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Periodic Wave Shock Solutions of Burgers Equations, A New Approach

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Abstract

In this paper we present a new approach for the study of Burgers equations. Our purpose is to describe the asymptotic behavior of the solution in the cauchy problem for viscid equation with small parametr ε and to discuss in particular the case of periodic wave shock. We show that the solution of this problem approaches the shock type solution for the cauchy problem of the inviscid burgers equation. The results are formulated in classical mathematics and proved with infinitesimal techniques of non standard analysis.

Keywords: non standard analysis, boundary problem, viscid Burgers equation, inviscid Burgers equation, heat equation, peroidic wave shock.
2010 MSC: 35XX ; 35LXX ; 58J45.

1. Introduction

One of the most important PDEs in the theory of non linear consevation laws is Burgers equation, she combining both nonlinear propagation effects and diffusive effect. This equation is the approximation for the one-dimensional propagation of weak shock waves in a fluid. It can also be used in the description of the variation in vehicle density in highway traffic. The equation is one of the fundamental model equations in fluid mechanics. Burgers introduced the equation to describe the behavior of shock waves, traffic flow and acoustic transmission. It is well known that Burgers equation plays a relevant role in many different areas of the mathematical physics, specially in Fluid Mechanics. Moreover the simplicity of its formulation, in contrast with the Navier-Stokes system, makes of the Burgers equation a suitable model equation to test different numerical algorithms and results of a varied nature [3], [4]. This equation is parabolic when the viscous term is included. It has the form [15] :

$$u_t + uu_x = \varepsilon u_{xx} \tag{1.1}$$

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If the viscous term is null the remaining equation is hyperbolic this is the inviscid Burgers equation

$$u_t + uu_x = 0 \tag{1.2}$$

If the viscous term is dropped from the Burgers equation, discontinuities may appaer in finite time, even if the initial condition is smooth they give rise to the phenomen of shock waves with important application in physics [10]. This properties make Burgers equation a proper model for testing numerical algorithms in flows where severe gradients or shocks are anticipated [3], [17], [18]. Recently, Kunjan and Twinkle [20] used mixture of new integral transform and Homotopy Perturbation Method to find the solution of Bugers' equation arising in the longitudinal dispersion phenomenon in fluid flow through porous media. Discretization methods are well-knoun techniques for sollving Burgers equation. Ascher and Mclachlan established many methods as multisymplectic box sheme. Olayiwola et *al.* [19], also presented the modified variational iteration method for the numerical solution of generalized Burger's-Huxley equation. Oleksandr A. Pocheketa ,and Roman .O.Popovch [2] used enhanced classification techniques for the numerical solution of generalized Burgers equation. .To establish numerical solutions for the modified Burgers equation; Ucar, Yagmurlu and Tasbozan,used in [1] finite difference methods. For the boundary value problem, SinaL [17] is intersted to the initial condition case : null on $\mathbb{R}^-$ and Brownian on $\mathbb{R}^+$. She, Aurell and Frich [16] with a numerical calculs particularly examine the initial conditions of Brownian fractionnair type.

This paper completes recent works on the study of boundary value problems of Burgers equations for different initial conditions [5], [6], [7]. In the presented work, our general purpose is to describe the asymptotic behavior of solutions in boundary value problem with a small parameter ε and to discuss in particular the case of periodic wave shocks with new techniques infinitesimal of Non Standard Analysis. We can conclude that the solution of the cauchy problem of inviscid equation is infinitely close to the solution of he cauchy problem of viscid equation as ε is a parameter positif sufficiently small. We introduce the infintesimal techniques to give a simple and direct formulation for the asymptotic behavior, unlike classical methods are based on search of solutions of the reduiced problem to deduce existence and asymptotic behavior of the solutions as ε tends to 0, the passasge of the limit is very complicated, but in general the limit exist and it's a solution for the reduced problem (when $\varepsilon = 0$). For ε small the solution $u(x,t)$ is approximated by this limit [10]. Other methods are based on a weak formulation of burgers equation seen as a conservation law satisfied on each of the computational domain called cell or finite volume. Stochastic particle method is so used for with different initial conditions It is worth noting that our contribution is an elegant combination of infinitesimal techniques of non standard analysis and the Van Den Berg method [5], [6], [14].

