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Nearly higher ternary derivations in Banach ternary algebras :An alternative fixed point approach

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Abstract

We say a functional equation (ξ) is stable if any function g satisfying the equation (ξ) approximately is near to true solution of (ξ). Using fixed point methods, we investigate approximately higher ternary derivations in Banach ternary algebras via the Cauchy functional equation

$$f(\lambda_1 x + \lambda_2 y + \lambda_3 z) = \lambda_1 f(x) + \lambda_2 f(y) + \lambda_3 f(z) \ .$$

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1. Introduction and preliminaries

A ternary algebra $\mathcal{A}$ is a real or complex linear space, endowed with a linear mapping, the so-called a ternary product $(x, y, z) \to [x, y, z]$ of $\mathcal{A} \times \mathcal{A} \times \mathcal{A}$ into $\mathcal{A}$ such that $[[x, y, z], w, v] = [x, [y, z, w], v] = [x, y, [z, w, v]]$ for all $x, y, z, w, v \in \mathcal{A}$.

If $(\mathcal{A}, \odot)$ is a usual (binary) algebra, then $[x, y, z] := (x \odot y) \odot z$ makes $\mathcal{A}$ into a ternary algebra. Hence the ternary algebra is a natural generalization of the binary case. In particular, if a ternary algebra $(\mathcal{A}, [\ ])$ has a unit, i.e., an element $e \in \mathcal{A}$ such that $x = [x, e, e] = [e, e, x]$ for all $x \in \mathcal{A}$, then $\mathcal{A}$ with the binary product $x \odot y := [x, e, y]$, is a usual algebra. By a normed ternary algebra

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we mean a ternary algebra with a norm $\|.\|$ such that $\|[x, y, z]\| \leq \|x\|\|y\|\|z\|$ for all $x, y, z \in \mathcal{A}$. A Banach ternary algebra is a normed ternary algebra such that the normed linear space with norm $\|.\|$ is complete.

A. Cayley [7] introduced the notion of cubic matrices and a generalization of the determinant, called the hyperdeterminant, then were found again and generalized by M. Kapranov, I. M. Gelfand and A. Zelevinskii in 1990 [28]. As an application in physics, the quark model inspired a particular brand of ternary algebraic systems. The so-called Nambu mechanics which has been proposed by Y. Nambu [33] in 1973, is based on such structures. There are also some applications, although still hypothetical, in the fractional quantum Hall effect, the non-standard statistics (the anyons), supersymmetric theories, Yang-Baxter equation, etc, (cf. [1], [29], [47]).

Throughout this paper, we assume that $\mathcal{A}$ and $\mathcal{B}$ are real or complex ternary algebras. For the sake of convenience, we use the same symbol [ ] (resp. $\|.\|$) in order to represent the ternary products (resp. norms) on ternary algebras $\mathcal{A}$ and $\mathcal{B}$.

A linear mapping $h : \mathcal{A} \to \mathcal{B}$ is said to be a ternary homomorphism if $h([x, y, z]) = [h(x), h(y), h(z)]$ holds for all $x, y, z \in \mathcal{A}$. A linear mapping $d : \mathcal{A} \to \mathcal{A}$ is said to be a ternary derivation if $d([x, y, z]) = [d(x), y, z] + [x, d(y), z] + [x, y, d(z)]$ holds for all $x, y, z \in \mathcal{A}$ (see [34]).

Let $\mathbb{N}$ be the set of natural numbers. For $m \in \mathbb{N} \cup \{0\} = \mathbb{N}_0$, a sequence $H = \{h_0, h_1, ..., h_m\}$ (resp. $H = \{h_0, h_1, ..., h_n, ...\}$) of linear mappings from $\mathcal{A}$ into $\mathcal{B}$ is called a higher ternary derivation of rank m (resp. infinite rank) from $\mathcal{A}$ into $\mathcal{B}$ if

