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COMMON FIXED POINTS OF FOUR MAPS USING GENERALIZED WEAK CONTRACTIVITY AND WELL-POSEDNESS

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ABSTRACT. In this paper, we introduce the concept of generalized ϕ-contractivity of a pair of maps w.r.t. another pair. We establish a common fixed point result for two pairs of self-mappings, when one of these pairs is generalized ϕ-contraction w.r.t. the other and study the well-posedness of their fixed point problem. In particular, our fixed point result extends the main result of a recent paper of Qingnian Zhang and Yisheng Song.

1. Introduction

The concept of the weak contraction was defined by Alber and Guerre-Delabriere [1] in 1997. Actually in [1], the authors defined such mappings for single-valued maps on Hilbert spaces and proved the existence of fixed points.

Definition 1.1. Let (X, d) be a metric space and S be self-mapping of X. Let $\phi : [0,\infty) \to [0,\infty)$ be a function such that $\phi(0) = 0$ and ϕ is positive on $(0,\infty)$. We say that T is a ϕ-weak contraction if we have

$$d(Tx, Ty) \leq d(fx, fy) - \phi(d(fx, fy)) \tag{1.1}$$

for all x, y in X

Rhoades [9] showed that most results of [1] are still true for any Banach space. Also Rhoades [9] proved the following important fixed point theorem which is one of generalizations of the Banach contraction principle [3], because it contains contractions as special case ($\phi(t) = (1 - k)t$).

Theorem 1.2. *(Rhoades [[9], Theorem 2]). Let (X, d) be a complete metric space, and let T be a ϕ-weak contraction on X. If $\phi : [0,\infty) \to [0,\infty)$ is a continuous and nondecreasing function such that $\phi(0) = 0$ and ϕ is positive on $(0,\infty)$, then T has a unique fixed point.*

Two generalizations of this result were given by I. Beg and M. Abbas in [4] and by A. Azam and M. Shakeel in [2].

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Recently, this theorem was recently extended by Qingnian Zhang and Yisheng Song (see [12]) to the context of generalized weak contractions. More precisely, the following result was established in [12].

Theorem 1.3. *([12]) Let (X, d) be a complete metric space and $S, T : X \to X$ be self-mappings of X such that*

$$d(Tx, Sy) \leq N(x, y) - \phi(N(x, y)), \quad \forall\ x, y \in X, \tag{1.2}$$

where $\phi : [0, \infty) \to [0, \infty)$ is a lower semi-continuous function with $\phi(t) > 0$ for all $t \in (0, \infty)$ and $\phi(0) = 0$ and

$$N(x, y) = \max\{d(x, y), d(Tx, x), d(Sy, y), \frac{1}{2}[d(y, Tx) + d(x, Sy)]\}.$$

Then there exists a unique point $u \in X$ such that $u = Tu = Su$.

In this paper, we introduce the concept of a pair of mappings which is generalized weakly contractive w.r.t. another pair of mappings by means of a function ϕ in the class Φ of functions considered in Theorem 1.3. We establish a common fixed point result for two pairs of self-mappings, when one of these pairs is generalized ϕ-contraction w.r.t. the other and study the well-posedness of their fixed point problem. In particular, our fixed point result (see Theorem 2.4 below) extends Theorem 1.3 of Qingnian Zhang and Yisheng Song (see [12]).

The main result of the second section is Theorem 2.4.

In the third section, we study the well-posedness of the common fixed point problem for two pairs of self-mappings of a metric space such that one of them is ϕ-weakly contractive w.r.t. the other. The main result of this section is Theorem 3.3.

2. Coincidence and common fixed points

We start with some definitions.

Definition 2.1. Let X be a nonempty set and S, T self-mappings on X.

A point $x \in X$ is called a coincidence point of S and T if $Sx = Tx$.

A point $w \in X$ is called a point of coincidence of S and T if there exists a coincidence point $x \in X$ of S and T such that $w = Sx = Tx$.

