Int. J. Nonlinear Anal. Appl. 3 (2012) No. 2, 59-66
ISSN: 2008-6822 (electronic)
http://www.ijnaa.semnan.ac.ir

Some Results on Maximal Open Sets

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(Communicated by A. Ebadian)

Abstract

In this paper, the notion of maximal m-open set is introduced and its properties are investigated. Some results about existence of maximal m-open sets are given. Moreover, the relations between maximal m-open sets in an m-space and maximal open sets in the corresponding generated topology are considered. Our results are supported by examples and counterexamples.

Keywords: Small Topology, Minimal Structure, Maximal Open Set, Cofinite Subset, Generated Topology.
2010 MSC: 54A05, 54B05.

1. Introduction

Generalized topological spaces and generalized open sets play a very important role in almost all branches of pure and applied mathematics, specially in General Topology, Real Analysis, Convex Analysis and etc. One of the most well-known notions and also an inspiration source are the notions of m-structure and m-space introduced by H. Maki, J. Umehara and T. Noiri [17].

The concepts of minimal open sets and maximal open sets in topological spaces are introduced and considered by F. Nakaoka and N. Oda in [18], [19] and [20]. More precisely, in 2001, Nakaoka and Oda [19] characterized minimal open sets and proved that any subset of a minimal open set is pre-open. Also, as an application of a theory of minimal open sets, they obtained a sufficient condition for a locally finite space to be a pre- Hausdorff space. The authors in [20] obtained fundamental properties

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Received: February 2012 *Revised:* June 2012

of maximal open sets such as decomposition theorem for a maximal open set and established basic properties of intersections of maximal open sets, such as the law of radical closure. By a dual concepts of minimal open sets and maximal open sets, the authors in [18] introduced the concepts of minimal closed sets and maximal closed sets and obtained results easily by dualizing the known results regarding minimal open sets and maximal open sets.

Several authors have used these new notions in many directions. For instance, maximal and minimal θ-open sets and their properties are considered by M. Caldas, S. Jafari and S. P. Moshokoa [11] and also θ-generalized open sets are investigated by M. Caldas, S. Jafari and T. Noiri [12]. The concept of minimal γ-open sets are introduced and considered by S. Hussain and B. Ahmad [15]. Moreover, S. Bhattacharya [8, 9] introduced the new concepts of generalized minimal closed sets and IF generalized minimal closed sets. Finally, S. Al Ghour [13, 14] have applied the notion of minimality and maximality of open sets to the fuzzy case.

The purpose of this paper is to introduce and investigate the concept of a new generalized class of maximal open sets in m- spaces, called maximal m-open set. Indeed, the concept of maximal m-open set is introduced and its properties are established. Some results about existence of maximal m-open sets are given. Further, the relations between maximal m-open sets in an m-space and maximal open sets in the corresponding generated topology are considered. We have supported our results by examples and counterexamples.

2. *m*-Structure and *m*-Space

The concept of m-structure and m-spaces, as a generalization of topology and topological spaces were introduced in [17]. For easy understanding of the material incorporated in this paper we recall some basic definitions and results. For details and more results on the following notions we refer to [1–7], [10, 16, 17, 21] and references cited therein.

Let $\mathcal{P}(X)$ denote the set of all nonempty subsets of X. A family $\mathcal{M} \subseteq \mathcal{P}(X)$ is said to be an *m-structure* on X if $\emptyset, X \in \mathcal{M}$. In this case $(X, \mathcal{M})$ is called an *m-space*. For examples in this setting see [16]. In an m-space $(X, \mathcal{M})$, $A \in \mathcal{P}(X)$ is said to be an *m-open set* if $A \in \mathcal{M}$ and also $B \in \mathcal{P}(X)$ is an *m-closed set* if $B^c \in \mathcal{M}$. We set $m\text{-}Int(A) = \bigcup\{U : U \subseteq A, U \in \mathcal{M}\}$ and $m\text{-}Cl(A) = \bigcap\{F : A \subseteq F, F^c \in \mathcal{M}\}$. For any $x \in X$, $N(x)$ is said to be an *m-neighborhood* of x, if for any $z \in N(x)$ there is an m-open subset $G_z \subseteq N(x)$ such that $z \in G_z$.