$$h_n([x, y, z]) = \sum_{i+j+k=n} [h_i(x), h_j(y), h_k(z)].$$

holds for each $n \in \{0, 1, ..., m\}$ (resp. $n \in \mathbb{N}_0$) and all $x, y, z \in \mathcal{A}$ (cf, see [25], [44]). The higher ternary derivation H from $\mathcal{A}$ into $\mathcal{B}$ is said to be onto if $h_0 : \mathcal{A} \to \mathcal{B}$ is onto. The higher ternary derivation H on $\mathcal{A}$ is called be strong if h_0 is an identity mapping on $\mathcal{A}$. Of course, a higher ternary derivation of rank 0 from $\mathcal{A}$ into $\mathcal{B}$ (resp. a strong higher ternary derivation of rank 1 on $\mathcal{A}$) is a ternary homomorphism (resp. a ternary derivation). So a higher ternary derivation is a generalization of both a ternary homomorphism and a ternary derivation.

We say a functional equation (ξ) is stable if any function g satisfying the equation (ξ) approximately is near to true solution of (ξ). We say that a functional equation is superstable if every approximately solution is an exact solution of it. The stability problem of functional equations originated from a question of Ulam [46] in 1940, concerning the stability of group homomorphisms. Let $(G_1, .)$ be a group and let $(G_2, *)$ be a metric group with the metric $d(., .)$. Given $\epsilon > 0$, does there exist a $\delta > 0$, such that if a mapping $h : G_1 \to G_2$ satisfies the inequality $d(h(x.y), h(x) * h(y)) < \delta$ for all $x, y \in G_1$, then there exists a homomorphism $H : G_1 \to G_2$ with $d(h(x), H(x)) < \epsilon$ for all $x \in G_1$? In the other words, under what condition does there exist a homomorphism near an approximate homomorphism? The concept of stability for functional equation arises when we replace the functional equation by an inequality which acts as a perturbation of the equation. In 1941, D. H. Hyers [23] gave the first affirmative answer to the question of Ulam for Banach spaces. Let $f : E \to E'$ be a mapping between Banach spaces such that

$$\|f(x + y) - f(x) - f(y)\| \leq \delta$$

for all $x, y \in E$, then there exists a unique additive mapping $T : E \to E'$ such that

$$\|f(x) - T(x)\| \leq \delta$$

for all $x \in E$. Moreover if $f(tx)$ is continuous in $t \in \mathbb{R}$ each fixed $x \in E$, then T is linear. In 1978, Th. M. Rassias [41] proved the following theorem.

Theorem 1.1. *Let $f : E \to E'$ be a mapping from a normed vector space E into a Banach space E'subject to the inequality*

$$\|f(x+y) - f(x) - f(y)\| \leq \epsilon(\|x\|^p + \|y\|^p), \tag{1.1}$$

for all $x, y \in E$, where ϵ and p are constants with $\epsilon > 0$ and $p < 1$. Then there exists a unique additive mapping $T : E \to E'$ such that

$$\|f(x) - T(x)\| \leq \frac{2\epsilon}{2 - 2^p}\|x\|^p \tag{1.2}$$

for all $x \in E$. If $p < 0$ then inequality (1.1) holds for all $x, y \neq 0$, and (1.2) for $x \neq$. Also, if the function $t \mapsto f(tx)$ from $\mathbb{R}$ into E' is continuous in real t for each fixed $x \in E$, then T is linear.

Since then, a great deal of work of Rassias type has been done by a number of authors (cf. [[27], [36], [37], [39]] and reference therein).

In 1949, D. G. Bourgin [6] proved the following result, which is sometimes called the superstability of ring homomorphisms: suppose that A and B are Banach algebras with unit. If $f : A \to B$ is a surjective mapping such that

$$\|f(x+y) - f(x) - f(y)\| \leq \varepsilon,$$

$$\|f(xy) - f(x)f(y)\| \leq \delta$$

for some $\varepsilon \geq 0$, $\delta \geq 0$ and all $x, y \in A$, then f is a ring homomorphism.

Recently, R. Badora [5] and T. Miura *et al.* [31] proved the Hyers-Ulam stability, the Isac and Rassias-type stability [24], the Hyers-Ulam-Rassias stability and the Bourgin-type superstability of ring derivations on Banach algebras.

