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Measuring the Efficiency of Banks' Performance Based on Economic Policies Using Data Envelopment Analysis

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Abstract

Due to long-term and all-compassing sanctions and the instability of political engagements with the other countries, Iran's economy more than ever should adjust its governance system to minimize economic Problems. As the most important financial institution, banks play an important role in the national economy. Considering the present and future economic conditions, the banks management always tries to improve banking services, marketing, innovation in service delivery, competition with other banks, considering the current and future economic conditions. Therefore, one of the goals of banks is to increase economic efficiency. Data envelopment analysis is a mathematical model to evaluate the relative efficiency of decision-making units. In this study, the efficiency of 83 first-class branches of Mellat Bank was analyzed using data envelopment analysis during 2016 and 2017 with an input-based approach and assuming a variable return to scale. In order to achieve efficiency, three inputs of effective deposits, loan, delayed debts, and an input of profit and loss were used. Finally, the efficiency of the branches was calculated using the Malmquist index. The research data were collected through the bank's annual report and analyzed using Lingo and Eviwes software. The present study is aimed to obtain the effectiveness of the bank's management using a three-stage data envelopment analysis. According to the results obtained after adjusting the environmental factors

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and statistical disturbance and comparing with the efficiency obtained before adjusting the inputs, it was found that environmental variables have a significant effect on the effectiveness of bank branches. According to the results of this research, it can be concluded that in many cases, the ineffectiveness of managers can be related to environmental and uncontrollable factors of managers, rather than to the manager's qualification.

Keywords: Data envelopment analysis, Efficiency, Economic policies.
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1. Introduction

Data envelopment analysis is generally used to evaluate the efficiency of units and pays less attention to units and decision centers. In fact, it states which unit is efficient and which is inefficient. It is only a relative criterion, that is, the decision centers measure a set only in relation to other units of the same set. Indeed, a decision unit may be efficient in terms of data envelopment analysis, but it is inefficient compared to similar units outside that organization. Also, for a decision set with a large number of inputs and outputs, it is not possible to determine which input and output is important, how important it is, and how much needs to be changed so that the decision unit becomes efficient. Today, the main issue in many organizations, especially in institutions such as banks with numerous branches, is that the performance evaluation method in them is not a logical and correct method. Conventional techniques used to evaluate performance are one-dimensional and cannot provide experts with adequate managerial information to identify the inefficient factors of inefficient units and to achieve the advantages and disadvantages of competitive strategies. However, multilevel data envelopment techniques can overcome this problem.

Today, productivity and efficiency are considered as culture and perspective in all areas of human labor and life and are the source of economic progress and development. This perspective is formed as by organizing the activities, the best result is achieved. One of the issues that need to be organized is banking industry, which is considered as one of the central activities in the economic development of any country, and the organization of this industry promotes its optimal performance. In fact, every activity requires capital and financial resources, so it also needs banks and credit institutions. Therefore, due to their effective role in economic activities, examining their performance and productivity is of great importance. The management of banks should considering improving banking services, marketing, budgeting, innovation in service, competitiveness with other financial and monetary firms, and finally increasing efficiency and productivity among the studied units based on the past the future economic conditions. One of the ways to fulfill this aim is to evaluate the inputs and outputs in their evaluation system. In a market-based economy, the banking system, which is one of the most important components of the country's economy, has a very heavy responsibility. We can say, the growth and prosperity of the country's economy is closely related to the procedure of banking institutes. The banking system offers services without which the country's economic system will not prosper well. Banks are a key element in guiding and managing the distribution of funds to production units and prevent the increase in economic turnover such as families, jobs, companies and government.

2. Review of literature

In a study, Hadian et al. [1] calculated the technical, assigned and economic efficiency of banks in the Iranian economy using data envelopment analysis method. In this study, the efficiency of 10 banks in the country over the period (1997-1999) has been studied. The results show that in the three years mentioned, by considering variable return to scale, the three Melli, agriculture and industry and mining banks, were efficient in terms of techniques, assignment and economy and Saderat Bank was only technically efficient. Azerbaijani et al. [2], after presenting the theoretical foundations of efficiency measurement, they discussed constant return to scale, Karafarin, Saman, housing and export development banks and assuming variable return, Saderat, Karafarin, Saman, Melli, Sepah, housing and Tose Saderat and in 2005, by assuming constant returns, Parsian, Karafarin and Saman banks, and assuming variable returns, Refah Kargaran, Saderat, Keshavarzi, Parsian, Karafarin and Saman have been efficient banks.

Hassanzadeh [3], in a study " Efficiency and effective factors on it in the Iranian banking system" calculated the efficiency index in the country's banking system and investigated the influence of functional and structural variables on the efficiency of the banking system of Iran on the data of 14 banks in 1996-2003 using non-parametric method of data envelopment analysis. The country's banking system used non-parametric data analysis methods. The results of the study showed that private banks have a higher degree of technical efficiency than state banks. In addition, the comparison of the technical efficiency index of Iranian banks compared to foreign banks in 2000-2002 is higher than the efficiency index of domestic banks, but it was decreased over the period 2002-2003. Hosseinzadeh Bahreini et al. [4] in another study "Comparison of economic performance of private and public Banks in Iran using data envelopment analysis "evaluated the economic performance of Private and public Banks by assuming variable returns to Scale. In this analysis, two intermediate attitudes with income approach and intermediate attitude with value-added approach have been used. The calculation of efficiency using the first method shows that the economic efficiency of state banks is higher than that of private banks. In the second approach, the result shows that the economic efficiency of private banks is higher than that of state banks. Ebrahimi et al. [5] examined the efficiency of Islamic banks in different countries using data envelopment analysis model. The results show that assuming constant returns to scale of Qatar Islamic International Bank, Sudan National Bank and Khyber Bank of Pakistan were efficient and assuming variable returns to scale, Qatar Islamic Bank, Qatar International Islamic Bank and Albanian Arabic Islamic Bank were more efficient than other banks. Alem Tabriz et al. [6], in a study "Evaluate the performance of fuzzy TOPSIS technique in improving the efficiency of banks' branches using data envelopment analysis method", examined the effects of considering the importance of inputs and outputs using fuzzy TOPSIS technique in the Data envelopment analysis. In this study, 51 branches were examined with inputs including number of bank counters, number of employed employees, administrative and personnel costs and other expenses and outputs including total revenues, resource equipping, receivables condition and total loans granted. A first, the efficiency of branches was obtained using the data envelopment analysis without considering the weight of inputs and outputs, the efficiency of 43 branches was achieved 1. However, when the efficiency evaluation process was used using the data envelopment analysis, the weight of inputs and outputs was reduced to 36 branches. Rajabi [7], in a study, evaluated the performance of Iranian commercial banks using the data envelopment analysis model over the period 1992-2009. The research results show that Melli Bank and Refah Bank are more efficient than other banks and show better performance. Namdari et al. [8], in a study examined the efficiency of ten state banks in Iran for the period 2003-2007 using data envelopment method with an approach to their human resources. The results show that Iran's state-owned banks are in the range of increased returns to scale. Karbasi Yazdi et al. [9] in a study "Determine the performance of banks based on data envelopment analysis model and Shannon entropy" using Shannon entropy model to select the most important inputs and outputs and DEA method to determine performance. In this study, 13 banks including private and public were examined. The results showed that out of 6 banks with an efficiency of 111%, 2 were private banks and 4 were state banks, which mean that private banks have a better performance than the state banks, despite their good history. In this study, it was shown that there is very little difference between super efficiency, weighting and unit prioritization by Shannon entropy method. Hamidi et al. [10] used the data envelopment analysis to measure the efficiency of bank branches and used the integration strategy to obtain inefficient branches. In this study, first the conceptual system of efficiency evaluation of bank branches was defined, then the inputs and outputs were determined using the bank's operational plan. In the next step, the efficiency of the branches of Melli Bank in Tehran province was calculated in a non-radial method using the data envelopment analysis model to identify inefficient branches. Then, based on the integration policies of the bank's branches, the integration clusters were determined, which merge into two each cluster as paired. Finally, through the non-radial model, the evaluation was performed using clustering method to measure their efficiency and compare it with the initial efficiency. Soheili et al. [11] examined and evaluated the efficiency of exchange and partnership contracts of 52 branches of Mellat Bank in Kermanshah province by data envelopment analysis. The results of contracts comparison indicate that 41% of the branches in estimation of exchange loan efficiency and 35% of branches in estimation of participative loan efficiency on efficiency frontier are efficient. Azar et al. [12], in a paper (evaluate the efficiency of bank branches using the network data envelopment analysis (One of the Banks of Gilan Province)) designed a model for measuring efficiency, effectiveness and productivity in the bank. In this regard, the number of employees and asset inputs, employee costs and general and administrative costs as intermediary variables and total income and total deposit were used as output. Finally, efficiency, effectiveness and productivity of the branches of one of the banks of Gilan province were measured during different processes and levels using the proposed model. One of the interesting results about this research is that the most efficient branch or the most effective branch is not necessarily the best branch from productivity aspects. Yaghoubi et al. [13], in a study, predicted the performance of decision-making units. Using the theory of credit and combining fuzzy data envelopment analysis and the main components analysis (PCA) examined the performance of decision units and finally, the results were compared with the Chen et al.'s Model. Khalili Damghani et al. [14], in a study "Presenting a combined approach based on multiple –criterion analysis of satisfaction and a three-stage data envelopment analysis to evaluate the efficiency of the services of Melli Bank branches of Iran" applied a three-stage network data envelopment analysis. The applied DEA model is of output-based CCR multiplier with constant returns to scale. The three-stage data envelopment analysis model measures the efficiency of customer expectations, customer satisfaction, and customer loyalty processes in branches. The results show that the average relative efficiency of the selected branches in the three sub-processes of "customer expectations approval", "customer satisfaction performance" and " operational results and customer and loyalty " are 94%, 82% and 91%, respectively. Also, the average efficiency in the overall process is 89% and only 4 branches (13% of the sample) were in the efficient frontier at all stages. According to the research findings, the branches that achieved 111% efficiency in the first process of "customer expectation approval" were placed on the efficient frontier in other sub-processes and total efficiency.