S and T are weakly compatible if they commute at their coincidence points, that is if $STx = TSx$, whenever $Sx = Tx$.

We recall that the concept of weak compatibility was introduced by Jungck and Rhoades [6].

Definition 2.2. Let Φ be the set of functions $\phi : [0, \infty) \to [0, \infty)$ satisfying the following properties:
(ϕ_1): ϕ is lower semi-continuous.
(ϕ_2): $\phi(0)$ and $\phi(t) > 0$ for all $t > 0$.

Definition 2.3. Let (X, d) be a metric space. Let $S, T, I, J : X \to X$ be four self-mappings of X.

Let $\phi \in \Phi$. The pair (S, T) is called generalized ϕ-weakly contractive with respect to the pair (I, J) if we have

$$d(Sx, Ty) \leq M(x, y) - \phi(M(x, y)), \tag{2.1}$$

for all x, y in X, where

$$M(x,y) := \max\{d(Ix,Jy), d(Ix,Sx), d(Jy,Ty), \frac{1}{2}[d(Ix,Ty)+d(Jy,Sx)]\}.$$

The pair (S,T) is called generalized weakly contractive with respect to the pair (I,J) if it is generalized ϕ-weakly contractive with respect to (I,J) with some $\phi \in \Phi$.

We observe that if $I = J = Id_X$ is the identity mapping, then $N(x,y) = M(x,y)$ for all $x, y \in X$.

The main result of this section reads as follows.

Theorem 2.4. *Let (X,d) be a metric space and let S, T, I, J be four self-mappings of X. Let $\phi \in \Phi$.*

We suppose that:

$(H1)$: *The pair (S,T) is generalized ϕ-weakly contractive with respect to the pair (I,J), that is*

$$d(Sx,Ty) \leq M(x,y) - \phi(M(x,y)), \tag{2.2}$$

for all x, y in X.

$(H2)$: *$S(X) \subset J(X)$ and $T(X) \subset I(X)$.*

$(H3)$: *One of the subsets $S(X)$, $T(X)$, $I(X)$ or $J(X)$ is a complete subspace of X.*

Then,

a) the pair $\{S,I\}$ has a point of coincidence,

b) the pair $\{T,J\}$ has a point of coincidence.

Moreover, if the pairs $\{S,I\}$ and $\{T,J\}$ are weakly compatible, then the mappings S, T, I and J have a unique common fixed point in X.

Proof. Let x_0 be an arbitrary point in X. Set $y_0 = Sx_0$. Since $S(X) \subset J(X)$, then we can find a point $x_1 \in X$ such that $y_0 = Sx_0 = Jx_1$. Set $y_1 = Tx_1$. Since $T(X) \subset I(X)$, then there exists a point $x_2 \in X$ such that $y_1 = Tx_1 = Ix_2$. By induction, we construct two sequences (x_n) and (y_n) in X satisfying for each nonnegative integer n,

$$y_{2n} = Sx_{2n} = Jx_{2n+1} \quad \text{and} \quad y_{2n+1} = Tx_{2n+1} = Ix_{2n+2} \tag{2.3}$$

To simplify notation, for each non negative integer n, we set $t_n := d(y_n, y_{n+1})$.

For all nonnegative integer n we have

$$\begin{aligned} t_{2n+1} &= d(y_{2n+2}, y_{2n+1}) = d(Sx_{2n+2}, Tx_{2n+1}) \\ &\leq M(x_{2n+2}, x_{2n+1}) - \phi(M(x_{2n+2}, x_{2n+1})) \\ &= \max\{t_{2n}, t_{2n+1}, \frac{1}{2}d(y_{2n}, y_{2n+2})\} - \phi(\max\{t_{2n}, t_{2n+1}, \frac{1}{2}d(y_{2n}, y_{2n+2})\}). \end{aligned} \tag{2.4}$$

