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Influences of magnetic field in viscoelastic fluid

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Abstract

This communication influences on magnetohydrodynamic flow of viscoelastic fluid with magnetic field induced by oscillating plate. General solutions have been found out for velocity and shear stress profiles using mathematical transformations (integral transforms). The governing partial differential equations have been solved analytically under boundary conditions $u(0,t) = A_0H(t)sin(\Omega t)$ and $u(0,t) = A_0H(t)cos(\Omega t)$ with $t \geq 0$. For the sake of simplicity of boundary conditions are verified on the analytical general solutions and similar solutions have been particularized under three limited cases namely (i) Maxwell fluid without magnetic field if $\gamma \neq 0, M = 0$ (ii) Newtonian fluid with magnetic field if $\gamma = 0, M \neq 0$ and (iii) Newtonian fluid with out magnetic field if $\gamma = 0, M = 0$. Finally various physical parameters with variations of fluid behaviors are analyzed and depicted graphical illustrations.

Keywords: MHD Maxwell fluid, Laplace and Fourier transforms, rheological Parameters.
2010 MSC: Primary 26A25; Secondary 39B62.

Nomenclature: M Magnetic parameter, A_0 Non-zero constant, γ Relaxation time, $H(t)$ Heaviside function, $\mathbf{Q}$ Cauchy stress tensor, $\mathbf{N}$ Extra-stress tensor,$\mathbf{B}$ Rivlin Ericksen tensor, $-p\mathbf{I}$ Spherical stress, Ω Frequency, ν Kinematic viscosity, μ dynamic Viscosity, ρ Density, t Time, T Transpose, η Fourier sine transform parameter, δ Laplace transform parameter, $u(y,t)$ Velocity Field, $\tau(y,t)$ Shear Stress.

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1. Introduction

The analysis of non-Newtonian fluid flows in magnetohydrodynamics (MHD) has diverted attention of mathematician, engineers and researchers. Because such phenomenon usually arises among various fields for instance, nuclear fuel debris treatment, the geothermal sources investigation, metal alloys, optimization of solidification processes of metals and many others. Newtonian fluids are subtle in contrast with non-Newtonian fluids. Due to this fact, most of resultant governing equations arise from non-Newtonian fluid when magnetohydrodynamics (MHD) flow is considered, these equations becomes very complex to solve due to their appearance of nonlinearity. Magnetohydrodynamic non-Newtonian fluid flows have been studies with various aspects which can be found in recent references [3], [10], [11], [12], [13]. Due to several applications in engineering and science, flow of electrically conducting (magnetohydrodynamics) viscoelastic fluids has diverted the interest of researchers. In geophysics, magnetohydrodynamics is applicable to study and measure the velocities and positions of frame of reference on the earth's surface that gets rotations towards the frame of inertial along with magnetic field. In the geophysical and astrophysical dynamics, magnetohydrodynamics is used for the analysis of inter planetary and inter stellar matter, solar storms and flares, stellar and solar structure and several others. MHD in engineering point of view finds its usefulness in industrial equipment such as MHD boundary layer control of reentry vehicles, MHD generators, MHD pumps, magnetic drug targeting, MHD bearings, ion propulsion and many others. Keeping the above motivations in mind, many researchers are busy for sharing valuable contributions regarding magnetohydrodynamics [1], [2], [4], [5], [6], [7], [8], [9], [14], [15], [16]. However, this article explores the influences on magnetohydrodynamic flow of viscoelastic fluid with and without magnetic field induced by oscillating plate. General solutions have been found out for velocity and shear stress profiles using mathematical transformations (Integral transforms). The governing partial differential equations have been solved analytically under boundary conditions $u(0,t) = A_0 H(t) sin(\Omega t)$ and $u(0,t) = A_0 H(t) cos(\Omega t)$ with $t \geq 0$. For the sake of simplicity of boundary conditions are verified on the analytical general solutions and similar solutions have been particularized under three limited cases namely (i) Maxwell fluid with out magnetic field if $\gamma \neq 0, M = 0$ (ii) Newtonian fluid with magnetic field if $\gamma = 0, M \neq 0$ and (iii) Newtonian fluid with out magnetic field if $\gamma = 0, M = 0$. Finally various physical parameters with variations of fluid behaviors are analyzed and depicted graphical illustrations.