Rosenmayer et al. [15] used a data envelopment analysis to evaluate the efficiency and adequacy of US banks. By this approach, he identified weaker banks and offered suggestions for improving them. Suggestions for improvement are actually based on a scenario, and the effect of each is unclear. John Laurent et al. [16] examined the performance of 47 Spanish banks using the dimensions of education, research dimensions and transfer knowledge efficiency and compared the DEA results with the results of the financial approaches used by the banks. They found that banks that did not have good financial indicators were lower than other banks in terms of efficiency. Hu et al. [17] used DEA and AHP to compare performance in Taiwanese universities and universities around the world and used data envelopment analysis to measure the input and output variables. In fact, they first selected efficient universities using data envelopment analysis tools and then, using the information obtained, selected the appropriate ranking criteria for universities. Schubert et al. [18] used data envelopment analysis to measure the effectiveness of the School of Mechanical Engineering in publishing scientific papers. They first analyzed the performance of different faculties and found that the most important factor that indicates the inefficiency of a faculty is the fluctuation in the number of papers in different years in a faculty. Royz et al. [19] presented a DEA model with the shortest distance to provide the closest purposes for use when the expert preferences in analysis were created to assess and determine the objectives and assessments of Spanish banks. Bonn et al. [20] compared the traditional data envelopment network analysis with traditional data envelopment method and concluded that the amount of efficiency obtained at each level reflects the efficiency of each level, which is absent in traditional data envelopment analysis. They considered the qualitative and quantitative aspects of the network structure of data envelopment analysis and concluded that the network method is superior to past researches in terms of efficiency. Sagra et al. [21] used a combination of traditional ranking and data envelopment analysis to measure the efficiency of 55 American universities over the period 2002-2012. They first selected efficient universities using data envelopment analysis and then ranked efficient universities using traditional methods. Gulati et al. [22] compared the data envelopment network analysis with the traditional data envelopment analysis method and concluded that the amount of efficiency at each level reflects the efficiency of each level, which is absent in the traditional data envelopment analysis method. Ruggiero et al. [23] examined the education system in German schools. They believed that adequacy is defined in education as a minimum set of outcome standards. Schools that are unable to achieve these goals are not providing an adequate education. This failure could arise from insufficient spending on productive resources and/or inefficient use of existing resources. They introduce a non-parametric measure of the cost of adequacy that controls for the socio-economic environment and resource prices. To do this, they use network data envelopment analysis method. They divided the social environment into favorable and harsh. Furthermore, the additional costs of achieving a higher standard are much higher for schools with a harsh environment than for schools with a favorable environment.

Gao et al. [24] used network data analytics to evaluate the efficiency of Chinese banks and used Lagenberg's productivity to calculate productivity. Imran et al. [25] evaluated the performance of 15 graduate schools in India. They identified efficient and inefficient points of the system between 2011-2012 using data envelopment analysis method and offered suggestions for proper use of resources to increase productivity.

In all of these papers, only efficient decision-making units are identified. In fact, data envelopment analysis is only used to identify efficient units, and how inefficient units can be efficient. These are important issues that can be investigated in data envelopment analysis.

3. Methodology

Considering the research problem, the present research is applied in terms of purpose and is descriptive-analytical in terms of data collection method. Data envelopment analysis has been extensively used to evaluate the performance of a set of DMUs with multiple inputs and outputs at the organizational level, such as banks, hospitals, and universities (Charnes et al. [26]). The extensive DEA technique is the CCR model developed by Charnes et al. [27]. In the present study, a three-stage DEA approach of Fried et al. [28] was used. A series of adjustments were made to the impact of environmental impacts and statistical disruption on sample bank branches to measure the actual managerial effectiveness of the branches. In this study, as the goal is to measure the efficiency of bank branch management, a three-stage data envelopment analysis method is used, which reflects managerial effectiveness more than other methods. Three-stage data envelopment analysis method: First, we obtain the efficiency of branches by using selected inputs and outputs, and secondly, due to various environmental effects and statistical disturbances that may affect the effectiveness of production, we can adjust the first stage inputs using SFA model to adjust the uncontrollable factors, and in the third step of this method it is necessary to obtain the management efficiency using the adjusted inputs obtained from the second method and achieve the first stage outputs. Finally, due to the use of cross-sectional data to measure effectiveness, it is necessary to use the Malmquist productivity index. Step 1: Technical efficiency and quantitative variables in the total data, the decision making unit regarding the variables within the data and the main output were calculated. The traditional DEA model for the kth decision-making unit can be written as follows:

$$
\begin{aligned}
&\mathrm{Min}\, \theta_{\mathbf{k}} - \varepsilon \left(\sum_{\mathbf{r}=1}^{s} S_{\mathrm{rk}}^{+} + \mathrm{S}_{\mathrm{ik}}^{-} \right) \\
&\text{s.t:} \ \sum_{j=1}^{n} \lambda_j X_{ij} + S_{ik}^{-} = \theta_k x_{ik}, \quad i = 1 \ldots m \\
&\sum_{j=1}^{n} \lambda_j Y_{rj} - S_{rk}^{+} = Y_{rk}, r = 1 \ldots s \\
&\sum_{j=1}^{n} \lambda_j = 1, j = 1 \ldots n \\
&\lambda_{\mathrm{j}}, \mathrm{S}_{\mathrm{rk}}^{+}, \mathrm{s}_{\mathrm{ik}}^{-} \geq 0
\end{aligned} \tag{3.1}
$$

Where Y_{rk} is the rth output, kth decision making unit; X_{ik} is ith input of DMU, kth; $\mathrm{S}_{\mathrm{rk}}^{+}$ the slack variable of the rth output of kth DMU; S_{ik} is the slack variable of rth output of kth DMU, ε ; is a small positive number in which no factor should be ignored, θ, TE are relative. Their effective values are 1 and $\mathrm{S}_{\mathrm{rk}}^{+}= \mathrm{S}_{\mathrm{ik}}= 0$, which forms the effectiveness frontier. Second stage: Slack variables in the first step are integrated and modified according to the main data inputs. If the DEA model is set as an input orientation model, it is assumed that both companies face the same production frontier and have the same technical level in production, so the inefficiency of banks is due to differences in management techniques and resources allocation. However, other factors that affect the effectiveness of bank production require such uncontrollable elements as economic volatilities, natural disasters, and accidents. Therefore, various environmental impacts and statistical disturbances may affect production efficiency. At this stage, regression analysis can be performed with a probability frontier approach to adjust uncontrollable factors; that is, the slack variable in the values input data, S_{ni}, is an independent variable in the SFA regression model. The independent variables in the SFA regression model are the t observable environmental variables t, $\mathrm{Z}_{\mathrm{i}}= \mathrm{Z}_{1\mathrm{i}} \ldots \mathrm{Z}_{\mathrm{ti}}$. The separate N of SFA regression models in step 2 can be expressed as follows:

$$
\mathrm{S}_{\mathrm{ni}} = \mathrm{f}^{\mathrm{n}} \left(\mathrm{Z}_{\mathrm{i}}; \beta^{\mathrm{N}} \right) + \mathrm{v}_{\mathrm{ni}} + \mu_{\mathrm{ni}}, \quad \mathrm{n} = 1 \ldots \mathrm{N}; \quad \mathrm{i} = 1 \ldots \mathrm{I} \tag{3.2}
$$

