Int. J. Nonlinear Anal. Appl.
Volume 11, Special Issue, Winter and Spring 2020, 395-411
ISSN: 2008-6822 (electronic)
http://dx.doi.org/10.22075/IJNAA.2020.4643

Game theory model for optimum pricing in a two level supply chain

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Abstract

In this research, we are looking to present a pricing model in the two-level supply chain using game theory, assuming there is advertising and in terms of uncertainty. In this research, it was tried to provide different models for different conditions in the supply chain, taking into account various supply chain competition strategies and conditions. The issue was designed with the goal of optimizing the pricing of two alternative products provided by two different manufacturers. The survey environment was considered highly competitive. The discussion of advertising in the chain and the uncertainty in the problem parameters were also considered in order to bring the model closer to real-world conditions. Then the model was solved for different strategies and optimal strategies for each side of the chain were identified. Finally, numerical issues were presented, solved and analyzed in different dimensions.

Keywords: pricing , two-level supply chain , game theory , advertising , uncertainty

1. Introduction

One of the basic arts in the management of manufacturing and servicing companies is pricing. Pricing which affects the sale rate and desirability of service, had been performed by rule of thumb or based on adding benefit to break-even point price. Nowadays, because of tight competition in business environments, a lot of contributing factors need to be considered for pricing and traditional approaches are not practical (Chaab and Rasti, 2016).

Numerous studies have been taking place about supply chain pricing and organizing in recent years. Supply chain organizing became researchers' interest recently. Berger (1972) is the first person who has studied advertising issues by using mathematical models. A common approach to compile the supply chain organizing is theory of games (Chaab and Rasti, 2016), which are developed in

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Received: August 2019 *Revised:* August 2020

static and dynamic categories. Dante and Berger (1996), Bergen and John (1997), Kim and Stiliin (1999), Huang and Lee (2001), Huang et. al. (2002), Lee et.al. (2002), and Carry and Zakur (2006) investigated static models, in which interactions between supply chain members is discussed. Georgenson and Zakur (2003), (Chaab and Rasti, 2016), Serinewasan et. al. (2017) and Ujla et. al. (2018) studied dynamic models. Although a lot of reports have been noted in the literature, more investigations are needed to minimize the difference between the models and real conditions. In a two level supply chain model, several producers and retailers exist, and products are transferred by vehicles from producers to retailers. One of the newest, fastest and cheapest ways to transfer products is unmanned vehicles (Torab Beigi et. al. 2019, Torab Beigi et. al. 2018, and Kin et. al. 2018). Although in such supply chain models different competitive conditions are considered for each chain members, they cannot simulate real business conditions, therefore more investigations to diminish defects of available models, is demanded. Mixed and new developing aspects have to be envisioned in studies in the field of supply chain (more specifically two level supply chains) in which competitive and advertising conditions are calculated.

In this research, three main subjects have to be noted. First is the matter of uncertainty in parameters of the problem, which is common in nowadays businesses. Second is competition and cooperation in the supply chain and in this research it is taken into account by several strategies. And last but not least is marketing and advertising in the supply chain. Advertising is not apart from competitive atmosphere, businesses and organizations use this tool to attract the attention of customers and increase their sales. In this study, a two level supply chain pricing model by using game theory and in advertisements and uncertainty conditions is evaluated. In this regard various models by considering strategies and different competitive conditions are presented.

2. Problem definition

The efficacy and quality of similar products become almost identical, thus such products are considered differentiated but substitutable. It means that two products which provide the same service, but are prepared by the two different producers (with different brands and prices), are recognized similarly. The pricing would be determined by sale costs (import, advertising, transfer,...), wholesale and retailers prices, while keeping the maximum benefit in mind. It has to be noted that there is uncertainty and in-transparency in production and distribution costs. The minimum and maximum costs of production and distribution could be estimated by name and history of a brand as well as advertising cost assumptions. Besides, power structure which plays an important role in product prices, is determined by different mixes and each mix makes a game. In this research three possible games would be modeled. Figure 1 shows a schematic of under investigation problem.

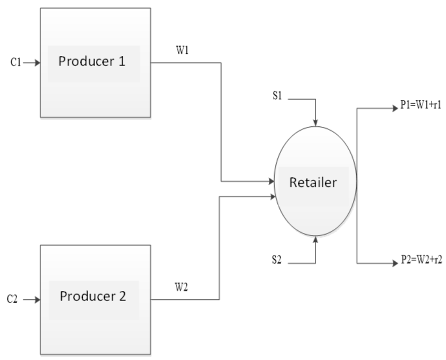

Figure 1: Schematic of the problem

Below are the conditions for modeling the problem:

1- The aim of problem is pricing two substitutable products from two different producers. To explain, among all products in the market the one which is most similar to targeted product, is known as a substitutable product.

2- Highly competitive condition exists in the market, in other words selling power is different for producers (brands, after sales services,…).

3- Producers use the same distribution channels.

