Int. J. Nonlinear Anal. Appl. 8 (2017) No. 1, 177-186
ISSN: 2008-6822 (electronic)
http://dx.doi.org/10.22075/ijnaa.2017.672.1107

Unsteady free convection oscillatory couette flow through a variable porous medium with concentration profile

Surjan Singh[a,*], Pawan Kumar[b], K.N. Rai[c], Nagendra Singh Tomar[d]

[a] *Department of Mathematics, Eternal University, Baru Sahib,Himachal Pradesh, 173101 India*
[b] *Department Of Applied Mathematics, Amity School Of Engineering And Technology, Delhi Palam Vihar Road, Bijwasan, New Delhi 110061, India*
[c] *Department Of Mathematical Sciences, IIT BHU, Varanasi, India, 221005*
[d] *Department Of Mathematics, Government College Hisar, Haryana, India*

(Communicated by Themistocles M. Rassias)

Abstract

In this paper we have studied the effect of free convection on the heat transfer and flow through variable porous medium which is bounded by two vertical parallel porous plates. In this study it is assume that free stream velocity oscillates with time about a constant mean. Periodic temperature is considered in the moving plate. Effect of different parameters on mean flow velocity, Transient velocity, Concentration profile and transient temperature studied in detail.

Keywords: Coutte flow; variable porous medium; concentration profile; oscillatory plates.
2010 MSC: Primary 76B75; Secondary 76N25.

1. Introduction

Density of the fluid changes according to temperature variation. Flow of fluids depends on the density of the fluid. In real life we can see that high density fluids flow slowly in comparison to low density fluids. Between two vertical parallel plates, free convective flow through a porous medium was studied by Singh [1]. Raptis [2] studied the problem related to unsteady flow through a porous medium. Constant suction and variable temperature considered in this study. Raptis and

*Corresponding author
Email addresses: surjan.singhbhu@gmail.com (Surjan Singh), drpawanksharma@yahoo.com (Pawan Kumar), knrai.apm@itbhu.ac.in (K.N. Rai), tomar_ns@rediffmail.com (Nagendra Singh Tomar)

Received: January 2015 *Revised:* February 2017

Peridikis [3] considered oscillatory type flow through a highly porous medium in presence of free convection. Nield [4] studied convection in a porous medium with inclined temperature gradient. Heat transfer response of laminar free convection boundary layer along vertical heated plate to surface temperature oscillation studied by Kelleher and yang [5]. Martynenko et al [6] studied the laminar free convection from a vertical plate. Sharma et al. [7] studied the unsteady free convection oscillatory coquette flow through a porous medium with periodic wall temperature, they also studied the effect of different parameters in detail. Ostrach [8] studied laminar natural convection flow and heat transfer of fluid, effect of presence and absence of heat source also studied. During the whole study ball temperature is considered constant. Free convection from a vertical plate in porous medium, subjected to a sudden change in surface temperature studied by Harris et al [9]. In this paper analytical solution investigated for the temperature and velocity in small time interval and numerical solution for boundary layer equations also obtained. Transient free convection flow past an infinite vertical plate with periodic temperature variation studied by Das et al [10]. Soundalgkar [11] studied viscous dissipation effects on unsteady free convective flow past an infinite vertical porous plate with constant suction. Temperature of the plates oscillates with small amplitude; it is assumed in this work.

In this study we have studied the effect of Concentration profile, permeability and convection on the velocity of oscillatory coquette flow through a highly porous medium , when moving plate is oscillating with free-stream velocity and wall temperature fluctuating with time.

2. Formulation of the problem

Unsteady coutte flow of a viscous incompressible fluid which is flow through a variable medium and bounded between two infinite vertical porous plates is considered in the study. One plate is quickly moved from rest position with a free stream velocity that oscillates with time about a constant mean. It is assumed that the permeability and temperature of moving plate fluctuates with time. We consider x*-axis along the moving vertical plate in the vertical upward direction and y*-axis is taken normal to this plate. The other stationary vertical plate is assumed to be situated at y* = b at temperature T_{s^*}. We consider the free-stream velocity as follows:

$$U^*(t^*) = U_0(1 + \varepsilon e^{i\omega^* t^*}) \tag{2.1}$$

Where U_0 is the mean constant free-stream velocity, ω^* is the frequency of oscillations and t^* is the time. The equations governing the problem are as follows