There are N firms and I input data; $\mathrm{f}^{\mathrm{n}}(\mathrm{Z}_{\mathrm{i}};\beta^{\mathrm{n}})$represents a frontier level; $\beta^{\mathrm{n}}$is the parameter vector of the environmental factor to be estimated; $(\mathrm{v}_{\mathrm{ni}}+ \mu_{\mathrm{ni}})\, is\ mixed\ error,\ residual$and v_{ni} and $\mu_{\mathrm{ni}}$are independent from each other, $\mathrm{v}_{\mathrm{ni}} \sim \mathrm{N}\ (0,\ \sigma_{\mathrm{vn}}^2)$ are statistical disturbance; and $\mu_{\mathrm{ni}}\sim \mathrm{N}\ (\mu^{\mathrm{n}},\ \sigma_{\mathrm{un}}^2) > 0$refers to managerial inefficiency and ineffectiveness. According to SFA results, the input data variables of each branch have been adjusted with the same environmental conditions and statistical disturbance.

The equation adjustment can be written as follows:

$$\begin{aligned} \mathrm{X}_{\mathrm{ni}}^{\mathrm{A}} = & \mathrm{x}_{\mathrm{ni}} + \left[\max_{\mathrm{i}}\left\{\mathrm{Z}_{\mathrm{i}}\widehat{\beta^{\mathrm{n}}}\right\} - \mathrm{Z}_{\mathrm{i}}\widehat{\beta^{\mathrm{n}}}\right] + \left[\max_{\mathrm{i}}\{\widehat{\mathrm{v}_{\mathrm{n1}}}\} - \widehat{\mathrm{v}_{\mathrm{n1}}}\right] \\ & \mathrm{n} = 1\ldots,\mathrm{N} \quad \mathrm{i} = 1\ldots\mathrm{I} \end{aligned} \tag{3.3}$$

Where $\mathrm{X}_{\mathrm{ni}}^{\mathrm{A}}$ is the adjustment level of observed input data. The first bracket states that all DMUs adjust to the same external environment. The second bracket adjusts the random errors of all DMUs in the same context so that each DMU faces the same operating environment and has the same luck. The managerial inefficiency is the last factor.

Third stage: The effectiveness of the required branches was evaluated in accordance to traditional DEA with the $\mathrm{X}_{\mathrm{ni}}^{\mathrm{A}}$ adjusted input data in the second stage and the main return of the first stage. The result is a mere managerial factor that is appeared when the environmental factors and statistical disturbance are eliminated.

Therefore, the effectiveness achieved in the third stage of the modified DEA reflects to a greater extent the managerial effectiveness. In the above effectiveness analysis, cross-sectional data were used to measure the relative effectiveness of DMUs. Data can only analyze static effectiveness at a single point in time. Therefore, the present study uses the Malmquist Productivity Index (MPI) approach to compare multi-period effectiveness to analyze the functional effectiveness and productivity change of bank branches during one or two annual periods. Fare, Grosskopf, Norris, and Zhang [29] used the Total Factor Productivity growth index, initially defined by Caves, D.W., Christensen, L.R. and Diewert [30], to calculate the distance function via DEA approach and to obtain MPI. The TFP index can be decomposed into TE (TEC) and technological change (TC). The TEC based on the assumption constant returns to scale, TE. If the possible production set is extended to VRS (Variable return to scale, then the change in TE related to VRS, ie Net TEC (PTEC) and change in SE (SEC) can be obtained. The present study uses the MPI (based on the assumption of constant returns to scale) to evaluate TEC, TC, and TFP defined by Fare et al., as follows:

$$\mathrm{MPI}_0^{\mathrm{t+1}}\left(\mathrm{x}^{\mathrm{t+1}},\mathrm{y}^{\mathrm{t+1}},\mathrm{x}^{\mathrm{t}},\mathrm{y}^{\mathrm{t}}\right) = \left[\frac{\mathrm{D}_0^{\mathrm{t}}\left(\mathrm{x}^{\mathrm{t+1}},\mathrm{y}^{\mathrm{t+}}|\mathrm{CRS}\right)}{\mathrm{D}_0^{\mathrm{t}}\left(\mathrm{x}^{\mathrm{t}},\mathrm{y}^{\mathrm{t}}|\mathrm{CRS}\right)} \times \frac{\mathrm{D}_0^{\mathrm{t+1}}\left(\mathrm{x}^{\mathrm{t+1}},\mathrm{y}^{\mathrm{t+1}}|\mathrm{CRS}\right)}{\mathrm{D}_0^{\mathrm{t+1}}\left(\mathrm{x}^{\mathrm{t}},\mathrm{y}^{\mathrm{t}}|\mathrm{CRS}\right)}\right]^{1/2} \tag{3.4}$$

Where $\mathrm{MPI}_0^{\mathrm{t+1}} = 1$ indicates stability in productivity, $\mathrm{MPI}_0^{\mathrm{t+1}} < 1$ a decrease in productivity. MPI can be distinguished by multiplying TEC and TC by assuming VRS. TEC, also known as catch-up effect, refers to the degree of progress or reduction of TE from a DMU. Tc is also known as the frontier effect of change or innovative effect; change in the effectiveness frontier, reflects two time periods. Two indicators can be defined as follows: showing the productivity progress trend $\mathrm{MPI}_0^{\mathrm{t+1}} > 1$.

$$\mathrm{TC} = \left[\frac{\mathrm{D}_0^{\mathrm{t}}\left((\mathrm{x}^{\mathrm{t+1}},\mathrm{y}^{\mathrm{t+1}}|\mathrm{CRS}\right)}{\mathrm{D}_0^{\mathrm{t+1}}\left(\mathrm{x}^{\mathrm{t+1}},\mathrm{y}^{\mathrm{t+1}}|\mathrm{CRS}\right)} \times \frac{\mathrm{D}_0^{\mathrm{t}}\left(\mathrm{x}^{\mathrm{t}},\mathrm{y}^{\mathrm{t}}|\mathrm{CRS}\right)}{\mathrm{D}_0^{\mathrm{t+1}}\left(\mathrm{x}^{\mathrm{t}},\mathrm{y}^{\mathrm{t}}|\mathrm{CRS}\right)}\right]^{1/2} \tag{3.5}$$

where TC> 1, TC indicates progress,
TC = 1, TC no change,
TC <1 regress in TC,
Besides,

$$\mathrm{TEC} = \frac{\mathrm{D}_0^{\mathrm{t+1}}\left(\mathrm{x}^{\mathrm{t+1}},\mathrm{y}^{\mathrm{t+1}}|\mathrm{CRS}\right)}{\mathrm{D}_0^{\mathrm{t}}\left(\mathrm{x}^{\mathrm{t}},\mathrm{y}^{\mathrm{t}}|\mathrm{CRS}\right)} \tag{3.6}$$

