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New analytical soliton solutions to the stochastic fractional long-short wave interaction system with multiplicative noise

A. Sarli, Ahmad Sharif*, Nemat A. Taghi-Nezhad

Department of Mathematics, Faculty of Sciences, Gonbad Kavous University, Gonbad, Iran

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Abstract

In the upcoming study, a modified Sardar sub-equation method has been studied to find the solutions to the stochastic fractional long–short-wave interaction system, which is a very useful equation in the field of mathematical physics. All kinds of answers have been obtained using the method in question, which shows the strong performance of this method.

Keywords: fractional long-short wave interaction system, modified Sardar sub-equation method, Exact solution
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1 Introduction

In this paper we consider the following stochastic fractional long–short wave interaction system (SFL-SWIS)

$$\begin{cases} is_t + \Im_x^{2\alpha} s - sr = i\gamma s W_t, \\ r_t + \Im_x^{\alpha} r + \Im_x^{\alpha}\left(|s|^2\right) = \gamma r W_t, \end{cases} \tag{1.1}$$

Nonlinear partial differential equations (NLPDEs) have been extensively used for determining natural propositions of applied mathematics and sciences which can be seen in the field of fluid dynamics, plasma physics, mathematical physics, natural sciences, electromagnetic theory, applied sciences, etc. The higher-order nonlinear evolution equations (NLEEs) examine the physical and complex behavior of solitary waves in terms of exact solutions including some arbitrary functional parameters and arbitrary constant parameters on the unpredictable and continuous background. It has been observed that most of the systems are nonlinear in nature and therefore differential equations play a vital role to explore the modeling of real physical systems. So far, many studies have been done to obtain the solutions of these equations, among which we can refer to [1]-[10].

2 The description of modified Sardar sub-equation method

This method is a powerful and robust approach that may be used to generate numerous types of soliton solutions, including dark, bright, W-shaped, mixed dark-bright, singular, mixed singular solitons, periodic, and other solutions,

*Corresponding author
Email addresses: sarli1402@gmail.com (A. Sarli), sharif@gonbad.ac.ir (Ahmad Sharif), n.taghinezhad@gmail.com (Nemat A. Taghi-Nezhad)

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for both the classical and fractional order NPDEs as compared to other methods in the literature. The fact that this approach overcomes the complications of the solitary wave ansatz method [18,46] is noteworthy. As a result, this section contains a full description of the MSSE approach. To use this approach, we assume the following general NPDE as:

$$G\left(u, u_t, u_x, u_{xx}, ...\right) = 0, \tag{2.1}$$

where G denotes a function of u and its derivatives. The complex wave transformation given as

$$u\left(x,t\right) = U\left(\xi\right)e^{i\left(\omega_2 x+\eta_2 t\right)}, \xi = \omega_1 x + \eta_1 t, \tag{2.2}$$

is proposed to reduce Eq. (2.1) into nonlinear ordinary differential equations (NODE):

$$N\left(U, U', U'', ...\right) = 0. \tag{2.3}$$

The solutions of Eq. (2.3) can be classify as:

$$U\left(\xi\right) = \delta_0 + \sum_{i=1}^{n} \delta_i Q^i\left(\xi\right), \qquad \delta_m \neq 0. \tag{2.4}$$

The function $Q\left(\xi\right)$in Eq. (2.4) satisfies

$$\left(Q'\left(\xi\right)\right)^2 = \upsilon_2 Q^4\left(\xi\right) + \upsilon_1 Q^2\left(\xi\right) + \upsilon_0, \tag{2.5}$$

where υ_0 , υ_1 , and υ_2 are constants. In addition, the family of solutions for Eq. (2.5) with constant are listed as below:

1. If $\upsilon_0 = 0, \upsilon_1 > 0$, and $\upsilon_2 = 0$, then

$$Q_1{}^{\pm}\left(\xi\right) = \pm\sqrt{-\frac{\upsilon_1}{\upsilon_2}}\sec h\left(\sqrt{\upsilon_1}\left(\xi+\varsigma\right)\right),$$

$$Q_2{}^{\pm}\left(\xi\right) = \pm\sqrt{-\frac{\upsilon_1}{\upsilon_2}}\csc h\left(\sqrt{\upsilon_1}\left(\xi+\varsigma\right)\right).$$