Momentum equation

$$\rho \frac{\partial u^*}{\partial t^*} = -\frac{\partial P}{\partial x^*} + \mu \frac{\partial^2 u^*}{\partial y^{*2}} - g\rho^* - \frac{u^* \mu}{k^*} \tag{2.2}$$

Energy equation

$$\frac{\partial T^*}{\partial t^*} = \alpha \frac{\partial^2 T^*}{\partial y^{*2}} \tag{2.3}$$

Mass flux equation (concentration equation)

$$\frac{\partial C^*}{\partial t^*} = D \frac{\partial^2 C^*}{\partial y^{*2}} \tag{2.4}$$

Boundary conditions are

$$\left.\begin{array}{llll} y^* = 0: & u^* = U_0(1 + \varepsilon e^{i\omega^* t^*}), & T^* & = T_n^* + \varepsilon(T_n^* - T_s^*)e^{i\omega^* t^*}, C^* = C_n^* \\ y^* = b: & & u^* = 0 & , T^* = T_n^*, C^* = C_s^* \end{array}\right\} \tag{2.5}$$

where u^* = velocity, U^* = free stream velocity, ρ = density, μ = viscosity, P = pressure, g = gravity, k^* = permeability parameter, α = thermal diffusivity, T^* = temperature of fluid in the boundary layer, T_n^* = temperature of the moving plate, T_s^* = temperature of stationary plate, C_s^* = Concentration at stationary plate, C^* = Concentration of fluid in boundary layer, C_n^* = Concentration of moving plate. Star $(*)$ indicates dimensional quantity.

Equation (2.2), for the free stream, is reduced to

$$\rho\frac{\partial U^*}{\partial t^*} = -\frac{\partial P}{\partial x^*} - g\rho_s^* - \frac{U^*\mu}{k^*}. \tag{2.6}$$

From (2.2) and (2.6) we get

$$\rho\frac{\partial u^*}{\partial t^*} = \rho\frac{\partial U^*}{\partial t^*} + \mu\frac{\partial^2 u^*}{\partial y^{*2}} + g(\rho_s^* - \rho^*) - \frac{(u - U^*)\mu}{k^*}.$$

Above equation reduced to

$$\rho\frac{\partial u^*}{\partial t^*} = \rho\frac{\partial U^*}{\partial t^*} + \mu\frac{\partial^2 u^*}{\partial y^{*2}} + g\beta\rho^*(T^* - T_s^*) + g\beta^\bullet\rho^*(C^* - C_s^*) - \frac{(1 + \varepsilon e^{i\omega^* t^*})(u^* - U^*)\mu}{k_0^*}. \tag{2.7}$$

Using the constitutive equation

$$g(\rho_s^* - \rho^*) = g\beta\rho^*(T^* - T_s^*) + g\beta^\bullet\rho * (C^* - C_s^*)$$

and $k^* = \frac{k_0^*}{(1+\varepsilon e^{i\omega^* t^*})}$,
where β = volumetric coefficient of thermal expansion, $\beta^\bullet$ = volumetric coefficient of Chemical expansion, ρ_s^* = density of fluid with temperature T_s^* and ρ^* = density of fluid with temperature T^*.

Introducing the following non-dimensional quantities

$$y = y^*/b, u = u^*/U_0, U = U^*/U_0, t = \omega^* t^*, \omega = \omega^* b^2/\nu, \theta = \frac{T^* - T_s^*}{T_n^* - T_s^*}, C = \frac{T^* - T_s^*}{T_n^* - T_s^*},$$

Gc(modified Grassoff number)= $\frac{g\beta^\bullet b^2(C_n^* - C_s^*)}{\nu_0 U_0}$, Gr(Grassoff number)= $\frac{g\beta b^2(T_n^* - T_s^*)}{\nu U_0}$, $K = K_0^*/b^2, \nu = \mu/\rho^*$, Pr (Prandtl number)= ν/α, Sc (Schmidt number)= v/D.