TEC> 1 refers to progress in TE, TEC = 1 indicates no change in TE; and TEC <1 refers to a decrease in TE. At the same time, TEC can be decomposed into PTEC and SEC, which are defined as follows:

$$\mathrm{TEC} = \mathrm{PTEC} \times \mathrm{SEC} \tag{3.7}$$

That

$$\begin{aligned} \mathrm{PTEC} &= \frac{\mathrm{D}_0^{\mathrm{t}+1}\left(\mathrm{x}^{\mathrm{t}+1}, \mathrm{y}^{\mathrm{t}+1} \mid \mathrm{VRS}\right)}{\mathrm{D}_0^{\mathrm{t}}\left(\mathrm{x}^{\mathrm{t}}, \mathrm{y}^{\mathrm{t}} \mid \mathrm{VRS}\right)} \\ \mathrm{SEC} &= \frac{\mathrm{D}_0^{\mathrm{t}+1}\left(\mathrm{x}^{\mathrm{t}+1}, \mathrm{y}^{\mathrm{t}+1} \mid \mathrm{CRS}\right) / \mathrm{D}_0^{\mathrm{t}+1}\left(\mathrm{x}^{\mathrm{t}+1}, \mathrm{y}^{\mathrm{t}+1} \mid \mathrm{VRS}\right)}{\mathrm{D}_0^{\mathrm{t}}\left(\mathrm{x}^{\mathrm{t}}, \mathrm{y}^{\mathrm{t}} \mid \mathrm{CRS}\right) / \mathrm{D}_0^{\mathrm{t}}\left(\mathrm{x}^{\mathrm{t}}, \mathrm{y}^{\mathrm{t}} \mid \mathrm{VRS}\right)} \end{aligned} \tag{3.8}$$

In the above equation: TC> 1 indicates progress in TC, TC = 1 indicates stability in TC; and TC <1 indicates decrease in TC. When SEC> 1, that is, compared to period t, period t + 1 is closer to constant returns to scale, DMU is closer to the optimal production scale. When SEC <1, it means that the t + 1 period is farther from the constant returns to scale compared to the period t; that is, the DMU is far from the optimal production scale [31]. According to the research literature and the interviews conducted during the research with the banking experts, we extract the input and output variables from the table below.

The first step in evaluating efficiency is to identify the factors and components needed to evaluate organizations. It seems that by studying the structure, limits of duties and responsibilities of banks, the goals of the country's development programs and also by studying the models presented to evaluate similar organizations inside and outside the country, effective factors can be evaluated to assess research choices. Taking these steps can attract the researcher's attention to many dimensions and levels as proposed indicators. Therefore, it is necessary for the proposed indicators to be compatible with the conditions and subject of the project to obtain a set of appropriate indicators to measure the relative efficiency of the units.

To provide the basics of black-box production technology, consider a production technology with an N-dimensional input vector x ∈ N, M-dimensional good output vector y ∈ M and L-dimensional bad output vector b ∈ L. The bank production possibility set T is denoted by:

$$T = \left\{(x, y, b) \in \mathfrak{R}_{+}^{N} \times \mathfrak{R}_{+}^{M} \times \mathfrak{R}_{+}^{L} \mid (y, b) \text{ can be produced from } x\right\} \tag{3.9}$$

In addition, T in Equation (3.9) can be expressed as the output probability correspondence (P: N + → P (x) as shown:

$$P(x) = \left\{(y, b) \in \mathfrak{R}_{+}^{M} \times \mathfrak{N}_{+}^{L} \mid (x, y, b) \in T\right\}, \quad x \in \mathfrak{R}_{+}^{N} \tag{3.10}$$

Where P (x) , called output probability set , represents all good and bad output vectors that can be produced from a certain level of inputs x ∈ N. Therefore, (x, y) ∈ T ⇔ y ∈ P (x). We assume strong disposability of inputs (SD x) and strong monotonicity of good outputs (SDy):

$$\begin{aligned} \mathrm{SD}^{x}: &\quad x' \geq x \in \mathfrak{R}_{+}^{N} \Rightarrow P(x) \subseteq P\left(x'\right) \subseteq \mathfrak{R}_{+}^{M} \times \mathfrak{R}_{+}^{L} \\ \mathrm{SD}^{y}: &\quad y \geq y' \in \mathfrak{R}_{+}^{M} \text{ and } (y, b) \in P(x) \subseteq \mathfrak{R}_{+}^{M} \times \mathfrak{R}_{+}^{L} \text{ for } x \in \mathfrak{R}_{+}^{N} \\ &\quad \Rightarrow \left(y', b\right) \in P(x) \end{aligned} \tag{3.11}$$

The SDx property states that if the input vector increases from x to x′, then the ban output possibility set P (x) will be contained in the resulting bank output possibility set P (x)′. The SDy property states that if the good output vector increases from y to ′ y, then y ′ can be produced for a certain

Table 1: Input and output indicators of Bank performance

Source	The applied indicators
Sigbjorn and Al te(2002)	Inputs: Deposit holding time, Total deposit demand, capital, Personnel Outputs: investment, housing loans, industrial and commercial loans, other loans
Rolf Fare, Daniel Primont(2002)	Inputs: The amount and measurement of personnel working hours, costs, installation and launching units, number of terminals, branch area Outputs: number of savings accounts, number of current accounts, number of personal accounts
Ferrier G. Knox Lovell C. A(1990)	Inputs: Number of employees, Job and equipment costs, Raw material costs (Resources) Outputs: number of requested deposits (long-term accounts), number of short-term requested deposits, number of housing loans, number of installment loans, number of commercial loans
Oral M. ve, O.R, Yolalan(1990)	a. From profiting dimension Inputs: Personnel costs, administrative cost, depreciation, deposits cost, sum of administrative costs and depreciation Outputs: the profit obtained from loan, income other than loan profit, sum of two previous cases b. From service satisfaction aspect: Number of personnel, number of connected computer networks, number of saving accounts, total accounts Outputs: Total service time, credit time, time of receiving deposit, time of foreign exchange deposits
Sherman David H, George Ladino(1995)	Inputs: Customer services, sale services, full time personnel, costs (except personnel and rent), four sides space of branch Outputs: Deposit of claims and cheque of money, bank cheque, travel cheque, sold bonds of branches deposits, loans, new accounts, long-term loan, deposit accounts
Avkiran K(1995)	Inputs: the number of full-time employees, behavior of employees, average household income, number of small industries and presence of competitors Outputs: total new despite accounts, total new loan accounts, total new referring of investment centers, fee income

Saha Asish, T.S Ravisankar(2000)	Inputs: Interest cost, movable cost, immovable costs, fixed assets Outputs: The deposit account of credits (loan), investment, interest-free income, gross earnings, total income
Fukuyama H; Weber .L(2002)	Inputs: Number of employees, capital, deposits Outputs: Loans granted, other investments
Hasan A, Siren K(2003)	Inputs: The number of employees, fixed assets, applied benefits for granting loan Outputs: The short-term loans, long-term loans, other productive assets
Shyu, Chiang, (2012)	Inputs: The number of operating employees, number of business employees, fee of running branch and performance costs Outputs: Net wage income and increasing income of net interest

Table 2: The most common variables used in the evaluation of banks' efficiency

No.	**Input variables**	**Output variables**	**Model features**
1	Resources, costs	Consumption, income	All resources are considered as inputs
2	Total cost, number of employees	Net loan, total income, total deposits	Considering human resources as important labor
3	Total deposits, total costs	Total consumption, total income	Emphasis on total consumption
4	Resources, total cost	Net loan, total income	Emphasis on loans as the most important consumption element
5	Total deposits, costs	Net loan, total income	Emphasis on deposits as the most important constituent element of resources

level of inputs and outputs. Now, we distinguish between two types of good output in relation to bad inputs. One type includes good outputs that jointly produced with bad outputs, and the other is the good outputs whose production is not jointly a weak disposable with bad outputs. We segregate, with respect to b, the good output vector y into two types: (1) the good output vector y $\in$ M, that causes production of bad output b, and (2) the good output vector y $\in$M, where M = M + $\ddot{M}$. That is $y = (\dot{y}, \ddot{y}) \in \Re_+^{\dot{M}} \times \Re_+^{\ddot{M}}$. According to Shephard [32] and Färe, Grosskopf [33], we assume that y and b are jointly weakly disposable:

$$\begin{aligned}&\text{JWD: } (\dot{y}, \ddot{y}, b) \in P(x) \subseteq \Re_+^M \times \Re_+^{\dot{M}} \times \Re_+^L \text{ for } x \in \Re_+^N \text{ and } 0 \leq \theta \leq 1\\ &\Rightarrow (\theta\dot{y}, \ddot{y}, \theta b) \in P(x)\end{aligned} \tag{3.12}$$

