 17137.06 $ and the highest cost of transportation relates to the class No. 3.

The lowest cost of ordering belongs to the product No. 7 that has only been ordered one time in the second month. The product No. 1 has the highest cost of ordering. The highest cost of purchase relates to the product No. 8 because it has the highest volume of order per year (3432 units).

Table 13 presents an abstract of all products and figure 4 shows an abstract of tables of these costs. The biggest sector of figure 4 relates to the purchase costs and equals 62%, and the smallest one relates to the order costs and equals 8%. It means that the purchase costs play the greatest role in the final costs. The holding cost is 18% and transportation cost in 12%. In the part industry, the level of inventory (88%) is larger than transportation cost (12%).

Table 10: optimal cost of ordering industrial parts

| $i \in I$ | $j \in J$ | | | | | | | | | | | | Ordering Cost |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | |
| 1 | 172.83 | 172.83 | 172.83 | 172.83 | 172.83 | 172.83 | 172.83 | 172.83 | 172.83 | 172.83 | 172.83 | 0 | 1901.08 |
| 2 | 425.93 | 425.93 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 851.86 |
| 3 | 275.90 | 275.90 | 0 | 275.90 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 827.70 |
| 4 | 48.38 | 48.38 | 0 | 48.38 | 0 | 48.38 | 48.38 | 48.38 | 0 | 0 | 0 | 0 | 241.89 |
| 5 | 502.27 | 502.27 | 0 | 502.27 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1506.82 |
| 6 | 191.14 | 191.14 | 0 | 191.14 | 0 | 191.14 | 191.14 | 0 | 0 | 0 | 0 | 0 | 955.70 |
| 7 | 0 | 207.08 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 207.08 |
| 8 | 540.98 | 540.98 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1081.96 |
| 9 | 260.43 | 260.43 | 260.43 | 260.43 | 0 | 260.43 | 260.43 | 260.43 | 0 | 260.43 | 0 | 0 | 2083.41 |
| 10 | 390.90 | 390.90 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 781.79 |
| | | | | | | | | | | | | | 10439.30 |

Table 11: optimal purchase cost $P(q_i^j)$ of industrial parts

| $i \in I$ | $j \in J$ | | | | | | | | | | | | Purchasing Cost |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | |
| 1 | 476.38 | 498.38 | 497.15 | 861.14 | 76.70 | 493.47 | 492.25 | 491.03 | 850.53 | 488.60 | 75.56 | 0 | 1901.08 |
| 2 | 96.82 | 11203.96 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 851.86 |
| 3 | 337.98 | 6941.31 | 0 | 19.73 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 827.70 |
| 4 | 180.74 | 811.32 | 0 | 331.90 | 0 | 330.26 | 0 | 186.52 | 0 | 0 | 0 | 0 | 241.89 |
| 5 | 215.85 | 8854.99 | 0 | 2242.01 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1506.82 |
| 6 | 237.03 | 3090.75 | 0 | 1243.01 | 0 | 689.33 | 647.18 | 0 | 0 | 0 | 0 | 0 | 955.70 |
| 7 | 0 | 5812.67 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 207.08 |
| 8 | 3795.53 | 14073.21 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1081.96 |
| 9 | 1512.53 | 1449.05 | 207.08 | 1501.36 | 0 | 746.97 | 745.12 | 1486.54 | 0 | 1347.36 | 0 | 0 | 2083.41 |
| 10 | 1354.51 | 10439 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 781.79 |
| | | | | | | | | | | | | | 10439.30 |

Table 12: optimal transportation cost $R_i^j(q_i^j;\ j \in C_v)$ of industrial parts

| $v \in V$ | $j \in J$ | | | | | | | | | | | | Transportation Cost |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | |
| 3 | 1224.73 | 8237.67 | 110.61 | 681.78 | 23.45 | 110.61 | 110.61 | 110.61 | 156.27 | 110.61 | 23.45 | 0 | 10900.40 |
| 4 | 164.33 | 4064.08 | 0 | 13.33 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 4241.75 |
| 7 | 217.47 | 537.28 | 50.03 | 320.28 | 0 | 217.47 | 217.47 | 217.47 | 0 | 217.47 | 0 | 0 | 1994.92 |
| | | | | | | | | | | | | | 17137.06 |

Table 13: comparison of industrial parts prices in the proposed model

| Transportation Cost | Inventory Cost | | | Total Cost |
|---|---|---|---|---|
| | Purchasing | Ordering | Holding | |
| 17137.06 | 87432.81 | 10439.30 | 25175.96 | 140185.12 |
| | 123048.06 | | | |

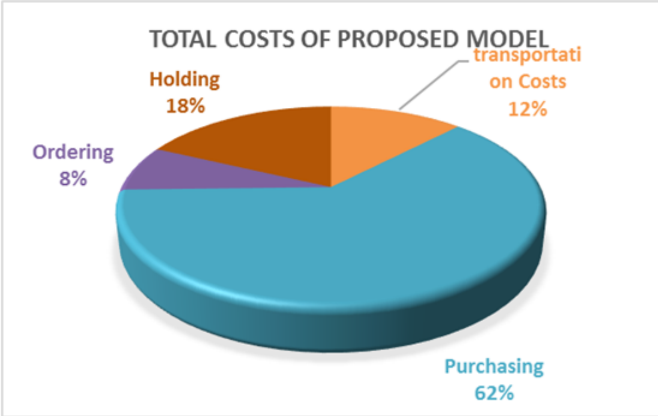


Figure 4: division of final costs of the proposed model

## 6 The comparative study

In this section, the optimal solutions of the research proposed model are compared with the results of just in time (JIT) model and equal order quantity (EOR) model and it shows that which one have a lower final cost.