Historically the subject non standard was developed by Robinson, Reeb, Lutz and Gose [12]. The idea to use Non Standard Analysis in perturbation theory of differential equations goes back to the 1970s with the Reebian school. Relative to this use, among many works we refer the interested reader, for instance, to [21]-[26] and the references therein. It has become today a wellestablished tool in asymptotic theory of differential equations. Among the famous discoveries of the nonstandard asymptotic theory of differential equations we can cite the canards which appear in slow-fast vector fields and are closely related to the stability loss delay phenomenon in dynamical bifurcations.

The paper is organised as follows : Section 2 concerns the boundary value problem of inviscid burgers equation, we start with the Fitting discontinuous shock then we describe the asymptotic behavoir of solutions for this problem in the periodic wave case. Section 3 concerns the boundary value problem of viscid burgers equation, it contains basic preliminaries results and deals with our main result about periodic wave shock case and its proof, we present it in a non standard form.

2. Inviscid Burgers Equation

We will focus first on equation (1.2). Specifically, we will deal with the initial value problem

$$\begin{cases} u_t + uu_x = 0 & , x \in \mathbb{R}, t > 0 \\ u(\xi, 0) = f(\xi) & , t = 0 \end{cases} \tag{2.1}$$

As it as has been suggested previously, although (2.1) seems to be a very innocent problem a priori it hides many unexpected phenomena.This problem does not admit the regular solutions but some weak solutions with certain regularity exist.

The Burgers equation on the whole line is known to possess traveling waves solutions. Using the characteristic method, the solution of the problem (2.1) may be given in a parametric form :

$$\begin{cases} u = f(\xi) \\ x = \xi + f(\xi)t \end{cases} \tag{2.2}$$

and shocks must be fitted in such that :

$$U = \frac{1}{2}(u_1 + u_2) = \frac{1}{2}(f(\xi_1) + f(\xi_2) \tag{2.3}$$

where $f : R \to R.$ is a standard continuous function. ξ_1 and ξ_2 are the values of ξ on the two sides of the shock [9].

According to (2.2), the solution at time t is obtained from the initial profile $u = f(\xi)$ by translating each point a distance $f(\xi)t$ to the right.The shock cuts out the part corresponding to $\xi_2 \geq \xi \geq \xi_1$. If the discontinuity line it is a straight line chord betewen the points $\xi = \xi_1$ and $\xi = \xi_2$ on the curve $f(\xi)$. Moreover since areas are preserved under the mapping, the equal area property still holds. The chord on the f curve cuts off lobes of equal area.

The shock determination can then be describe entirely on the fixe $f(\xi)$ curve by drawing all the chords with the equal area propperty can be written analytically as

$$\frac{1}{2}\left\{(f(\xi_1) + f(\xi_2)\right\}(\xi_1 - \xi_2) = \int_{\xi_2}^{\xi_1} f(\xi) d\xi \tag{2.4}$$

This is the differential equation for the shock line chord wich verifies the entropic condition such as [9]. Since the left hand side is the area under the chord and the right hand side is the area under the f curve. If the shock is at $x = s(t)$ at time t, we also have

$$s(t) = \xi_1 + f(\xi_1)t \tag{2.5}$$

$$s(t) = \xi_2 + f(\xi_2)t \tag{2.6}$$

From (2.5) and (2.6), we have

$$t = \frac{\xi_1 - \xi_2}{f(\xi_1) - f(\xi_2)} \tag{2.7}$$

2.1. Single Hump

To describe the solutions of the problem (2.1), we assume that the initial condition f verify the following assumptions:

(H_1) : f is equal to a constant u_0 outside the range $0 < \xi < L$.

(H_2) : $f(\xi) > u_0$ in the range.

The theorem bellow gives the assymptotic behavior to the solution behind the shock and at the shock.

Theorem 2.1. *Suppose that* (H_1) *and* (H_2) *are satisfied, the solution of the problem* (2.1) *is given by* (2.2) *with* $0 < \xi < \xi_2$ *and the asymptotic form is*

$$u \sim \frac{x}{t}, \quad \textit{for} \;\; u_0 t < x < u_0 t + \sqrt{2At} \tag{2.8}$$

Proof . We consider the problem (2.1) and we suppose that (H_1) and (H_2) are satisfied. Equation (2.4) may be written as

$$\frac{1}{2}\left[f(\xi_1) + f(\xi_2) - 2u_0\right](\xi_1 - \xi_2) = \int_{\xi_1}^{L} (f(\xi) - u_0) d\xi \tag{2.9}$$