2. Formulation of Flow Equations

The electrically conducting flows of an incompressible fluid due to body forces are

$$\nabla.\mathbf{W} = 0, \nabla.\mathbf{Q} = \{\mathbf{W}_t + (\mathbf{W}.\nabla)\mathbf{W}\}\rho + \sigma M_0^2 \mathbf{W}, \tag{2.1}$$

where, $\mathbf{W}$ is the velocity field, ∇ represents the dell operator, $\mathbf{Q}$ is Cauchy stress, ρ is density of fluid, M_0 is the applied magnetic field and t is the time. The assumption of Reynolds number for small magnetic field is induced uniformly. The Cauchy stress $\mathbf{Q}$ in an incompressible Maxwell fluid is given by

$$\mathbf{Q} = -pI + \mathbf{N}, \mathbf{N} + \lambda(\dot{\mathbf{N}} - \mathbf{PN} - \mathbf{NP}^T) = \mu\mathbf{B}, \tag{2.2}$$

where, $\mathbf{Q}$ is Cauchy stress,$-pI$ denotes the indeterminate spherical stress, $\mathbf{N}$ is extra-stress tensor, γ is relaxation time, $\mathbf{P}$ is the velocity gradient, $\mathbf{B} = \mathbf{P} + \mathbf{P}^T$ is the first Rivlin Ericksen tensor, μ is the dynamic viscosity of the fluid, the superscript $\mathbf{T}$ indicates the transpose operation. Velocity is assumed as $\mathbf{W}$ and having an extra-stress tensor $\mathbf{N}$

$$\mathbf{W} = \mathbf{W}(y,t) = u(y,t)i, \mathbf{N} = \mathbf{N}(y,t). \tag{2.3}$$

If we consider the fluid is at rest up to the moment $t = 0$, then

$$\mathbf{W} = (y,0) = 0, \mathbf{N} = (y,0) = 0, \tag{2.4}$$

and equations (2.2), (2.3) and (2.4) imply $N_{yz} = N_{yy} = N_{xz} = N_{zz} = 0$,

$$\left(\gamma\frac{\partial}{\partial t} + 1\right)\tau(y,t) - \mu\frac{\partial u(y,t)}{\partial t} = 0. \tag{2.5}$$

Without body forces, the balance of linear momentum lessens to

$$\frac{\partial p}{\partial x} + \sigma M_0^2 u(y,t) - \frac{\partial \tau(y,t)}{\partial t} + \rho\frac{\partial u(y,t)}{\partial t} = 0, \sigma M_0^2 u(y,t) = -\frac{\partial p}{\partial y}, \sigma M_0^2 u(y,t) = -\frac{\partial p}{\partial z}. \tag{2.6}$$

Eliminating τ between equations(2.5) and (2.6), then the equation are

$$\left(\gamma\frac{\partial}{\partial t} + 1\right)\frac{\partial u(y,t)}{\partial t} = -\left(\gamma\frac{\partial}{\partial t} + 1\right)\frac{1}{\rho}\frac{\partial p}{\partial x} + \nu\frac{\partial^2 u(y,t)}{\partial t^2} - \frac{\sigma M_0^2}{\rho}\left(\gamma\frac{\partial}{\partial t} + 1\right)u(y,t); y,t > 0. \tag{2.7}$$

The governing equations corresponding to an incompressible MHD Maxwell fluid are

$$\left(\gamma\frac{\partial}{\partial t} + 1\right)\frac{\partial u(y,t)}{\partial y} + \nu\frac{\partial^2 u(y,t)}{\partial t^2} + M\left(\gamma\frac{\partial}{\partial t} + 1\right)u(y,t) = 0, \tag{2.8}$$

$$\left(\gamma\frac{\partial}{\partial t} + 1\right)\tau(y,t) = \mu\frac{\partial u(y,t)}{\partial y}, \tag{2.9}$$

where $M = \frac{\sigma M_0^2}{\rho}$.

3. Initial and Boundary Conditions of the Probelem

Let incompressible MHD Maxwell fluid possessing the space lying over an infinitely oscillating plane which is positioned in the xz plane and perpendicular to the y-axis. Initially, the fluid is at rest and at the moment $t = 0^+$ the plane is impulsively brought to velocity $u(0,t) = A_0 H(t) sin(\Omega t)$ or $u(0,t) = A_0 H(t) cos(\Omega t)$ in its own plane. Due to the shear, the fluid above the plane is gradually moved. Its velocity is of the form (2.4) and appropriate initial and boundary conditions are as sketched in Fig. 1.

$$u(y,0) = \frac{\partial u(y,0)}{\partial t} = 0, \tau(y,0) = 0, y > 0, \tag{3.1}$$

$$u(0,t) = A_0 H(t) sin(\Omega t), u(0,t) = A_0 H(t) cos(\Omega t), t \geq 0, \tag{3.2}$$

naturally

$$u(y,t), \frac{\partial u(y,t)}{\partial t} \to 0, \ as\ y \to \infty\ and\ t > 0, \tag{3.3}$$

have to be also satisfied.