Tables 14 and 15 show the order quantities and level of inventories in JIT model. To understand how table 15 has been calculated, we consider the product No. 6 to the time point 3. From table 1, we have:

$$l_6^3 = l_6^2 - d_6^2 + q_3^b = l_6^0 - d_6^0 + l_1^b - d_6^1 + q_2^6 - d_6^2 + q_3^6 = d_6^3$$

Table 14: the order quantity $q_i^j$ of industrial parts in JIT model

| $j \in J$ | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | $\sum_{j=1}^{12} q_i^j$ | $\sum_{j=0}^{12} d_i^j - l_i^0$ | Total Cost |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 1 | 41 | 43 | 43 | 43 | 43 | 43 | 43 | 43 | 43 | 43 | 43 | 0 | 471 | 471 | |
| 2 | 5 | 145 | 0 | 145 | 0 | 145 | 0 | 145 | 0 | 145 | 0 | 0 | 730 | 730 | |
| 3 | 17 | 0 | 117 | 0 | 0 | 117 | 0 | 0 | 117 | 0 | 0 | 0 | 368 | 368 | |
| 4 | 20 | 17 | 20 | 17 | 20 | 17 | 20 | 17 | 20 | 17 | 20 | 0 | 205 | 205 | |
| 5 | 22 | 334 | 0 | 284 | 0 | 290 | 0 | 274 | 0 | 287 | 0 | 0 | 1491 | 1491 | |
| 6 | 17 | 56 | 41 | 51 | 51 | 37 | 34 | 34 | 41 | 52 | 39 | 0 | 453 | 453 | |
| 7 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 364 | 0 | 728 | 728 | |
| 8 | 700 | 83 | 500 | 300 | 0 | 500 | 300 | 83 | 500 | 300 | 83 | 0 | 3432 | 3432 | |
| 9 | 202 | 101 | 101 | 101 | 0 | 101 | 101 | 101 | 101 | 101 | 101 | 0 | 1212 | 1212 | |
| 10 | 122 | 128 | 0 | 105 | 0 | 122 | 128 | 0 | 121 | 122 | 120 | 0 | 1087 | 1087 | |
| | | | | | | | | | | | | | | | 140899.91 |

Total holding cost in JIT model is less than holding costs of the model proposed in this paper. It seems that in JIT model, products are ordered as per demand at each time point; so ordering cost in JIT model is more than ordering cost in our proposed model and this is due to higher number of replications in JIT model.

Table 15: level of inventory $l_i^j$ of industrial parts in JIT model

| $i \in \mathcal{I}$ | $j \in \mathcal{J}$: 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 1 | 45 | 43 | 43 | 43 | 43 | 43 | 43 | 43 | 43 | 43 | 43 | 43 | 0 |
| 2 | 140 | 0 | 145 | 0 | 145 | 0 | 145 | 0 | 145 | 0 | 145 | 0 | 0 |
| 3 | 100 | 0 | 0 | 117 | 0 | 0 | 117 | 0 | 0 | 117 | 0 | 0 | 0 |
| 4 | 17 | 20 | 17 | 20 | 17 | 20 | 17 | 20 | 17 | 20 | 17 | 20 | 0 |
| 5 | 300 | 0 | 334 | 0 | 284 | 0 | 290 | 0 | 274 | 0 | 287 | 0 | 0 |
| 6 | 40 | 19 | 56 | 41 | 51 | 51 | 37 | 34 | 34 | 41 | 52 | 39 | 0 |
| 7 | 0 | 0 | 0 | 0 | 0 | 364 | 0 | 0 | 0 | 0 | 0 | 364 | 0 |
| 8 | 100 | 300 | 83 | 500 | 300 | 83 | 500 | 300 | 83 | 500 | 300 | 83 | 0 |
| 9 | 0 | 101 | 101 | 101 | 101 | 101 | 101 | 101 | 101 | 101 | 101 | 101 | 0 |
| 10 | 128 | 124 | 124 | 0 | 105 | 119 | 122 | 128 | 0 | 121 | 122 | 120 | 0 |

Table 16: holding costs $H(q_i^j)$ of industrial parts in JIT model

| $i \in \mathcal{I}$ | $j \in \mathcal{J}$: 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | Holding Cost |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 1 | 33.24 | 31.76 | 31.76 | 31.76 | 31.76 | 31.76 | 31.76 | 31.76 | 31.76 | 31.76 | 31.76 | 31.76 | 0 | 382.63 |
| 2 | 150.67 | 0 | 156.05 | 0 | 156.05 | 0 | 156.05 | 0 | 156.05 | 0 | 156.05 | 0 | 0 | 930.91 |
| 3 | 65.52 | 0 | 0 | 76.65 | 0 | 0 | 76.65 | 0 | 0 | 76.65 | 0 | 0 | 0 | 295.48 |
| 4 | 4.02 | 4.72 | 4.02 | 4.72 | 4.02 | 4.72 | 4.02 | 4.72 | 4.02 | 4.72 | 4.02 | 4.72 | 0 | 52.45 |
| 5 | 184.48 | 0 | 205.39 | 0 | 174.64 | 0 | 178.33 | 0 | 168.49 | 0 | 176.49 | 0 | 0 | 1087.83 |
| 6 | 33.95 | 16.13 | 47.53 | 34.80 | 43.29 | 43.29 | 31.40 | 28.86 | 28.86 | 34.80 | 44.14 | 33.10 | 0 | 420.14 |
| 7 | 0 | 0 | 0 | 0 | 0 | 227.25 | 0 | 0 | 0 | 0 | 0 | 227.25 | 0 | 454.49 |
| 8 | 35.39 | 106.18 | 29.38 | 176.97 | 106.18 | 29.38 | 176.97 | 106.18 | 29.38 | 176.97 | 106.18 | 29.38 | 0 | 1108.57 |
| 9 | 0 | 57.37 | 57.37 | 57.37 | 57.37 | 57.37 | 57.37 | 57.37 | 57.37 | 57.37 | 57.37 | 57.37 | 0 | 631.07 |
| 10 | 90.34 | 87.51 | 90.34 | 0 | 74.10 | 83.98 | 86.10 | 90.34 | 0 | 85.40 | 86.10 | 84.69 | 0 | 858.89 |
| | | | | | | | | | | | | | | 6222.46 |

