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Wavelet collocation solution of non-linear Fin problem with temperature dependent thermal conductivity and heat transfer coefficient

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Abstract

In this paper, Wavelet Collocation Method has been used to solve nonlinear fin problem with temperature dependent thermal conductivity and heat transfer coefficient. Thermal conductivity of fin materials varies any type so that we consider thermal conductivity as the general function of temperature. Here we consider three particular cases, where we assume that thermal conductivity is constant, linear and exponential function of temperature. In each case efficiency of fin is evaluated. The whole analysis is presented in dimensionless form and the effect of variability of fin parameter, exponent and thermal conductivity parameter on temperature distribution and fin efficiency is shown graphically and discussed in detail.

Keywords: Collocation; conductivity; fin, temperature; transfer; wavelet.
MSC: Primary 34B15; Secondary 34G20.

1. Introduction and preliminaries

Nomenclature:

A_c = cross section area of the fin (m^2)
exp = exponential term
h = heat transfer coefficient ($Wm^{-2}K^{-1}$)
h_b = the heat transfer coefficient at fin base ($Wm^{-2}K^{-1}$)
k = thermal conductivity of fin material ($Wm^{-1}K^{-1}$)
k_a = thermal conductivity at ambient temperature($Wm^{-1}K^{-1}$)
L = length of fin (m)
n = exponent
P = perimeter of the fin (m)
T = local temperature on the fin surface (K)
T_a = environment (ambient) temperature (K)

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T_b = base temperature of the fin (K)
x = dimensional space coordinate (m)
λ = the slope of the thermal conductivity-temperature curve (K^{-1})
η = fin efficiency

Dimensionless variables and similarity criteria

$N = \sqrt{\frac{h_b P L^2}{k_a A_c}}$ fin parameter
$X = \frac{x}{L}$ space coordinate,
$\theta = (\frac{T-T_a}{T_b-T_a})$ temperature of the fin
$\beta = \lambda(T_b - T_a)$,

Abbreviation

ADM = Adomian Decomposition Method
HAM = Homotopy Analysis Method
HPM = Homotopy Perturbation Method
WCM = Wavelet Collocation Method

Heat exchangers are known as fins. In the study of heat transfer, fin is the surface made by metallic material which is used to increase the rate of heat transfer to environment. The rate of heat transfer depends on the surface area of the fin. Heat transfer is most important aspect in industrial processes. In industry, heat is removed for smooth functioning of machine, for that it is either added or transfered from one medium to another.

Medium chosen may be liquid, solid or gas. Fin or heat exchangers are widely used in industries, for instance in sugar plants, chemical plants, power plants, refineries, Automobile industry, food, medical science, petroleum, air conditioning, refrigeration, chilling plant, cold storage etc. and electrical devices like motors and transformers in which the generated heat can be efficiently transferred. Typically, the fin material has a high thermal conductivity. The most common fin materials are aluminium alloys. Aluminium alloys 6061 and 6063 are commonly used, with thermal conductivity values of 166 and 201 W/m K, respectively. Copper has around twice the conductivity of aluminium, and five times more expensive than aluminium. Diamond is another heat sink material and its thermal conductivity is 2000 W/mK which exceeds copper by five-folds. In contrast to metals, where heat is conducted by delocalized electrons, lattice vibrations are responsible for diamond's very high thermal conductivity. Composite materials can also be used; for example a copper-tungsten pseudo-alloy, AlSiC (silicon carbide in aluminium), Dym-alloy (diamond in copper-silver alloy) etc. For production of heat exchangers copper, cast steel, aluminium metals are generally used. Silver has highest thermal conductivity of all the known metals at room temperature.

A. Rajabi [16], studied the problem using HPM to evaluate the temperature distribution and efficiency of straight fins. According to him this method provides simple approximate exact solution which converges very fast. Schematic diagram of fin is presented in Figure1. Ching-Huang Chiu and Chao-Kuang Chen [4] solve the fin problem and evaluate the efficiency using ADM, in which nonlinear problems were treated in a manner similar to linear problems. The ADM provides an analytical solution in the form of an infinite power series. They compare results with numerical solution, Perturbation solution and Galerkin solution, and conclude that the decomposition method gives faster convergence and higher accuracy in comparison to other methods. The accuracy of the ADM depends upon the number of terms used in solution. Cihat Arslanturk [2] studied the same problem using ADM to evaluate the efficiency of straight fins and to determine temperature distribution within the fin. For constant thermal conductivity he compared exact and ADM results. Davood et al. [11] used HPM to evaluate the temperature distribution of annular fin with temperature dependent thermal conductivity. The annular fin heat transfer rate with temperature dependent thermal conductivity has been obtained as a function of thermo geometric fin parameter and the thermal conductivity parameter describing the variation of the thermal conductivity. Ebrahim Momoniat [14] provided a formulation of the fin efficiency based on Newtons law of cooling and compared with a formulation based on a ratio of heat transferred from the fin surface to the surrounding fluid to the heat conduction through the base. This paper shows that first formulation of the fin efficiency contains approximate errors so approximate efficiency has been determined. The second formulation of fin efficiency is exact when the first integral can be evaluated. F. Khani et al.[12] used HAM to determine the analytical approximate solution and to obtain efficiency and temperature distribution in fin. According to them, by choosing the auxiliary parameter h in a suitable way, one can obtain reasonable solution for large values of N and β. ADM and HPM solutions fail when N increases to a large number but the HAM solution remains accurate. G. Domairry and N. Fazeli [8] used HAM to solve many differential equations and obtained the fin efficiency of convective straight fin with temperature dependent thermal conductivity. The result thus obtained is compared with exact solution and ADM. I.N. Dulkin and G.I. Garasko [9, 10] obtained a closed form inverse solution of the one dimensional heat conduction problem for a single fin or spine of constant cross section with an insulated tip is generalized to account for the effect of the tip heat loss.