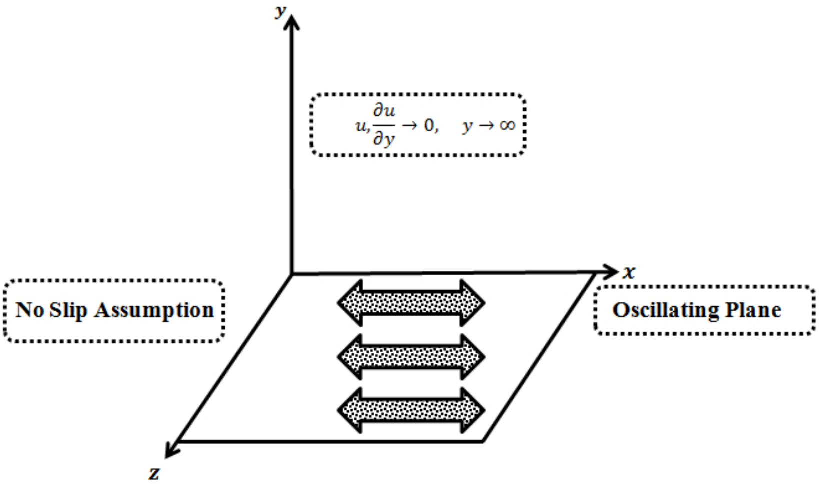

Figure 1: Geometry of the problem under the oscillations of a plate.

4. Solution of Sine Oscillations

4.1. Velocity Field

Employing Fourier sine transform on (2.8) and taking conditions (3.1), (3.2), (3.3). We have

$$\frac{\partial u_s(\eta,t)}{\partial t}+\gamma\frac{\partial^2 u_s(\eta,t)}{\partial t^2}=-\nu\eta^2 u_s(\eta,t)+\nu\eta\sqrt{\frac{2}{\pi}}A_0H(t)cos(\Omega t)-M\left(\gamma\frac{\partial}{\partial t}+1\right)u_s(\eta,t), \quad (4.1)$$

where $u_s(\eta,t)$ is Fourier sine transform, $H(t)$ is the Heaviside function and Fourier sine transform is defined by

$$u_s(\eta,t)=\sqrt{\frac{2}{\pi}}\int_0^\infty sin(y\eta)u(y,t)dy. \quad (4.2)$$

The following initial conditions are satisfied by Fourier sine transform,

$$u_s(\eta,0)=\frac{\partial u_s(\eta,0)}{\partial t}=0, \eta>0. \quad (4.3)$$

Applying the Laplace transform to (4.1), we find that

$$\bar{u}_s(\eta,\delta)=\sqrt{\frac{2}{\pi}}\frac{A_0\ \nu\ \eta\ \Omega}{(\delta^2+\Omega^2)[\gamma\delta^2+(1+\gamma M)\delta+M+\nu\eta^2]}, \quad (4.4)$$

breaking (4.4) in below expression as in equivalent form

$$\bar{u}_s(\eta,\delta)=\frac{A_0\ \nu\ \eta\ \Omega}{(M+\nu\eta^2)}\sqrt{\frac{2}{\pi}}\left[\frac{1}{(\delta^2+\Omega^2)}-\frac{\delta(1+\gamma\delta+M\gamma)}{(\delta^2+\Omega^2)[\gamma\delta^2+(1+\gamma M)\delta+M+\nu\eta^2]}\right]. \quad (4.5)$$

Inverting (4.5) by Fourier sine transformation, we can write $\bar{u}_s(\eta,\delta)$

$$\bar{u}(y,\delta) \quad (4.6)$$

$$=\frac{2A_0\nu\eta\Omega}{\pi}\sqrt{\frac{2}{\pi}}\int_0^\infty\frac{\eta sin(y\eta)}{(M+\nu\eta^2)}\left[\frac{1}{(\delta^2+\Omega^2)}-\frac{\delta(1+\gamma\delta+M\gamma)}{(\delta^2+\Omega^2)[\gamma\delta^2+(1+\gamma M)\delta+M+\nu\eta^2]}\right]. \quad (4.7)$$

Finally we apply the inverse Laplace transform and its convolution theorem to (4.6), having the below fact

$$\int_0^\infty \frac{\eta sin(y\eta)}{(q^2+\eta^2)} = \frac{2}{\pi}e^{-qy}, q > 0. \tag{4.8}$$

Velocity field is expressed in multiple integral form as,

$$u(y,t) = A_0H(t)sin(\Omega t)e^{-\sqrt{\frac{M}{\nu}}y} - \frac{2A_0H(t)\Omega\nu}{\pi\gamma(\delta_1-\delta_2)}\int_0^\infty\int_0^t \frac{\eta sin(y\eta)}{(M+\nu\eta^2)}cos\Omega(t-z)$$

$$\times\left[(1+\gamma\delta_1)e^{\delta_1}z - (1+\gamma\delta_2)e^{\delta_2}z\right] d\eta\ dz + \frac{2A_0H(t)\Omega\nu M}{\pi(\delta_1-\delta_2)}\int_0^\infty\int_0^t \frac{\eta sin(y\eta)}{(M+\nu\eta^2)}$$

$$\times cos\Omega(t-z)(e^{\delta_1}t - e^{\delta_2}t)\ d\eta\ dz, \tag{4.9}$$

where, $\delta_1, \delta_2 = \frac{-(1+\gamma M)\pm\sqrt{(1+\gamma M)^2-4\gamma(M+\nu\eta^2)}}{2\gamma}$ are the roots of the algebraic equation $\gamma\delta^2 + (1+\gamma M)\delta + (M+\nu\eta^2) = 0$.