Equations (2.7), (2.3) and 2.4) become as follows in dimensionless form:

$$\omega\frac{\partial u}{\partial t} = \omega\frac{\partial U}{\partial t} + \frac{\partial^2 u}{\partial y^2} + Gr\theta + GcC - \frac{(1 + \varepsilon e^{it})(u - U)}{k} \tag{2.8}$$

$$\omega Pr\frac{\partial \theta}{\partial t} = \frac{\partial^2 \theta}{\partial y^2} \tag{2.9}$$

$$\omega Sc\frac{\partial C}{\partial t} = \frac{\partial^2 C}{\partial y^2}. \tag{2.10}$$

Corresponding boundary conditions are as follows:

$$y = 0 : u = 1 + \varepsilon e^{it}, \theta = 1 + \varepsilon e^{it}, C = 1 + \varepsilon e^{it},$$

$$y = 1 : u = 0, \theta = 0, C = 0.$$

3. Solution of the problem

Amplitude of the free stream velocity and temperature variation $\varepsilon(<< 1)$ are very small, now assume the solution is of the following form:

$$\left.\begin{aligned} u(y,t) &= u_0(y) + \varepsilon u_1(y)e^{it}, \\ \theta(y,t) &= \theta_0(y) + \varepsilon\theta_1(y)e^{it}, \\ C(y,t) &= C_0(y) + \varepsilon C_1(y)e^{it} \end{aligned}\right\} \tag{3.1}$$

and free stream velocity is

$$U = 1 + \varepsilon e^{it}.$$

Substituting equation (3.1) and free-stream velocity in equation (2.8), (2.9), (2.10), we get

$$u_0^{''} - \frac{u_0}{k} = -Gr\theta_0 - GcC_0 - \frac{1}{k} \tag{3.2}$$

$$\theta_0^{''} = 0 \tag{3.3}$$

$$u_1^{''} - (i\omega + \frac{1}{k})u_1 = -Gr\theta_1 - GcC_1 - i\omega - \frac{2}{k} + \frac{u_0}{k} \tag{3.4}$$

$$\theta_1^{''} - i\omega Pr\theta_1 = 0 \tag{3.5}$$

$$C_0^{''} = 0 \tag{3.6}$$

$$C_1^{''} - i\omega ScC_1 = 0 \tag{3.7}$$

and the corresponding boundary conditions are

$$\left.\begin{aligned} y = 0 : u_0 = 1, u_1 = 1, \theta_0 = 1, \theta_1 = 1, C_0 = 1, C_1 = 1 \\ y = 1 : u_0 = 0, u_1 = 0, \theta_0 = 0, \theta_1 = 0, C_0 = 0, C_1 = 0 \end{aligned}\right\} \tag{3.8}$$

Solving these equations under the corresponding boundary conditions, we get

$$\theta_0(y) = 1 - y \tag{3.9}$$

$$C_0(y) = 1 - y \tag{3.10}$$

$$\theta_1(y) = Ae^{\lambda(1+i)y} + Be^{-\lambda(1+i)y} \tag{3.11}$$

$$C_1(y) = A_1e^{\lambda_1(1+i)y} + B_1e^{-\lambda_1(1+i)y} \tag{3.12}$$

$$u_0 = A_2e^{a_1y} + B_2e^{-a_1y} + Grk(1-y) + Gck(1-y) + 1 \tag{3.13}$$

$$\begin{aligned} u_1 = A_3e^{(a_2+ia_3)y} + B_3e^{-(a_2+ia_3)y} - AA_4e^{\lambda(1+i)y} - BA_4e^{-\lambda(1+i)y} - A_1A_5e^{\lambda_1(1+i)y} \\ -B_1A_5e^{-\lambda_1(1+i)y} + A_2S_1e^{a_1y} + B_2S_1e^{-a_1y} - (1-y)S_2 - (1-y)S_3 + 1 \end{aligned} \tag{3.14}$$

where

$$\lambda = \sqrt{\frac{\omega Pr}{2}};\ \lambda_1 = \sqrt{\frac{\omega Sc}{2}};\ A = -\frac{e^{-\lambda(1+i)}}{e^{\lambda(1+i)} - e^{-\lambda(1+i)}};\ B = \frac{e^{-\lambda(1+i)}}{e^{\lambda(1+i)} - e^{-\lambda(1+i)}};$$