As time goes on ξ_1 increase and eventually exceed L. At this stage $f(\xi_1) = u_0$ and the shock is moving into the constant region $u = u_0$. The function $\xi_1(t)$ can then be eliminated for we have

$$\frac{1}{2}\left[(f(\xi_2) - u_0)(\xi_1 - \xi_2)\right] = \int_{\xi_2}^{L} (f(\xi) - u_0) d\xi, \quad t = \frac{\xi_1 - \xi_2}{f(\xi_2) - u_0} \tag{2.10}$$

There for

$$\frac{1}{2}(f(\xi_2) - u_0)^2 t = \int_{\xi_2}^{L} (f(\xi) - u_0) d\xi \tag{2.11}$$

At this stage the shock position and the value of u just behind the shock are given by

$$\begin{cases} u = f(\xi_2) \\ s(t) = \xi_2 + f(\xi_2)t \end{cases} \tag{2.12}$$

where ξ_2 satisfie (2.11). As t is infinitely large we have ξ_2 infinitesimal and $f(\xi_2)$ approach u_0, hence the equation for $\xi_2(t)$ takes the limiting form

$$\frac{1}{2}(f(\xi_2) - u_0)^2 t \sim A \tag{2.13}$$

where

$$A = \int_{0}^{L} (f(\xi) - u_0) d\xi \tag{2.14}$$

is the area of the hump above the undisturbed value u_0.We have ξ_2 infinitesimal and

$$f(\xi_2) \sim u_0 + \sqrt{2A/t} \tag{2.15}$$

Therefore the asymptotic formulas for $s(t)$ and u in (2.12) are

$$s(t) \sim u_0 t + \sqrt{2At} \tag{2.16}$$

$$u - u_0 \sim \sqrt{2A/t} \tag{2.17}$$

at the shock. The shock curve is asymptotically parabolic. The solution behind the shock is given by (2.2) with $0 < \xi < \xi_2$. Since ξ_2 is small enouhg as t is small enouhg, all the relevant values of ξ also small enouhg and the asymptotic form is

$$u \sim \frac{x}{t}, \quad \text{for } u_0 t < x < u_0 t + \sqrt{2At}$$

The asymptotic solution and the corresponding (x, t) diagram are shown in Figure.1

□

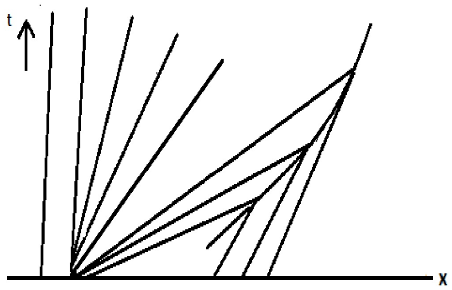

Figure 1: Asymptotic triangular wave.

2.2. Periodic Wave

Another intersting problem is that of an initial distribution

$$f(\xi) = u_0 + a.\sin\left(\frac{2\pi\xi}{\lambda}\right)$$

In this case, the shock equations (2.4) simplify considerably for all times t. Consider one period $0 < \xi < \lambda$ as in Figure 2. Relations (2.4) becomes

$$(\xi_1 - \xi_2)\sin\left(\frac{\pi}{\lambda}(\xi_1+\xi_2)\right)\cos\left(\frac{\pi}{\lambda}(\xi_1-\xi_2)\right) = \frac{\lambda}{\pi}\sin\left(\frac{\pi}{\lambda}(\xi_1-\xi_2)\right)\sin\left(\frac{\pi}{\lambda}(\xi_1+\xi_2)\right) \tag{2.18}$$

and the relevant choice is the trivial one

$$\sin\left(\frac{\pi}{\lambda}(\xi_1+\xi_2)\right) = 0 \quad \text{that is,} \quad \xi_1 + \xi_2 = \lambda$$

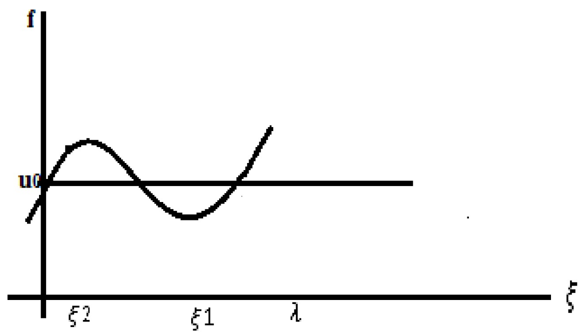

Figure 2: Shock construction for a periodic wave.