Table 17: ordering costs $Q(q_i^j)$ of industrial parts in JIT model

| $i \in \mathcal{I}$ | $j \in \mathcal{J}$: 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | Ordering Cost |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 1 | 172.83 | 172.83 | 172.83 | 172.83 | 172.83 | 172.83 | 172.83 | 172.83 | 172.83 | 172.83 | 172.83 | 0 | 1901.08 |
| 2 | 425.93 | 425.93 | 0 | 425.93 | 0 | 425.93 | 0 | 425.93 | 0 | 425.93 | 0 | 0 | 851.86 |
| 3 | 275.90 | 0 | 275.90 | 0 | 0 | 275.90 | 0 | 0 | 275.90 | 0 | 0 | 0 | 827.70 |
| 4 | 48.38 | 48.38 | 48.38 | 48.38 | 48.38 | 48.38 | 48.38 | 48.38 | 48.38 | 48.38 | 48.38 | 0 | 241.89 |
| 5 | 502.27 | 502.27 | 0 | 502.27 | 0 | 502.27 | 0 | 502.27 | 0 | 502.27 | 0 | 0 | 1506.82 |
| 6 | 191.14 | 191.14 | 191.14 | 191.14 | 191.14 | 191.14 | 191.14 | 191.14 | 191.14 | 191.14 | 191.14 | 0 | 955.70 |
| 7 | 0 | 0 | 0 | 0 | 207.08 | 0 | 0 | 0 | 0 | 0 | 207.08 | 0 | 207.08 |
| 8 | 540.98 | 540.98 | 540.98 | 540.98 | 540.98 | 540.98 | 540.98 | 540.98 | 540.98 | 540.98 | 540.98 | 0 | 1081.96 |
| 9 | 260.43 | 260.43 | 260.43 | 260.43 | 260.43 | 260.43 | 260.43 | 260.43 | 260.43 | 260.43 | 260.43 | 0 | 2083.41 |
| 10 | 390.90 | 390.90 | 0 | 390.90 | 390.90 | 390.90 | 390.90 | 0 | 390.90 | 390.90 | 390.90 | 0 | 781.79 |
| | | | | | | | | | | | | | 10439.30 |

Table 19 shows a summary of transportation, purchasing, ordering, and holding costs in JIT model. In figure 5, the main sector belongs to purchasing cost. As seen, the smallest sector belongs to the holding costs.

In the equal order quantity (EOQ) model, the factor owner (producer) orders a certain and equal quantity at any time point. To understand how the EOQ model acts, consider that annual order quantity of product No. 8 equals 3432. The producer has 12 methods for ordering product No. 8 in one year. Table 20 shows these possible methods. Each method has a specific total cost. After comparing the final cost, the optimal solution is to order 858 units of product No. 8 per every three months.

Table 18: purchasing costs $P(q_i^j)$ of industrial parts in JIT model

| $i \in \mathcal{I}$ | $j \in \mathcal{J}$ | | | | | | | | | | | | Purchasing Cost |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | |
| 1 | 476.38 | 498.38 | 497.15 | 495.92 | 494.70 | 493.47 | 492.25 | 491.03 | 489.81 | 488.60 | 487.38 | 0 | 1901.08 |
| 2 | 96.82 | 2352.83 | 0 | 2341.22 | 0 | 2329.65 | 0 | 2318.12 | 0 | 2306.63 | 0 | 0 | 851.86 |
| 3 | 337.98 | 0 | 2314.65 | 0 | 0 | 2297.52 | 0 | 0 | 2280.47 | 0 | 0 | 0 | 827.70 |
| 4 | 180.74 | 153.25 | 179.85 | 152.49 | 178.96 | 151.74 | 178.08 | 150.99 | 177.19 | 150.24 | 176.31 | 0 | 241.89 |
| 5 | 215.85 | 2797.56 | 0 | 2367.03 | 0 | 2405.09 | 0 | 2261.15 | 0 | 2356.68 | 0 | 0 | 1506.82 |
| 6 | 237.03 | 764.45 | 558.30 | 692.76 | 691.04 | 500.10 | 458.42 | 457.28 | 550.06 | 695.90 | 520.63 | 0 | 955.70 |
| 7 | 0 | 0 | 0 | 0 | 3350.14 | 0 | 0 | 0 | 0 | 0 | 3300.59 | 0 | 207.08 |
| 8 | 3795.53 | 513.06 | 3083.12 | 1845.30 | 509.27 | 3060.30 | 1831.63 | 505.49 | 3037.59 | 1818.03 | 501.74 | 0 | 1081.96 |
| 9 | 1512.53 | 780.41 | 778.49 | 776.56 | 774.64 | 772.73 | 770.81 | 768.90 | 766.99 | 765.09 | 763.19 | 0 | 2083.41 |
| 10 | 1354.51 | 1417.62 | 0 | 1184.07 | 1338.62 | 1337.86 | 1400.18 | 0 | 1317.05 | 1324.64 | 1329.91 | 0 | 781.79 |
| | | | | | | | | | | | | | 10439.30 |

Table 19: comparison of costs of industrial parts in JIT model

| Transportation Cost | Inventory Cost | | | Total Cost |
|---|---|---|---|---|
| | Purchasing | Ordering | Holding | |
| 17594.39 | 93126.74 | 23956.31 | 6222.46 | 140899.91 |
| | 123305.51 | | | |

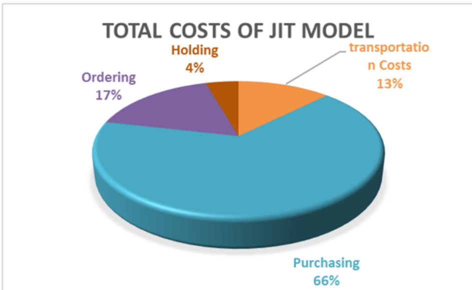


Figure 5: division of total costs of JIT model

Table 20: all possible methods for ordering product No. 8 in the equal order quantity (EOQ) model