2. For constants A_1 andA_2 . Let$\upsilon_2 = \pm 4A_1A_2$, $\upsilon_1 > 0$,, and $\upsilon_0 = 0$, we have

$$Q_3{}^{\pm}\left(\xi\right) = \frac{4\sqrt{\upsilon_1}A_1}{\left(4A_1^2-\upsilon_2\right)\cosh\left(\sqrt{\upsilon_1}\left(\xi+\xi_0\right)\right) \pm \left(4A_1^2+\upsilon_2\right)\sinh\left(\sqrt{\upsilon_1}\left(\xi+\xi_0\right)\right)}.$$

3. If $\upsilon_0 = \frac{\upsilon_1^2}{4\upsilon_2}, \upsilon_1 < 0, \upsilon_2 > 0$, with constants B_1 , and B_2 , then

$$Q_3{}^{\pm}\left(\xi\right) = \pm\sqrt{-\frac{\upsilon_1}{2\upsilon_2}}\tanh\left(\sqrt{-\frac{\upsilon_1}{2}}\left(\xi+\varsigma\right)\right),$$

$$Q_4{}^{\pm}\left(\xi\right) = \pm\sqrt{-\frac{\upsilon_1}{2\upsilon_2}}\coth\left(\sqrt{-\frac{\upsilon_1}{2}}\left(\xi+\varsigma\right)\right),$$

$$Q_5{}^{\pm}\left(\xi\right) = \pm\sqrt{-\frac{\upsilon_1}{2\upsilon_2}}\left(\tanh\left(\sqrt{-2\upsilon_1}\left(\xi+\varsigma\right)\right) \pm i\sec h\left(\sqrt{-2\upsilon_1}\left(\xi+\varsigma\right)\right)\right),$$

$$Q_6{}^{\pm}\left(\xi\right) = \pm\sqrt{-\frac{\upsilon_1}{8\upsilon_2}}\left(\tanh\left(\sqrt{-\frac{\upsilon_1}{8}}\left(\xi+\varsigma\right)\right) \pm \coth\left(\sqrt{-\frac{\upsilon_1}{8}}\left(\xi+\varsigma\right)\right)\right),$$

$$Q_7{}^{\pm}\left(\xi\right) = \pm\sqrt{-\frac{\upsilon_1}{2\upsilon_2}}\left(\frac{\pm\sqrt{B_1^2+B_2^2} - B_1\cosh\left(\sqrt{-2\upsilon_1}\left(\xi+\varsigma\right)\right)}{B_1\sinh\left(\sqrt{-2\upsilon_1}\left(\xi+\varsigma\right)\right)+B_2}\right),$$

$$Q_9{}^{\pm}\left(\xi\right) = \pm\sqrt{-\frac{\upsilon_1}{2\upsilon_2}}\left(\frac{\cosh\left(\sqrt{-2\upsilon_1}\left(\xi+\varsigma\right)\right)}{\sinh\left(\sqrt{-2\upsilon_1}\left(\xi+\varsigma\right)\right) \pm i}\right).$$

4. If $\upsilon_0 = 0, \upsilon_1 < 0, \upsilon_2 \neq 0$, then

$$Q_{10}{}^{\pm}(\xi) = \pm\sqrt{-\frac{\upsilon_1}{\upsilon_2}}\sec\left(\sqrt{-\upsilon_1}(\xi+\varsigma)\right),$$

$$Q_{11}{}^{\pm}(\xi) = \pm\sqrt{-\frac{\upsilon_1}{\upsilon_2}}\csc\left(\sqrt{-\upsilon_1}(\xi+\varsigma)\right).$$