Min-Hsing Chang [3] used ADM to analyze the thermal characteristics of a straight rectangular annular fin for all possible types of heat transfer. This method provides a simple approximate solution. Operational matrix of integration presented by M. Razzaghi et al.[15]. M.S.H. Chowdhury et al.[5] the power-law fin-type problem was solved by the Homotopy Analysis Method and the result thus obtained is compared with the exact solution and decomposition method. The obtained solutions are more accurate with easily computable terms. Sazzad Hossien Chowdhury [6] used HPM to evaluate temperature distribution in fin, where heat transfer coefficient is considered to vary with a power-law type function of temperature. He compared his results for thirteen-terms ADM solution and six-term modified HPM, here HPM gives better result. Rafael Cortell [7] used numerical analysis to obtain the temperature distribution within a single fin. It is assumed that the heat transfer coefficient depends on temperature and is considered as power law type form. S. Abbasbandy and E. Shivanian [1] found an exact analytical solution in implicit form of a nonlinear equation for different values of n; and obtained dual solutions for $n = -3/2, N = \pm 0.9$ and $n = -3, N = \pm 0.4$. Sin Kim and Huang [13] provide an extension of a series solution of this nonlinear fin problem. It has been shown that the Adomian decomposition solution is just an approximation of the series solution given in their work. The accuracy of approximate solution is verified with the numerical solution. In difficult nonlinear moving fin problem, Legendre wavelet basis functions and WCM method is used by [17]. In [17], thermal conductivity of fin is considered as constant or a linear function of temperature. No solution is available when thermal conductivity of fin varies with temperature in general.

In this study we consider the heat transfer in a fin whose thermal conductivity varies in general with temperature and the heat transfer coefficient is expressed as a power law type form. To solve this nonlinear boundary value problem, we use Wavelet Collocation Method (WCM).

As WCM require less computation and provide better result in comparison to other methods. Three particular cases namely when thermal conductivity is (I) constant, (II) linear and (III) exponential function of temperature are discussed in detail.

2. Formulation of the problem

We consider straight one dimensional fin with a constant cross section area A_c. The fin, with perimeter P and length L, is attached to a base surface of temperature T_b and extends into a fluid of temperature T_a. It is assumed that the amount of heat transfer through the tip end is negligibly small. The one dimensional steady-state heat balance equation can be written as

$$A_c \frac{d}{dx}\left(k(T)\frac{dT}{dx}\right) - Ph(T - T_a) = 0,\ 0 < x < L, \tag{2.1}$$

with conditions

$$x = 0, \frac{dT}{dx} = 0$$
$$x = L, T = T_b$$

where h is the heat transfer coefficient and may be non-uniform along the fin.
The heat transfer coefficient may depend on the temperature and usually can be expressed as a power form

$$h = h(T) = h_b\left(\frac{T - T_a}{T_b - T_a}\right)^n, \tag{2.2}$$

where h_b is the heat transfer coefficient at the base temperature. The exponent n depends on the heat transfer mode. Typical values of n are -1/4 for laminar film boiling or condensation, 1/4 for laminar natural convection, 1/3 for turbulent natural convection, 2 for nucleate boiling, 3 for radiation and 0 for constant heat transfer coefficient. In Eq. (2.1), K is the thermal conductivity. We consider thermal conductivity as the general function of temperature i.e.

$$k(T) = k_a f\left(\frac{T - T_a}{T_b - T_a}\right).$$

Introducing the dimensionless variables

$$\theta = \frac{T - T_a}{T_b - T_a}, X = \frac{x}{L}, \beta = \lambda(T_b - T_a), N^2 = \frac{h_b P L^2}{k_a A_c}. \tag{2.3}$$

The heat balance Eq. (2.1) can be written as

$$f(\theta)\frac{d^2\theta}{dX^2}+g(\theta)\left(\frac{d\theta}{dX}\right)^2-N^2\theta^{n+1}=0. \tag{2.4}$$

The Eq. (2.4) is subjected to

$$\theta'(0)=0 \tag{2.5}$$

$$\theta(1)=1 \tag{2.6}$$

3. Solution of the problem

We solve this problem using Wavelet Collocation Method. Continuous Wavelets is defined by the following formula

$$\psi_{a,b}(X)=|a|^{\frac{-1}{2}}\psi\left(\frac{X-b}{a}\right),a,b\in R,a\neq 0, \tag{3.1}$$

where a is dilation parameter and b is a translation parameter.
Legendre wavelets is defined on the interval $(0,1)$ by [15]

$$\psi_{n,m}(X)=\begin{cases}\sqrt{(m+1/2)}2^{k/2}P_m\left(2^kX-\hat{n}\right) & , \quad \frac{\hat{n}-1}{2^k}\leq X\leq\frac{\hat{n}+1}{2^k}\\ 0 & , \quad otherwise\end{cases}, \tag{3.2}$$

where $m=0,1,\ldots,M-1$ and $n=1,2,\ldots,2^{k-1}$. Here $P_m(X)$ is the well known Legendre polynomials of order m.