| Possible Ways to order item 8 | $j \in J$ | | | | | | | | | | | | $\sum_{j=1}^{12} q_i^j$ | $\sum_{j=0}^{12} d_i^j - I_i^0$ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | | |
| 1 | 286 | 286 | 286 | 286 | 286 | 286 | 286 | 286 | 286 | 286 | 286 | 286 | 3432 | 3432 |
| 2 | 572 | 0 | 572 | 0 | 572 | 0 | 572 | 0 | 572 | 0 | 572 | 0 | 3432 | 3432 |
| 3 | 858 | 0 | 0 | 858 | 0 | 0 | 858 | 0 | 0 | 858 | 0 | 0 | 3432 | 3432 |
| 4 | 1144 | 0 | 0 | 0 | 1144 | 0 | 0 | 0 | 1144 | 0 | 0 | 0 | 3432 | 3432 |
| 5 | 1430 | 0 | 0 | 0 | 0 | 1430 | 0 | 0 | 0 | 0 | 572 | 0 | 3432 | 3432 |
| 6 | 1716 | 0 | 0 | 0 | 0 | 0 | 1716 | 0 | 0 | 0 | 0 | 0 | 3432 | 3432 |
| 7 | 2002 | 0 | 0 | 0 | 0 | 0 | 0 | 1430 | 0 | 0 | 0 | 0 | 3432 | 3432 |
| 8 | 2288 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1144 | 0 | 0 | 0 | 3432 | 3432 |
| 9 | 2574 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 858 | 0 | 0 | 3432 | 3432 |
| 10 | 2860 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 572 | 0 | 3432 | 3432 |
| 11 | 3146 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 286 | 3432 | 3432 |
| 12 | 3432 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 3432 | 3432 |

Tables 23, 24, 25 and 26 show holding, ordering, and purchasing costs, and summary of all costs in the equal order

Table 21: optimal order quantities in the equal order quantity model for all products

| $j \in J$ | | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | $\sum_{j=1}^{12} q_i^j$ | $\sum_{j=0}^{12} d_i^j - l_i^0$ | Total Cost |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | 1 | 39 | 39 | 39 | 39 | 39 | 39 | 39 | 39 | 39 | 40 | 40 | 40 | 471 | 471 | |
| | 2 | 121 | 0 | 121 | 0 | 122 | 0 | 122 | 0 | 122 | 0 | 122 | 0 | 730 | 730 | |
| | 3 | 92 | 0 | 0 | 92 | 0 | 0 | 92 | 0 | 0 | 92 | 0 | 0 | 368 | 368 | |
| | 4 | 17 | 17 | 17 | 17 | 17 | 17 | 17 | 17 | 17 | 17 | 17 | 18 | 205 | 205 | |
| | 5 | 248 | 0 | 248 | 0 | 248 | 0 | 249 | 0 | 249 | 0 | 249 | 0 | 1491 | 1492 | |
| | 6 | 37 | 37 | 37 | 38 | 38 | 38 | 38 | 38 | 38 | 38 | 38 | 38 | 453 | 454 | |
| | 7 | 121 | 0 | 121 | 0 | 122 | 0 | 121 | 0 | 121 | 0 | 122 | 0 | 728 | 728 | |
| | 8 | 858 | 0 | 0 | 858 | 0 | 0 | 858 | 0 | 0 | 858 | 0 | 0 | 3432 | 3432 | |
| | 9 | 202 | 101 | 101 | 101 | 101 | 101 | 101 | 101 | 101 | 101 | 101 | 0 | 1212 | 1212 | |
| | 10 | 90 | 90 | 90 | 90 | 91 | 91 | 91 | 91 | 90 | 91 | 91 | 91 | 1087 | 1087 | |
| | | | | | | | | | | | | | | | | 140736.67 |

Table 22: the level of inventory $l_i^j$ in the equal order quantity (EOQ) model

| $i \in \mathcal{I}$ | $j \in \mathcal{J}$: 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 1 | 45 | 41 | 37 | 33 | 29 | 25 | 21 | 17 | 13 | 9 | 6 | 3 | 0 |
| 2 | 140 | 116 | 116 | 92 | 92 | 69 | 69 | 46 | 46 | 23 | 23 | 0 | 0 |
| 3 | 100 | 75 | 75 | 75 | 50 | 50 | 50 | 25 | 25 | 25 | 0 | 0 | 0 |
| 4 | 17 | 17 | 14 | 14 | 11 | 11 | 8 | 8 | 5 | 5 | 2 | 2 | 0 |
| 5 | 300 | 226 | 226 | 140 | 140 | 104 | 104 | 63 | 63 | 38 | 38 | 0 | 0 |
| 6 | 40 | 39 | 57 | 38 | 35 | 22 | 9 | 10 | 14 | 18 | 15 | 1 | 0 |
| 7 | 0 | 121 | 121 | 242 | 242 | 364 | 0 | 121 | 121 | 242 | 242 | 364 | 0 |
| 8 | 100 | 458 | 158 | 75 | 433 | 133 | 50 | 408 | 108 | 25 | 383 | 83 | 0 |
| 9 | 0 | 101 | 101 | 101 | 101 | 101 | 101 | 101 | 101 | 101 | 101 | 101 | 0 |
| 10 | 128 | 92 | 58 | 20 | 110 | 96 | 68 | 37 | 0 | 90 | 60 | 29 | 0 |

quantity model. Holding and warehousing costs in this model are less than other two models, and both final ordering costs and final purchasing costs are more than those of the model proposed in this paper. Figure 6 shows percentage of quantitative costs. As shown by this figure, the highest rate relates to purchasing cost like two prior models. The lowest cost relates to holding and warehousing like JIT model.

Table 23: holding and warehousing cost $H(q_i^j)$ of industrial parts in the equal order quantity model