5. If $\upsilon_0 = \frac{\upsilon_1^2}{4\upsilon_2}, \upsilon_1 > 0, \upsilon_2 > 0$,and$B_1^2 - B_2^2 > 0$, then

$$Q_{12}{}^{\pm}(\xi) = \pm\sqrt{-\frac{\upsilon_1}{2\upsilon_2}}\tan\left(\sqrt{\frac{\upsilon_1}{2}}(\xi+\varsigma)\right), \quad Q_{13}{}^{\pm}(\xi) = \pm\sqrt{\frac{\upsilon_1}{2\upsilon_2}}\cot\left(\sqrt{\frac{\upsilon_1}{2}}(\xi+\varsigma)\right),$$

$$Q_{14}{}^{\pm}(\xi) = \pm\sqrt{\frac{\upsilon_1}{2\upsilon_2}}\left(\tan\left(\sqrt{2\upsilon_1}(\xi+\varsigma)\right) \pm \sec\left(\sqrt{2\upsilon_1}(\xi+\varsigma)\right)\right),$$

$$Q_{15}{}^{\pm}(\xi) = \pm\sqrt{\frac{\upsilon_1}{8\upsilon_2}}\left(\tan\left(\sqrt{\frac{\upsilon_1}{8}}(\xi+\varsigma)\right) - \cot\left(\sqrt{\frac{\upsilon_1}{8}}(\xi+\varsigma)\right)\right),$$

$$Q_{16}{}^{\pm}(\xi) = \pm\sqrt{\frac{\upsilon_1}{2\upsilon_2}}\left(\frac{\pm\sqrt{B_1^2 - B_2^2} - B_1\cos\left(\sqrt{2\upsilon_1}(\xi+\varsigma)\right)}{B_1\sin\left(\sqrt{-2\upsilon_1}(\xi+\varsigma)\right) + B_2}\right),$$

$$Q_{17}{}^{\pm}(\xi) = \pm\sqrt{\frac{\upsilon_1}{2\upsilon_2}}\left(\frac{\cos\left(\sqrt{2\upsilon_1}(\xi+\varsigma)\right)}{\sin\left(\sqrt{2\upsilon_1}(\xi+\varsigma)\right) \pm 1}\right).$$

6. If $\upsilon_0 = 0$ and $\upsilon_1 > 0$, then

$$Q_{18}{}^{\pm}(\xi) = \pm\frac{4\upsilon_1 e^{\pm\sqrt{\upsilon_1}(\xi+\varsigma)}}{\upsilon_1 e^{\pm 2\sqrt{\upsilon_1}(\xi+\varsigma)} - 4\upsilon_1\upsilon_2}, \quad Q_{19}{}^{\pm}(\xi) = \pm\frac{\pm 4\upsilon_1 e^{\pm\sqrt{\upsilon_1}(\xi+\varsigma)}}{1 - 4\upsilon_1 e^{\pm 2\sqrt{\upsilon_1}(\xi+\varsigma)}}.$$

7. If $\upsilon_0 = \upsilon_1 = 0$and $\upsilon_2 > 0$, then

$$Q_{20}{}^{\pm}(\xi) = \pm\frac{1}{\sqrt{\upsilon_2}(\xi+\varsigma)}.$$

8. If $\upsilon_0 = \upsilon_1 = 0$ and $\upsilon_2 < 0$, then

$$Q_{21}{}^{\pm}(\xi) = \pm\frac{i}{\sqrt{-\upsilon_2}(\xi+\varsigma)}.$$

3 MSSEM application to the SFL-SWIS

In the following, we first consider the stochastic fractional long–short wave interaction system as follows

$$\begin{cases} is_t + \Im_x^{2\alpha}s - sr = i\gamma sW_t, \\ r_t + \Im_x^{\alpha}r + \Im_x^{\alpha}\left(|s|^2\right) = \gamma rW_t, \end{cases}$$

where $s(x,t)$represents a complex function, $r(x,t)$ is a real function, $\Im_x^{\alpha}$is the Jumarie modified Riemann–Liouville derivatives for $0 < \alpha \leq 1, \gamma$, is a constant representing noise strength, and $W_t = \frac{dW}{dt}$, $W = W(t)$is the standard Brownian motion. In order We consider the following wave transformation

$$s(x,t) = \hbar(\xi)e^{\left(i\theta+\gamma W(t)-\frac{\gamma^2}{2}t\right)}, r(x,t) = g(\xi)e^{\left(\gamma W(t)-\frac{\gamma^2}{2}t\right)}, \tag{3.1}$$

$$\xi = \frac{x^{\alpha}}{\Gamma(1+\alpha)} - 2kt, \theta = \frac{kx^{\alpha}}{\Gamma(1+\alpha)} + \lambda t, \tag{3.2}$$