$$P_0(X)=1,P_1(X)=X,P_{m+1}(X)=\frac{2m+1}{m+1}XP_m(X)-\frac{m}{m+1}P_{m-1}(X),m=1,2,3,\ldots,M-1. \tag{3.3}$$

A function f(X) defined in domain [0,1] can be expressed as

$$f(X)=\sum_{n=1}^{\infty}\sum_{m=0}^{\infty}c_{n,m}\psi_{n,m}\left(X\right), \tag{3.4}$$

where $c_{n,m}=<f(X),\psi_{n,m}\left(X\right)>$ in which $<.,.>$ denotes the inner product.
If we take some terms of infinite series, then Eq. (3.4)can be written as

$$f(X)=\sum_{n=1}^{2^{k-1}}\sum_{m=0}^{M-1}c_{n,m}\psi_{n,m}\left(X\right)=C^T\psi(X), \tag{3.5}$$

where C and $\psi(X)$ *are* $M\times 1$ Matrices given by

$$C=[c_{1,0},c_{1,1},\ldots,c_{1,M-1},c_{2,0},c_{2,1},\ldots,c_{2,M-1},c_{2^{k-1},0}c_{2^{k-1},1},\ldots,c_{2^{k-1},M-1}]^T. \tag{3.6}$$

$$\psi\left(X\right)=[\psi_{1,0}(X),\psi_{1,1}(X),\ldots,\psi_{1,M-1}(X),\psi_{2,0}(X),\ldots,\psi_{2,M-1}(X),\ldots \\ \psi_{2^{k-1},0}(X)\psi_{2^{k-1},1}(X),\ldots,\psi_{2^{k-1},M-1}(X)]^T. \tag{3.7}$$

(i)Property of the product of two Legendre wavelets
If E is a given wavelets vector then we have the property

$$E^T\psi\psi^T=\psi^T\hat{E}, \tag{3.8}$$

where $\hat{E}$ *is* $M\times M$ matrices depend on the wavelet vector E.
(ii) Operational matrix of integration
The integration of the wavelets $\psi(X)$ which is defined in Eq. (3.2) can be obtained as

$$\int_0^X\psi(s)ds=P\psi(X),\ X\in[0,1], \tag{3.9}$$

where P is $M \times M$ operational matrix of integration is given by

$$P = \frac{1}{2}\begin{pmatrix} 1 & \frac{1}{\sqrt{3}} & 0 & \cdots & 0 \\ \frac{-1}{\sqrt{3}} & 0 & \frac{1}{\sqrt{15}} & \cdots & 0 \\ 0 & \frac{-1}{\sqrt{15}} & 0 & \cdots & 0 \\ & \vdots & & \ddots & \vdots \\ 0 & 0 & 0 & \cdots & \frac{\sqrt{2M-3}}{(2M-3)\sqrt{2M-1}} \\ 0 & 0 & 0 & \cdots & \frac{-\sqrt{2M-3}}{(2M-3)\sqrt{2M-1}} \; 0 \end{pmatrix}. \tag{3.10}$$

3.1. Wavelet Collocation Method

Let

$$\theta''(X) = C^T\psi(X). \tag{3.11}$$

Integrating from 0 to X, we get

$$\theta'(X) = \theta'(0) + C^T P\psi(x),$$

$$\Rightarrow \theta'(X) = C^T P\psi(X). \tag{3.12}$$

Again integrating above equation and using boundary conditions Eq. (2.5)and Eq. (2.6), we get

$$\theta(X) = \theta(0) + C^T P^2\psi(X). \tag{3.13}$$

Putting X = 1 in Eq. (3.13), we get

$$\theta(0) = 1 - C^T P^2\psi(1).$$

Substituting $\theta(0)$ in Eq. (3.13), we get

$$\theta(X) = 1 - C^T P^2\psi(1) + C^T P^2\psi(X). \tag{3.14}$$

Substituting the value of $\theta''(X)$ and $\theta(X)$ in Eq. (2.4), we obtain

$$f(\theta)\, C^T\psi(X) + g(\theta)(C^T P\psi(X))^2 - N^2\left\{1 - C^T P^2\psi(1) + C^T P^2\psi(X)\right\}^{n+1}$$

$$= R(X, c_1, c_2, \ldots, c_n) = 0. \tag{3.15}$$

As $\theta(X)$ is an approximate solution of system Eq. (2.4) to Eq. (2.6). Choosing n collocation points $X_i, i = 1, 2, 3, \ldots, n$ in the interval $(0, 1)$, at which residual $R(X, c_1, c_2, \ldots, c_n)$ equal to zero. The number of such points must be equal to the number of coefficients $c_i, i = 1, 2, 3, \ldots, n$. Thus, we get $R(X, c_1, c_2, \ldots, c_n) = 0, i = 1, 2, 3, \ldots, n$.