| $i \in \mathcal{I}$ | $j \in \mathcal{J}$: 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | Holding Cost |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 1 | 33.24 | 30.29 | 27.33 | 24.38 | 21.42 | 18.47 | 15.51 | 12.56 | 9.60 | 6.65 | 4.43 | 2.22 | 0 | 206.09 |
| 2 | 150.67 | 124.84 | 124.84 | 99.01 | 99.01 | 74.26 | 74.26 | 49.51 | 49.51 | 24.75 | 24.75 | 0 | 0 | 895.40 |
| 3 | 65.52 | 49.14 | 49.14 | 49.14 | 32.76 | 32.76 | 32.76 | 16.38 | 16.38 | 16.38 | 0 | 0 | 0 | 360.34 |
| 4 | 4.02 | 4.02 | 3.31 | 3.31 | 2.60 | 2.60 | 1.89 | 1.89 | 1.18 | 1.18 | 0.47 | 0.47 | 0 | 26.93 |
| 5 | 184.48 | 138.98 | 138.98 | 86.09 | 86.09 | 63.95 | 63.95 | 38.74 | 38.74 | 23.37 | 23.37 | 0 | 0 | 886.75 |
| 6 | 33.95 | 33.10 | 48.38 | 32.25 | 29.71 | 18.67 | 7.64 | 8.49 | 11.88 | 15.28 | 12.73 | 0.85 | 0 | 252.93 |
| 7 | 0 | 75.54 | 75.54 | 151.08 | 151.08 | 227.25 | 0 | 75.54 | 75.54 | 151.08 | 151.08 | 227.25 | 0 | 1360.98 |
| 8 | 35.39 | 162.11 | 55.92 | 26.55 | 153.26 | 47.08 | 17.70 | 144.41 | 38.23 | 8.85 | 135.56 | 29.38 | 0 | 854.43 |
| 9 | 0 | 57.37 | 57.37 | 57.37 | 57.37 | 57.37 | 57.37 | 57.37 | 57.37 | 57.37 | 57.37 | 57.37 | 0 | 631.07 |
| 10 | 90.34 | 64.93 | 40.93 | 14.11 | 77.63 | 67.75 | 47.99 | 26.11 | 0 | 63.52 | 42.34 | 20.47 | 0 | 556.13 |
| | | | | | | | | | | | | | | 6031.04 |

Table 27 shows a summary of all costs of every three models. Final cost of the model proposed in this paper is very lower than final costs of JIT and EOQ models. This means that this model proposes a set of order quantities. Holding and warehousing costs of the proposed model are more than those of other two models; but ordering, purchasing, and transportation costs of this model are less than those of other two models. JIT model has the highest final cost, ordering cost, and purchasing cost. And EOQ model has the highest transportation cost. Figure 7 shows the relation between purchasing and transportation costs in all the models. These two costs have the highest effect among the supply chain.

Table 24: ordering cost $Q(q_i^j)$ of industrial parts in the equal order quantity model

| $i \in I$ | $j \in J$ | | | | | | | | | | | | Ordering Cost |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | |
| 1 | 172.83 | 172.83 | 172.83 | 172.83 | 172.83 | 172.83 | 172.83 | 172.83 | 172.83 | 172.83 | 172.83 | 172.83 | 2073.91 |
| 2 | 425.93 | 0 | 425.93 | 0 | 425.93 | 0 | 425.93 | 0 | 425.93 | 0 | 425.93 | 0 | 2555.59 |
| 3 | 275.90 | 0 | 0 | 275.90 | 0 | 0 | 275.90 | 0 | 0 | 275.90 | 0 | 0 | 1103.60 |
| 4 | 48.38 | 48.38 | 48.38 | 48.38 | 48.38 | 48.38 | 48.38 | 48.38 | 48.38 | 48.38 | 48.38 | 48.38 | 580.53 |
| 5 | 502.27 | 0 | 502.27 | 0 | 502.27 | 0 | 502.27 | 0 | 502.27 | 0 | 502.27 | 0 | 3013.64 |
| 6 | 191.14 | 191.14 | 191.14 | 191.14 | 191.14 | 191.14 | 191.14 | 191.14 | 191.14 | 191.14 | 191.14 | 191.14 | 2293.67 |
| 7 | 207.08 | 0 | 207.08 | 0 | 207.08 | 0 | 207.08 | 0 | 207.08 | 0 | 207.08 | 0 | 1242.51 |
| 8 | 540.98 | 0 | 0 | 540.98 | 0 | 0 | 540.98 | 0 | 0 | 540.98 | 0 | 0 | 2163.93 |
| 9 | 260.43 | 260.43 | 260.43 | 260.43 | 260.43 | 260.43 | 260.43 | 260.43 | 260.43 | 260.43 | 260.43 | 0 | 2864.68 |
| 10 | 390.90 | 390.90 | 390.90 | 390.90 | 390.90 | 390.90 | 390.90 | 390.90 | 390.90 | 390.90 | 390.90 | 390.90 | 4690.76 |
| | | | | | | | | | | | | | 22582.81 |

Table 25: purchasing cost $P(q_i^j)$ of industrial parts in the equal order quantity model

| $i \in I$ | $j \in J$ | | | | | | | | | | | | Purchasing Cost |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | |
| 1 | 453.14 | 452.02 | 450.19 | 449.79 | 448.68 | 447.57 | 446.46 | 445.35 | 444.25 | 454.51 | 453.38 | 452.25 | 5398.30 |
| 2 | 2030.74 | 0 | 2020.73 | 0 | 2027.37 | 0 | 2017.34 | 0 | 2007.34 | 0 | 1997.38 | 0 | 12100.89 |
| 3 | 1829.08 | 0 | 0 | 1815.57 | 0 | 0 | 1802.12 | 0 | 0 | 1788.74 | 0 | 0 | 7235.51 |
| 4 | 153.63 | 153.25 | 152.87 | 152.49 | 152.12 | 151.74 | 151.36 | 150.99 | 150.61 | 150.24 | 149.87 | 158.29 | 1827.46 |
| 5 | 2241.17 | 0 | 2230.12 | 0 | 2219.11 | 0 | 2217.04 | 0 | 2206.05 | 0 | 2195.10 | 0 | 13308.59 |
| 6 | 506.33 | 505.08 | 503.83 | 516.17 | 514.89 | 513.62 | 512.35 | 511.08 | 509.81 | 508.54 | 507.28 | 506.02 | 6115 |
| 7 | 1187.20 | 0 | 1181.35 | 0 | 1185.23 | 0 | 1169.70 | 0 | 1163.90 | 0 | 1167.70 | 0 | 7055.08 |
| 8 | 4652.23 | 0 | 0 | 4617.86 | 0 | 0 | 4583.66 | 0 | 0 | 4549.62 | 0 | 0 | 18403.37 |
| 9 | 1512.53 | 780.41 | 778.49 | 776.56 | 774.64 | 772.73 | 770.81 | 768.90 | 766.99 | 765.09 | 763.19 | 0 | 9230.34 |
| 10 | 1022.47 | 1019.95 | 1017.43 | 1014.91 | 1023.65 | 1021.12 | 1018.59 | 1016.06 | 1002.41 | 1011.03 | 1008.51 | 1006.01 | 12182.14 |
| | | | | | | | | | | | | | 92856.70 |