Definition 2.1. *We say that, the m-space $(X, \mathcal{M})$ enjoys the*

(a) *property $\mathfrak{I}$, if any finite intersection of m-open sets is m-open;*

(b) *property $\mathfrak{F}$, if any finite union of m-open sets is m-open;*

(c) *property $\mathfrak{U}$, if any arbitrary union of m-open sets is m-open.*

Proposition 2.2. *[21] For an m-structure $\mathcal{M}$ on a set X, the following are equivalent.*

(a) *$\mathcal{M}$ has the property $\mathfrak{U}$.*

(b) *If $m\text{-}Int(A) = A$, then $A \in \mathcal{M}$.*

(c) *If $m\text{-}Cl(B) = B$, then $B^c \in \mathcal{M}$.*

Proposition 2.3. [16] *For any two sets A and B,*

(a) $m\text{-}Int(A) \subseteq A$ *and* $m\text{-}Int(A) = A$ *if* A *is an* m*-open set.*

(b) $A \subseteq m\text{-}Cl(A)$ *and* $A = m\text{-}Cl(A)$ *if* A *is an* m*-closed set.*

(c) $m\text{-}Int(A) \subseteq m\text{-}Int(B)$ *and* $m\text{-}Cl(A) \subseteq m\text{-}Cl(B)$ *if* $A \subseteq B$.

(d) $m\text{-}Int(A \cap B) \subseteq (m\text{-}Int(A)) \cap (m\text{-}Int(B))$ *and* $(m\text{-}Int(A)) \cup (m\text{-}Int(B)) \subseteq m\text{-}Int(A \cup B)$.

(e) $m\text{-}Cl(A \cup B) \supseteq (m\text{-}Cl(A)) \cup (m\text{-}Cl(B))$ *and* $m\text{-}Cl(A \cap B) \subseteq (m\text{-}Cl(A)) \cap (m\text{-}Cl(B))$.

(f) $m\text{-}Int(m\text{-}Int(A)) = m\text{-}Int(A)$ *and* $m\text{-}Cl(m\text{-}Cl(B)) = m\text{-}Cl(B)$.

(g) $(m\text{-}Cl(A))^c = m\text{-}Int(A^c)$ *and* $(m\text{-}Int(A))^c = m\text{-}Cl(A^c)$.

3. Maximal *m*-Open Sets

Definition 3.1. *Let $(X, \mathcal{M})$ be an m-space. A nonempty proper m-open subset A of X is said to be maximal m-open if any m-open set which contains A is X or A. We denote the set of all maximal m-open sets of an m-space $(X, \mathcal{M})$ by* $\max(X, \mathcal{M})$.

First, we represent an existence theorem of maximal m-open sets in a special case. Recall that a *cofinite* subset is a subset which it's complement is finite.

Theorem 3.2. *Let $(X, \mathcal{M})$ be an m-space and B a nonempty proper cofinite m-open set. Then there exists at least one (cofinite) maximal m-open set A such that $B \subseteq A$.*

Proof . If B is a maximal m-open set, put $A = B$. Otherwise, there exists an (cofinite) m-open set B_1 in which $B \subsetneq B_1 \neq X$. If B_1 is a maximal m-open set, we may put $A = B_1$. If B_1 is not maximal m-open, then there exists an (cofinite) m-open set B_2 such that $B \subsetneq B_1 \subsetneq B_2 \neq X$. By continuing this process, we have a sequence of m-open sets

$$B \subsetneq B_1 \subsetneq B_2 \subsetneq \cdots \subsetneq B_k \subsetneq \cdots \subsetneq X.$$

Since B is a cofinite set, this process will stop somewhere. Then, finally we will find a maximal m-open set $A = B_n$ for some $n \in \mathbb{N}$. □

Example 3.3. *Let $X = \mathbb{N}$, $B = \{1, 3\}$ and $A = \{1, 3, 5\}$. Set $\mathcal{M} = \{\emptyset, A, B, \mathbb{N}\} \cup \{C_n : n \in \mathbb{N}\}$, where $C_n = \{2, 4, 6, ..., 2n\}$. Clearly, B is not cofinite, while it has a maximal m-open extension A. This shows that a set which is not cofinite may has a maximal m-open extension. Moreover, C_n's are not cofinite and also they do not have any maximal m-open extension. This means that* Theorem 3.2 *may not hold, when the set is not cofinite.*

For a nonempty proper cofinite m-open set in an m-space, the maximal m-open extension is not always unique. As in the following example, it is possible that an m-open set has many maximal m-open extensions.