By substituting (3.1)-(3.2) in eq. (1.1) we obtain

$$\hbar'' - \left(k^2+\lambda\right)\hbar + \frac{1}{1-2k}\hbar^3 e^{\gamma W(t)-\frac{\gamma^2}{2}t} = 0. \tag{3.3}$$

Because $W(t)$ is a standard Gaussian process, the mathematical expectation is

$$e^{\gamma W(t)} = e^{\frac{\gamma^2}{2}t}. \tag{3.4}$$

Considering the expectation on both sides of Eq. (3.3), we obtain:

$$\hbar'' - \frac{1}{2k-1}\hbar^3 - \left(k^2+\lambda\right)\hbar = 0. \tag{3.5}$$

Substituting (3.1)-(3.2) into (1.1), we obtain:

$$(1-2k)\,g' + 2\hbar\hbar' e^{\gamma W(t)-\frac{\gamma^2}{2}t} = 0. \tag{3.6}$$

Applying the mathematical expectation of Brownian motion, Eq. (3.6) can be rewritten as

$$(1-2k)\,g' + 2\hbar\hbar' = 0. \tag{3.7}$$

Integrating Eq. (3.7) once yields

$$(1-2k)\,g + \hbar^2 = c. \tag{3.8}$$

$$g = \frac{c-\hbar^2}{(1-2k)}. \tag{3.9}$$

Therefore, the exact solution of SFL-SWIS (1.1) can be constructed by solving Eqs. (3.5) and (3.9). Due to the relationship (3.9), we must focus on the bifurcation and the solution of Eq. (3.5). For this aim we consider Eq. (2.4) for Eq. (3.5).

$$h(\xi) = \delta_0 + \delta_1 Q(\xi), \qquad \delta_1 \neq 0,$$
$$\delta_1 = \frac{(2k-1)}{3}\sqrt{\frac{-6v_2}{2k-1}}, \qquad \delta_0 = \frac{\sqrt{3}\,(2k-1)}{3}\sqrt{\frac{-k^2-\lambda+v_1}{2k-1}}. \tag{3.10}$$

So solutions of equation (1.1) as follows:

3-1. If $v_0 = 0, v_1 > 0$, and $v_2 = 0$, then

$$s_1(x,t) = \tfrac{\sqrt{3}(2k-1)}{3}\sqrt{\tfrac{-k^2-\lambda+v_1}{2k-1}}\,e^{\left(i\left(\frac{kx^\alpha}{\Gamma(1+\alpha)}+\lambda t\right)+\gamma W(t)-\frac{\gamma^2}{2}t\right)} +$$
$$\left(\tfrac{(2k-1)}{3}\sqrt{\tfrac{-6v_2}{2k-1}}\sqrt{-\tfrac{v_1}{v_2}}\,\sec h\left(\sqrt{v_1}\left(\tfrac{x^\alpha}{\Gamma(1+\alpha)}-2kt+\varsigma\right)\right)\right)e^{\left(i\theta+\gamma W(t)-\frac{\gamma^2}{2}t\right)},$$

$$s_2(x,t) = \tfrac{\sqrt{3}(2k-1)}{3}\sqrt{\tfrac{-k^2-\lambda+v_1}{2k-1}}\,e^{\left(i\left(\frac{kx^\alpha}{\Gamma(1+\alpha)}+\lambda t\right)+\gamma W(t)-\frac{\gamma^2}{2}t\right)} +$$
$$\left(\tfrac{(2k-1)}{3}\sqrt{\tfrac{-6v_2}{2k-1}}\sqrt{-\tfrac{v_1}{v_2}}\,\csc h\left(\sqrt{v_1}\left(\tfrac{x^\alpha}{\Gamma(1+\alpha)}-2kt+\varsigma\right)\right)\right)e^{\left(i\left(\frac{kx^\alpha}{\Gamma(1+\alpha)}+\lambda t\right)+\gamma W(t)-\frac{\gamma^2}{2}t\right)}.$$