Since $\frac{1}{2}d(y_{2n}, y_{2n+2}) \leq \frac{1}{2}(t_{2n} + t_{2n+1})$, then

$$\max\{t_{2n}, t_{2n+1}, \frac{1}{2}d(y_{2n}, y_{2n+2})\} = \max\{t_{2n}, t_{2n+1}\}.$$

Suppose that $t_{2n} < t_{2n+1}$. Then by (2.4) we obtain

$$0 < t_{2n+1} \leq t_{2n+1} - \phi(t_{2n+1}) < t_{2n+1},$$

a contradiction. Thus $t_{2n} \geq t_{2n+1}$, and

$$0 < t_{2n+1} \leq t_{2n} - \phi(t_{2n}).$$

By similar arguments, we obtain

$$t_{2n+2} \leq t_{2n+1} - \phi(t_{2n+1}) \leq t_{2n+1}.$$

We conclude that for all nonnegative integer n, we have

$$t_{n+1} \leq t_n - \phi(t_n) \leq t_n. \tag{2.5}$$

The sequence $\{t_n\}$ is nonincreasing, so it converges to a limit (say) $t \geq 0$. Since ϕ is lower semi-continuous, then

$$\phi(t) \leq \liminf_{n\to\infty} \phi(t_n) \leq \lim_{n\to\infty}(t_n - t_{n+1}) = 0.$$

Thus $0 \leq \phi(t) \leq 0$, which implies that $\phi(t) = 0$. By property (ϕ_2), we obtain $t = 0$.

Let us show that $\{y_n\}$ is a Cauchy sequence. Since $\lim_{n\to\infty} d(y_n, y_{n+1}) = 0$, then we need only to show that $\{y_{2n}\}$ is a Cauchy sequence. To get a contradiction, let us suppose that there is a number $\epsilon > 0$ and two sequences $\{2n(k)\}$, $\{2m(k)\}$ with $2k \leq 2m(k) < 2n(k)$, $(k \in \mathbb{N})$ verifying

$$d(y_{2n(k)}, y_{2m(k)}) > \epsilon. \tag{2.6}$$

For each integer k, we shall denote $2n(k)$ the least even integer exceeding $2m(k)$ for which (2.6) holds. Then we have

$$d(y_{2m(k)}, y_{2n(k)-2}) \leq \epsilon \quad \text{and} \quad d(y_{2m(k)}, y_{2n(k)}) > \epsilon. \tag{2.7}$$

For each integer k, we set $p_k := d(y_{2m(k)}, y_{2n(k)})$, then we have

$$\begin{aligned}
\epsilon < p_k &= d(y_{2m(k)}, y_{2n(k)}) \\
&\leq d(y_{2m(k)}, y_{2n(k)-2}) + d(y_{2n(k)-2}, y_{2n(k)-1}) + d(x_{2n(k)-1}, y_{2n(k)}) \\
&\leq \epsilon + t_{2n(k)-2} + t_{2n(k)-1}.
\end{aligned} \tag{2.8}$$

Since the sequence $\{t_n\}$ converges to 0, we deduce from (2.8) that $\{p_k\}$ converges to ϵ. For every integer $k \in \mathbb{N}$ we set

$$\begin{aligned}
q_k &:= d(y_{2m(k)+1}, y_{2n(k)+2}), & r_k &:= d(y_{2m(k)}, y_{2n(k)+1}), \\
s_k &:= d(y_{2m(k)+1}, y_{2n(k)+1}), & v_k &:= d(y_{2m(k)}, y_{2n(k)+2}).
\end{aligned}$$

By using the triangle inequality, for all integer k, we obtain the following estimates:

$$\begin{aligned}
| \, r_k - p_k \, | &\leq t_{2n(k)} \leq t_k, \\
| \, r_k - s_k \, | &\leq t_{2m(k)} \leq t_k, \\
| \, s_k - q_k \, | &\leq t_{2n(k)+1} \leq t_k, \\
| \, v_k - q_k \, | &\leq t_{2m(k)} \leq t_k.
\end{aligned}$$

Since the sequence $\{t_n\}$ converges to 0, we deduce that the sequences: $\{q_k\}$, $\{r_k\}$, $\{s_k\}$ and $\{v_k\}$ converge to ϵ.