4- Production, sale and demand costs are uncertain and are modeled by fuzzy variables.

5- Demand volume relation with the retail price for each product and market basic price (possible demand of the market when the prices are zero) is linear.

6- Advertising cost is an effective factor in product selling to each representative (retailers) and is an imprecise fuzzy variable. Other costs of retailers are neglected.

7- All imprecise variables are considered independent and positive.

8- Full information assumption, which means all the chain members (producers and retailers) have comprehensive information about the demand volume and other expenses.

9- Risk neutral assumption, which says the risk for producers and retailer is neutral, and they try to maximize their benefit.

10- Positive assumption, which means the costs of producers and retailers are less than wholesale and retail prices.

11- All product representatives are considered equal, and they entitled as retailers.

3. Modeling

3.1. Model parameters

In the following, parameters and symbols are defined:

$i = 1, 2$: Index of product or producer of the product.
$\tilde{c}_i$: The cost of production of one unit product i, which is a fuzzy variable.
w_i: Wholesale price of one unit product i (by producer i), which is producer i decision variable.
$\tilde{s}_i$: The cost of retailer advertising for one unit of product i, which is a fuzzy variable.
r_i: The cost that the retailer adds to the wholesale price of one unit product i to sell it, which is dependent on the retailers decision.
$p_i = w_i + r_i$: The retail price of one unit product i (end user price).
$\tilde{d}_i$: Market basic price of the product i (the possible demand volume of product i when the price of all products in the market is zero), which is a fuzzy variable.
$\pi_{m_i=(w_i-\tilde{c}_i)}$: Producers benefit function for product i.
$\pi_r = \Sigma_{i=1}^{2}(r_i - \tilde{s}_i)q_i$: Retailer's benefit function (product representatives).
δ: Sensitivity of demand volume in one unit of product for marginal customer parameter.
γ: Sensitivity of demand volume to qualifications and price of product for switching customer parameter.
$\beta = \delta + \gamma$:Substitute parameter in demand modeling.
$E[\ ]$: Expected value for the variable or target function.

3.2. Modeling of each product demand volume in the market

To model each product demand volume, the common demand linear function was applied; the function was first presented by Anderson and Bao in 2010. They divided customers in the market into two categories of switching and marginal customers. Switching customers definitely buy a product, but they decide after analyzing the product price and qualification. On the other hand, marginal customers buy a product if it costs less than a specific price. In order to model these two types of customer behavior, they also defined sensitivity parameters for customer behavior to product volume demand. These parameters determine that how changing one unit of each product price affect volume demand of each type of customer. To simplify the condition of the problem, the value of each parameter considered identical. The demand model is presented in equations 1 and 2.

$$q_i = d_i - \delta_{p_i} + \sigma^2_{j=1,j\neq i}(p_j - p_i), i = 1,2 \quad \text{(Eq.1)}$$

$$q_i = d_i - (\delta + \gamma)p_i + \gamma p_{3-i}, i = 1,2 \quad \text{(Eq.2)}$$

By applying non-cooperative game theory (there is no cooperation between the members), three power structures for chain members would be defined, and sale prices would be determined with the aim of maximizing benefit function (Huang and Ke, 2017).

3.3. Producer dominant structure

3.3.1. Determining benefit function

In this structure, chain members' competition is based on the Stackelberg leadership model, in which producers as leaders, determine the prices on the basis of maximum benefit, while retailers declare their prices after producers pricing and based on maximum benefit. As mentioned earlier, these games are non- cooperative, which means there is no cooperation for pricing and in this particular structure, each producer individually determines wholesale price conditional on the optimum sale response (retailer) to producers pricing as well as maximizing expected benefit. Then, the retailer represents most beneficiary retail price for two products after determining producer's pricing.

Producers announce their wholesale price by taking their maximum benefit into account and the assumption of retailer's price. This could be formulated in first two equations of relation 3. On the other hand, the last equation of relation 3 is resulted from retailers pricing, explained earlier. To simplify, both producers and retailers compete about pricing in a way that each of them maximize their benefits. Final prices are the equilibrium point of the benefit functions of producers and retailers, and none of members would pass over this point, since increasing or decreasing the prices would decline their expected benefit (Nash,?).

It has to be noted that costs (production and sale) should not be more than wholesale and retail prices and demand volume should not be negative.

$$\begin{aligned}
&\max_{w_1} E[\pi_{m_1}] = E[(w_1 - \tilde{c}_1)(\tilde{d}_1 - \beta(r_1^* + w_1) + \gamma(r_2^* + w_2))]\\
&\max_{w_2} E[\pi_{m_2}] = E[(w_2 - \tilde{c}_2)(\tilde{d}_2 - \beta(r_2^* + w_2) + \gamma(r_1^* + w_1))]\\
&\max_{r_1,r_2} E[\pi_r] = E[\Sigma_{i=1}^2 (r_i - \tilde{s}_i)(\tilde{d}_i - \beta(r_i + w_i) + \gamma(r_{3-i} + w_{3-i}))] \quad \text{(Relation 3)}
\end{aligned}$$

As indicated in the first two equations of relation 3, to determine optimum wholesale prices of producers 1 and 2, first retail prices (r_1^* andoptimizing the ,calculated. Thus have to)r_2^* last equation of relation 3 has to be done for r_1 and r_2.