The JWD property states that in proportion to the reduced interconnected good outputs and bad inputs, they are technically feasible given a constant level of outputs and non-joint good outputs Relative to (3.10), the bank directional output distance function $D_O(x, y, b; g) = D_O\left(x, \dot{y}, \ddot{y}, b; g^{\dot{y}}, g^{\ddot{y}}, g^b\right)$ is shown as follows:

$$\begin{aligned}&\vec{D}_0\left(x, \dot{y}, \ddot{y}, b; g^{\dot{y}}, g^{\bar{y}}, g^b\right)\\ &\quad = \max\left\{\beta | \left(\dot{y} + \beta g^y, \ddot{y} + \beta g^{\bar{y}}, b - \beta g^b\right) \in P(x)\right\}\end{aligned} \tag{3.13}$$

where $g^{\dot{y}} = (g_1^y, g_2^y, \ldots, g_M^y) \in \Re_+^{\dot{M}}, g^{\tilde{y}} = \left(g_1^{\tilde{y}}, g_2^{\tilde{y}}, \ldots, g_M^{\tilde{y}}\right) \in \Re_+^{\bar{M}}$ and $g^b = \left(g_1^b, g_2^b, \ldots, g_L^b\right) \in \Re_+^L$ are directional vectors for good and bad outputs, respectively. Note that $g = \left(g^j, g^j, g^b\right)$. Using (3.9) as the production technology, the bank's revenue function is denoted as follows: (3.11)

$$\begin{aligned}R(x, \dot{p}, \ddot{p}, v) &= \min_{y, \bar{y}, b}\{\dot{p}\dot{y} + \ddot{p}\ddot{y} - vb | (\dot{y}, \ddot{y}, b) \in P(x)\}, \quad \text{or}\\ R(x, p, v) &= \min_{y, b}\{py - vb| \quad (y, b) \in P(x)\}\end{aligned} \tag{3.14}$$

where $p = (\dot{p}_1, \dot{p}_2, \ldots, \dot{p}_{\dot{M}}; \ddot{p}_1, \ddot{p}_2, \ldots, \ddot{p}_M) \in \Re_{++}^{\dot{M}} \times \Re_{++}^{\ddot{M}}$ is a positive good output price vector and v ∈L is a positive bad input price vector. The domestic products , py and p$\ddot{y}$, represent revenues from joint and non-joint good outputs, respectively. The domestic product vb denotes the cost associated to bad outputs. p$\dot{y}$ + $\ddot{p}\ddot{y}$ .v b Equation (3.14) is interpreted as an effective net income or benefit. The bank revenue function is a bank revenue extension of Färe, Grosskopf et al. [33] environmental revenue function, which is defined as connected and unconnected outputs.

Similar to the dual relationship created by Färe, Grosskopf and Weber [33], the bank directional distance functions (3.13) and (3.14) can be recovered from each other as follows:

$$\begin{aligned}R(x, \dot{p}, \ddot{p}, v) &= \max_{yy, b}\{\dot{p}\dot{y} + \ddot{p}\ddot{y} - vb | D_o(x, \dot{y}, \ddot{y}, b : g) \geq 0\}\\ D_O(x, \dot{y}, \ddot{y}, b; g) &= \min_{p, p, v}\left\{\frac{R\left(x_0, \dot{p}, \ddot{p}, v\right) - (\dot{p}\dot{y} + \ddot{p}\ddot{y} - vb)}{\dot{p}g^y + \ddot{p}g^g + vg^y}\right\}\end{aligned} \tag{3.15}$$

$$D_0(x, \dot{y}, \ddot{y}, b; g) \leq \frac{R\left(x_o, \dot{p}, \ddot{p}, v\right) - (\dot{p}\ddot{y} + \ddot{p}\ddot{y} - vb)}{\dot{p}g^{\dot{y}} + \ddot{p}g^{\dot{y}} + vg^b} \tag{3.16}$$

which is the basis for Nerlovian efficiency decomposions, presented in the next section.

In this section, we have developed a non-parametric or DEA two-stage network bank technology framework, in which the term t"wo-stage network bank technology" indicates that all outputs of a bank's first stage enter into a second stage. Assume that J banks, j (j = 1, . . . , J banks with exogenous inputs, M = M + $\ddot{M}$ final good inputs, and Q intermediate products). Define the observed amounts of exogenous inputs, joint good outputs, non-joint good inputs, bad inputs and intermediate products of banks j by $0(n = 1, \ldots, N), \dot{y}_{mj} > 0(\dot{m} = 1, \ldots, \dot{M}), \bar{y}_{\ddot{m}j} > 0(\ddot{m} = 1, \ldots, \bar{M}), b_{hj} > 0(h = 1, \ldots, \bar{H})$ and $z_{qj} > 0(q = 1, \ldots, Q)$. Define the intensity vectors for the two stages as $\lambda^1 = (\lambda_1^1, \ldots, \lambda_1^1) \in$ and $\lambda^2 = \left(\lambda_1^2, \ldots, \lambda_j^2\right) \in \Re_+^J$ for the purpose of considering the two-stage structure in a non-parametric DEA framework (Fig. 1).

The stage one technology is represented by

$$T^1 = \{(x,z)|x_n \geq \sum_{j=1}^{J} x_{nj}\lambda_j^1(\forall n), \sum_{j=1}^{J} z_{qj}\lambda_j^1 \geq z_q(\forall q) \\ \left.\sum_{j=1}^{J}\lambda_j^1 = 1, \quad \lambda_j^1 \geq 0(\forall j)\right\} \tag{3.17}$$

and stage 2 technology as

$$T^2 = \left\{ (z,y,\bar{y},b)| \begin{array}{c} \sum_{j-1}^{J} z_{qj}\lambda_j^2 \leq z_q(\forall q), \dot{y}_m \leq \sum_{j-1}^{J} \dot{y}_{mj}\theta_j\lambda_j^2(\forall \dot{m}) \\ \bar{y}_m \leq \sum_j^J \bar{y}_{mj}\lambda_j^2(\forall \bar{m}) \\ b_l = \sum_{j=1}^{J} b_{lj}\theta_j\lambda_j^2(\forall l), \sum_{j=1}^{J}\lambda_j^2 = 1, \lambda_j^2 \geq 0(\forall j) \\ 1 \geq \theta_j \geq 0(\forall j) \end{array} \right\} \tag{3.18}$$

Each stage represents variable returns to scale. Note that with good outputs y is produced jointly by the bad inputs b, which are executed using an abatement factor θj $\in$ [0, 1. See Kuosmanen [34] for this abatement factor in a general setting. Note that Shephard [32] and Färe and Grosskopf [33] assumed θj = θ($\forall$ j). Recently, Epure and Lafuente (2015) differentiated between $\dot{y}$ and $\ddot{y}$ and a black box DEA production model. They developed for the bank.

The constraints associated with the intermediate product q are shown by $\sum_{j=1}^{J} z_{qj}\lambda_j^1 \geq z_q$ and $\sum_{j=1}^{J} z_{qj}\lambda_j^2 \leq z_q$ are in stages 1 and 2, respectively under the assumption of strong disposability. By combining these constraints, we can have J j=1 z qj λ1 j $\geq$z q $\geq$J j=1 z qj λ2 j $\Leftrightarrow$ J j=1 z qj (λ1 j $-\lambda$2 j) $\geq$0 under the assumption that all the intermediate products are produced and consumed in a bank. Note that some intermediate products in stage 1 can be wasted in a bank. The two-stage network production possibility set is presented as follows:

$$NT = \left\{ \begin{array}{c} x_n \geq \sum_{j-1}^{J} x_{nj}\lambda_j^1(\forall n) \\ y_m \leq \sum_{j-1}^{J} y_{mj}\theta_j\lambda_j^2(\forall in), \vec{y}_{in} \leq \sum_{j-1}^{J} \ddot{y}_{inj}\lambda_j^2(\forall m) \\ b_1 = \sum_{j-1}^{J} b_{1j}\theta_j\lambda_j^2(\forall l), \sum_{j-1}^{J} z_{qj}\left(\lambda_j^1 - \lambda_j^2\right) \geq 0 \\ \sum_{j=1}^{J}\lambda_j^1 = 1, \sum_{j-1}^{J}\lambda_j^2 = 1 \\ \lambda_j^1 \geq 0, \lambda_j^2 \geq 0(\forall j), 1 \geq \theta_j \geq 0(\forall j) \end{array} \right\} \tag{3.19}$$