$$a_1 = \frac{1}{\sqrt{k}};\ a_2 = \frac{1}{\sqrt{2}}[\sqrt{(}\omega^2 + \frac{1}{k^2}) + \frac{1}{k}]^{1/2};\ a_3 = \frac{1}{\sqrt{2}}[\sqrt{(}\omega^2 + \frac{1}{k^2}) - \frac{1}{k}]^{1/2};$$

$$A_1 = -\frac{e^{-\lambda_1(1+i)}}{e^{\lambda_1(1+i)} - e^{-\lambda_1(1+i)}};\ B_1 = \frac{e^{\lambda_1(1+i)}}{e^{\lambda_1(1+i)} - e^{-\lambda_1(1+i)}};A_2 = -\frac{1 - Grke^{-a_1} - Gcke^{-a_1}}{e^{a_1} - e^{-a_1}};$$

$$B_2 = \frac{1 - Grke^{a_1} - Gcke^{a_1}}{e^{a_1} - e^{-a_1}};\ A_4 = \frac{Gr}{\lambda^2(1+i)^2 - (i\omega + \frac{1}{k})};$$

$$A_5 = \frac{Gc}{\lambda_1^2(1+i)^2 - (i\omega + \frac{1}{k})};\ S_1 = \frac{i}{\omega k};\ S_2 = \frac{Gr}{i\omega + \frac{1}{k}};\ S_3 = \frac{Gc}{i\omega + \frac{1}{k}};$$

$$\begin{aligned} A_3 = & \frac{AA_4[e^{-(a_2+ia_3)} - e^{\lambda(1+i)}] + BA_4[e^{-(a_2+ia_3)} - e^{-\lambda(1+i)}] + B_1A_5[e^{-(a_2+ia_3)} - e^{-\lambda_1(1+i)}]}{e^{-(a_2+ia_3)} - e^{(a_2+ia_3)}} \\ & + \frac{A_1A_5[e^{-(a_2+ia_3)} - e^{\lambda_1(1+i)}] + A_2S_1[e^{a_1} - e^{-(a_2+ia_3)}] + B_2S_1[e^{-a_1} - e^{-(a_2+ia_3)}]}{e^{-(a_2+ia_3)} - e^{(a_2+ia_3)}} \\ & + \frac{(S_2+S_3)e^{-(a_2+ia_3)} + 1}{e^{-(a_2+ia_3)} - e^{(a_2+ia_3)}} \end{aligned}$$

$$\begin{aligned} B_3 = & -\frac{AA_4[e^{(a_2+ia_3)} - e^{\lambda(1+i)}] + BA_4[e^{(a_2+ia_3)} - e^{-\lambda(1+i)}] + B_1A_5[e^{(a_2+ia_3)} - e^{-\lambda_1(1+i)}]}{e^{-(a_2+ia_3)} - e^{(a_2+ia_3)}} \\ & + \frac{A_1A_5[e^{(a_2+ia_3)} - e^{\lambda_1(1+i)}] + A_2S_1[e^{a_1} - e^{(a_2+ia_3)}] + B_2S_1[e^{-a_1} - e^{(a_2+ia_3)}] + (S_2+S_3)e^{(a_2+ia_3)} + 1}{e^{-(a_2+ia_3)} - e^{(a_2+ia_3)}}. \end{aligned}$$

4. Result and discussion

In this study we have studied the effect of Concentration profile, permeability and convection on the velocity, when moving plate is oscillating with free-stream velocity and wall temperature fluctuating with time. We obtain following results. Results calculated for different values of Pr, Gr, Gc, ω and k. Two particular values of Pr considered in this study that is pr = 0.71 for air and $pr = 7.0$ for water at 20^oC. The values of Gr, Gc, ω , and K are selected arbitrarily.

Mean flow Mean flow velocity of the fluid is represented by equation (3.13). Effect of permeability on mean flow velocity is presented in figure 1 and 2. In Figure 1 and 2 mean flow velocity u_0 is taken on y-axis. Figure 1 is drawn for fixed values of Pr = 0.71 (air) and $\omega = 5$. Upper two curves shows the effect of variability of permeability parameter k. We observe that as value of permeability parameter increases, velocity decrease. This happened due to porous material, since porous material resists the flow of fluid. In lower two curves effect of variability of Gr is shown for fix values of Gc, K and Sc. Here we observe that as value of Gr increases velocity also increases. From figure 1 and 2 we observe that both figures are approximately same. Thus we conclude that mean flow velocity profile is approximately same for Pr = 0.71 (air) and Pr = 7.0 (water) while we change the value of Gr, Gc, K and Sc.