In this case we have

$$r_1(x,t) = \tfrac{c}{(1-2k)}e^{\left(\gamma W(t)-\frac{\gamma^2}{2}t\right)} - \tfrac{1}{(1-2k)}\left(\tfrac{\sqrt{3}(2k-1)}{3}\sqrt{\tfrac{-k^2-\lambda+v_1}{2k-1}}+\right.$$
$$\left.\tfrac{(2k-1)}{3}\sqrt{\tfrac{-6v_2}{2k-1}}\sqrt{-\tfrac{v_1}{v_2}}\,\sec h\left(\sqrt{v_1}\left(\tfrac{x^\alpha}{\Gamma(1+\alpha)}-2kt+\varsigma\right)\right)\right)^2 e^{\left(\gamma W(t)-\frac{\gamma^2}{2}t\right)}$$

$$r_2(x,t) = \tfrac{c}{(1-2k)}e^{\left(\gamma W(t)-\frac{\gamma^2}{2}t\right)} - \tfrac{1}{(1-2k)}\left(\tfrac{\sqrt{3}(2k-1)}{3}\sqrt{\tfrac{-k^2-\lambda+v_1}{2k-1}}+\right.$$
$$\left.\tfrac{(2k-1)}{3}\sqrt{\tfrac{-6v_2}{2k-1}}\sqrt{-\tfrac{v_1}{v_2}}\,\csc h\left(\sqrt{v_1}\left(\tfrac{x^\alpha}{\Gamma(1+\alpha)}-2kt+\varsigma\right)\right)\right)^2 e^{\left(\gamma W(t)-\frac{\gamma^2}{2}t\right)}$$

3-2. If $v_0 = \frac{v_1^2}{4v_2}, v_1 < 0, v_2 > 0$, with constants B_1, and B_2, then

$$s_3(x,t) = \tfrac{\sqrt{3}(2k-1)}{3}\sqrt{\tfrac{-k^2-\lambda+v_1}{2k-1}}\,e^{\left(i\left(\frac{kx^\alpha}{\Gamma(1+\alpha)}+\lambda t\right)+\gamma W(t)-\frac{\gamma^2}{2}t\right)} +$$
$$\left(\tfrac{(2k-1)}{3}\sqrt{\tfrac{-6v_2}{2k-1}}\sqrt{-\tfrac{v_1}{2v_2}}\,\tanh\left(\sqrt{-\tfrac{v_1}{2}}\left(\tfrac{x^\alpha}{\Gamma(1+\alpha)}-2kt+\varsigma\right)\right)\right)e^{\left(i\left(\frac{kx^\alpha}{\Gamma(1+\alpha)}+\lambda t\right)+\gamma W(t)-\frac{\gamma^2}{2}t\right)},$$

$$s_4(x,t)=\frac{\sqrt{3}(2k-1)}{3}\sqrt{\frac{-k^2-\lambda+\upsilon_1}{2k-1}}e^{\left(i\left(\frac{kx^\alpha}{\Gamma(1+\alpha)}+\lambda t\right)+\gamma W(t)-\frac{\gamma^2}{2}t\right)}+\left(\frac{(2k-1)}{3}\sqrt{\frac{-6\upsilon_2}{2k-1}}\sqrt{-\frac{\upsilon_1}{2\upsilon_2}}\coth\left(\sqrt{-\frac{\upsilon_1}{2}}\left(\frac{x^\alpha}{\Gamma(1+\alpha)}-2kt+\varsigma\right)\right)\right)e^{\left(i\left(\frac{kx^\alpha}{\Gamma(1+\alpha)}+\lambda t\right)+\gamma W(t)-\frac{\gamma^2}{2}t\right)},$$

and in this case we obtain

$$r_3(x,t)=\frac{c}{(1-2k)}e^{\left(\gamma W(t)-\frac{\gamma^2}{2}t\right)}-\frac{1}{(1-2k)}\left(\frac{\sqrt{3}(2k-1)}{3}\sqrt{\frac{-k^2-\lambda+\upsilon_1}{2k-1}}+\frac{(2k-1)}{3}\sqrt{\frac{-6\upsilon_2}{2k-1}}\sqrt{-\frac{\upsilon_1}{2\upsilon_2}}\tanh\left(\sqrt{-\frac{\upsilon_1}{2}}\left(\frac{x^\alpha}{\Gamma(1+\alpha)}-2kt+\varsigma\right)\right)\right)^2e^{\left(\gamma W(t)-\frac{\gamma^2}{2}t\right)}$$