Here we consider three particular cases as follows:

Case I Thermal conductivity is constant,$k = k_a$ i.e $f(\theta) = 1, g(\theta) = 0$. Eq. (3.15) reduces to

$$C^T\psi(X) - N^2\{1 - C^T P^2\psi(1) + C^T P^2\psi(X)\}^{n+1} = R(X, c_1, c_2, \ldots, c_n). \tag{3.16}$$

Case II Thermal conductivity is the linear function of temperature,

$$k = k_a\{1 + \lambda(T - T_a)\} i.e. f(\theta) = 1 + \beta\theta, g(\theta) = \beta.$$

Eq. (3.15) takes the form

$$\left[1 + \beta\left\{1 - C^T P^2\psi(1) + C^T P^2\psi(X)\right\}\right] C^T\psi(X) + \beta\left\{C^T P\psi(X)\right\}^2$$
$$-N^2\{1 - C^T P^2\psi(1) + C^T P^2\psi(X)\}^{n+1} = R(X, c_1, c_2, \ldots, c_n). \tag{3.17}$$

Case III Thermal conductivity is exponential function of temperature,

$$k = k_a e^{\lambda(T - T_a)} i.e. f(\theta) = e^{\beta\theta}, g(\theta) = \beta e^{\beta\theta},$$

Eq. (3.15) becomes

$$e^{\beta\{1 - C^T P^2\psi(1) + C^T P^2\psi(X)\}} C^T\psi(X) + \beta e^{\beta\{1 - C^T P^2\psi(1) + C^T P^2\psi(X)\}} C^T\psi(X)\left\{C^T P\psi(X)\right\}^2$$
$$-N^2\{1 - C^T P^2\psi(1) + C^T P^2\psi(X)\}^{n+1} = R(X, c_1, c_2, \ldots, c_n). \tag{3.18}$$

Solving system of equations separately Eq. (3.16), Eq. (3.17) and Eq. (3.18) for Case I, Case II and Case III respectively and using Newton-Raphson method, the coefficients c_i, i=1, 2, 3, ... ,n are evaluated in each case and temperature can be determined from Eq. (3.14).

4. Fin Efficiency

The fin efficiency is the ratio of actual heat transferred from the fin surface to the surrounding fluid; to the heat conducted through the base at $X = L$ $(orX = 1)$. The efficiency can be written as

$$\eta = \frac{kA_C \frac{dT}{dx}|x = L}{Ph(T_b - T_a)L}. \tag{4.1}$$

In Case I efficiency

$$\eta = \frac{1}{N^2}\theta'(1), \ \ forN \neq 0, N = 1, 2, 3, 4, 5. \tag{4.2}$$

In Case II efficiency

$$\eta = \frac{1+\beta}{N^2}\theta'(1). \tag{4.3}$$

In Case III efficiency

$$\eta = \frac{e^{\beta}}{N^2}\theta'(1). \tag{4.4}$$

5. Results and discussion

The value of exponent n depends on the heat transfer mode. We choose three values $(-1/4, 1/3, 3)$ of exponent n for laminar film boiling or condensation, turbulent natural convection and radiation respectively. Temperature distribution in fin depends on fin parameter N, β and exponent n. We use nine Legendre wavelet basis functions in computation. For constant thermal conductivity $(\beta = 0)$ temperature distribution in fin decreases as N increases, shown in Figure 2. In Table 1 we compare wavelet collocation method and exact results, both are same for $N = 0.5$ and $N = 1$ and it proves the accuracy of the method.

The result thus obtained is correct up to ten decimal places for $N = 0.5$ and 1. In Case II and III the dimensionless temperature distributions along the fin surface when β vary from -0.4 to 0.4 and n vary from -1/4, 1/3 to 3 are presented in Figures 3 to 14 for $N = 1$ and $N = 2$. For laminar film boiling or condensation $(n = -1/4)$ and $N = 1$ temperature in fin increases as β increases in Case II and III both, but temperature in Case III is always higher than Case II as shown in Figures3 and 4. For $n = -1/4$ and $N = 2$ temperature distribution in fin increases as β increases in Case II and III which is presented in Figures 5 and 6. For turbulent natural convection $(n = 1/3)$ and $N = 1, 2$ temperature distribution in fin increases as β increases in Case II and III both and is presented by Figures 7, to10. In turbulent natural convection we observe that for $N = 1, n = 1/3$ and $\beta = 0.4$, temperature at $X = 0$ in fin is 0.7343, 0.7439 in Case II and III respectively. For $N = 2, n = 1/3$ and $\beta = 0.4$, temperature at $X = 0$ in fin is 0.3958, 0.4041 in Case II and III respectively. In radiation mode $(n = 3)$ temperature at $X = 0$ in fin for $N = 1$ is 0.8134, 0.8191 and for $N = 2$ is 0.6400, 0.6457 in Case II and III respectively. Thus, temperature in fin at $X = 0$ increases for laminar film boiling (or condensation) to turbulent natural convection and from turbulent natural convection to radiation. It has been observed that the heat transfer through fin is highest in radiation mode. In Case III, temperature distribution in fin is always higher than Case II as shown in Figures 3 to 14. Summary of minimum and maximum temperature distribution in fin at $X = 0$ is given in Table2 for Case I, II and III.

Fin efficiency is presented in Figures15 to 18 for different values of n, β, and N. In Case I effect of n and N on fin efficiency is shown in Figure15. It is clear from this figure that fin efficiency decreases as the value of n increase from -1/4 to 3 and N increases from 0 to 3. In this case the fin efficiency is higher for laminar film boiling or condensation $(n = -1/4)$.