4.2. Shear Stress

Applying Laplace transform to equation (2.9) the expression takes place as,

$$\bar{\tau}(y,\delta) = \frac{\mu}{(1+\gamma\delta)}\frac{\partial\bar{u}(y,\delta)}{\partial y}. \tag{4.10}$$

Solving (4.6) for partial differentiation, with respect to y, we get

$$\frac{\partial\bar{u}(y,\delta)}{\partial y} = \frac{2A_0\nu\Omega}{\pi}\int_0^\infty \frac{\eta^2 cos(y\eta)}{(M+\nu\eta^2)}\left[\frac{1}{(\delta^2+\Omega^2)} - \frac{\delta(1+\gamma\delta+M\gamma)}{(\delta^2+\Omega^2)[\gamma\delta^2+(1+\gamma M)\delta+M+\nu\eta^2]}\right] d\eta. \tag{4.11}$$

Substituting equation (4.11) in (4.10), we have

$$\bar{\tau}(y,\delta) = \frac{\mu}{(1+\gamma\delta)} \times\left[\frac{2A_0\nu\Omega}{\pi}\sqrt{\frac{2}{\pi}}\int_0^\infty \frac{\eta^2 cos(y\eta)}{(M+\nu\eta^2)}\left(\frac{1}{(\delta^2+\Omega^2)} - \frac{\delta(1+\gamma\delta+M\gamma)}{(\delta^2+\Omega^2)[\gamma\delta^2+(1+\gamma M)\delta+M+\nu\eta^2]}\right)\right] d\eta. \tag{4.12}$$

Simplifying (4.12) for suitable expression of shear stress as

$$\bar{\tau}(y,\delta) = -\frac{\mu}{(1+\gamma\delta)}\sqrt{\frac{M}{\nu}}\frac{A_0\Omega}{(\delta^2+\Omega^2)}e^{-\sqrt{\frac{M}{\nu}}y} - \frac{2A_0\nu\Omega\mu}{\pi}\int_0^\infty \frac{\eta^2 cos(y\eta)}{(M+\nu\eta^2)}\frac{\delta}{(\delta^2+\Omega^2)}$$

$$\times\left(\frac{1}{\delta-\delta_1} - \frac{1}{\delta-\delta_2}\right)d\eta + \frac{2MA_0\mu\Omega\nu}{\pi(\delta_1-\delta_2)}\int_0^\infty \frac{\eta^2 cos(y\eta)}{(M+\nu\eta^2)}\frac{\delta}{(\delta^2+\Omega^2)}$$

$$\times\left(\frac{1}{(\delta-\delta_1)(1+\delta_1\gamma)} - \frac{1}{(\delta-\delta_2)(1+\delta_2\gamma)} + \frac{\gamma^2(\delta_1-\delta_2)}{(1+\delta_1\gamma)(1+\delta_2\gamma)(1+\delta\gamma)}\right) d\eta. \tag{4.13}$$

Finally, applying inverse Laplace transform on equation (4.13), we get shear stress in integral form

$$\tau(y,t) = -\frac{A_0H(t)\mu}{\gamma}\sqrt{\frac{M}{\nu}}e^{-\sqrt{\frac{M}{\nu}}y}\int_0^t sin\Omega(t-z)e^{\frac{z}{\gamma}}dz - \frac{2A_0H(t)\nu\Omega\mu}{\pi\gamma(\delta_1-\delta_2)}\int_0^\infty \frac{\eta^2 cos(y\eta)}{(M+\nu\eta^2)}$$
$$\times cos\Omega(t-z)(e^{\delta_1 z}-e^{\delta_2 z})\ d\eta\ dz + \frac{2MA_0H(t)\mu\Omega\nu}{\pi(\delta_1-\delta_2)}\int_0^\infty \frac{\eta^2 cos(y\eta)}{(M+\nu\eta^2)}cos\Omega(t-z)$$
$$\times\left(\frac{e^{\delta_1 z}}{(1+\delta_1\gamma)} - \frac{e^{\delta_2 z}}{(1+\delta_2\gamma)} + \frac{\gamma^2(\delta_1-\delta_2)e^{\frac{-z}{\gamma}}}{(1+\delta_1\gamma)(1+\delta_2\gamma)}\right)\ d\eta\ dz. \tag{4.14}$$