From the difference and sum of (2.5) and (2.6), we have

$$t = \frac{\xi_1 - \xi_2}{2a\sin\left(\frac{\pi}{\lambda}(\xi_1-\xi_2)\right)}$$

$$s = u_0 t + \frac{\lambda}{2}$$

Respectively, the discontinuity in u at the shock is

$$\begin{aligned} u_2 - u_1 &= a\sin\left(\frac{2\pi\xi_1}{\lambda}\right) - a\sin\left(\frac{2\pi\xi_2}{\lambda}\right) \\ &= 2a\sin\left(\frac{\pi}{\lambda}(\xi_1-\xi_2)\right) \end{aligned}$$

If we introduce

$$\xi_1 - \xi_2 = \frac{\lambda\theta}{\pi}, \quad \xi_1 + \xi_2 = \lambda$$

we have

$$\begin{aligned} t &= \frac{\lambda}{2\pi a}.\frac{\theta}{\sin\theta} \\ s &= u_0 t + \frac{\lambda}{2} \\ \frac{u_2-u_1}{u_0} &= \frac{2a}{u_0}\sin\theta \end{aligned} \tag{2.19}$$

The shock has constant velocity u_0 and this result could have been deduced in advance from the symmetry of the problem. The shock starts with zero strength corresponding to $\theta = 0$ at time $t = \dfrac{\lambda}{2\pi a}$. It reaches a maximum strength of $\dfrac{2a}{u_0}$ for $\theta = \dfrac{\pi}{2}$, $t = \dfrac{\lambda}{4a}$ and decays ultimately with θ approach π, when t is infinitly large

$$\frac{u_2-u_1}{u_0} \sim \frac{\lambda}{u_0 t} \tag{2.20}$$

It is intersting that the final decay formula does not even depend explicitly on the amplitude a, however the condition for its application is $t >> \dfrac{\lambda}{a}$. For any periodic sinusoidal $f(\xi)$ or not $\xi_1 - \xi_2 \to \lambda$ as t infinitely large; thence from (2.7).

$$\frac{u_2-u_1}{u_0} = \frac{f(\xi_2)-f(\xi_1)}{u_0} \sim \frac{\lambda}{u_0 t} \tag{2.21}$$

Between successive shocks, the solution for u is linear in x with slope $\dfrac{1}{t}$ as before, and the asymptotic form of the entire profile is the sawtooth shown in Figure 3.

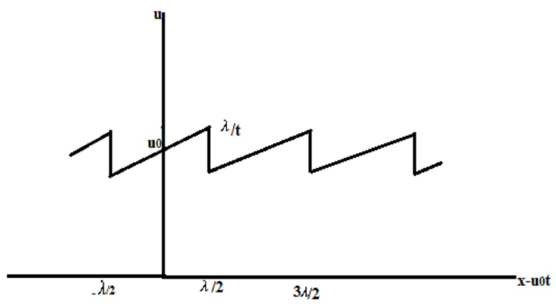

Figure 3: Asymptotic form of a periodic wave.

3. Viscid Burgers Equation

In this section we shall present and prove our main result, we discuss the periodic wave case in the boundary value problem of viscid Burgers equation

$$\begin{cases} u_t + uu_x = \varepsilon u_{xx} & x \in \mathbb{R}, \quad t > 0 \\ u(\xi, 0) = f(\xi) & t = 0 \end{cases} \tag{3.1}$$

Before going further in this case we need the following proposition and lemma.

3.1. The Cole-Hopf Transformation

Cole and Hopf noted the remarkable result[15] that the viscid burgers equation (1.1) may be reduced to the linear heat equation

$$\varphi_t = \varepsilon \varphi_{xx} \tag{3.2}$$

by the non linear transformation

$$u = -2\varepsilon[\log \varphi]_x \tag{3.3}$$

It is again convenient to do the trasformation in two steps. Firstly are introduced

$$u = \psi_x$$

So that (1.1) may be integrated to

$$\psi_t + \frac{1}{2}\psi_x^2 = \varepsilon \psi_{xx}$$

Then we introduce

$$\psi = -2\varepsilon[\log \varphi]$$

to obtain (3.2).