Figures16 and 17 shows fin efficiencies in three cases for $n = -1/4$ and $n = 3$ respectively. When we compare these two figures we observe that the fin efficiency increases as value of β increases from -0.4 to 0.4, also we observe that the fin efficiency in exponential case is always better in comparison to constant and linear thermal conductivity cases. When we compare efficiencies for $n = -1/4$ and $n = 3$; we observe that efficiency is higher for $n = -1/4$ than $n = 3$. Effect of N and β on fin efficiency in Case II and III is presented by Figure 18. In this figure, we observe that fin efficiency decreases as β decreases from 0 to -0.4, and fin efficiency increases as β increases from 0 to 0.4. Fin efficiency decreases as value of fin parameter N increases. MATLAB-2011 software is used in computation.

Table 1: The dimensionless temperature distribution in case of constant thermal conductivity ($\beta = 0$)

X	N=0.5 Exact	N=0.5 WCM	N=1 Exact	N=1 WCM
0.0	0.8868188840	0.8868188840	0.6480542737	0.6480542737
0.1	0.8879276385	0.8879276385	0.6512972462	0.6512972462
0.2	0.8912566747	0.8912566747	0.6610586204	0.6610586204
0.3	0.8968143168	0.8968143168	0.6774360915	0.6774360915
0.4	0.9046144618	0.9046144618	0.7005935707	0.7005935707
0.5	0.9146766141	0.9146766141	0.7307628258	0.7307628259
0.6	0.9270259345	0.9270259345	0.7682458010	0.7682458010
0.7	0.9416933025	0.9416933025	0.8134176383	0.8134176383
0.8	0.9587153943	0.9587153943	0.8667304327	0.8667304327
0.9	0.9781347739	0.9781347739	0.9287177566	0.9287177566
1.0	1.0000000000	1.0000000000	1.0000000000	1.0000000000

Table 2: Temperature distribution in fin at $X = 0$ in case I, II and III

Case I	N = 1	N =2		
n = 0	0.6480542737	0.2658022407		
	N=1	N=1	N=2	N=2
Case II	Max. ($\beta = 0.4$)	Min. ($\beta = -0.4$)	Max. ($\beta = 0.4$)	Min.($\beta = -0.4$)
n = -1/4	0.6995789245	0.5184892278	0.2528167206	0.1371376455
n = 1/3	0.7342877986	0.5884638509	0.3958008725	0.2810137813
n = 3	0.8133692964	0.7294440889	0.6400902871	0.5592446283
Case III	Max. ($\beta = 0.4$)	Min.($\beta = -0.4$)	Max. ($\beta = 0.4$)	Min.($\beta = -0.4$)
n = -1/4	0.7113533561	0.5353633325	0.2617901041	0.1438727994
n = 1/3	0.7439619588	0.6026561643	0.4041066087	0.2886948088
n = 3	0.8191027640	0.7384888834	0.6457481124	0.5663486619

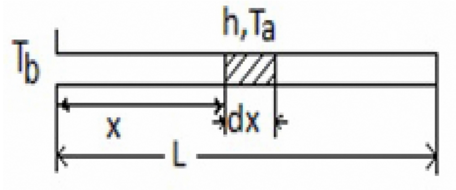

Figure 1: Geometry of a straight fin.

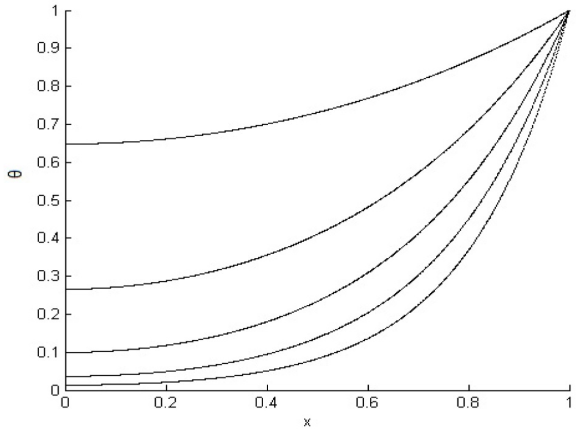

Figure 2: Case I, temperature distributions in fin by WCM for $n = 0, N = 1, 2, 3, 4, 5$ top to bottom respectively.

6. Conclusion

1. According to [12] ADM and HPM solutions fails when N increases to a large number but the WCM gives sufficient accuracy for large values of N.

2. In case I, II and III as we increase the value of fin parameter N, the thermal conductivity decreases and temperature in fin decreases rapidly.

3. Temperature in fin increases with n as given in Figures 3 to 14. The heat transfer through fin is highest in radiation mode ($n = 3$) and lowest in laminar film boiling or condensation ($n = -1/4$).

4. In Case II and III, for $n = -1/4$ fin efficiency decreases as β decreases from 0 to - 0.4 and increases as β increases from 0 to 0.4.

5. The fin efficiency increases from Case I to II and II to III (see Figure16). It seems that fin efficiency increases as thermal conductivity of fin material increases with temperature.