On the other hand, C. Park [34] has contributed works on the stability problem of ternary homomorphisms and ternary derivations.

Cădariu and Radu applied the fixed point method to the investigation of stability of the functional equations. (see also [[2]- [35]]).

There are some examples of approximately higher ternary derivations which are not exactly higher ternary derivations in Banach ternary algebras. The remark 1.1 [38] is a slight modification of an example given by P. Semrl [45] which is due to B. E. Johnson [26] (see also [[31], Example 1.1]).

In this paper, we will adopt the fixed point alternative of Cădariu and Radu to show the existence of an exact higher ternary derivation near to an approximately higher ternary derivation by investigating the Hyers-Ulam stability for higher ternary derivations in Banach ternary algebras. Furthermore, we are going to examine the Isac and Rassias-type stability [24] and the Bourgin-type superstability for higher ternary derivations in Banach ternary algebras.

2. main results

Before proceeding to the main results, we will state the following theorem.

Theorem 2.1. *(the alternative of fixed point [11]). Suppose that we are given a complete generalized metric space (Ω, d) and a strictly contractive mapping $T : \Omega \to \Omega$ with Lipschitz constant L. Then for each given $x \in \Omega$, either $d(T^m x, T^{m+1}x) = \infty$ for all $m \geq 0$, or other exists a natural number m_0 such that*

- ($\star$) $d(T^m x, T^{m+1}x) < \infty$ *for all* $m \geq m_0$;
- ($\star$) *the sequence* $\{T^m x\}$ *is convergent to a fixed point* y^* *of* T.
- ($\star$) y^* *is the unique fixed point of* T *in the set* $\Lambda = \{y \in \Omega : d(T^{m_0}x, y) < \infty\}$;
- ($\star$) $d(y, y^*) \leq \frac{1}{1-L} d(y, Ty)$ *for all* $y \in \Lambda$.

By a similar to in [4], we first obtain the Hyers-Ulam stability result.

Theorem 2.2. *Let $\mathcal{A}$ be a normed ternary algebra and B a Banach ternary algebra. Suppose that $F = \{f_0, f_1, ..., f_n, ...\}$ is a sequence of mappings from $\mathcal{A}$ into $\mathcal{B}$ such that for some $\delta \geq 0$, $\varepsilon \geq 0$ and each $n \in \mathbb{N}_0$,*

$$\|f_n(\lambda_1 x + \lambda_2 y + \lambda_3 z) - \lambda_1 f_n(x) - \lambda_2 f_n(y) - \lambda_3 f_n(z)\| \leq \varepsilon \tag{2.1}$$

and

$$\|f_n([x, y, z]) - \sum_{i+j+k=n} [f_i(x), f_j(y), f_k(z)]\| \leq \delta \tag{2.2}$$

hold for all $x, y, z \in \mathcal{A}$ and all $\lambda_1, \lambda_2, \lambda_3 \in \mathbb{U} = \{z \in \mathbb{C} : |z| = 1\}$. Then there exists a unique higher ternary derivation $H = \{h_0, h_1, ..., h_n, ...\}$ of any rank from $\mathcal{A}$ into $\mathcal{B}$ such that for each $n \in \mathbb{N}_0$,

$$\|f_n(x) - h_n(x)\| \leq \frac{\varepsilon}{2} \tag{2.3}$$

holds for all $x \in \mathcal{A}$. Moreover, we have

$$\sum_{i+j+k=n} [h_i(x), h_j(y), \{h_k(z) - f_k(z)\}] = 0 \tag{2.4}$$

for each $n \in \mathbb{N}_0$ and all $x, y, z \in \mathcal{A}$.

Proof . Putting $\lambda_1 = \lambda_2 = \lambda_3 = 1$ and $x = y = z$ in (2.1) implies

$$\|\frac{1}{3} f_n(3x) - f_n(x)\| \leq \frac{\varepsilon}{3} \tag{2.5}$$

for each $n \in \mathbb{N}_0$ and all $x \in \mathcal{A}$.