Solution for cosine oscillations: Solution of cosine oscillation is obtained by utilization of similar algorithm

$$u(y,t) = A_0H(t)cos(\Omega t)e^{-\sqrt{\frac{M}{\nu}}y} - \frac{2A_0H(t)\Omega\nu}{\pi\gamma(\delta_1-\delta_2)}\int_0^\infty\int_0^t \frac{\eta sin(y\eta)}{(M+\nu\eta^2)}sin\Omega(t-z)$$
$$\times\left[(1+\gamma\delta_1)e^{\delta_1}z - (1+\gamma\delta_2)e^{\delta_2}z\right]\ d\eta\ dz + \frac{2A_0H(t)\Omega\nu M}{\pi(\delta_1-\delta_2)}\int_0^\infty\int_0^t \frac{\eta sin(y\eta)}{(M+\nu\eta^2)}$$
$$\times sin\Omega(t-z)(e^{\delta_1}z - e^{\delta_2}z)\ d\eta\ dz, \tag{4.15}$$

$$\tau(y,t) = -\frac{A_0H(t)\mu}{\gamma}\sqrt{\frac{M}{\nu}}e^{-\sqrt{\frac{M}{\nu}}y}\int_0^t cos\Omega(t-z)e^{\frac{z}{\gamma}}dz - \frac{2A_0H(t)\nu\Omega\mu}{\pi}\int_0^\infty \frac{\eta cos(y\eta)}{(M+\nu\eta^2)}$$
$$\times sin\Omega(t-z)\left(e^{\delta_1 z}-e^{\delta_2 z}\right)\ d\eta\ dz + \frac{2MA_0H(t)\mu\Omega\nu}{\pi(\delta_1-\delta_2)}\int_0^\infty \frac{\eta^2 cos(y\eta)}{(M+\nu\eta^2)}sin\Omega(t-z)$$
$$\times\left(\frac{e^{\delta_1 z}}{(1+\delta_1\gamma)} - \frac{e^{\delta_2 z}}{(1+\delta_2\gamma)} + \frac{\gamma^2(\delta_1-\delta_2)e^{\frac{-z}{\gamma}}}{(1+\delta_1\gamma)(1+\delta_2\gamma)}\right)\ d\eta\ dz. \tag{4.16}$$

5. Particular Cases

5.1. Solutions of Maxwell Fluid $M = 0$ (Absence of Magnetic Field)

Solutions of Maxwell fluid are obtained when limit $M \to 0$ into equations (4.9), (4.14), (4.15) and (4.16)

$$u_{MS}(y,t) = A_0H(t)sin(\Omega t) - \frac{2A_0H(t)\Omega\nu}{\pi(\delta_1-\delta_2)}\int_0^\infty\int_0^t \frac{sin(y\eta)}{\eta}cos\Omega(t-z)$$
$$\times[(1+\gamma\delta_1)e^{\delta_1}z - (1+\gamma\delta_2)e^{\delta_2}z]\ d\eta\ dz, \tag{5.1}$$

$$\tau_{MS}(y,t) = -\frac{2A_0H(t)\nu\Omega}{\pi(\delta_1-\delta_2)}\int_0^\infty\int_0^t cos(y\eta)cos\Omega(t-z)[e^{\delta_1 z}-e^{\delta_2 z}]\ d\eta\ dz, \tag{5.2}$$

$$u_{MC}(y,t) = A_0H(t)cos(\Omega t) - \frac{2A_0H(t)\Omega\nu}{\pi(\delta_1-\delta_2)}\int_0^\infty\int_0^t \frac{sin(y\eta)}{\eta}sin\Omega(t-z)$$
$$\times[(1+\gamma\delta_1)e^{\delta_1}z - (1+\gamma\delta_2)e^{\delta_2}z]\ d\eta\ dz, \tag{5.3}$$

$$\tau_{MC}(y,t) = -\frac{2A_0H(t)\nu\Omega}{\pi}\int_0^\infty\int_0^t \frac{cos(y\eta)}{\eta}sin\Omega(t-z)[e^{\delta_1 z}-e^{\delta_2 z}]\ d\eta\ dz. \tag{5.4}$$

5.2. MHD Newtonian Fluid $\gamma = 0$ (Presence of Magnetic Field)

Solutions of MHD Newtonian fluid are obtained when limit $\gamma = 0$ into equations (4.9),(4.14),(4.15) and (4.16) along with usage of following facts

$\lim_{\gamma\to 0}\delta_1 = -(M+\nu\eta^2)$, $\lim_{\gamma\to 0}\delta_2 = \infty$, $\lim_{\gamma\to 0}\gamma(\delta_1-\delta_2) = 1$

$$u_{MNS}(y,t) = A_0H(t)sin(\Omega t)e^{-\sqrt{\frac{M}{\nu}}y} - \frac{2A_0H(t)\Omega\nu}{\pi}\int_0^\infty\int_0^t \frac{\eta sin(y\eta)}{(M+\nu\eta^2)}cos\Omega(t-z)e^{-(M+\nu\eta^2)z}\,d\eta\,dz,$$