A nonlinear straight fin with variable thermal conductivity analysed using the wavelet collocation method. The

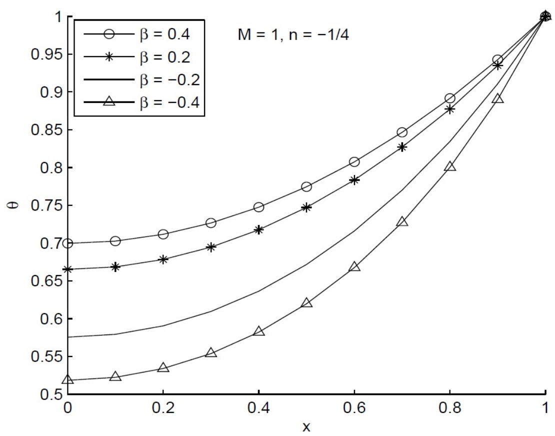

Figure 3: Case II, temperature distributions with $N = 1$ and $n = -1/4$ for $\beta = \pm 0.4$ and $\beta = \pm 0.2$.

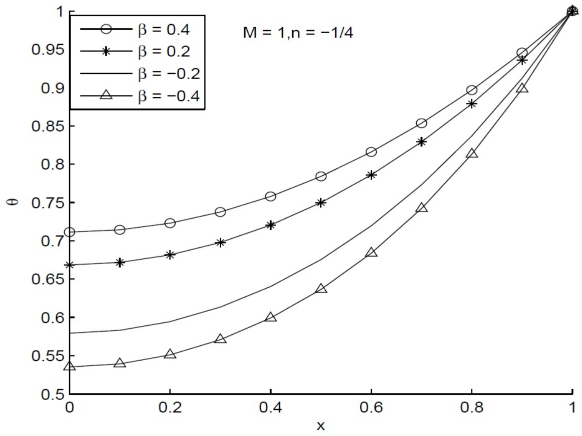

Figure 4: Case III, temperature distributions with $N = 1$ and $n = -1/4$ for $\beta = \pm 0.4$ and $\beta = \pm 0.2$.

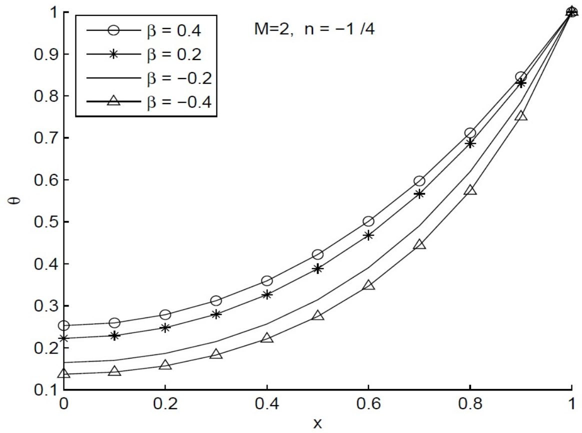

Figure 5: Case II, temperature distributions with $N = 2$ and $n = -1/4$ for $\beta = \pm 0.4$ and $\beta = \pm 0.2$.

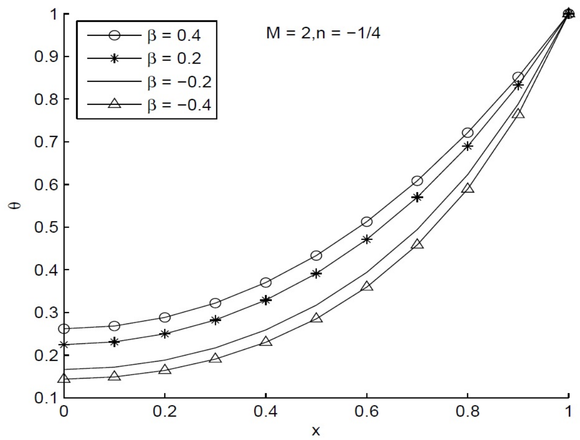

Figure 6: Case III, temperature distributions with $N = 2$ and $n = -1/4$ for $\beta = \pm 0.4$ and $\beta = \pm 0.2$.

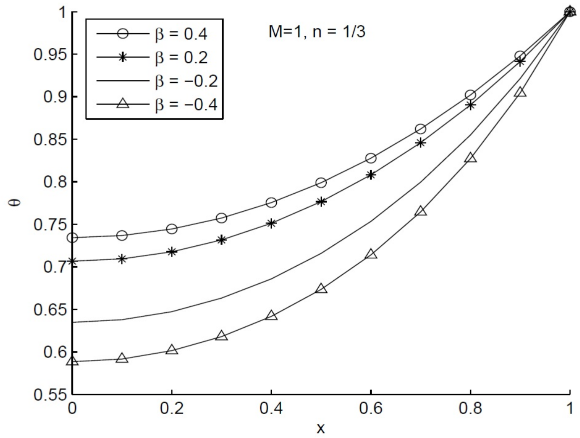

Figure 7: Case II, temperature distributions with $N = 1$ and $n = 1/3$ for $\beta = \pm 0.4$ and $\beta = \pm 0.2$.