Consider the set $X_n := \{g \mid g : \mathcal{A} \to \mathcal{B}\}$ and introduce the generalized metric on X_n for all $n \in \mathbb{N}_0$:

$$d(h, g) := inf\{c \in \mathbb{R}^+ : \|g(x) - h(x)\| \leq c\varepsilon, \quad \forall x \in \mathcal{A}\}.$$

It is easy to show that (X_n, d) is complete for all $n \in \mathbb{N}_0$. Now we define the linear mapping $J : X_n \to X_n$ for all $n \in \mathbb{N}_0$ by

$$J(h)(x) = \frac{1}{3}h(3x)$$

for all $x \in \mathcal{A}$. By Theorem 3.1 of [11],

$$d(J(g), J(h)) \leq \frac{1}{3}d(g, h)$$

for all $g, h \in X_n$.

It follows from (2.5) that

$$d(f, J(f)) \leq \frac{1}{3}.$$

By Theorem 2.1, J has a unique fixed point in the set $X_{n_1} := \{h \in X : d(f, h) < \infty\}$ for all $n \in \mathbb{N}_0$. Let h_n be the fixed point of J for all $n \in \mathbb{N}_0$. h_n is the unique mapping with

$$h_n(3x) = 3h_n(x)$$

for all $x \in \mathcal{A}$ and for all $n \in \mathbb{N}_0$ satisfying there exists $c \in (0, \infty)$ such that

$$\|h_n(x) - f_n(x)\| \leq c\varepsilon$$

for all $x \in \mathcal{A}$ and for all $n \in \mathbb{N}_0$. On the other hand we have $lim_m d(J^m(f_n), h_n) = 0$ for all $n \in \mathbb{N}_0$. It follows that

$$\lim_{m\to\infty} \frac{1}{3^m} f_n(3^m x) = h_n(x) \tag{2.6}$$

for all $x \in \mathcal{A}$ and for all $n \in \mathbb{N}_0$. It follows from $d(f_n, h_n) \leq \frac{1}{1-\frac{1}{3}}d(f_n, J(f_n))$, that

$$d(f_n, h_n) \leq \frac{1}{2}$$

for all $n \in \mathbb{N}_0$. This implies the inequality (2.3).

Let $\lambda_1 = \lambda_2 = \lambda_3 = 1$ in (2.1) and then let us replace x by $3^m x$, y by $3^m y$ and z by $3^m z$. By dividing the result by 3^m and taking $m \to \infty$, we see that for each $n \in \mathbb{N}_0$, h_n satisfies the functional equation $f(x + y + z) - f(x) - f(y) - f(z) = 0$ for all $x, y, z \in \mathcal{A}$. Note that the functional equation $f(x + y + z) - f(x) - f(y) - f(z) = 0$ is equivalent to the Cauchy additive functional equation $f(x + y) - f(x) - f(y) = 0$. Hence h_n is additive for each $n \in \mathbb{N}_0$.

The rest of the proof is similar to the proof of Theorem 2.1 in [38]. □

Let $\mathbb{R}^+$ be the set of positive real numbers. G. Isac and Th. M. Rassias [24] generalized the Hyers theorem by introducing a mapping $\psi : \mathbb{R}^+ \to \mathbb{R}^+$ subject to the conditions

$$\lim_{t\to\infty} \frac{\psi(t)}{t} = 0, \tag{2.7}$$

$$\psi(ts) \leq \psi(t)\psi(s) \quad forall\ t, s \in \mathbb{R}^+, \tag{2.8}$$

$$\psi(t) < t \quad forall\ t > 1. \tag{2.9}$$

Here we obtain the Isac and Rassias-type stability result for higher ternary derivations which is a generalization of Theorem 2.2.