$$\tau_{MNS}(y,t) = -\mu A_0H(t)\sqrt{\frac{M}{\nu}}\,sin(\Omega t)\,e^{-\sqrt{\frac{M}{\nu}}y} - \frac{2A_0H(t)\nu\Omega\mu}{\pi}\int_0^\infty\int_0^t \frac{\eta^2 cos(y\eta)}{(M+\nu\eta^2)} \times cos\Omega(t-z)e^{-(M+\nu\eta^2)z}\,d\eta\,dz,$$

$$u_{MNC}(y,t) = A_0H(t)cos(\Omega t)e^{-\sqrt{\frac{M}{\nu}}y} - \frac{2A_0H(t)\Omega\nu}{\pi}\int_0^\infty\int_0^t \frac{\eta sin(y\eta)}{(M+\nu\eta^2)}sin\Omega(t-z)e^{-(M+\nu\eta^2)z}\,d\eta\,dz,$$

$$\tau_{MNC}(y,t) = \frac{2A_0H(t)\nu\Omega\mu}{\pi}\int_0^\infty\int_0^t \frac{\eta cos(y\eta)}{(M+\nu\eta^2)}sin\Omega(t-z)e^{-(M+\nu\eta^2)z}\,d\eta\,dz.$$

5.3. Newtonian Fluid $\gamma = 0$ and $M = 0$ (Absence of Magnetic Field)

Solutions of Newtonian fluid are obtained when limit $\gamma = 0$ and $M = 0$ into equations (4.9), (4.14), (4.15) and (4.16)

$$u_{NS}(y,t) = A_0H(t)sin(\Omega t) - \frac{2A_0H(t)\Omega}{\pi}\int_0^\infty\int_0^t \frac{sin(y\eta)}{\eta}cos\Omega(t-z)e^{-\nu\eta^2 z}\,d\eta\,dz,$$

$$\tau_{NS}(y,t) = -\frac{2A_0H(t)\nu\Omega\mu}{\pi}\int_0^\infty\int_0^t cos(y\eta)cos\Omega(t-z)e^{-\nu\eta^2 z}\,d\eta\,dz,$$

$$u_{NC}(y,t) = A_0H(t)cos(\Omega t) - \frac{2A_0H(t)\Omega}{\pi}\int_0^\infty\int_0^t \frac{sin(y\eta)}{\eta}sin\Omega(t-z)e^{-\nu\eta^2 z}\,d\eta\,dz,$$

$$\tau_{NC}(y,t) = -\frac{2A_0H(t)\Omega\mu}{\pi}\int_0^\infty\int_0^t cos(y\eta)cos\Omega(t-z)e^{-\nu\eta^2 z}\,d\eta\,dz.$$

It is also worth noted that when $\gamma = 0$ then solutions can be recovered for second grade fluid investigated by [1]. In continuation, When $\gamma = 0$ and $M = 0$ then solutions can also be reduced for Newtonian fluid traced out by [1] (see equations 40-43).

6. Conclusion

In this portion, the characteristics of magnetohydrodynamic flow of viscoelastic fluid with and without magnetic field induced by oscillating plate are shown. General solutions have been found out for velocity and shear stress profiles using mathematical transformations (Integral transforms) under the sine and cosine boundary conditions $u(0,t) = A_0 H(t) sin(\Omega t)$ and $u(0,t) = A_0 H(t) cos(\Omega t)$ with $t \geq 0$. For the sake of simplicity of boundary conditions are verified on the analytical general solutions and similar solutions have been particularized under three limited cases namely (i). Maxwell fluid with out magnetic field if $\gamma \neq 0$ and $M = 0$ (ii). Newtonian fluid with magnetic field if $\gamma = 0$ and $M \neq 0$ and (iii). Newtonian fluid with out magnetic field if $\gamma = 0$ and $M = 0$. The bunch of graphs has been prepared with typical values at different situations for rheological parameters to reveal some relevant physical aspects. Finally various outcomes are discussed below:
The influence on fluid motion is displayed in Fig. 2 for the sine and cosine oscillations. It is noticed that by increasing various values of time $t = 2.0, 2.2, 2.4, 2.6$ the velocity of fluid is increasing on the entire boundary region which indicates that as time progresses fluid velocity is monotonically enhanced.

Fig. 3 represents the relaxation γ phenomenon of fluid between $1 \leq \gamma \leq 4$, both velocity field as well as shear stress have strong effects on decreasing fluid behavior as expected.

The viscous effects ν are shown in Fig. 4 in which velocity field is thickening and shear stress is scattering when viscosity? increases, such phenomenon is termed as shear thinning and shear thickening.

Impacts of magnetic field M on fluid motion are displayed in Fig. 5 in which velocity field and shear stress decreases when magnetic parameter increases. As we expected in MHD flow, wall regions will be balanced if fluid velocity decreases.

Fig. 6 is prepared to analyze the impact of oscillations, It is noted that increase in fluid oscillations Ω approaches to zero and decays away from the oscillating plate. Meanwhile, Fig. 7 show the behavior of plate in terms of helical oscillations.