Table 26: comparison of industrial parts prices in the equal order quantity (EOQ) model

| Transportation Cost | Inventory Cost | | | Total Cost |
|---|---|---|---|---|
| | Purchasing | Ordering | Holding | |
| 19266.12 | 92856.70 | 22582.81 | 6031.04 | 140736.67 |
| | 121470.56 | | | |

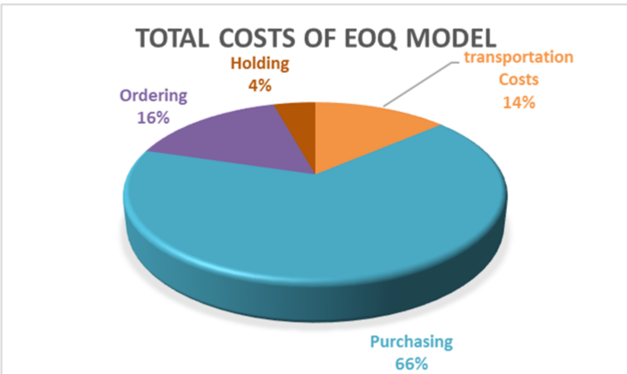


Figure 6: division of total costs of industrial parts in the equal order quantity model

As figure 7 shows, over 7550 $ is saved in these two costs in the model proposed in this paper, and as mentioned earlier, this saving will be further more in Iran with regard to the rate of exchange to rial.

Table 27: comparison of total costs of these three models.

| | Transportation | Purchasing | Ordering | Holding | Total |
|---|---|---|---|---|---|
| JIT | 17594.39 | 93126.74 | 23956.31 | 6222.46 | 140899.91 |
| EOQ | 19266.12 | 92856.70 | 22582.81 | 6031.04 | 140736.67 |
| Proposed Model | 17137.06 | 87432.81 | 10439.30 | 25175.96 | 140185.12 |

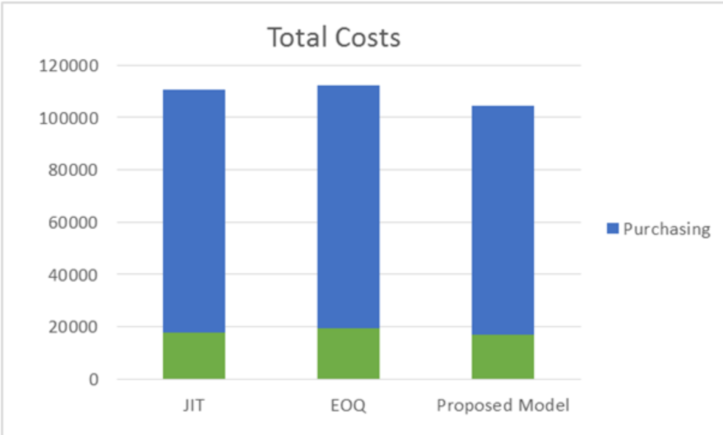


Figure 7: comparison of purchasing and transportation costs of three models

Table 28 and figure 8 shows the relation between holding and warehousing costs in every three models. Holding cost of the model proposed in this model is more than other two models. For all products, EOQ model has the lowest cost for holding and warehousing except for products No. 3 and 7. For these two products, JIT model has the lowest costs. Product No. 9 has equal holding and warehousing costs in two EOQ and JIT models because their order quantity is equal. Product No. 8 has the highest holding and warehousing costs in the proposed model of this paper and product No. 4 has the lowest holding and warehousing costs in EOQ model.

Table 28: comparison of holding costs in the three models

| | $\sum_{j\in J} H(q_i^j)$ | | |
|---|---|---|---|
| $i \in I$ | JIT | EOQ | Proposed Model |
| 1 | 382.63 | 206.09 | 464.62 |
| 2 | 930.91 | 895.40 | 4051.89 |
| 3 | 295.48 | 360.34 | 1214.02 |
| 4 | 52.45 | 26.93 | 168.68 |
| 5 | 1087.83 | 886.75 | 4242.49 |
| 6 | 420.14 | 252.93 | 1467.51 |
| 7 | 454.49 | 1360.98 | 3181.44 |
| 8 | 1108.57 | 854.43 | 5353.83 |
| 9 | 631.07 | 631.07 | 927.57 |
| 10 | 858.89 | 556.13 | 4103.90 |
| Total | 6222.46 | 6031.04 | 25175.96 |

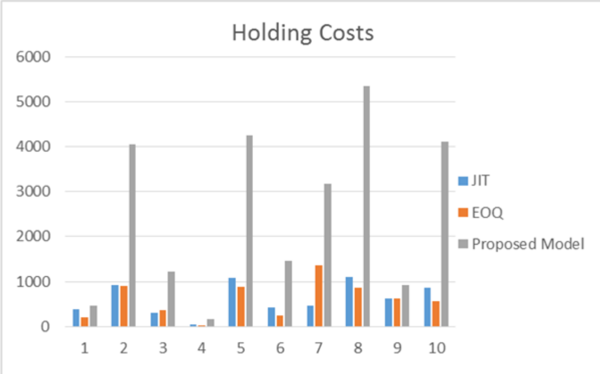


Figure 8: comparison of holding costs in the three models

Table 29 and figure 9 shows ordering costs of all models for all products. The proposed model of this paper has the lowest ordering cost for all products compared to other models, and EOQ model has the highest ordering costs except for products No. 8 that has been ordered 11 times in JIT model and only 4 times in EOQ model. Products No. 2, 3, 5 and 9 have equal holding costs in EOQ and JIT models. In figure 10, the highest ordering cost relates to the product No. 8 in JIT model and the lowest one relates to the product No. 7 in the proposed model that has been ordered only one time per year.