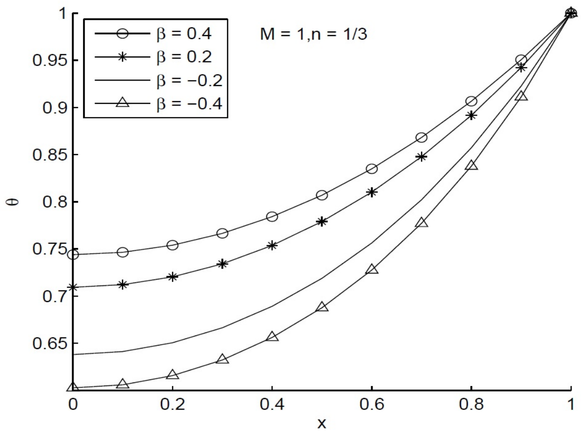

Figure 8: Case III, temperature distributions with $N = 1$ and $n = 1/3$ for $\beta = \pm 0.4$ and $\beta = \pm 0.2$

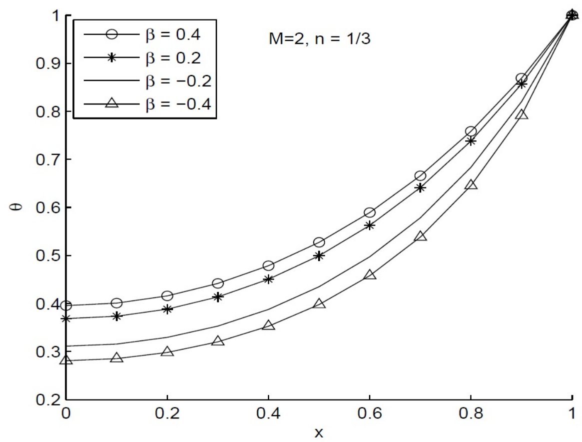

Figure 9: Case II, temperature distributions with $N = 2$ and $n = 1/3$ for $\beta = \pm 0.4$ and $\beta = \pm 0.2$

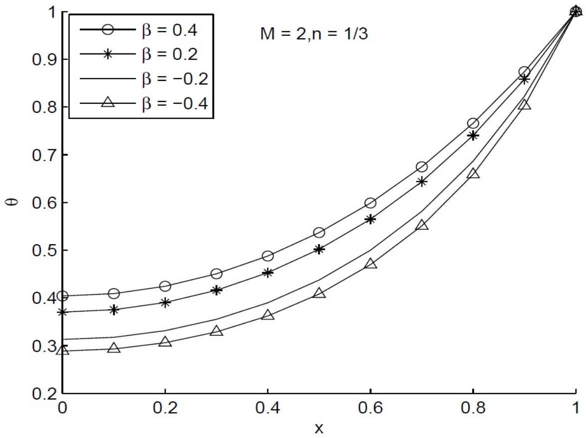

Figure 10: Case III, temperature distributions with $N = 2$ and $n = 1/3$ for $\beta = \pm 0.4$ and $\beta = \pm 0.2$

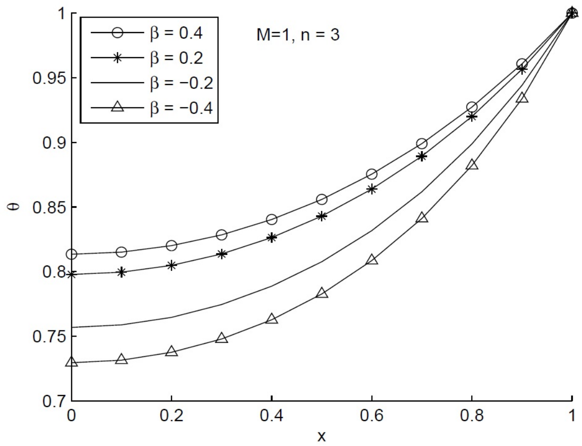

Figure 11: Case II, temperature distributions with $N = 1$ and $n = 3$ for $\beta = \pm 0.4$ and $\beta = \pm 0.2$.

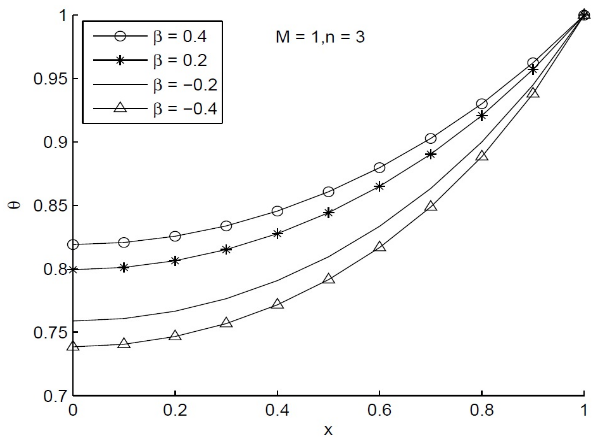

Figure 12: Case III, temperature distributions with $N = 1$and $n = 3$ for $\beta = \pm 0.4$ and $\beta = \pm 0.2$.

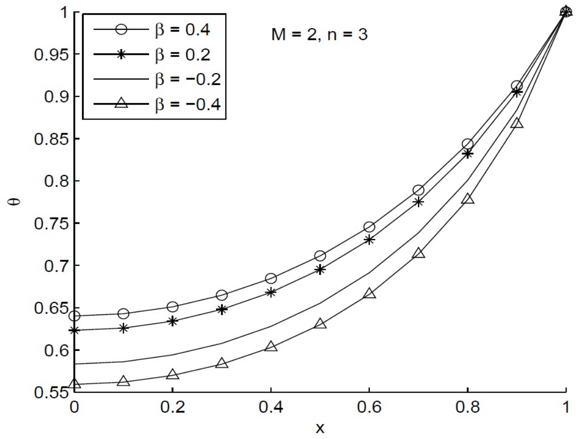

Figure 13: Case II, temperature distributions with $N = 2$ and $n = 3$ for $\beta = \pm 0.4$ and $\beta = \pm 0.2$.