$$r_4(x,t)=\frac{c}{(1-2k)}e^{\left(\gamma W(t)-\frac{\gamma^2}{2}t\right)}-\frac{1}{(1-2k)}\left(\frac{\sqrt{3}(2k-1)}{3}\sqrt{\frac{-k^2-\lambda+\upsilon_1}{2k-1}}+\frac{(2k-1)}{3}\sqrt{\frac{-6\upsilon_2}{2k-1}}\sqrt{-\frac{\upsilon_1}{2\upsilon_2}}\coth\left(\sqrt{-\frac{\upsilon_1}{2}}\left(\frac{x^\alpha}{\Gamma(1+\alpha)}-2kt+\varsigma\right)\right)\right)^2e^{\left(\gamma W(t)-\frac{\gamma^2}{2}t\right)}.$$

3-3. If $\upsilon_0=0,\upsilon_1<0,\upsilon_2\neq 0$, then

$$s_5(x,t)=\frac{\sqrt{3}(2k-1)}{3}\sqrt{\frac{-k^2-\lambda+\upsilon_1}{2k-1}}e^{\left(i\left(\frac{kx^\alpha}{\Gamma(1+\alpha)}+\lambda t\right)+\gamma W(t)-\frac{\gamma^2}{2}t\right)}+\left(\frac{(2k-1)}{3}\sqrt{\frac{-6\upsilon_2}{2k-1}}\sqrt{-\frac{\upsilon_1}{\upsilon_2}}\sec\left(\sqrt{-\upsilon_1}\left(\frac{x^\alpha}{\Gamma(1+\alpha)}-2kt+\varsigma\right)\right)\right)e^{\left(i\left(\frac{kx^\alpha}{\Gamma(1+\alpha)}+\lambda t\right)+\gamma W(t)-\frac{\gamma^2}{2}t\right)},$$

$$s_6(x,t)=\frac{\sqrt{3}(2k-1)}{3}\sqrt{\frac{-k^2-\lambda+\upsilon_1}{2k-1}}e^{\left(i\left(\frac{kx^\alpha}{\Gamma(1+\alpha)}+\lambda t\right)+\gamma W(t)-\frac{\gamma^2}{2}t\right)}+\left(\frac{(2k-1)}{3}\sqrt{\frac{-6\upsilon_2}{2k-1}}\sqrt{-\frac{\upsilon_1}{\upsilon_2}}\csc\left(\sqrt{-\upsilon_1}\left(\frac{x^\alpha}{\Gamma(1+\alpha)}-2kt+\varsigma\right)\right)\right)e^{\left(i\left(\frac{kx^\alpha}{\Gamma(1+\alpha)}+\lambda t\right)+\gamma W(t)-\frac{\gamma^2}{2}t\right)}.$$

In this case we have

$$r_5(x,t)=\frac{c}{(1-2k)}e^{\left(\gamma W(t)-\frac{\gamma^2}{2}t\right)}-\frac{1}{(1-2k)}\left(\frac{\sqrt{3}(2k-1)}{3}\sqrt{\frac{-k^2-\lambda+\upsilon_1}{2k-1}}+\frac{(2k-1)}{3}\sqrt{\frac{-6\upsilon_2}{2k-1}}\sqrt{-\frac{\upsilon_1}{\upsilon_2}}\sec\left(\sqrt{-\upsilon_1}\left(\frac{x^\alpha}{\Gamma(1+\alpha)}-2kt+\varsigma\right)\right)\right)^2e^{\left(\gamma W(t)-\frac{\gamma^2}{2}t\right)}$$

$$r_6(x,t)=\frac{c}{(1-2k)}e^{\left(\gamma W(t)-\frac{\gamma^2}{2}t\right)}-\frac{1}{(1-2k)}\left(\frac{\sqrt{3}(2k-1)}{3}\sqrt{\frac{-k^2-\lambda+\upsilon_1}{2k-1}}+\frac{(2k-1)}{3}\sqrt{\frac{-6\upsilon_2}{2k-1}}\sqrt{-\frac{\upsilon_1}{\upsilon_2}}\csc\left(\sqrt{-\upsilon_1}\left(\frac{x^\alpha}{\Gamma(1+\alpha)}-2kt+\varsigma\right)\right)\right)^2e^{\left(\gamma W(t)-\frac{\gamma^2}{2}t\right)}.$$