Theorem 2.3. *Let $\mathcal{A}$ be a normed ternary algebra, $\mathcal{B}$ a Banach ternary algebra and $\psi : \mathbb{R}^+ \to \mathbb{R}^+$ a mapping with properties (2.7), (2.8) and (2.9). In addition, let $\varphi : \mathbb{R}^+ \to \mathbb{R}^+$ be a mapping satisfying the condition*

$$\lim_{t\to\infty} \frac{\varphi(t)}{t} = 0. \tag{2.10}$$

Suppose that $F = \{f_0, f_1, ..., f_n, ...\}$ is a sequence of mappings from $\mathcal{A}$ into $\mathcal{B}$ such that for some $\varepsilon \geq 0$ and each $n \in \mathbb{N}_0$,

$$\|f_n(\lambda_1 x + \lambda_2 y + \lambda_3 z) - \lambda_1 f_n(x) - \lambda_2 f_n(y) - \lambda_3 f_n(z)\| \leq \varepsilon\{\psi(\|x\|) + \psi(\|y\|) + \psi(\|z\|)\} \tag{2.11}$$

and

$$\|f_n([x, y, z]) - \sum_{i+j+k=n} [f_i(x), f_j(y), f_k(z)]\| \leq \varphi(\|x\|\|y\|\|z\|) \tag{2.12}$$

hold for all $x, y, z \in \mathcal{A}$ and all $\lambda_1, \lambda_2, \lambda_3 \in \mathbb{U} = \{z \in \mathbb{C} : |z| = 1\}$. Then there exist a unique higher ternary derivation $H = \{h_0, h_1, ..., h_n, ..\}$ of any rank from $\mathcal{A}$ into $\mathcal{B}$ and a constant $k \in \mathbb{R}$ such that for each $n \in \mathbb{N}_0$ and all $x \in \mathcal{A}$,

$$\|f_n(x) - h_n(x)\| \leq k\varepsilon\psi(\|x\|). \tag{2.13}$$

Moreover, the relation (2.4) is fulfilled.

Proof . Putting $\lambda_1 = \lambda_2 = \lambda_3 = 1$ and $x = y = z$ in (2.11) yields

$$\|\frac{1}{3} f_n(3x) - f_n(x)\| \leq \varepsilon\psi(\|x\|) \tag{2.14}$$

for each $n \in \mathbb{N}_0$ and all $x \in \mathcal{A}$. Consider the set $X_n := \{g \mid g : \mathcal{A} \to \mathcal{B}\}$ and introduce the generalized metric on X_n for all $n \in \mathbb{N}_0$:

$$d(h, g) := inf\{c \in \mathbb{R}^+ : \|g(x) - h(x)\| \leq c\psi(\|x\|), \quad \forall x \in \mathcal{A}\}.$$

It is easy to show that (X_n, d) is complete for all $n \in \mathbb{N}_0$. Now we define the linear mapping $J : X_n \to X_n$ for all $n \in \mathbb{N}_0$ by

$$J(h)(x) = \frac{1}{3} h(3x)$$

for all $x \in \mathcal{A}$. By Theorem 3.1 of [11],

$$d(J(g), J(h)) \leq \frac{1}{3} d(g, h)$$

for all $g, h \in X_n$.

It follows from (2.14) that

$$d(f, J(f)) \leq \varepsilon.$$

By Theorem 2.1, J has a unique fixed point in the set $X_{n_1} := \{h \in X : d(f, h) < \infty\}$ for all $n \in \mathbb{N}_0$. Let h_n be the fixed point of J for all $n \in \mathbb{N}_0$. h_n is the unique mapping with

$$h_n(3x) = 3h_n(x)$$

for all $x \in \mathcal{A}$ and for all $n \in \mathbb{N}_0$ satisfying there exists $c \in (0, \infty)$ such that

$$\|h_n(x) - f_n(x)\| \le c\psi(\|x\|)$$

for all $x \in \mathcal{A}$ and for all $n \in \mathbb{N}_0$. On the other hand we have $lim_m d(J^m(f_n), h_n) = 0$ for all $n \in \mathbb{N}_0$. It follows that

$$\lim_{m\to\infty} \frac{1}{3^m} f_n(3^m x) = h_n(x)$$

for all $x \in \mathcal{A}$ and for all $n \in \mathbb{N}_0$. It follows from $d(f_n, h_n) \le \frac{1}{1-\varepsilon} d(f_n, J(f_n))$, that

$$d(f_n, h_n) \le \frac{\varepsilon}{1-\varepsilon}$$

for all $n \in \mathbb{N}_0$. This implies the inequality (2.13).