For all nonnegative integer k, we have

$$\begin{aligned}
&M(x_{2n(k)+2}, x_{2m(k)+1}) \\
&= \max\{d(y_{2n(k)+1}, y_{2m(k)}), d(y_{2n(k)+1}, y_{2n(k)+2}), \\
&d(y_{2m(k)}, y_{2m(k)+1}), d(y_{2n(k)+1}, y_{2m(k)+1}), d(y_{2m(k)}, y_{2n(k)+2})\} \\
&= \max\{r_k, t_{2n(k)+1}, t_{2m(k)}, s_k, v_k\}.
\end{aligned}$$

Then, by using the condition (2.1), for every non negative integer k, we have the following estimates:

$$\begin{aligned} q_k = d(y_{2n(k)+2}, y_{2m(k)+1}) &= d(Sx_{2n(k)+2}, Tx_{2m(k)+1}) \\ &\leq M(x_{2n(k)+2}, x_{2m(k)+1}) - \phi(M(x_{2n(k)+2}, x_{2m(k)+1})) \\ &\leq \max\{r_k, t_{2n(k)+1}, t_{2m(k)}, s_k, v_k\} - \phi(\max\{r_k, t_{2n(k)+1}, t_{2m(k)}, s_k, v_k\}). \end{aligned}$$

Then, we obtain

$$\phi(\max\{r_k, t_{2n(k)+1}, t_{2m(k)}, s_k, v_k\}) \leq \max\{r_k, t_{2n(k)+1}, t_{2m(k)}, s_k, v_k\} - q_k.$$

Letting k tend to ∞ and using the lower semicontinuity of ϕ, we get

$$\phi(\epsilon) \leq \liminf_{k\to\infty} \phi(\max\{r_k, t_{2n(k)+1}, t_{2m(k)}, s_k, v_k\})$$

$$\leq \lim_{k\to\infty}(\max\{r_k, t_{2n(k)+1}, t_{2m(k)}, s_k, v_k\} - q_k) = 0,$$

which implies $\phi(\epsilon) = 0$ a contradiction to property (ϕ_2). Thus $\{y_n\}$ is a Cauchy sequence.

Suppose that $J(X)$ is a complete subspace of X, Since M is complete, then the sequence $\{y_n\}$ converges to a point (say) $z \in J(X)$. Thus we have

$$z = \lim_{n\to\infty} Sx_{2n} = \lim_{n\to\infty} Jx_{2n+1} = \lim_{n\to\infty} Tx_{2n+1} = \lim_{n\to\infty} Ix_{2n}. \tag{2.9}$$

Let $u \in X$ such that $z = Ju$. By inequality (2.1), we obtain

$$\begin{aligned} d(y_{2n}, Tu) &= d(Sx_{2n}, Tu) \\ &\leq M(x_{2n}, u) - \phi(d(x_{2n}, u)) \\ &= \max\{d(Ix_{2n}, z), d(Ix_{2n}, Sx_{2n}), d(z, Tu), \frac{1}{2}[d(Ix_{2n}, Tu) + d(z, Sx_{2n})]\} \\ &\quad - \phi(\max\{d(Ix_{2n}, z), d(Ix_{2n}, Sx_{2n}), d(z, Tu), \frac{1}{2}[d(Ix_{2n}, Tu) + d(z, Sx_{2n})]\}), \end{aligned}$$

from which, we get

$$\phi(\max\{d(Ix_{2n}, z), d(Ix_{2n}, Sx_{2n}), d(z, Tu), \frac{1}{2}[d(Ix_{2n}, Tu) + d(z, Sx_{2n})]\})$$

$$\leq \max\{d(Ix_{2n}, z), d(Ix_{2n}, Sx_{2n}), d(z, Tu), \frac{1}{2}[d(Ix_{2n}, Tu) + d(Sx_{2n}, z)]\} - d(y_{2n}, Tu).$$