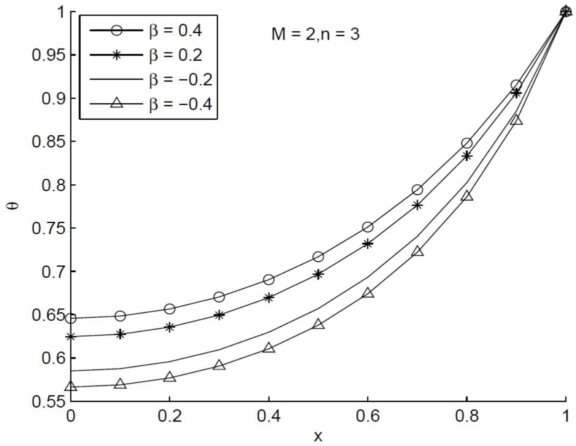

Figure 14: Case III, temperature distributions with $N = 2$ and $n = 3$ for $\beta = \pm 0.4$ and $\beta = \pm 0.2$.

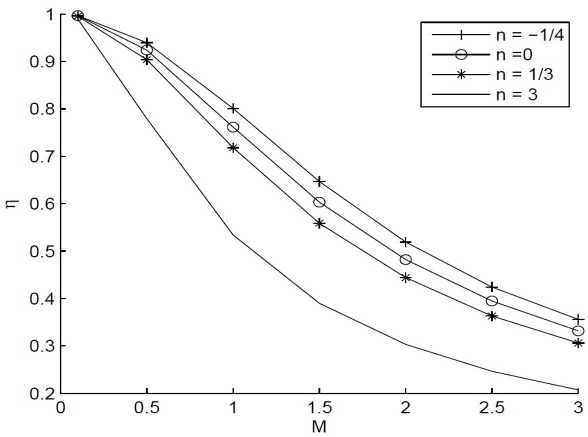

Figure 15: Fin efficiency for constant thermal conductivity

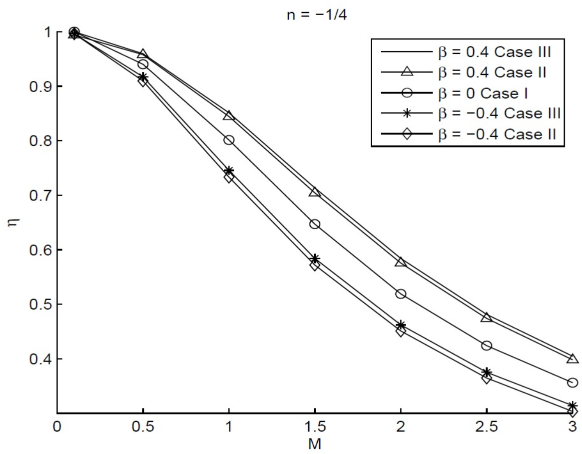

Figure 16: Fin efficiency comparison for $n = -1/4$ in Case I, II and III, variation with N and β.

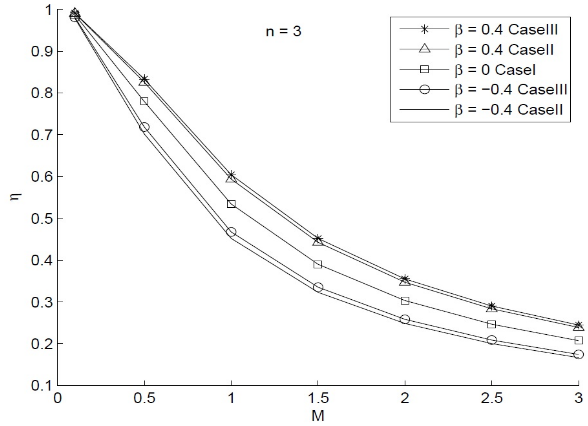

Figure 17: Fin efficiency comparison for $n = 3$ in Case I, II and III, variation with N and β.

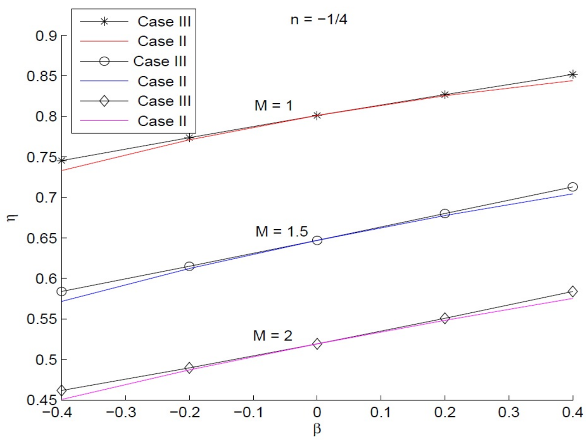

Figure 18: Fin efficiency in Case II and III for N = 1, 1.5 and 2.

wavelet collocation method is reliable, gives higher accuracy, faster convergence and easy computation. Temperature distribution and efficiency of fin increase from case I to II and case II to III. If we assume the value of n as -1/4, 1/3, 3 temperature in fin increases as given in Figure3 to 14. This method can be applied in difficult nonlinear heat conduction problems, and useful to engineers to analyse nonlinear system.

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