Relative to (3.19), the directional bank output distance function of the directional for the following bank is shown as follows:

$$N\vec{D}\left(x_0, \dot{y}_0, \vec{y}_0, b_0; g\right) = \max \left\{ \begin{array}{l} |\ \ x_{no} \geq \sum_{j-1}^{J} x_{nj}\lambda_j^1(\forall n) \\ \ \ \sum_{j-1}^{J} z_{qj}\left(\lambda_j^1 - \lambda_j^2\right) \geq 0 \\ p_{mo} + \beta g_m^j \leq \sum_{j-1}^{J} y_{mj}\theta_j\lambda_j^2(\forall m) \\ \ddot{y}_{m0} + \beta g_m^y \leq \sum_{j-1}^{J} \ddot{y}_{mj}\lambda_j^2(\forall \ddot{m}), \\ b_{l0} - \beta g_l^0 = \sum_{j-1}^{J} b_{lj}\theta_j\lambda_j^2(\forall l) \\ \sum_{j-1}^{J}\lambda_j^1 = 1, \sum_{j-1}^{J}\lambda_j^2 = 1 \\ \lambda_j^1 \geq 0, \lambda_j^2 \geq 0(\forall j) \\ 1 \geq \theta_j \geq 0(\forall j), \beta: \text{ free} \end{array} \right\} \tag{3.20}$$

Where g$\dot{y}\dot{m}$ ($\forall$ $\dot{m}$) , g$\ddot{y}\ddot{m}$ ($\forall\ddot{m}$) and g l b ($\forall$ l indicate an inefficient measurement directions for linked, unlinked outputs, and bad inputs, respectively. As (3.20) is a nonlinear program, we transform it to a linear program by Kuosmanen's [34] method. That is, settings j = λ1 j ($\forall$ j) , γj = (1 $-\theta$j) λ2 j ($\forall$j) , j = θj λ2 j ($\forall$ j, Equation (3.20) can be expressed as follows:

$$N\vec{D}\left(x_0, \dot{y}_0, \vec{y}_0, b_0; g\right) = \max \left\{ \begin{array}{l} x_{no} \geq \sum_{j-1}^{J} x_{nj}\Lambda_j^1 (\forall n) \\ \sum_{j=1}^{J} z_{qj}\left(\Lambda_j^1 - \Lambda_j^2 - \gamma_j\right) \geq 0 \\ y_{m0} + \beta g_m^y \leq \sum_{j-1}^{J} \dot{y}_{mj}\Lambda_j^2 (\forall in) \\ \ddot{y}_{m0} + \beta g_m^y \leq \sum_{j-1}^{J} \ddot{y}_{mj}\left(\Lambda_j^2 + \gamma_j\right) (\forall \bar{m}) \\ b_{l0} - \beta g_l^0 = \sum_{j-1}^{J} b_{lj}\Lambda_j^2 (\forall l) \\ \sum_{j-1}^{J} \Lambda_j^1 = 1, \sum_{j-1}^{J}\left(\Lambda_j^2 + \gamma_j\right) = 1 \\ \Lambda_j^1 \geq 0, \Lambda_j^2 \geq 0 (\forall j) \\ \gamma_j \geq 0 (\forall j), \beta: \text{ free} \end{array} \right\} \tag{3.21}$$

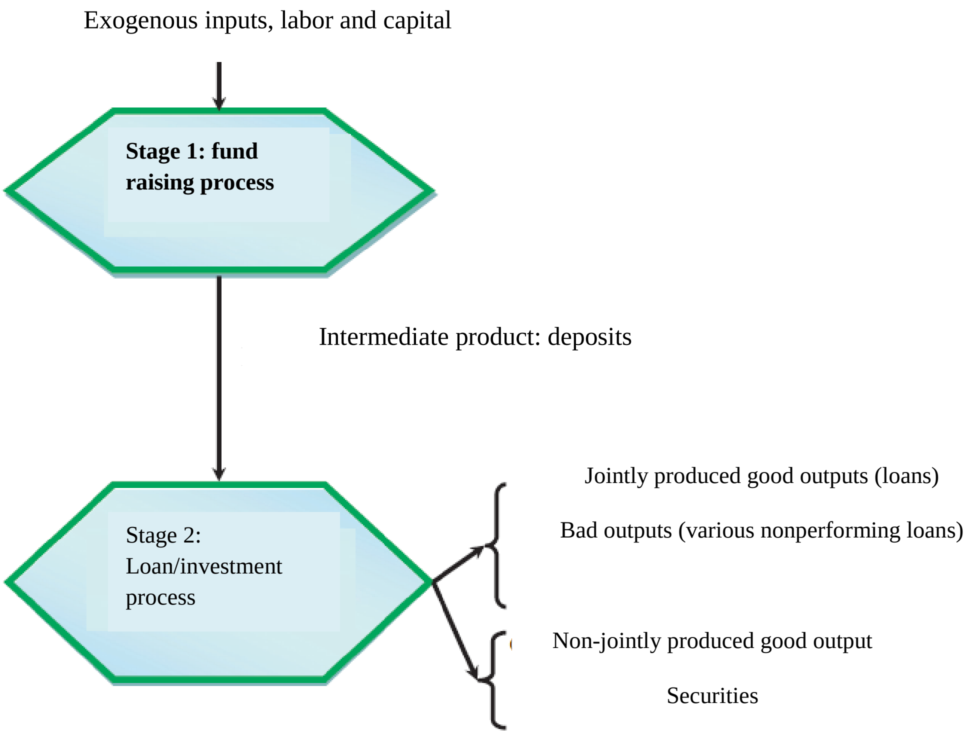

Figure 1: Two-stage network process for Mellat Bank branches.

Using Equation (3.19) as the network technology, the network revenue function for the bank under the assessment is considered as follows:

$$\mathrm{NR}\left(x_0, \dot{p}, \bar{p}, v\right) = \max \left\{ \begin{array}{c|c} x_{n0} \geq \sum_{j-1}^{J} x_{nj}\lambda_j^1 (\forall n) & \\ \sum_{j-1}^{J} z_{qj}\left(\lambda_j^1 - \lambda_j^2\right) \geq 0 & \\ \dot{y}_m \leq \sum_{j-1}^{J} \dot{y}_{mj}\theta_j\lambda_j^2 (\forall \dot{m}) & \\ + \sum_{m-1}^{M} \dot{p}_m \dot{y}_m & \bar{y}_m \leq \sum_{j-1}^{J} \ddot{y}_{mj}\lambda_j^2 (\forall \bar{m}) \\ - \sum_{l-1}^{L} v_l b_l & \lambda_j^j \geq 0, \lambda_j^2 \geq 0 (\forall j) \\ & 1 \geq \theta_j \geq 0 (\forall j) \\ \dot{y}_m \geq 0 & (\forall m), \bar{y}_m \geq 0 (\forall \bar{m}) \\ b_l \geq 0 & (\forall l) \end{array} \right\} \tag{3.22}$$

It can be thought of as a two-stage network DEA version of the parametric revenue function of Färe, Grosskopf and Weber. The Bank's network revenue function (NR (x o ,$\dot{p}$,$\ddot{p}$, v is also an example of the two-stage network cost function of Fukuyama [39], which does not include bad outputs. Similar to the transformation based on the directional bank output distance function, we set j = λ1 j (∀ j) , γj = (1 −θj) λ2 j (∀ j) and j = θj λ2 j (∀ j) to obtain a linear program equivalent to (3.22) as:

$$\mathrm{NR}\left(x_0, \dot{p}, \bar{p}, v\right)=\max \left\{\begin{array}{cc} \sum_{\sum_{m=1}^{M} \dot{p}_m \dot{y}_m} & \begin{array}{c} x_{no} \geq \sum_{j-1}^{J} x_{nj} \Lambda_j^1\left(\psi_n\right) \\ \sum_{j=1}^{J} z_{qj}\left(\Lambda_j^1-\Lambda_j^2-\gamma_j\right) \geq 0 \\ \dot{y}_m \leq \Sigma_{j-1}^{J} \dot{y}_{mj} \Lambda_j^2(\forall m) \\ \mid \dot{y}_m \leq \sum_{j-1}^{J} \ddot{y}_{mj}\left(\Lambda_j^2+\gamma_j\right)(\forall m) \\ -\Sigma_{1m-1}^{M} \ddot{p}_m \bar{y}_m, b_l=\Sigma_{j-1}^{J} b_{lj} \Lambda_j^2(\forall l) \\ -\Sigma_{l-1}^{L} v_l b_l, \sum_{j=1}^{J} \Lambda_j^1=1, \sum_{j-1}^{J}\left(\Lambda_j^2+\gamma_j\right)=1 \\ \Lambda_j^1 \geq 0, \Lambda_j^2 \geq 0, \gamma_j \geqslant 0(\forall j) \\ \dot{y}_m \geq 0(\forall m), \bar{y}_m \geq 0(\forall \bar{m}), b_l \geq 0(\forall l) \end{array} \end{array}\right\} \tag{3.23}$$