By letting n tend to infinity and using lower semi-continuity, we obtain

$$\begin{aligned} &\phi(d(z, Tu)) \\ &\leq \liminf_{n\to\infty} \phi(\max\{d(Ix_{2n}, z), d(Ix_{2n}, Sx_{2n}), d(z, Tu), \frac{1}{2}[d(Ix_{2n}, Tu) + d(z, Sx_{2n})]\}) \\ &\leq \phi(d(z, Tu)) - d(z, Tu), \end{aligned}$$

which implies that $d(z, Tu)$. Hence we have $z = Ju = Tu$. Since $T(X) \subset I(X)$, then there exists $w \in X$ such that $z = Tu = Iw$. By using inequality (2.1), we have

$$d(Sw, z) = d(Sw, Tu) \leq M(w, u) - \phi(M(w, u)).$$

Since

$$\begin{aligned}&M(w,u)\\&=\max\{d(Iw,Ju),d(Iw,Sw),d(Ju,Tu),\frac{1}{2}[d(Iw,Tu)+d(Ju,Sw)]\}\\&=\max\{0,d(z,Sw),0,\frac{1}{2}[d(z,Sw)]\}\\&=d(z,Sw).\end{aligned}$$

We deduce that

$$d(Sw,z)\leq d(z,Sw)-\phi(d(z,Sw)),$$

from which, we get $\phi(d(z,Sw))=0$, which implies that $d(Sw,z)=0$, thus $z=Sw=Iw$. We conclude that

$$Sw=Iw=z=Ju=Tu. \tag{2.10}$$

So the conclusions a) and b) are obtained. By similar arguments, the same conclusions will be obtained if we suppose that one of $S(X),T(X)$ or $I(X)$ is a complete subspace of X.

Suppose that the pairs $\{S,I\}$ and $\{T,J\}$ are weakly compatible, then by (2.10), we have

$$Sz=Iz \quad \text{and} \quad Tz=Jz.$$

Since

$$\begin{aligned}&M(w,z)\\&=\max\{d(Iw,Jz),d(Iw,Sw),d(Jz,Tz),\frac{1}{2}[d(Iw,Tz)+d(Jz,Sw)]\}\\&=\max\{d(z,Jz),0,0,\frac{1}{2}[d(z,Tz)+d(Jz,z)]\}\\&=d(z,Tz),\end{aligned}$$

then by inequality (2.1), we obtain

$$d(z,Tz)=d(Sw,Tz)\leq M(w,z)-\phi(M(w,z))=d(z,Tz)-\phi(d(z,Tz)),$$

which implies that $\phi(d(z,Tz))=0$. Thus, by property (ϕ_2), we obtain $d(z,Tz)=0$. So we have $z=Tz=Jz$.

Again, by inequality (2.1), we obtain

$$d(Sz,z)=d(Sz,Tz)\leq M(z,z)-\phi(M(z,z))=d(Sz,z)-\phi(d(Sz,z)).$$

Hence $\phi(d(Sz,z))=0$, which by property (ϕ_2), implies that $d(Sz,z)=0$. So we have $z=Sz=Iz$. Thus z is a common fixed point of the mappings S,T,I and J.

Let q be another common fixed point of the mappings S,T,I and J. Then, by using the inequality (2.1), we obtain

$$d(z,q)=d(Sz,Tq)\leq M(z,q)-\phi(d(z,q))=d(z,q)-\phi(d(z,q)),$$

which gives $\phi(d(z,q))=0$. By property (ϕ_2), we conclude that $z=q$. This completes the proof. □

3. Well-posedness

The notion of well-posednes of a fixed point problem has evoked much interest to a several mathematicians, for examples, F.S. De Blasi and J. Myjak (see [5]), S. Reich and A. J. Zaslavski (see [8]), B.K. Lahiri and P. Das (see [7]) and V. Popa (see [10] and [11]).