Figs. 8 and 9 are depicted for comparison of four kinds of models i-e Maxwell fluid in presence of magnetic field, Maxwell fluid in absence of magnetic field, Newtonian fluid in presence of magnetic field and Newtonian fluid in absence of magnetic field, the motion of fluid flow is contrasting, i-e velocity field is increasing and shear stress is scattering with respect to different increasing effects of time parameter t.

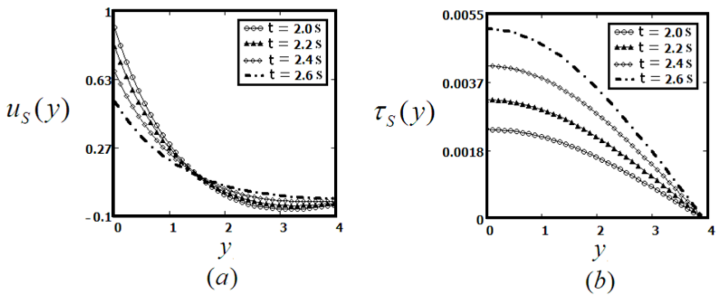

Figure 2: Profiles of the velocity field $u(y,t)$ and the shear stress $\tau(y,t)$ for MHD Maxwell fluid given by equations (4.8) and (4.13), for $A_0 = 1,\ \nu = 0.63,\ \mu = 1.52,\ \gamma = 2,\ M = 0.5,\ \Omega = 1$ and different values of t.

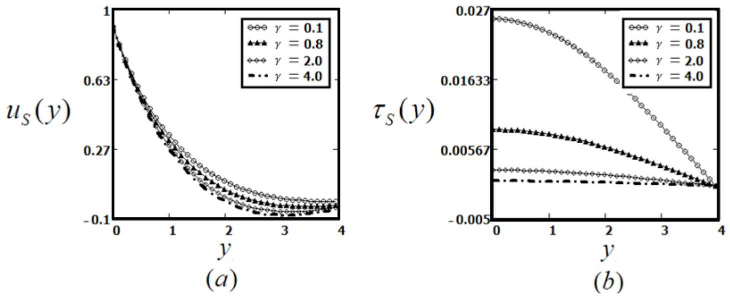

Figure 3: Profiles of the velocity field $u(y,t)$ and the shear stress $\tau(y,t)$ for MHD Maxwell fluid given by equations (4.8) and (4.13), for $A_0 = 1,\ \nu = 0.63,\ \mu = 1.52,\ t = 2s,\ M = 0.5,\ \Omega = 1$ and different values of γ.

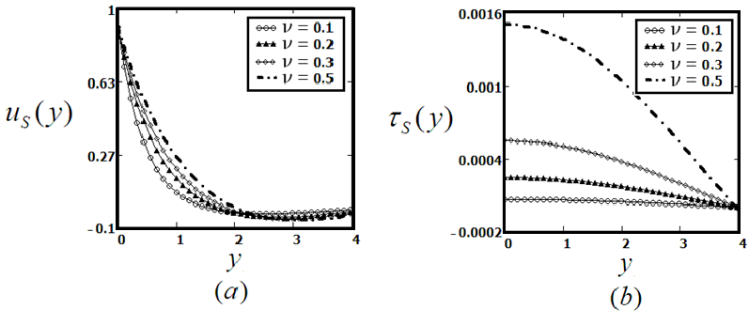

Figure 4: Profiles of the velocity field $u(y,t)$ and the shear stress $\tau(y,t)$ for MHD Maxwell fluid given by equations (4.8) and (4.13), for $A_0 = 1,\ \rho = 2.41,\ \mu = 1.52,\ \gamma = 2,\ M = 0.5,\ \Omega = 1$ and different values of ν.

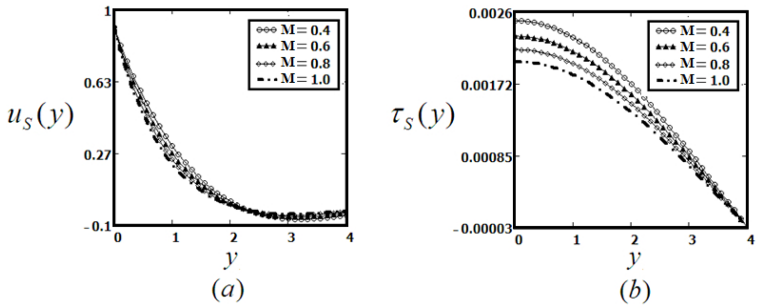

Figure 5: Profiles of the velocity field $u(y,t)$ and the shear stress $\tau(y,t)$ for MHD Maxwell fluid given by equations (4.8) and (4.13), for $A_0 = 1,\ \nu = 0.63,\ \mu = 1.52,\ \gamma = 2,\ t = 2s,\ \Omega = 1$ and different values of ν.