The transient velocity is given by equation (3.14). This velocity component is presented in figure 3. From this figure we conclude that when we increase value of Gc and Gr initially transient velocity increases up to cross points. After these cross points transient velocity decreases. Physical meaning of this figure is, when we flow a fluid from parallel oscillatory plates initially transient velocity increases until the fluid particles spread on whole plate. When fluid cover whole plate transient velocity decreases due to oppose by concentrated fluid contact in plates. We describe flow of fluid with the help of figure 4. In this figure layer 1 of concentrated fluid which is in contact with plate

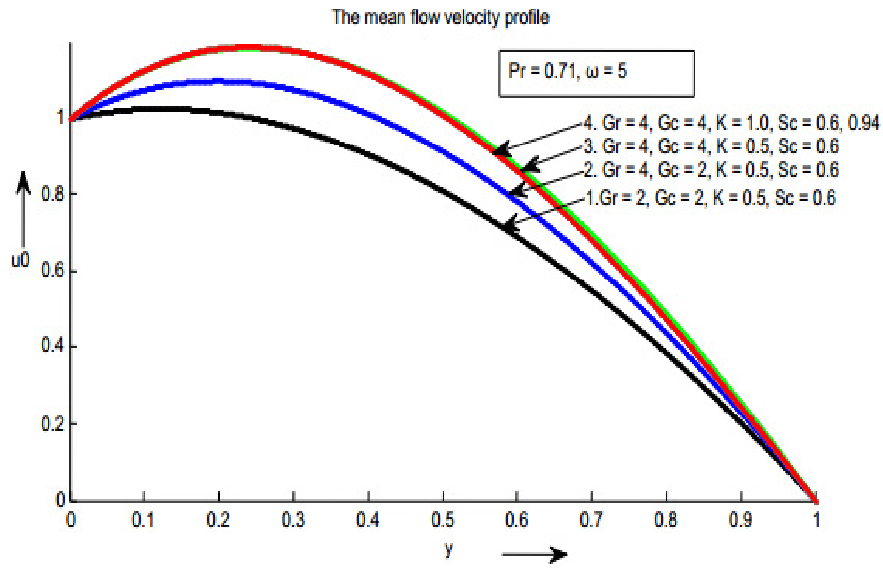

Figure 1: Effect of permeability on mean flow velocity for air

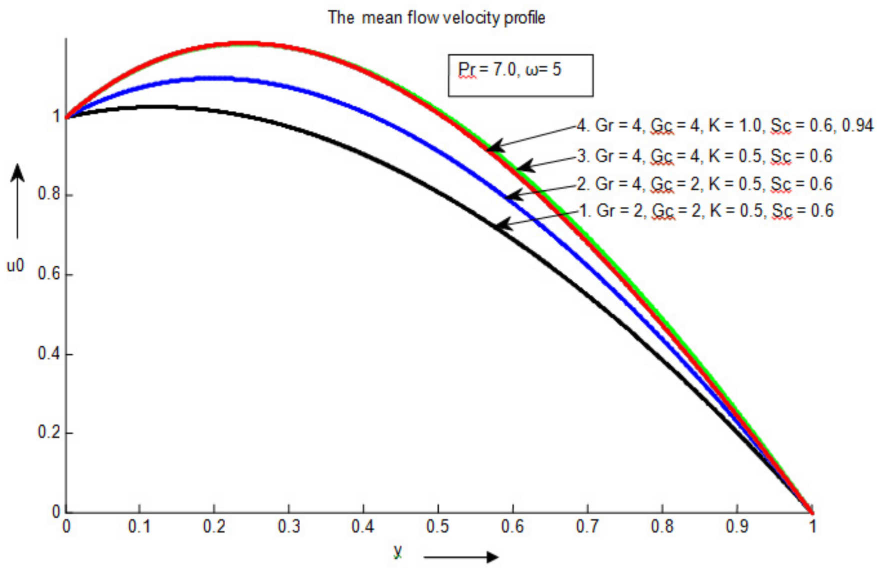

Figure 2: Effect of permeability on mean flow velocity for water

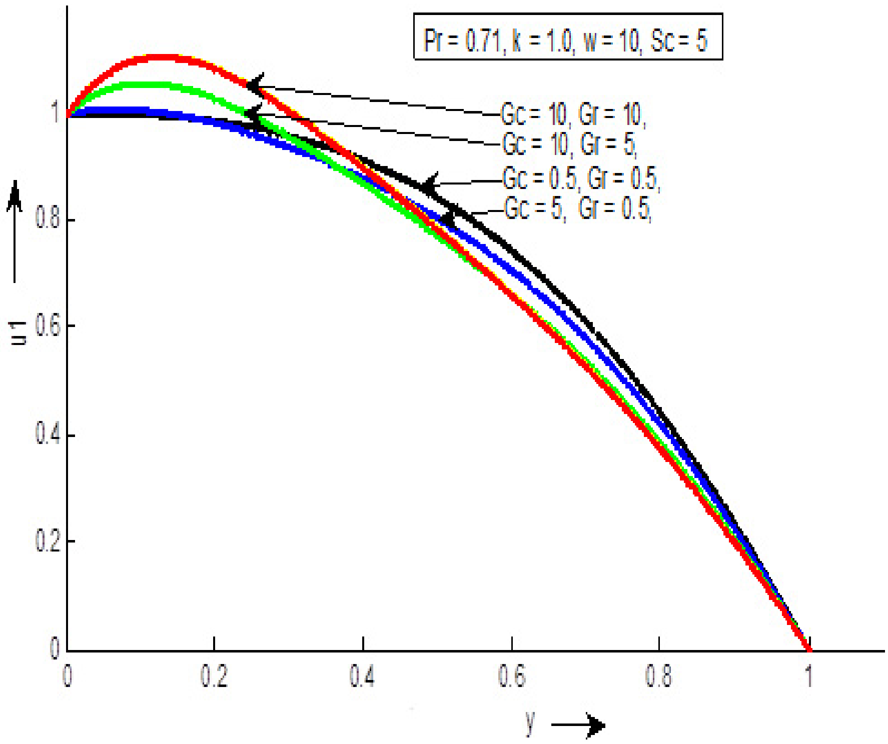

Figure 3: Effect of Gc and Gr on transient velocity profile

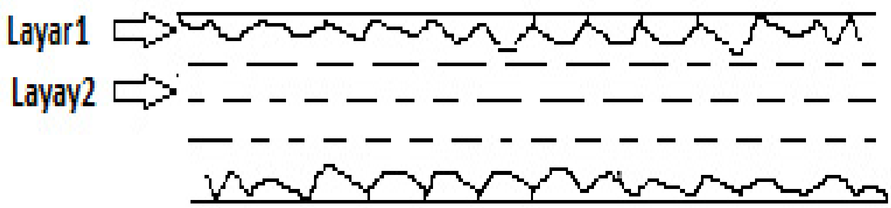

Figure 4: Flow of the fluid

makes obstacle in flow of Layar 2. Thus we conclude that due to concentration profile, transient velocity of the fluid decreases rapidly.

In figure 5 we see that trasient velocity decreases as increase permeability parameter k and decreasing frequency of oscillation ω. Naturally we see that porosity of medium makes obstacle in flow of fluid.