The non linear transformation just eliminates the nonlinear term. The general solution of the heat equation (3.2) is well khown and can be handled by a variety of methods. The basic problem considered in section 2 is the initial value problem

$$u = f(\xi), \quad \text{at} \quad t = 0$$

This is transformed by (3.3) to the initial value problem

$$\varphi = \phi(x) = \exp\left\{-\frac{1}{2\varepsilon}\int_0^x f(\eta)d\eta\right\}, \quad t = 0 \tag{3.4}$$

For the heat equation the solution for φ is

$$\varphi = \frac{1}{\sqrt{4\pi\varepsilon t}}\int_{-\infty}^{+\infty}\phi(\eta)\exp\left\{-\frac{(x-\eta)^2}{4\varepsilon t}\right\}d\eta \tag{3.5}$$

Through (3.3) the solution for u is

$$u(x,t) = \frac{\int_{-\infty}^{+\infty}\frac{x-\eta}{t}e^{-\frac{G}{2\varepsilon}}d\eta}{\int_{-\infty}^{+\infty}e^{-\frac{G}{2\varepsilon}}d\eta} \tag{3.6}$$

where

$$G(\eta, x, t) = \int_0^\eta f(\nu)d\nu + \frac{(x-\eta)^2}{2t} \tag{3.7}$$

3.2. The Behavior of solutions as ε small enough

The behavior of the exact solution (3.6) is now considered as ε is small enough. For x, t and $f(x)$ are held fixed as ε is small enough, the dominant contributions to the integrals in (3.6) come from the neighborhood of the stationnary points of G. A stationary point is where

$$\frac{\partial G}{\partial \eta} = f(\eta) - \frac{x-\eta}{t} = 0$$

Let $\eta = \xi(x,t)$ be such a point that $\xi(x,t)$ is difined as a solution of

$$f(\xi) - \frac{x-\xi}{t} = 0 \tag{3.8}$$

The contribution from the neighborhood of a stationary point $\eta = \xi$ in an integral is given with the lemma 3.2.

Lemma 3.1. *(The Van. Den. Berg lemma [14]). Let G be a standard function definied and increasing on $[0,+\infty[$ such that $G(v) = av^r\ (1+\delta)$ for $v \simeq 0$ and $G(v) > m(v)^q$. Let φ be an intern function definied on $]0,+\infty[$ such that : $\varphi(v) = bv^s(1+\delta)$ for $v \approx 0$ and such that $\forall\ d > 0$, $\exists$ standard k and c such that : $|\varphi(v)| < k\exp(\cosh(v))$ for $v > d$. Then*

$$\int_0^\infty \varphi(v)\exp\left(-\frac{G(v)}{2\varepsilon}\right)dv = \frac{b\Gamma\left(\dfrac{s+1}{r}\right)}{ra^{\frac{s+1}{r}}}\frac{1}{\left(\dfrac{1}{2\varepsilon}\right)^{\frac{s+1}{r}}} \tag{3.9}$$

where a and r are positifs standard, m and q are the both positifs. δ is a positif real small enough. b and s are standard, $b \neq 0$ and $s > -1$.

Lemma 3.2. *(The Nonstandard formula of the method of steepest descents). Let ε be a positif rael small enough and let φ and G be two standard functions such that : G, is a C^2 class function verifie the lemma 3.1, and admits on the ξ point an unique absolute minimum ($G'(\xi) = 0$ et $G''(\xi) > 0$). $\varphi(\xi) \neq 0$, it is S- continuous on ξ and satisfie the conditions of the lemma 3.1 in the two sens. Then*

$$\int_{-\infty}^{+\infty} \varphi(\eta).e^{-\frac{G}{2\varepsilon}}d\eta = \varphi(\xi)\frac{\sqrt{4\pi\varepsilon}}{\sqrt{G''(\xi)}}.e^{-\frac{G}{2\varepsilon}}(1+\delta) \tag{3.10}$$

δ is a positif real small enouhg.