Developing the dual results of Chambers et al. [35] and Färe, Grosskopf et al. [33], we can obtain the following inequality:

$$\begin{array}{l} NR\left(x_o, \dot{p}, \ddot{p}, v\right)-\left(\sum_{m=1}^{M} \dot{p}_m \dot{y}_m+\sum_{m=1}^{\bar{M}} \ddot{p}_{\bar{m}} \ddot{y}_{\bar{m}}-\sum_{l=1}^{L} v_l b_l\right) \\ \quad \sum_{m=1}^{M} \dot{p}_m g_m^y+\sum_{m=1}^{M} \ddot{p}_{\dot{m}} g_m^{\bar{y}}+\sum_{l=1}^{L} v_l g_l^b \\ \geq ND\left(x_o, \dot{y}_o, \bar{y}_o, b_o ; g\right) \end{array} \tag{3.24}$$

The left side of Equation (3.24) is the Nerlovian term of a two-stage network version of revenue inefficiency (see Equation 8), and the right side of the equation is a two-stage network version of technical inefficiency. Determining the deviation between left and right side of Equation (3.24) as the allocative inefficiency AE, we obtain the following equation:

$$\begin{array}{l} NR\left(x_0, \dot{p}, \ddot{p}, v\right)-\left(\sum_{m=1}^{M} \dot{p}_m \dot{y}_{\dot{m}}+\sum_{\bar{m}=1}^{M} \ddot{p}_{\bar{m}} \ddot{y}_{\bar{m}}-\sum_{l=1}^{L} v_1 b_l\right) \\ \quad \sum_{m=1}^{M} \dot{p}_{\dot{m}} g_{\dot{m}}^{\dot{y}}+\sum_{\bar{m}=1}^{\bar{M}} \ddot{p}_{\bar{m}} g_{\dot{m}}^{\bar{y}}+\sum_{l=1}^{L} v_l g_l^b \\ = ND\left(x_o, \dot{y}_o, \ddot{y}_o, b_o ; g\right)+\text{ Alneff} \end{array} \tag{3.25}$$

Which states that the normal revenue inefficiency is broken down into the directional output inefficiency and the allocative inefficiency. To the right side of the equation (3.25) is the revenue inefficiency expression.

4. Data analysis

Our analysis period was much longer than previously published research studies. Experimental studies employ two approaches when measuring bank outputs and costs, and these approaches are especially discussed in the studies of Berger et al. [36], Sealey and Lindley [37], and a recent study by Berger et al. [36] and Kauk [38]. The production approach considers that banks create accounts of different sizes by processing deposits and loans, incurring capital and labor costs. The intermediation approach defines banks as transformers of deposits and funds purchased in the form of loans and other assets. The application of these two approaches usually depends on providing data and the purpose of the study. Here, we used an innovative intermediation approach (Fukuyama et al. [39]). In a bank domain, we need to decide whether deposits (or funds) are input or output, because there are differences of opinion in this regard in the bank efficiency literature (3.12). However, conventional efficiency analysis uses deposits as inputs or outputs.

Defined Inputs, outputs and prices:

x1 Labor = number of workers

x2 Physical capital = real estate
z1 Intermediate product = deposits
$\dot{y}$ Performing loans (jointly produced good output) linked to performing loans
$\ddot{y}$ Good output not linked to bad loans = securities investment
B Bad output = Nonperforming loans
$\dot{p}$ Price of y = interest income
$\ddot{p}$ Price of y = interest income
V Price of b = reserve for possible loan losses

Table 3: Definition of research variables

Score of employees	A combination of history, age, knowledge, training hour and received wage of employees
Paid profit	The profit paid for the deposits paid for the customers
Sum of four deposits	Four deposits of current, short-term, long-term and non-profit
Other resources	Some resources except four above deposits
Loans	Including different types of loans presented by the bank to customers
Received profit	The amount of profit received for the loans granted to the custoemrs
Received fee	For the service activities presented by the bank to the customers.
Delayed debts	The amount of debt the bank receives from the customers for non- timely payment of the installment.

Based on the selected approaches in determining the model input and output, the variables will be changed. But in general, the variables of this research include the total of effective deposits, granting loan, delayed debts and profit and loss. The central indices including mean, maximum, and minimum variables and deviation scattering indices are calculated for different variable as shown in the Table below.
In this section, we calculate the Pearson correlation coefficient, which is used to determine the amount of relationship, type and direction of the relationship between the two variables. The Pearson correlation coefficient ranges between 1 and -1. 1 indicates a complete direct relationship between the two variables, a direct or positive relationship means that if one of the variables increases (decreases), the other increases (decreases). If it is -1, it is the complete inverse relationship between the two variables. Based on the information we obtained, it can be concluded that in 2016, the relationship between profit and loss with the input variable of effective deposit is a direct relationship, which means that the more effective the deposit, the higher the bank's profit. Also, there is an inverse relationship between the input variable of loans and receivables with the output variable of profit and loss, which means that with increasing loans and receivables, due to the negative correlation coefficient of bank profit decreases and the Pearson Correlation coefficient in 2017 is the same.
In the first stage, the model is considered with one output of profit and loss and 3 inputs of total effective deposits, granted loans and delayed debts. Considering these variables, the total number of input and output variables is 4, which according to the statistical sample, which is 83 branches, the condition of the branches {Number of branches $\geq$3 (input +output)} which is necessary to perform data envelopment analysis model is observed. At this stage, the data of the total effective deposits,

Table 4: Some descriptive parameters related to the selected variables in 2016

Descriptive statistics				
2016	**Input**			**Output**
Figures in Million Rls				
	Effective deposit	**Granted loan**	**Delayed debts**	**Profit/loss**
Number of branches	83	83	83	83
Median	561369.0000	-90623.0000	-9471.0000	10124.0000
SD	385563.13211	231399.07418	569546.40410	118415.89470
Min	58694.00	-1628103.00	-4165581.00	-658303.00
Max	2620979.00	-2735.00	5.00	746243.00

Table 5: Some descriptive parameters related to the selected variables in 2017

Descriptive statistics				
2017	**Input**			**Output**
Figures in Million Rls				
	Effective deposit	**Granted loan**	**Delayed debts**	**Profit/loss**
Number of branches	83	83	83	83
Median	873564.2688	-146681.2366	-160089.9462	-52849.7163
SD	619714.98207	220340.95001	561405.39871	118826.91049
Min	75052.00	-1537067.00	-4292826.00	-946109.00
Max	4815605.00	122732.00	.00	310096.00

granted loans, delayed debts and profit and loss have been obtained from the General Directorate of studies-planning and risk management. To determine the efficiency of banks, it is necessary to determine the input and output variables. Therefore, first, important indicators are mentioned in the banks, some of which are considered as inputs and some as outputs. Among 83 branches of Mellat

Table 6: Pearson correlation coefficient

Pearson Correlations				
2016	**Effective (X_1)deposit**	**Granted (X_2)loans**	**Delayed debts (X_3)**	**/ Profit (Y_1)Loss**
X_1	1			
X_2	-0.71	1		
X_3	-0.09	0.162	1	
Y_1	0.87	-0.01	-0.51	1
2017	**Effective (X_1)deposit**	**Granted (X_2)loans**	**Delayed debts (X_3)**	**/ Profit (Y_1)Loss**
X_1	1			
X_2	-0.13	1		
X_3	-0.42	-0.09	1	
Y_1	-0.128	-0.180	0.631	1

Bank, about 29 branches in 2016 have efficiency 1and in other words, they have BCC efficiency. According to the results in the above table, Branch 47 has the lowest efficiency among all branches with an efficiency of 0.13.