Example 3.4. *Let $X = \mathbb{N}$, $C_1 = \{1,3,5\}$, $C_2 = \{1,3\}$, $C_3 = \{2,4,6\}$, $C_4 = \{2\}$, $C_5 = \{4\}$ and $C_6 = \{6\}$. Set $\mathcal{M} = \{\emptyset, B_1, A_2, B_3, A_4, A_5, A_6, \mathbb{N}\}$, where $B_i = \mathbb{N} \setminus C_i$ for $i = 1,3$ and $A_j = \mathbb{N} \setminus C_j$ for $j = 2,4,5,6$. Evidently, B_1 and B_3 are cofinite and B_1 has a unique maximal m-open extension A_2 whereas B_3 has three maximal m-open extensions A_4, A_5 and A_6.*

Theorem 3.5. *Suppose that $(X, \mathcal{M})$ is an m-space with the property of $\mathfrak{F}$. Let S be a nonempty proper m-open set such that each element of it's complement is contained in a finite m-closed set. Then there exists at least one (cofinite) maximal m-open set A with $S \subseteq A$.*

Proof . Since S is a proper subset of X, so there exists an element x of S^c. By the assumption there exists a finite m-closed set F such that $x \in F$. One can easily check that $S \cup F^c$ is a nonempty proper cofinite m-open set. Therefore, by Theorem 3.2 we can find a maximal m-open set A satisfying $S \cup F^c \subseteq A$. Evidently A is a (cofinite) maximal m-open extension of S. □

Example 3.6. *Let $X = \mathbb{N}$ and $U_{2n+1} = \mathbb{N} \setminus \{2n+1, 2n+3, \ldots\}$ for each $n \in \mathbb{N}$. Consider the m-structure $\mathcal{M} = \{\emptyset, \mathbb{N} \setminus \{1\}, \mathbb{N} \setminus \{2\}, \mathbb{N} \setminus \{1,2\}, U_3^c, \mathbb{N}\} \cup \{U_{2n+1} : n \in \mathbb{N}\}$ on X. Clearly $(X, \mathcal{M})$ is an m-space with the property of $\mathfrak{F}$. Set $S_1 = \mathbb{N} \setminus \{1,2\}$. It is easy to see that S_1 satisfies in all conditions of* Theorem 3.5 *and so it has two extensions $\mathbb{N} \setminus \{1\}$ and $\mathbb{N} \setminus \{2\}$. Now, imagine $S_2 = U_3$, for $5 \in U_3^c$ there is no finite m-closed set containing 5. Note that S_2 does not have any maximal m-open extension, because of the following chain.*

$$U_3 \subsetneq U_5 \subsetneq \cdots \subsetneq U_{2n+1} \subsetneq \cdots \subsetneq \mathbb{N}.$$

Finally, set $S_3 = U_3^c$. For $4 \in S_3^c$, there is no finite m-closed set containing 4 while S_3 has two maximal extensions $\mathbb{N} \setminus \{1\}$ and $\mathbb{N} \setminus \{2\}$.

Corollary 3.7. *Suppose that $(X, \mathcal{M})$ is an m-space with the property of $\mathfrak{F}$. Let each element of X is contained in a finite m-closed set. Then for any nonempty proper m-open set S there exists at least one (cofinite) maximal m-open set A with $S \subseteq A$.*

Proof . It is an immediate consequence of Theorem 3.5. □

Remark 3.8. *Let X, $\mathcal{M}$ and U_{2n+1} be the same as in* Example 3.6. *One can find that* Theorem 3.5 *is stronger than* Corollary 3.7.

Proposition 3.9. *Let $(X, \mathcal{M})$ be an m-space with the property of $\mathfrak{F}$ and let $A, B \in \max(X, \mathcal{M})$ and W be an m-open set. Then*

(a) *$A \cup W = X$ or $W \subseteq A$,*

(b) *$A \cup B = X$ or $A = B$.*

Proof . Suppose W is an m-open set such that $A \cup W \neq X$. Since A is a maximal m-open set, $A \subseteq A \cup W$ and the space enjoys the property of $\mathfrak{F}$, we have $A \cup W = A$. Therefore $W \subseteq A$; (a) is proved. For (b), let $A \cup B \neq X$. Since $A, B \in \max(X, \mathcal{M})$, by (a) we have $A \subseteq B$ and $B \subseteq A$. Therefore $A = B$. This is the end of proof. □

If the space has not the property of $\mathfrak{F}$, last proposition is not true in general as you can see in following example.