Table 29: comparison of ordering costs of the three models

| | $\sum_{j\epsilon J} O(q_i^j)$ | | |
|---|---|---|---|
| $i \in I$ | JIT | EOQ | Proposed Model |
| 1 | 1901.08 | 2073.91 | 1901.08 |
| 2 | 2555.59 | 2555.59 | 851.86 |
| 3 | 1103.60 | 1103.60 | 827.70 |
| 4 | 532.15 | 580.53 | 241.89 |
| 5 | 3013.64 | 3013.64 | 1506.82 |
| 6 | 2102.53 | 2293.67 | 955.70 |
| 7 | 414.17 | 1242.51 | 207.08 |
| 8 | 5950.80 | 2163.93 | 1081.96 |
| 9 | 2864.68 | 2864.68 | 2083.41 |
| 10 | 3518.07 | 4690.76 | 781.79 |
| **Total** | **23956.31** | **22582.81** | **10439.30** |

Table 30 and figure 10 shows purchasing costs. Products No. 3 and 4 have the lowest purchasing costs in EOQ model; but for other products, the proposed model of this paper has the lowest purchasing cost. Table 31 shows transportation costs of all types of NMFC in all models. Transportation cost in the proposed model of this paper is less than other two models. For $v = 3$ and $v = 7$, transportation cost of the proposed model is the lowest one; and for $v = 4$, JIT model has the lowest transportation cost.

## 7 Analysis of sensitivity

Analysis of sensitivity is used for considering effective parameters on the model. When a parameter changes while other parameters are constant, parameters that are analyzed are namely, holding and warehousing cost ($h_i$), ordering cost ($O_i$), transportation cost ($V_X^v$), and rate of interest.

### 7.1 Holding and warehousing cost of product i in the time unit $h_i$

Studying $h_i$ helps us to consider the effect of holding and warehousing cost $H(q_i^j)$ on the total cost. Five possible states for $h_i$ include decrease 80%, decrease 40%, increase 80%, increase 40%, and remaining without change.


Figure 9: comparison of ordering costs of the three models

Table 30: comparison of purchasing costs of the three models

| | $\sum_{j\in J} P(q_i^j)$ | | |
|---|---|---|---|
| $i \in I$ | JIT | EOQ | Proposed Model |
| 1 | 5405.08 | 5398.30 | 5352.56 |
| 2 | 11745.28 | 12100.89 | 11300.79 |
| 3 | 7230.63 | 7235.51 | 7299.02 |
| 4 | 1829.84 | 1827.46 | 1840.73 |
| 5 | 12403.36 | 13308.59 | 11312.86 |
| 6 | 6125.96 | 6115 | 5907.30 |
| 7 | 6650.73 | 7055.08 | 5812.67 |
| 8 | 20501.07 | 18403.37 | 17868.74 |
| 9 | 9230.34 | 9230.34 | 8996 |
| 10 | 12004.46 | 12182.14 | 11793.52 |
| **Total** | **93126.74** | **92856.70** | **87484.17** |


Figure 10: comparison of purchasing costs of the three models

A set of optimal order quantities will change by $h_i$ change. The attached tables show that by increase in $h_i$, order quantity of products 2, 3, 5, 7, 8 and 10 will not change which is due to higher holding and warehousing costs of these products compared to the rest ones. So, 40% or 80% change in holding and warehousing costs will make no change in the method of ordering these products. For products No. 8, 4, 6 and 9, change in holding cost $(h_i)$ makes a significant

Table 31: comparison of transportation costs of the three models

| | $\sum_{j\in\mathcal{J}} R(q_i^j)$ | | |
|---|---|---|---|
| $v \in \mathcal{V}$ | JIT | EOQ | Proposed Model |
| 3 | 342844.39 | 334179.77 | 327011.91 |
| 4 | 115353.82 | 135230.83 | 127252.36 |
| 7 | 69633.64 | 108572.94 | 59847.51 |
| **Total** | **527831.85** | **577983.54** | **514111.79** |


Figure 11: comparison of transportation costs of the three models

difference in the method of ordering. By increase in the holding and warehousing cost, the number of products is reduced in the warehouse and number of orders is increased. For example, product No. 9 is ordered 8 times per year when $h_i$ is reduced 80%. When $h_i$ is increased 80%, the number of orders will be 10 which will increase the ordering cost and reduce holding and warehousing cost.

Table 32: comparison of all costs with different $h_i$

| $h_i$ | Transportation | Purchasing | Ordering | Holding | Total |
|---|---|---|---|---|---|
| Decrease 80% | 16975.84 | 87265.45 | 10487.67 | 5073.30 | **119802.27** |
| Decrease 40% | 17045.07 | 87262.25 | 10015.20 | 15191.04 | **129513.57** |
| 0 | 17137.06 | 87432.25 | 10439.30 | 25175.96 | **140185.12** |
| Increase 40% | 17314.57 | 87507.20 | 11178.76 | 34995.17 | **150995.69** |
| Increase 80% | 17469 | 87580.17 | 11296.42 | 44878.23 | **161223.82** |

Table 32 shows a comparison of price in the sets of order quantity. And it indicates that by increase in $h_i$, transportation, purchasing, and ordering costs increase to keep balance of total cost when holding and warehousing cost is constant.

### 7.2 Ordering costs of product i per time unit $o_i$

Studying ordering costs ($o_i$) helps us to analyze the role of $O(q_i^j)$ in the total cost. Any of these changes has a different volume of optimal order quantity which has been shown in the attached tables. When the ordering costs increase, the proposed model of this paper tries to reduce the order replication. For example, product No. 6 is ordered 7 times per year when $O_i$ is decreased 40% and it is ordered 5 times per year when $O_i$ is increased 80%.

Table 33 compares costs of orders quantities. When the ordering costs increase, transportation, purchasing, holding, and warehousing costs are decreased. By reduction of ordering costs, purchasing, and transportation costs do not comply with a certain model. When the ordering costs are decreased 40%, transportation costs increase; but when the ordering costs are reduced 80%, purchasing costs are reduced and transportation costs will be increased which is due to keeping balance in the final cost.