Proof . Suppose first that there is only one stationary point $\xi(x,t)$ wich satisfies (3.8), then

$$\int_{-\infty}^{+\infty} \frac{x-\eta}{t}e^{-\frac{G}{2\varepsilon}}d\eta = \frac{x-\eta}{t}\frac{\sqrt{4\pi\varepsilon}}{\sqrt{G''(\xi)}}e^{-\frac{G}{2\varepsilon}}(1+\delta) \tag{3.11}$$

$$\int_{-\infty}^{+\infty} e^{-\frac{G}{2\varepsilon}}d\eta = \frac{\sqrt{4\pi\varepsilon}}{\sqrt{G''(\xi)}}e^{-\frac{G}{2\varepsilon}}(1+\delta) \tag{3.12}$$

and in (3.6) we have

$$u \simeq \frac{x-\xi}{t} \tag{3.13}$$

where $\xi(x,t)$ is difinied by (3.8).This asymptotic solution may be rewriten as

$$\begin{cases} u = f(\xi) \\ x = \xi + f(\xi)t \end{cases}$$

It is exactly the solution of (2.1) witch was discussed in section 2. The stationary point $\xi(x,t)$ becomes the characteristic variable. $\square$

3.3. Periodic Wave. The Main Result

A periodic solution may be obtained by taking for φ a distribution of heat sources spaced a distance λ apart. Then

$$\varphi = \frac{1}{\sqrt{4\pi\varepsilon t}} \sum_{n=-\infty}^{\infty} \exp\left\{-\frac{(x-n\lambda)^2}{4\varepsilon t}\right\} \tag{3.14}$$

The main result of this work is :

Theorem 3.3. *Assume that the initial data f has a profile shown in Figure 2, when $\frac{\lambda^2}{4\varepsilon t} >> 1$, the problem* (3.1) *admit an unique solution for $t > 0$ given by*

$$u \sim \frac{x-m\lambda}{t}, \quad (m-\frac{1}{2})\lambda < x < (m+\frac{1}{2})\lambda$$

Such solution is the periodic wave chock, and for ε small enough, this solution is infinitely close to the solution of the inviscid problem (2.1) *given in section 2.*

Proof . To obtain a periodic wave for u, we choose for φ a distribution of heat sources spaced a distance λ given by (3.14). Then the corresponding solution for u is

$$u = -2\varepsilon\frac{\varphi_x}{\varphi} = \frac{\sum\limits_{n=-\infty}^{+\infty}\left(\frac{x-n\lambda}{t}\right)\exp\left\{-\frac{(x-n\lambda)^2}{4\varepsilon t}\right\}}{\sum\limits_{n=-\infty}^{+\infty}\exp\left\{-\frac{(x-n\lambda)^2}{4\varepsilon t}\right\}} \tag{3.15}$$

For $\frac{\lambda^2}{4\varepsilon t} >> 1$ this implies that $\sqrt{\varepsilon t} << \frac{\lambda}{2}$, and $\sum\limits_{n=-\infty}^{+\infty} = 2\sum\limits_{n=0}^{+\infty}$, then for $n = m$ we have

$$\frac{(x-n\lambda)^2}{4\varepsilon t} = \frac{(x-m\lambda)^2}{\varepsilon t} << 1$$

wich gives $|x - m\lambda| << \sqrt{\varepsilon t}$. and the exponential with the minimum value of $\frac{(x-n\lambda)^2}{4\varepsilon t}$ will dominate over all the others.Therefore the term witch will dominate for

$$(m-\frac{1}{2})\lambda < x < (m+\frac{1}{2})\lambda$$

and (3.15) is approximately

$$u \sim \frac{x-m\lambda}{t}, \quad \text{for } (m-\frac{1}{2})\lambda < x < (m+\frac{1}{2})\lambda$$

This is a sawtooth wave with a periodic set of shocks a distance λ apart, and u jumps from $-\frac{\lambda}{2t}$ to $\frac{\lambda}{2t}$ at each shock . The result agrees with he inviscid solution given by (2.20). □

4. Conclusions

This paper completes recent works on the study of boundary value problems of Burgers equations for different initial conditions [5]-[7]. Our general purpose is to describe the asymptotic behavior of solutions in boundary value problem with a small parameter ε and to discuss in particular the case of periodic wave shocks. The originality of this work consists in introducing new infinitesimal techniques of non-standard analysis. We can conclude that the solution of the cauchy problem of inviscid equation is infinitely close to the solution of he cauchy problem of viscid equation as ε is a parameter positif sufficiently small. We introduce the infintesimal techniques to give a simple formulation for the asymptotic behaviour. It is worth noting that our contribution is an elegant combination of infinitesimal techniques of non standard analysis and the Van Den Berg method [5], [6], [14].

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