Example 3.10. *Let* $X = \{x, y, z, t\}$, $A = \{x, y\}$, $B = \{y, z\}$ *and* $W = \{z\}$. *Put* $\mathcal{M} = \{\emptyset, A, B, W, X\}$, *then clearly* $A, B \in \max(X, \mathcal{M})$ *and* W *is an* m*-open set whereas*

(a) $A \cup W \neq X$ *and* $W \nsubseteq A$.

(b) $A \cup B \neq X$ *and* $A \neq B$.

Also, it is possible that Proposition 3.9 holds whereas the space does not enjoy the property of $\mathfrak{F}$. The following example shows that.

Example 3.11. *Let* $X = \{x, y, z, t\}$, $A = \{x, y, z\}$, $B = \{y, z, t\}$, $W_1 = \{x\}$ *and* $W_2 = \{t\}$. *Put* $\mathcal{M} = \{\emptyset, A, B, W_1, W_2, X\}$, *then clearly* $A, B \in \max(X, \mathcal{M})$. *It is not hard to check that the results of* Proposition 3.9 *hold here whereas the space does not enjoy the property of* $\mathfrak{F}$.

Corollary 3.12. *Let* $(X, \mathcal{M})$ *be an* m*-space with the property of* $\mathfrak{F}$ *and* $A \in \max(X, \mathcal{M})$. *If* $x \in A$, *then* $A \cup W = X$ *or* $W \subseteq A$ *for any* m*-open neighborhood* W *of* x.

Proof . The result is obvious by part (a) of Proposition 3.9. □

Proposition 3.13. *Let* $(X, \mathcal{M})$ *be an* m*-space with the property of* $\mathfrak{F}$, $A_1, A_2, A_3 \in \max(X, \mathcal{M})$ *such that* $A_1 \neq A_2$. *If* $A_1 \cap A_2 \subseteq A_3$, *then* $A_1 = A_3$ *or* $A_2 = A_3$.

Proof . It is clear that

$$\begin{aligned}
A_1 \cap A_3 &= A_1 \cap (A_3 \cap X) \\
&= A_1 \cap (A_3 \cap (A_1 \cup A_2)) \qquad (by part (b) of Proposition 3.9) \\
&= A_1 \cap ((A_3 \cap A_1) \cup (A_3 \cap A_2)) \\
&= (A_1 \cap A_3) \cup (A_1 \cap A_3 \cap A_2) \\
&= (A_1 \cap A_3) \cup (A_1 \cap A_2) \qquad (by assumption) \\
&= A_1 \cap (A_2 \cup A_3).
\end{aligned}$$

That is $A_1 \cap A_3 = A_1 \cap (A_2 \cup A_3)$. If $A_2 \neq A_3$, then by part (b) of Proposition 3.9 we have $A_2 \cup A_3 = X$ and hence we can say $A_1 \cap A_3 = A_1$ which means that $A_1 \subseteq A_3$. Since A_1 and A_3 are maximal m-open sets, we get $A_1 = A_3$. □

Example 3.14. *Let* $X = \{x, y, z, t\}$, $A_1 = \{x, y\}$, $A_2 = \{y, z\}$ *and* $A_3 = \{z, t\}$. *Put* $\mathcal{M} = \{\emptyset, A_1, A_2, A_3, X\}$, *so* $A_1, A_2, A_3 \in \max(X, \mathcal{M})$, $A_1 \neq A_2$, $A_1 \cap A_2 \subseteq A_3$ *while neither* $A_1 = A_3$ *nor* $A_2 = A_3$. *This shows that* Proposition 3.13 *may not hold when the space does not enjoy the property of* $\mathfrak{F}$.