Definition 3.1. Let (X, d) be a metric space and $T : (X, d) \to (X, d)$ a mapping. The fixed point problem of T is said to be well posed if:
(a) T has a unique fixed point z in X;
(b) for any sequence $\{x_n\}$ of points in X such that $\lim_{n\to\infty} d(Tx_n, x_n) = 0$, we have $\lim_{n\to\infty} d(x_n, z) = 0$.

For a set of mappings, it is natural to introduce the following definition.

Definition 3.2. Let (X, d) be a metric space and let $\mathcal{T}$ be a set of self-mappings of X. The fixed point problem of $\mathcal{T}$ is said to be well-posed if:
(a) $\mathcal{T}$ has a unique fixed point z in X;
(b) for any sequence $\{x_n\}$ of points in X such that

$$\lim_{n\to\infty} d(Tx_n, x_n) = 0, \quad \forall T \in \mathcal{T},$$

we have $\lim_{n\to\infty} d(x_n, z) = 0$.

Concerning the well-posedness of the common fixed point problem for four mappings satisfying the conditions of Theorem 2.4, we have the following result.

Theorem 3.3. *Let (X, d) be a metric space and let S, T, I, J be four self-mappings of X. Let $\phi \in \Phi$.*
We suppose that:
$(H1)$: *The pair (S, T) is ϕ-weakly contractive with respect to the pair (I, J), that is*

$$d(Sx, Ty) \leq M(x, y) - \phi(M(x, y)), \tag{3.1}$$

for all x, y in X.
$(H2)$: *$S(X) \subset J(X)$ and $T(X) \subset I(X)$.*
$(H3)$: *The pairs $\{S, I\}$ and $\{T, J\}$ are weakly compatible.*
$(H4)$: *One of the subsets $S(X)$, $T(X)$, $I(X)$ or $J(X)$ is a complete subspace of X.*
$(H5)$: *The function ϕ is nondecrasing on $[0, \infty)$.*

Then, the common fixed point problem for the set of mappings $\{S, T, I, J\}$ is well-posed.

Proof. We know, by Theorem 2.4, that the mappings S, T, I and J have a unique common fixed point (say) $z \in X$. Let $\{x_n\}$ of points in X such that

$$\lim_{n\to\infty} d(Sx_n, x_n) = \lim_{n\to\infty} d(Tx_n, x_n) = \lim_{n\to\infty} d(Ix_n, x_n) = \lim_{n\to\infty} d(Tx_n, x_n) = 0. \tag{3.2}$$

We observe that for all nonnegative integer n, we have

$$\begin{aligned} & M(z, x_n) \\ &= \max\{d(z, Jx_n), d(Jx_n, Tx_n), \tfrac{1}{2}[d(z, Tx_n) + d(Jx_n, z)]\} \\ &\leq d(z, x_n) + d(x_n, Jx_n) + d(x_n, Tx_n). \end{aligned}$$

By the triangle inequality and inequality (3.1), we have

$$\begin{aligned} d(z,x_n) &\le d(Sz,Tx_n)+d(Tx_n,x_n)\\ &\le M(z,x_n)-\phi(M(z,x_n))+d(Tx_n,x_n)\\ &\le d(z,x_n)+d(x_n,Jx_n)+2d(Tx_n,x_n)-\phi(M(z,x_n)). \end{aligned}$$