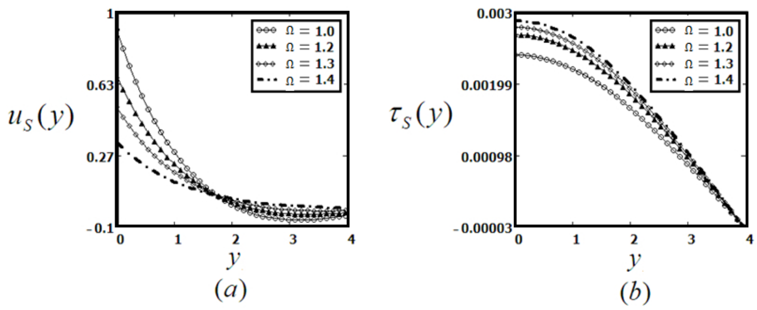

Figure 6: Profiles of the velocity field $u(y,t)$ and the shear stress $\tau(y,t)$ for MHD Maxwell fluid given by equations (4.8) and (4.13), for $A_0 = 1,\ \nu = 0.63,\ \mu = 1.52,\ \gamma = 2,\ t = 2s,\ t = 2s$ and different values of Ω.

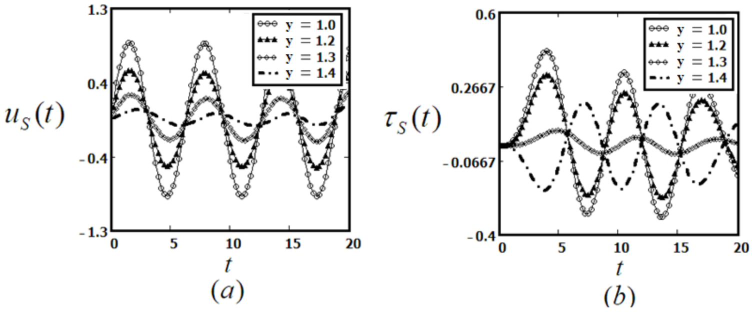

Figure 7: Profiles of the velocity field $u(y,t)$ and the shear stress $\tau(y,t)$ for MHD Maxwell fluid given by equations (4.8) and (4.13), for $A_0 = 1,\ \nu = 0.63,\ \mu = 1.52,\ \gamma = 2,\ M = 0.5, \Omega = 1,$ and different values of y.

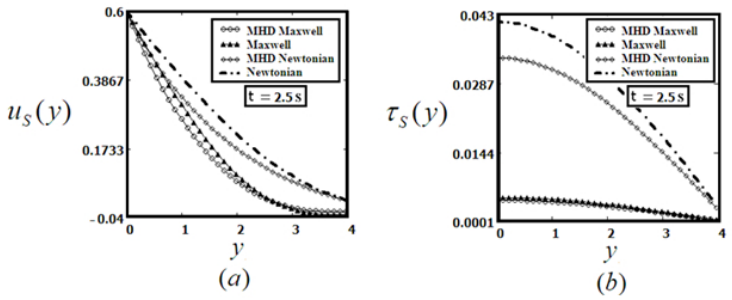

Figure 8: Profiles of the velocity field $u(y,t)$ and the shear stress $\tau(y,t)$ for MHD Maxwell fluid, Maxwell fluid, MHD Newtonian fluid and Newtonian fluid given by equations (4.8), (4.13), (5.1), (5.2), (5.5), (5.6), (5.9) and (5.10) for $A_0 = 1,\ \nu = 0.63,\ \mu = 1.52,\ \gamma = 2,\ M = 0.5, \Omega = 1,$ and fixed value for $t = 2.5s$.

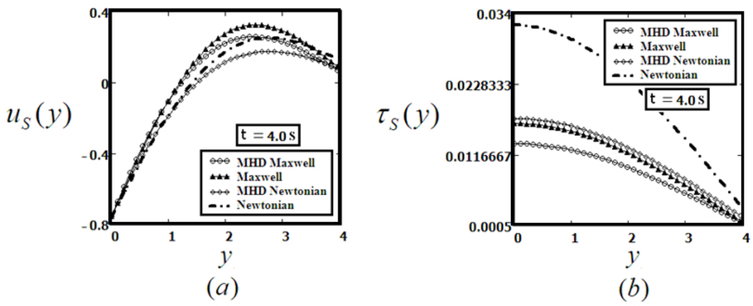

Figure 9: Profiles of the velocity field $u(y,t)$ and the shear stress $\tau(y,t)$ for MHD Maxwell fluid, Maxwell fluid, MHD Newtonian fluid and Newtonian fluid given by equations (4.8), (4.13), (5.1), (5.2), (5.5), (5.6), (5.9) and (5.10) for $A_0 = 1,\ \nu = 0.63,\ \mu = 1.52,\ \gamma = 2,\ M = 0.5, \Omega = 1,$ and fixed value for $t = 4.0s$.

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