Example 3.15. *Let* $X = \{x, y, z, t\}$ *and* $\mathcal{M} = \{\emptyset, \{x\}, \{y\}, \{x, y\}, \{y, z\}, X\}$. *Let* $A_1 = \{x, y\}$ *and* $A_2 = A_3 = \{y, z\}$. *Clearly,* $A_1 \neq A_2$, $A_1 \cap A_2 \subseteq A_3$ *and* $A_2 = A_3$ *whereas the space does not enjoy the property of* $\mathfrak{F}$. *This represents it is possible that* Proposition 3.13 *holds even the space does not enjoy the property of* $\mathfrak{F}$.

Theorem 3.16. *Let $(X, \mathcal{M})$ be an m-space with the property of $\mathfrak{F}$, $A_1, A_2, A_3 \in \max(X, \mathcal{M})$ which are different from each others. Then $A_i \cap A_j \nsubseteq A_i \cap A_k$, where $\{i, j, k\} = \{1, 2, 3\}$.*

Proof . Suppose this is not the case. Thus $A_i \cap A_j \subseteq A_i \cap A_k$, so

$$(A_i \cap A_j) \cup (A_j \cap A_k) \subseteq (A_i \cap A_k) \cup (A_j \cap A_k).$$

Hence

$$A_j \cap (A_i \cup A_k) \subseteq (A_i \cup A_j) \cap A_k.$$

Since $A_i \cup A_j = A_j \cup A_k = X$, we get $A_j \subseteq A_k$. Now, it follows from Definition 3.1 that $A_j = A_k$, which contradicts with our assumption. □

Example 3.17. *Let $X = \{x, y, z, t\}$, $A_1 = \{x, t\}$, $A_2 = \{y, t\}$, $A_3 = \{z, t\}$. Put $\mathcal{M} = \{\emptyset, A_1, A_2, A_3, X\}$, then $A_1 \cap A_2 \subseteq A_1 \cap A_3$. This shows that* Theorem 3.16 *may not hold when the space does not enjoy the property of $\mathfrak{F}$.*

Example 3.18. *Let $X = \{x, y, z, t\}$, $A_1 = \{x, y, t\}$, $A_2 = \{y, z, t\}$, $A_3 = \{x, z\}$. Put $\mathcal{M} = \{\emptyset, \{y\}, A_1, A_2, A_3, X\}$. It is easy to see that $A_i \cap A_j \nsubseteq A_i \cap A_k$, where $\{i, j, k\} = \{1, 2, 3\}$. This shows that it is possible that* Theorem 3.16 *holds even the space does not enjoy the property of $\mathfrak{F}$.*

Proposition 3.19. *Let $(X, \mathcal{M})$ be an m-space with the property of $\mathfrak{F}$, $A \in \max(X, \mathcal{M})$ and $x \in A$. Then*

$$A = \cup\{W : W isanm{-}openneighborhoodofxinwhich \quad A \cup W \neq X\}.$$

Proof . Since the maximal m-open set A is an m-open neighborhood of x, Corollary 3.12 implies that

$$A \subseteq \cup\{W : W isan\mathrm{m}{-}openneighborhoodof\mathrm{xin}which \quad A \cup W \neq X\} \subseteq A.$$

So, we have done. □

Example 3.20. *Let $X = \{x, y, z, t\}$, $A_1 = \{x, y\}$, $A_2 = \{t\}$ and $W = \{x, z\}$. Put $\mathcal{M} = \{\emptyset, A_1, A_2, W, X\}$. Clearly $A_1, A_2 \in \max(X, \mathcal{M})$. One can easily verify that A_2 satisfies in the conclusion of* Proposition 3.19 *while A_1 dose not satisfy it. Note that the space dose not have the property of $\mathfrak{F}$.*

4. Maximal Open Sets in Generated Topology

We know that any topology on a set X is an m-structure on it and also the converse is not true in general. But, always there exists a smallest topology containing the m-structure. In the rest of this paper τ is denoted for the topology generated by $\mathcal{M}$. In the next results we consider relationships between $\max(X, \tau)$ and $\max(X, \mathcal{M})$.

Proposition 4.1. *Let* $(X, \mathcal{M})$ *be an* m*-space and* τ *be the topology generated by* $\mathcal{M}$*. Then* $\max(X,\tau) \cap \mathcal{M} \subseteq \max(X, \mathcal{M})$.