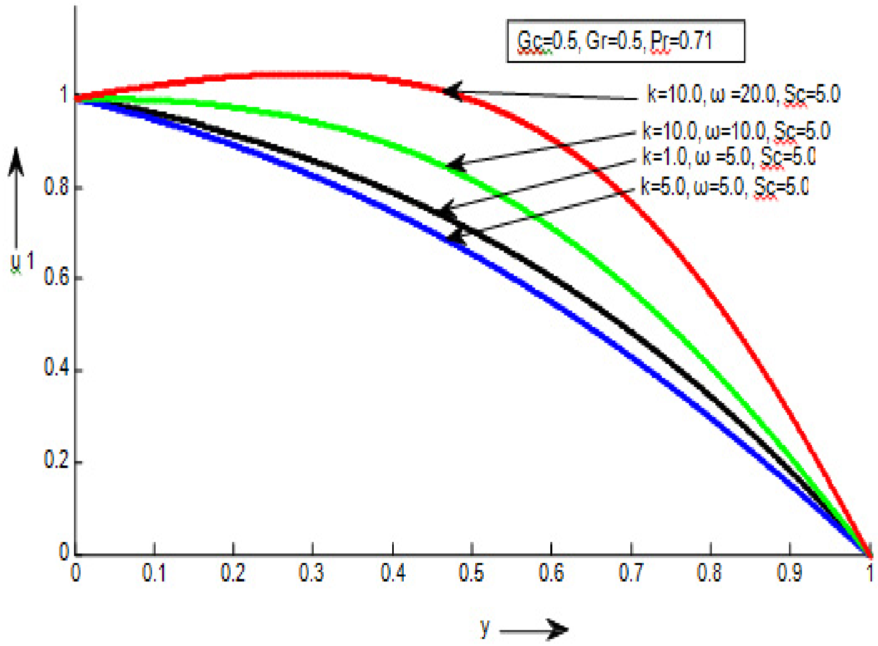

Figure 5: Effect on transient velocity of different parameters

Figure 6 illustrates the effect of Sc and ω on concentration profile C1 when Pr = 0.71, Gr = 2, Gc = 2 and k = 5. It is observed that concentration profile C1 is a decreasing as value of y increasing. From curve 1,2 and 3 we see that when we increase value of Sc, concentration profile decreases. From curve 4 it is clear that when we increase value of ω (frequency of oscillations) concentration profile C1 decreases rapidly.

The transient temperature is given by equation 3.11. This transient temperature is presented in figure 7. It is observed that from curve 1 and 2 transient temperature profile decreases slowly as we increase value of Pr and ω. From curve 1 and 3 it is observed that when we increase value of Pr transient temperature profile decreases rapidly. We use software- Matlab10 for calculations and figures.

5. Conclusion

Behaviour of mean flow velocity is similar for air and water. We observe that as value of permeability parameter k increases, mean flow velocity decrease. Transient velocity decreases as increase permeability parameter k and decreasing frequency of oscillation ω. This shows that the porous material resists the flow of fluid. Concentration profile decreases as value of Sc and ω increases. Transient temperature decreases from air to water with increasing frequency of oscillation.

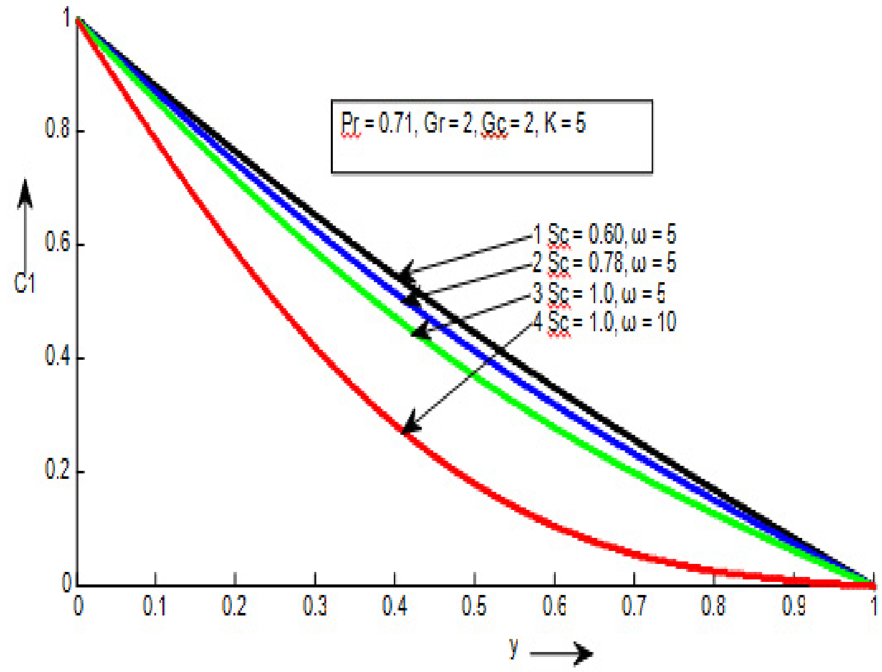

Figure 6: Effect of Sc and ω on concentration profile when other parameters fixed