Table 33: comparison of all costs with different $O_i$s

| $O_i$ | Transportation | Purchasing | Ordering | Holding | Total |
|---|---|---|---|---|---|
| Decrease 80% | 17158.38 | 87352.62 | 2149.62 | 25113.55 | **131774.17** |
| Decrease 40% | 17231.17 | 87468.08 | 6707.25 | 25091.31 | **136497.82** |
| 0 | 17137.06 | 87432.81 | 10439.30 | 25175.96 | **140185.12** |
| Increase 40% | 17109.40 | 87429.37 | 15018.07 | 25134.11 | **144690.95** |
| Increase 80% | 17221.66 | 87373.66 | 19520.74 | 25115.32 | **149231.38** |

### 7.3 Rate of interest, $r$

Changes in $r$ reduce purchasing costs $P(q_i^j)$ and total costs. Purchasing costs include expenses of buying the product and payment of loan. Loan payment is in the form of monthly payment. For a loan with the interest rate

$$r = \frac{\text{Annual Interest rate}}{12}$$

The annual interest rate is 6% for our case study and has been compared with 1%, 4%, and 11% values. Any change in r has different sets of optimal order quantities that have been shown in the attached tables No. $A_9$ to $A_n$. order quantity of products No. 2, 3, 5, 7, 8 and 10 is constant with different values of r. the order quantity of other products does not change for the balance of total price. If r increases, the ordering, holding and warehousing costs increase and transportation cost is reduced. For example, product No. 4 with the annual interest rate 11% has been ordered 7 times but this rate has been reduced to 5 times in the annual interest rate 1% which will increase ordering, holding and warehousing costs and reduce transportation costs.

Table 34: comparison of all costs with different $r$ values

| Annual Interest Rate | $r$ | Transportation | Purchasing | Ordering | Holding | Total |
|---|---|---|---|---|---|---|
| Decrease 5% | 0.0033 | 17208.84 | 85460.07 | 10439.30 | 25121.30 | **138229.51** |
| Decrease 2% | 0.00083 | 17055.89 | 86616.82 | 10227.25 | 25218.42 | **139118.38** |
| 0 | 0.0050 | 17137.06 | 87432.81 | 10439.30 | 25175.95 | **140185.12** |
| Increase 5% | 0.00917 | 16999.73 | 89408.85 | 10536.05 | 25231.68 | **142176.31** |

### 7.4 Transportation costs per 100 pound in NMFC unit and for class $v$ in the weight range $k$, $V_k^v$

Change of $V_k^v$ will influence transportation cost $R_v^j(q_i^j; l \in C_v)$ and final cost. Any of these changes will make a different set of order quantities which have been shown in tables $A_{12}$ to $A_{15}$. Products No. 2, 3, 5, 7, 8 and 10 are constant in different values of $V_k^j$. Order quantity of other products will be variable for keeping balance in the final cost. By increase in the transportation cost, purchasing and ordering costs are reduced and holding and warehousing costs are increased. Table 35 shows the details.

When the transportation cost increases, the proposed model tries to keep the final cost constant; so minimizing the number helps to reduce transportation cost. By reduction of number and cost of transportation, holding and warehousing cost increases. So, when the holding and warehousing cost is less than sum of ordering, purchasing, and transportation costs, the proposed model justifies increase in the holding and warehousing costs relative to other costs.

Table 35: comparison of all costs with different values of $V_k^v$

| $\mathcal{V}_{\mathcal{K}}^{v}$ | Transportation | Purchasing | Ordering | Holding | Total |
|---|---|---|---|---|---|
| Decrease 80% | 3518.52 | 87578.92 | 11556.85 | 24876.37 | **127530.66** |
| Decrease 40% | 10431.37 | 87501.48 | 11248.04 | 25011.66 | **134192.56** |
| 0 | 17137.06 | 87432.81 | 10439.30 | 25175.96 | **140185.12** |
| Increase 40% | 23861.76 | 87439.68 | 10727.19 | 25174.28 | **147202.90** |
| Increase 80% | 30543.98 | 87396.38 | 10727.19 | 25258.23 | **153925.78** |

### 7.5 Conclusion of sensitivity analysis

Table 36 presents a summary of the results of all parameters. This table shows that by increase in $h_i$, transportation, purchasing, and ordering costs increase and holding and warehousing cost decreases. This means that if $h_i$ increases, products are ordered more. so, the ordering cost increases because the number increases.

The next row of the table ($O_i$) in increasing, changes in the transportation price are uncertain and are reduced by change and this will lead to the reduction of transportation cost. For the parts of automobile industry, there is a different relation between holding costs and ordering costs and this makes the result complicated. Finally by increase in $O_i$, purchasing and holding costs are reduced and ordering cost and final cost increase.

The third row indicates that when the interest rate is increasing, the model tries to order less numbers. So, holding costs increase and transportation cost decreases. The final row relates to $V_k^v$ whose changes are exactly contrary to $h_i$; because when transportation cost is increasing, less products are ordered and so ordering, purchasing, and transportation costs are reduced and holding cost increases.

Table 36: summary of sensitivity analysis

| By Increasing | Transportation Cost | Purchasing Cost | Ordering Cost | Holding Cost | Total Cost |
|---|---|---|---|---|---|
| $h_i$ | Increase | Increase | Increase | Decrease | Increase |
| $O_i$ | Unknown | Decrease | Increase | Decrease | Increase |
| $r$ | Decrease | Decrease | Increase | Increase | Increase |
| $\mathcal{V}_{\mathcal{K}}^{v}$ | Decrease | Decrease | Decrease | Increase | Increase |

## 8 Conclusions

The synthetic algorithm used in this paper sought to use genetic algorithm mechanism to find the primary solution and use it in the simulated annealing algorithm and so get closer to the optimal solution. The structure of the used synthetic algorithm shows that its convergence has been proven because it has used two known algorithms in its mechanism. The reason is clear because the genetic algorithm searches the problem solving space to find the solution and so examines some points of justified problem space that is considered less and this increases chance of finding the primary proper solution. Then the synthetic algorithm gives this solution to the simulated annealing algorithm so as to move towards optimal solution by using neighboring solutions and examining surrounding spaces. Algorithm four indicates the above contents in a simple language.

Optimizing investment and transportation costs helps the producer and transportation sector to demand an appropriate rate of order quantity at the best time. The model presented in this paper will provide the optimal rate of order quantity given by a supplier to the producer. NMFC model covers drop of prices and finance costs. Furthermore, the flexible time horizon planning permits the producer to use this model in different time lags like hour, day, and month.

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