Branch 20, with an efficiency of 0.09, has the lowest efficiency of all branches. As we know, the most important task of DEA is to build efficient virtual banks, which means that if we assume that one of the bank's branches, for example Branch 2, is inefficient, this model introduces other banks with more outputs with the same data or the same output with a fewer data. Each of these banks, referred to as reference banks, is effective with one degree intensity or special effect in building the virtual unit and this intensity degree in DEA is denoted by λ. In the tables extracted from Lingo software for each of the inefficient banks, the model banks are listed according to their degree of severity. The great advantage of the DEA method is that if a decision-making unit is ineffective with the DEA method, it is not efficient by any other method, and the weakness of this method is that if the decision-making unit is considered efficient because evaluation is done in the best possible conditions, in some cases, the efficiency of the unit is questionable. But the question that arises is that if in evaluation, some decision making units have full efficiency, which unit has better performance. In other words, among the efficient units, which one is better than the others and how can the efficient units be ranked? Ranking is a discussion of efficient units because the inefficient units can be ranked according to the methods mentioned. On the other hand, in the continuous interval, the probability of the efficiency of the two inefficient units is zero. Therefore, there is no theory for events where the probability of their occurrence is zero. There are several ways to rank efficient units, each with its own characteristics.

4.1. Ranking results of efficient units before adjusting inputs

In the previous section, we saw that the results were such that among 30 branches out of 83 branches of Mellat Bank in 2016 and 26 branches out of 83 branches in 2017, we could not distinguish and the efficiency of all of them was calculated as one. One of the proposed ways to differentiate between units is the Anderson-Peterson model. For this purpose, we first wrote and executed Anderson-Peterson models for efficient branches using Lingo software. Stochastic frontier function model is one of the types of parametric efficiency measurement methods. In the first step, we calculated the input-based BCC model of the efficiency of the given bank branches using the data envelopment analysis method. Some branches were efficient and some were inefficient. Because we are in a dynamic and constantly changing environment, there are many uncontrollable variables in the environment that affect the effectiveness of banks. In the present study, this model is used to eliminate uncontrollable elements that are beyond the control of bank branch management. Among the many uncontrollable elements , branch managers can not change and select these variables. According to the opinion of the supervisor and the advisor and with the survey of the bank's experts, four variables including inflation rate, economic growth rate, branch area and the years of operation of the branches were selected. Therefore, at this stage, we use the SFA model to adjust the bank's input variables, in other words, we should eliminate the effects of environmental variables and statistical disturbances so that we can calculate efficiency in the next stage without the intervention of environmental and uncontrollable factors. In the third step, like the first step, we solve the model with one output (profit and loss) and 3 inputs (total effective deposits, granted loans and delayed debts), with the difference that in this step, we adjusted the first stage inputs using the SFA model and consider the same output as the first step and solve the model. At this step, using the determined inputs and outputs, by assuming the variable returns to scale, we performed the data envelopment analysis using Lingo software. According to the results, Branch 63 with efficiency of 0.24 has the lowest efficiency among all branches. At this stage, the number of efficient branches has increased compared to the first stage, and the elimination of uncontrollable factors can be considered as the main reason for this increase in efficiency. According to the results, Branch 21 has the lowest efficiency among all branches with an efficiency of 0.33. At this stage, the number of efficient branches has increased compared to the first stage, and the elimination of uncontrollable factors can be considered as the main reason for this increase in efficiency. Approximately 78% of the branches became efficient. The present study uses the Malmquist Productivity Index (MPI) approach to compare multi-period effectiveness to analyze the functional effectiveness and productivity change of bank branches during the two annual periods (2016 and 2017). THe total productivity of factors using the Malmquist productivity index is defined as the changes in the technical efficiency multiplied by technological changes. Technical Efficiency Change (TEC) is equal to the multiplication of Pure Technical Efficiency Change (PTEC) in Scale Efficiency Change (SEC), which in this study emphasizes and focuses more on technical efficiency. We obtain the three TE, PTE and SE variables using the given software.

$$\text{TEC} = \text{PTEC} \times \text{SEC}$$

After adjusting the inputs, the efficiency of the branches also changed, so that in 2017, about 80% of the branches reached the efficient frontier, so it is necessary to calculate and compare the MPI after adjusting the inputs.

Evaluating the performance of organizations plays a key role in guiding their future strategic decisions. In this regard, the efficiency and productivity of organizations should be calculated in order to plan the process of their economic growth in future decision-making. Efficiency is one of the most important economic units that can provide suitable conditions for growth and development in

Table 7: Average variables obtained from the Malmquist index

	TE	PTE	SE
2016	0.24	0.46	0.55
2017	0.26	0.59	0.45

Table 8: Average variables obtained from the Malmquist index

	TE	PTE	SE
2016	0.24	0.46	0.55
2017	0.26	0.59	0.45

various sectors of a country's economy via extensive banking operations. Therefore, just as sound and efficient banks can help the country's economic growth, their improper performance can also create crises. The banking system also plays a very important role in Iran's economy. Determining the efficiency and performance of branches without considering the location and potential of the region and only relying on intra-organizational data can lead to incorrect results. This paper has contributed to a recent study on the efficiency of the bank by introducing an advanced two-stage model to estimate the efficiency of the bank's revenue function. The proposed model is exclusively a combination of past maturity loans with a bank revenue function. This paper improved the methodological concept of the two-stage model that was previously introduced in the bank's efficiency sources (Fukuyama et al. [39]). The Applied Methodology approach in this paper allows us to compare optimal income levels, past maturity loans, and bank output with real levels. Therefore, we can identify banks that work at their optimal capacity. Most studies in the field of efficiency of Kermanshah banks are mainly focused on allocative efficiency. This paper deals with the concept of revenue index. The inclusion of past maturity loans in our model was justified by some recent studies, such as Asef et al. [40], Fuji et al. [41] and so on.

5. Conclusion

Due to long-term, all-encompassing sanctions and the instability of political engagements with the other countries, Iran's economy more than ever should adjust its governance system to minimize economic Problems. As the most important financial institution, banks play an important role in the national economy. Considering the present and future economic conditions, the banks management always tries to improve banking services, marketing, innovation in service delivery, competition with other banks, considering the current and future economic conditions.

Performance appraisal has always been a vital issue for organizations. Every organization needs to continuously evaluate its performance to determine, modify, and make changes to its long-term and short-term goals. Achieving macroeconomic stability and paving the way for sustainable economic growth requires increasing the efficiency of monetary policy instruments and increasing the independence of the central bank in using those tools. Another important point in this regard is that the monetary policy and resources of the central bank should not compensate for the inefficiency of financial policies and the imbalances formed in the economy. Commitments such as direct support to banks and firms, compensation for government budget deficits, purchase of foreign exchange from the government, compensation for depositors, and the like, need to be eliminated from the responsibility of monetary policymaker, and the central bank can only commit to its inherent responsibilities and core missions.

Evaluating the performance of organizations plays a key role in guiding their future strategic decisions. In this regard, the efficiency and productivity of organizations should be calculated in order to plan their future economic growth in future decisions. Efficiency is one of the most important economic units that can provide suitable conditions for growth and development in various sectors of a country's economy through extensive banking operations. Therefore, just as sound and efficient banks can contribute to the economic growth of the country, their improper performance can also create crises. The banking system also plays a very important role in Iran's economy. Determining the efficiency and performance of branches without considering the location and potential of the region and only relying on intra-organizational data can result into incorrect outcomes. In this research, by introducing environmental factors such as local population, household cost and number of competing branches within one kilometer of the branch, it was attempted to determine the efficiency of the branches according to their spatial characteristics. The common approach to measuring the efficiency of bank branches focused on indicators that measured the statistic performance of the units, but based on the efforts of Laplanthe and Paradi [42], dynamic approaches were introduced to analyze the growth potential of branches, which were used in this paper. We measure the performance of the bank using the economic efficiency of the banks and by stochastic data envelopment analysis. After collecting data from 31 branches of Mellat Bank in Kermanshah province, after implementing the model, we will evaluate the results. These results will calculate the efficiency of the first and second levels. The following results are as follows:

This paper has contributed to a recent study on the efficiency of the bank by introducing an advanced two-state model to estimate the efficiency of the bank's revenue. The proposed model is exclusively a combination of past maturity loans with a bank revenue function. This paper helped to improve the methodological concept of the two-stage model that was previously introduced in the bank's efficiency resources (Fukuyama et al. [39]). The Applied Methodology approach in this paper allows us to compare optimal revenue levels, past maturity loans, and bank output with actual levels. Therefore, we can identify banks that operate below their optimal capacity. Most studies on the efficiency of Kermanshah banks focus mainly on allocative efficiency. This paper deals with the concept of revenue index. The inclusion of past maturity loans in our model was justified by some recent studies such as those conducted by Asef et al. [40], Fuji et al. [41], and so on.

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