We deduce that

$$\phi(M(z,x_n))\le d(x_n,Jx_n)+2d(Tx_n,x_n). \tag{3.3}$$

Thus we have

$$\lim_{n\to\infty}\phi(M(z,x_n))=0. \tag{3.4}$$

To get a contradiction, let us suppose that the sequence $\{x_n\}$ does not converge to z. Then the sequence $\{Jx_n\}$ does not converge to z. Then, there exists a positive number $\epsilon>0$ and a subsequence $\{x_{n_k}\}$ such that

$$d(z,Jx_{n_k})\ge\epsilon,\quad \text{for all integer } k. \tag{3.5}$$

Since ϕ is nondecreasing, from (3.3) and (3.5), we obtain

$$\phi(\epsilon)\le\phi(d(z,Jx_{n_k}))\le\phi(M(z,Jx_{n_k}))\le d(x_{n_k},Jx_{n_k})+2d(Tx_{n_k},x_{n_k}).$$

By letting k to infinity, we get

$$\phi(\epsilon)=0,$$

a contradiction to the property (ϕ_2). This completes the proof. □

As a consequence, we have the following improvement to Theorem 1.3 of [12].

Corollary 3.4. *Let (X,d) be a complete metric space and $S,T:X\to X$ be self mappings of X such that*

$$d(Tx,Sy)\le N(x,y)-\phi(N(x,y)),\quad \forall\, x,y\in X. \tag{1.2}$$

where $\phi:[0,\infty)\to[0,\infty)$ is a lower semi-continuous function with $\phi(t)>0$ for all $t\in(0,\infty)$ and $\phi(0)=0$ and

$$N(x,y)=\max\{d(x,y),d(Tx,x),d(Sy,y),\frac{1}{2}[d(y,Tx)+d(x,Sy)]\}.$$

Then, there exists a unique point $u\in X$ such that $u=Tu=Su$.

Moreover, if ϕ is nondcreasing then the common fixed point problem for the pair $\{S,T\}$ is well-posed.

References

1. Ya. I. Alber and S. Guerre-Delabriere, *Principle of weakly contractive maps in Hilbert spaces,* New Results in Operator Theory and Its Applications (I. Gohberg and Yu. Lyubich, eds.), Oper. Theory Adv. Appl., vol. 98, Birkhuser, Basel, (1997) 7-22.
2. A. Azam and M. Shakeel, *Weakly contractive maps and common fixed points,* Mat. Vesnik, 60 (2008), no. 4, 101-106.
3. S. Banach, *Sur les opérations dans les ensembles abstraits et leur applications aux équations intégrales,* Fundamenta Math., (3) (1922), 133-181.
4. I. Beg and M. Abbas, *Coincidence point and invariant approximation for mappings satisfying generalized weak contractive condition,* Fixed Point Theory Appl., vol 2006 (2006), 1-7.

5. F.S. De Blasi and J. Myjak, *Sur la porosité des contractions sans point fixe,* C. R. Acad. Sci. Paris, (308), (1989) 51-54.
6. G. Jungck and B. E. Rhoades, *Fixed points for set valued functions without continuity,* Indian J. Pure Appl. Math., 29 (3) (1998), 227-238.
7. B.K. Lahiri and P. Das, *Well-posednes and porosity of certain classes of operators,* Demonstratio Math., 38 (2005), 170-176.
8. S. Reich and A.J. Zaslavski, *Well-posednes of fixed point problems,* Far East J. Math. Sci., Special volume 2001, Part III (2001) 393-401.
9. B. E. Rhoades, *Some theorems on weakly contractive maps,* Nonlinear Analysis, 47 (4) (2001), 2683-2693.
10. V. Popa, *Well-posedness of fixed point problem in orbitally complete metric spaces,* Stud. Cerc. St. Ser. Mat. Univ. **16** (2006), Supplement. Proceedings of ICMI 45, Bacău, Sept. 18-20 (2006), 209-214.
11. V. Popa, *Well-posedness of fixed point problem in compact metric spaces,* Bul. Univ. Petrol-Gaze, Ploiesti, Sec. Mat. Inform. Fiz. 60 (1) (2008), 1-4.
12. Qingnian Zhang, Yisheng Song, *Fixed point theory for generalized ϕ-weak contractions,* Applied Mathematics Letters, 22 (2009), 75-78.

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