Proof . If $\max(X,\tau) \cap \mathcal{M} = \emptyset$, there is nothing to prove. Assume that $A \in \max(X,\tau) \cap \mathcal{M}$. On the contrary suppose $A \notin \max(X, \mathcal{M})$ then there exists $U \in \mathcal{M}$ such that $A \subsetneq U \subsetneq X$ which is a contradiction since $\mathcal{M} \subseteq \tau$. □

Example 4.2. *Let* $X = \{x, y, z, t\}$, $U = \{x\}$, $V = \{y\}$, $A = \{x, y\}$, $B = \{x, z, t\}$, $\mathcal{M} = \{\emptyset, U, V, B, X\}$*. It is easy to see that* $\tau = \{\emptyset, U, V, A, B, X\}$, $\max(X, \mathcal{M}) = \{V, B\}$ *and* $\max(X, \tau) = \{A, B\}$.

Proposition 4.3. *Let* $(X, \mathcal{M})$ *be an* m*-space with property* $\mathfrak{F}$*. Then* $\max(X, \mathcal{M}) \subseteq \max(X, \tau)$.

Proof . If $\max(X, \mathcal{M}) = \emptyset$, there is nothing to prove. Let $A \in \max(X, \mathcal{M})$ be such that $A \subsetneq U$ for some $U \in \tau$. Then for $x \in U \setminus A$ by the structure of the generated topology τ there exist $U_1, U_2, \ldots, U_m \in \mathcal{M}$ with $x \in U_1 \cap U_2 \cap \cdots \cap U_m \subseteq U$. It follows from Proposition 3.9 that $U_i \cup A = X$ for each $i = 1, 2, \ldots, m$. Therefore, $A \cup (U_1 \cap U_2 \cap \cdots \cap U_m) = X$ which it means that $U = X$ and so $A \in \max(X, \tau)$. □

Proposition 4.4. *Let* $(X, \mathcal{M})$ *be an* m*-space with property* $\mathfrak{U}$*. Then* $\max(X, \tau) \subseteq \mathcal{M}$.

Proof . If $\max(X, \tau) = \emptyset$, there is nothing to prove. Let $A \in \max(X, \tau)$. According to the structure of the generated topology τ one can consider

$$A = (U_1 \cap U_2 \cap \cdots \cap U_n) \cup (V_1 \cap V_2 \cap \cdots \cap V_m) \cup \cdots$$

where $U_1, U_2, \ldots, U_m, V_1, V_2, \ldots, V_n, \ldots \in \mathcal{M}$. We claim that $U_i \cup (V_1 \cap V_2 \cap \ldots \cap V_m) \cup \cdots \subsetneq X$ for some $i \in \{1, 2, \ldots, n\}$. Because, the equality $U_i \cup (V_1 \cap V_2 \cap \ldots \cap V_m) \cup \cdots = X$ for each $i = 1, 2, \ldots, n$, implies that $A = X$ which is impossible. Now suppose for example $A \subseteq U_1 \cup (V_1 \cap V_2 \cap \cdots \cap V_m) \cup \cdots \subsetneq X$. Then maximality of A and the property of $\mathfrak{U}$ imply that $A = U_1 \cup (V_1 \cap V_2 \cap \cdots \cap V_m) \cup \cdots$. Similarly, we get $A = (U_1 \cap U_2 \cap \cdots \cap U_m) \cup V_1 \cup \cdots$. Hence $A = U_1 \cup V_1 \cup \cdots$, so $A \in \mathcal{M}$. □

Theorem 4.5. *Let* $(X, \mathcal{M})$ *be an* m*-space with property* $\mathfrak{U}$*. Then* $\max(X, \mathcal{M}) = \max(X, \tau)$.

Proof . It follows from propositions 4.1, 4.3 and 4.4. □

Example 4.6. *Let* $X = \mathbb{N}$, $A_n = \{2, 4, \ldots, 2n\}$ *for each* $n \in \mathbb{N}$ *and* A *be the set of all even numbers. Clearly* $\mathcal{M} = \{\emptyset, \mathbb{N}\} \cup \{A_n : n \in \mathbb{N}\}$ *is an* m*-structure on* X *with the property* $\mathfrak{F}$ *whereas it does not have the property of* $\mathfrak{U}$*. Evidently,* $A \in \max(X, \tau)$ *while* $A \notin \mathcal{M}$ *as a result* $A \notin \max(X, \mathcal{M})$*. Indeed,* $\mathcal{M}$ *does not have any maximal* m*-open set.*

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