3-4. If $\upsilon_0=\frac{\upsilon_1^2}{4\upsilon_2},\upsilon_1>0,\upsilon_2>0,\text{and}B_1^2-B_2^2>0$, then

$$s_7(x,t)=\frac{\sqrt{3}(2k-1)}{3}\sqrt{\frac{-k^2-\lambda+\upsilon_1}{2k-1}}e^{\left(i\left(\frac{kx^\alpha}{\Gamma(1+\alpha)}+\lambda t\right)+\gamma W(t)-\frac{\gamma^2}{2}t\right)}+\left(\frac{(2k-1)}{3}\sqrt{\frac{-6\upsilon_2}{2k-1}}\sqrt{-\frac{\upsilon_1}{2\upsilon_2}}\tan\left(\sqrt{\frac{\upsilon_1}{2}}\left(\frac{x^\alpha}{\Gamma(1+\alpha)}-2kt+\varsigma\right)\right)\right)e^{\left(i\left(\frac{kx^\alpha}{\Gamma(1+\alpha)}+\lambda t\right)+\gamma W(t)-\frac{\gamma^2}{2}t\right)},$$

$$s_8(x,t)=\frac{\sqrt{3}(2k-1)}{3}\sqrt{\frac{-k^2-\lambda+\upsilon_1}{2k-1}}e^{\left(i\left(\frac{kx^\alpha}{\Gamma(1+\alpha)}+\lambda t\right)+\gamma W(t)-\frac{\gamma^2}{2}t\right)}+\left(\frac{(2k-1)}{3}\sqrt{\frac{-6\upsilon_2}{2k-1}}\sqrt{-\frac{\upsilon_1}{2\upsilon_2}}\cot\left(\sqrt{\frac{\upsilon_1}{2}}\left(\frac{x^\alpha}{\Gamma(1+\alpha)}-2kt+\varsigma\right)\right)\right)e^{\left(i\left(\frac{kx^\alpha}{\Gamma(1+\alpha)}+\lambda t\right)+\gamma W(t)-\frac{\gamma^2}{2}t\right)}.$$

In this case we have

$$r_7(x,t)=\frac{c}{(1-2k)}e^{\left(\gamma W(t)-\frac{\gamma^2}{2}t\right)}-\frac{1}{(1-2k)}\left(\frac{\sqrt{3}(2k-1)}{3}\sqrt{\frac{-k^2-\lambda+\upsilon_1}{2k-1}}+\frac{(2k-1)}{3}\sqrt{\frac{-6\upsilon_2}{2k-1}}\sqrt{-\frac{\upsilon_1}{2\upsilon_2}}\tan\left(\sqrt{\frac{\upsilon_1}{2}}\left(\frac{x^\alpha}{\Gamma(1+\alpha)}-2kt+\varsigma\right)\right)\right)^2e^{\left(\gamma W(t)-\frac{\gamma^2}{2}t\right)}$$

$$r_8(x,t)=\frac{c}{(1-2k)}e^{\left(\gamma W(t)-\frac{\gamma^2}{2}t\right)}-\frac{1}{(1-2k)}\left(\frac{\sqrt{3}(2k-1)}{3}\sqrt{\frac{-k^2-\lambda+\upsilon_1}{2k-1}}+\frac{(2k-1)}{3}\sqrt{\frac{-6\upsilon_2}{2k-1}}\sqrt{-\frac{\upsilon_1}{2\upsilon_2}}\cot\left(\sqrt{\frac{\upsilon_1}{2}}\left(\frac{x^\alpha}{\Gamma(1+\alpha)}-2kt+\varsigma\right)\right)\right)^2e^{\left(\gamma W(t)-\frac{\gamma^2}{2}t\right)}$$