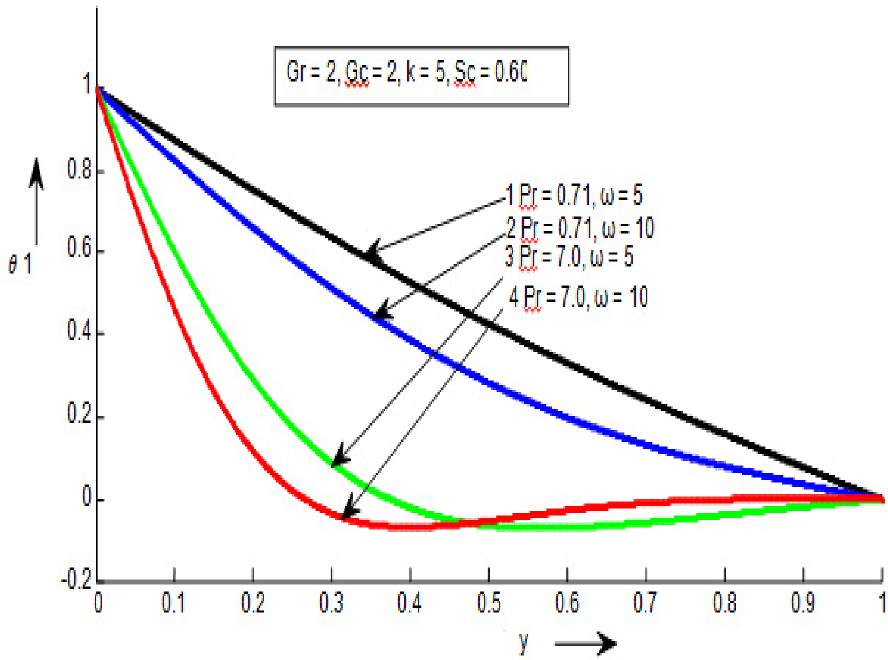

Figure 7: Effect on transient temperature of different parameters

References

[1] A.K. Singh, *MHD free convective flow through a porous medium between two vertical parallel plates*, Ind. J. Pure Appl. Phys. 40 (2002) 709–713.

[2] A. Raptis, *Unsteady free convection flow through a porous medium*, Int. J. Engin. Sci. 21 (1983) 345–348.

[3] A. Raptis and C.P. Pericikis, *oscillatory flow through a porous medium by the presence of free convective flow*, Int. J. Engin. Sci. 23 (1985) 51–55.

[4] D.A. Nield, *Convection in a porous medium with inclined temperature gradient: An additional results*, Int. J. Heat Mass Transfer 37 (1994) 3021–3025.

[5] M.D. Kelleher and K.T. Yang, *Heat transfer response of laminar free convection boundary layer along vertical heated plate to surface temperature oscillation* ZAMP 19 (1968) 31–44.

[6] O.G. Martynenko, A.A. Berezovsky and Yu. A Sokovishin, *Laminar free convection from a vertical plate*, Int. J. Heat Mass Transfer 27 (1984) 869–881.

[7] P. K. Sharma, B.K. Sharma and R.C. Chaudhary, *Unsteady free convetion oscillatory coquette flow through a porous medium with periodic wall temperature*, Tamkang Journal Of Mathematics Vol 38 Number 1 (2007) 93–102.

[8] S. Ostrach, *Laminar natural convection flow and heat transfer of fluids with and without heat sources in channels with constant wall temperature*, NACA TN (1952) 2863.

[9] S.D. Harris, D.B. Ingham and I. Pop, *Free convection from a vertical plate in porous medium subjected to a sudden change in surface temperature*, Int. Comm. Heat Mass Transfer 24 (1997) 543–552.

[10] U.N. Das, R.K. Deka and V.M. Soundalgekar, *Transient free convection flow past an infinite vertical plate with periodic temperature variation*, J. Heat Transfer (ASME) 121 (1999) 1091–1094.

[11] V.M. Soundalgkar, *Viscous Dissipation effects on unsteady free convective flow past an infinite vertical porous plate with constant suction*, Int. J. Heat Mass Transfer 15 (1972) 1253–1261.