Let $\lambda_1 = \lambda_2 = \lambda_3 = 1$ in (2.11) and then let us replace x by $3^m x$, y by $3^m y$ and z by $3^m z$. Let us divide the result by 3^m and utilize (2.8). Now, if we take $m \to \infty$, then we see that for each $n \in \mathbb{N}_0$, h_n satisfies the functional equation $f(x + y + z) - f(x) - f(y) - f(z) = 0$ for all $x, y, z \in \mathcal{A}$. So, h_n is additive for each $n \in \mathbb{N}_0$.

The rest of the proof is similar to the proof of Theorem 2.2 in [38]. □

Remark 2.4. *The typical example of the mapping ψ fulfilling (2.7), (2.8) and (2.9) is given by $\psi(t) = t^p$, where $p < 1$. The example of the mapping φ satisfying (2.10) is $\varphi(t) = t^q$, where $q < 1$. If we intend to consider the case of $p, q > 1$, then we adopt the method given by Z. Gajda in [20] to obtain the Isac and Rassias-type stability result for the mapping $\psi : \mathbb{R}^+ \to \mathbb{R}^+$ fulfilling the conditions*

$$\lim_{t\to\infty} \frac{\psi(t)}{t} = 0, \tag{2.15}$$

$$\psi(ts) \le \psi(t)\psi(s) \quad forall t, s \in \mathbb{R}^+, \tag{2.16}$$

$$\psi(t) < t \quad forall t \in (0, 1). \tag{2.17}$$

In the proof of Theorem 2.2, if we replace (2.6) by

$$h_n(x) = \lim_{m\to\infty} 3^m f_n(\frac{1}{3^m} x)$$

and (2.9) in [38] by

$$\lim_{m\to\infty} 3^m \triangle_n(\frac{1}{3^m} x, y, z) = 0,$$

then Theorem 2.3 is still true under the conditions (2.15), (2.16) and (2.17). As consequences of Theorem 2.2, we get the following Bourgin-type super- stability.

Corollary 2.5. *(Corollary 2.4 of [38]) Let $\mathcal{A}$ be a Banach ternary algebra with unit e and $\mathcal{B}$ a Banach ternary algebra with unit e^*. Suppose that $F = \{f_0, f_1, ..., f_n, ...\}$ is a sequence of mappings from $\mathcal{A}$ into $\mathcal{B}$ satisfying (2.1) and (2.2), where f_0 is onto and $f_0(e) = e^*$. Then $F = \{f_0, f_1, ..., f_n, ...\}$ is a higher ternary derivation of any rank from $\mathcal{A}$ onto $\mathcal{B}$.*

Corollary 2.6. *(Corollary 2.6 of [38]) Let A be a Banach ternary algebra with unit. Suppose that $F = \{f_0, f_1, ..., f_n, ...\}$ is a sequence of mappings on $\mathcal{A}$ satisfying (2.1) and (2.2), where f_0 is an identity mapping on $\mathcal{A}$. Then $F = \{f_0, f_1, ..., f_n, ...\}$ is a strong higher ternary derivation of any rank on $\mathcal{A}$.*

Remark 2.7. *As in Theorem 2.3 and Remark 2.4, we can generalize our results by substituting another functions satisfying appropriate conditions (see, for instance, [21]) for the bounds ε and δ of the inequalities corresponding to the functional equations*

$$f_n(\lambda_1 x + \lambda_2 y + \lambda_3 z) = \lambda_1 f_n(x) + \lambda_2 f_n(y) + \lambda_3 f_n(z),$$

$$f_n([x, y, z]) = \sum_{i+j+k=n} [f_i(x), f_j(y), f_k(z)].$$

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