3-5. If $\upsilon_0 = 0$ and $\upsilon_1 > 0$, then

$$s_9(x,t) = \frac{\sqrt{3}(2k-1)}{3}\sqrt{\frac{-k^2-\lambda+\upsilon_1}{2k-1}}e^{\left(i\left(\frac{kx^\alpha}{\Gamma(1+\alpha)}+\lambda t\right)+\gamma W(t)-\frac{\gamma^2}{2}t\right)} + \left(\frac{(2k-1)}{3}\sqrt{\frac{-6\upsilon_2}{2k-1}}\frac{4\upsilon_1 e^{\pm\sqrt{\upsilon_1}\left(\frac{x^\alpha}{\Gamma(1+\alpha)}-2kt+\varsigma\right)}}{\upsilon_1 e^{\pm 2\sqrt{\upsilon_1}\left(\frac{x^\alpha}{\Gamma(1+\alpha)}-2kt+\varsigma\right)}-4\upsilon_1\upsilon_2}\right)e^{\left(i\left(\frac{kx^\alpha}{\Gamma(1+\alpha)}+\lambda t\right)+\gamma W(t)-\frac{\gamma^2}{2}t\right)},$$

$$s_{10}(x,t) = \frac{\sqrt{3}(2k-1)}{3}\sqrt{\frac{-k^2-\lambda+\upsilon_1}{2k-1}}e^{\left(i\left(\frac{kx^\alpha}{\Gamma(1+\alpha)}+\lambda t\right)+\gamma W(t)-\frac{\gamma^2}{2}t\right)} + \left(\frac{(2k-1)}{3}\sqrt{\frac{-6\upsilon_2}{2k-1}}\frac{\pm 4\upsilon_1 e^{\pm\sqrt{\upsilon_1}\left(\frac{x^\alpha}{\Gamma(1+\alpha)}-2kt+\varsigma\right)}}{1-4\upsilon_1 e^{\pm 2\sqrt{\upsilon_1}\left(\frac{x^\alpha}{\Gamma(1+\alpha)}-2kt+\varsigma\right)}}\right)e^{\left(i\left(\frac{kx^\alpha}{\Gamma(1+\alpha)}+\lambda t\right)+\gamma W(t)-\frac{\gamma^2}{2}t\right)}.$$

So we have

$$r_9(x,t) = \frac{c}{(1-2k)}e^{\left(\gamma W(t)-\frac{\gamma^2}{2}t\right)} - \frac{1}{(1-2k)}\left(\frac{\sqrt{3}(2k-1)}{3}\sqrt{\frac{-k^2-\lambda+\upsilon_1}{2k-1}} + \frac{(2k-1)}{3}\sqrt{\frac{-6\upsilon_2}{2k-1}}\frac{4\upsilon_1 e^{\pm\sqrt{\upsilon_1}\left(\frac{x^\alpha}{\Gamma(1+\alpha)}-2kt+\varsigma\right)}}{\upsilon_1 e^{\pm 2\sqrt{\upsilon_1}\left(\frac{x^\alpha}{\Gamma(1+\alpha)}-2kt+\varsigma\right)}-4\upsilon_1\upsilon_2}\right)^2 e^{\left(\gamma W(t)-\frac{\gamma^2}{2}t\right)}$$

$$r_{10}(x,t) = \frac{c}{(1-2k)}e^{\left(\gamma W(t)-\frac{\gamma^2}{2}t\right)} - \frac{1}{(1-2k)}\left(\frac{\sqrt{3}(2k-1)}{3}\sqrt{\frac{-k^2-\lambda+\upsilon_1}{2k-1}} + \frac{(2k-1)}{3}\sqrt{\frac{-6\upsilon_2}{2k-1}}\frac{\pm 4\upsilon_1 e^{\pm\sqrt{\upsilon_1}\left(\frac{x^\alpha}{\Gamma(1+\alpha)}-2kt+\varsigma\right)}}{1-4\upsilon_1 e^{\pm 2\sqrt{\upsilon_1}\left(\frac{x^\alpha}{\Gamma(1+\alpha)}-2kt+\varsigma\right)}}\right)^2 e^{\left(\gamma W(t)-\frac{\gamma^2}{2}t\right)}.$$

4 Conclusion

Many methods have been proposed so far to obtain the solutions of equations with partial non-linear derivatives, most of them have a lot of complexity for calculations and often give us limited solutions. In addition to its simplicity, this method gives us a wide range of answers, some of which are mentioned in the article.

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