3.3.2. Optimum prices for retailers

After optimizing last equation of relation 3, the optimum amount for r_1^* anddetermined. is r_2^* are maximum points, both necessary (r_1, r_2)) function and if r_1 and r_2 is a two variable $(E[\pi_r])$ and sufficient conditions have to be established:
Necessary condition: the first derivative of the function has to be zero at maximum point, which is shown in equation 4.

$$\frac{\partial E[\pi_r(r_1, r_2)]}{\partial r_1} = 0 \tag{Eq.4}$$

Sufficient condition: Hessian matrix of the function should be negative semi- definite, which means the elements on the main diagonal are negative and matrix determinant is positive.

$$\begin{vmatrix} \frac{\partial^2 E[\pi_r(r_1, r_2)]}{\partial r_1^2} & \frac{\partial^2 E[\pi_r(r_1, r_2)]}{\partial r_1 \partial r_2} \\ \frac{\partial^2 E[\pi_r(r_1, r_2)]}{\partial r_2 \partial r_1} & \frac{\partial^2 E[\pi_r(r_1, r_2)]}{\partial r_2^2} \end{vmatrix} = \begin{vmatrix} -2\beta & 2\gamma \\ 2\gamma & -2\beta \end{vmatrix} \tag{Relation 5}$$

Relation 5 indicates Hessian matrix of the function. If H_1 is a negative semi- definite matrix, relation 6 has to be established between β and γ:

$$\beta > \gamma > 0 \tag{Relation 6}$$

To find maximum point of $E[\pi_r(r_1, r_2)]$ function, its first derivative has to be calculated at r_1 and r_2 , and then equalized to zero. After solving the equations, optimum values of r_1^* and r_2^* at which $E[\pi_r(r_1, r_2)]$ is maximized, would be resulted. Equation 7 and 8 verify this explanation. By solving system of two equations with two unknowns (7 and 8), optimum values of r_1^* and r_2^* would be calculated according to 9 and 10 equations.

$$\frac{\partial E[\pi_r(r_1, r_2)]}{\partial r_1} = -2\beta r_1 + 2\gamma r_2 - \beta w_1 + \gamma w_2 + E[\tilde{d}_1] + \beta E[\tilde{s}_1] - \gamma E[\tilde{s}_2] = 0 \tag{Eq.7}$$

$$\frac{\partial E[\pi_r(r_1, r_2)]}{\partial r_2} = -2\beta r_2 + 2\gamma r_1 - \beta w_2 + \gamma w_1 + E[\tilde{d}_2] + \beta E[\tilde{s}_2] - \gamma E[\tilde{s}_1] = 0 \tag{Eq.8}$$

$$r_2^*(w_1, w_2) = \frac{\beta E[\tilde{d}_2] - \gamma E[\tilde{d}_1] + (\beta^2 - \gamma^2) E[\tilde{s}_2]}{2(\beta^2 - \gamma^2)} - \frac{w_2}{2} \tag{Eq.9}$$

$$r_1^*(w_1, w_2) = \frac{\beta E[\tilde{d}_1] - \gamma E[\tilde{d}_2] + (\beta^2 - \gamma^2) E[\tilde{s}_1]}{2(\beta^2 - \gamma^2)} - \frac{w_1}{2} \tag{Eq.10}$$

3.3.3. Optimum wholesale prices

First, similar to equations 4-6, necessary and sufficient conditions have to be distinguished for producers benefit function, using retailers optimum prices for two products and supposing the assumption of rationality. The optimum wholesale prices of two producers would be determined by equations 7-10.

Necessary condition:

$$\frac{\partial E[\pi_{m_1}]}{\partial w_1} = 0 \tag{Eq.11}$$

$$\frac{\partial E[\pi_{m_2}]}{\partial w_2} = 0 \tag{Eq.12}$$

Sufficient condition:

$$\begin{vmatrix} \dfrac{\partial^2 E[\pi_{m_1}(w_1,w_2)]}{\partial w_1^2} & \dfrac{\partial^2 E[\pi_{m_1}(w_1,w_2)]}{\partial w_1 \partial w_2} \\ \dfrac{\partial^2 E[\pi_{m_2}(w_1,w_2)]}{\partial w_2^2} & \dfrac{\partial^2 E[\pi_{m_2}(w_1,w_2)]}{\partial w_2 \partial w_1} \end{vmatrix} \begin{vmatrix} -\beta & \dfrac{\gamma}{2} \\ \dfrac{\gamma}{2} & -\beta \end{vmatrix} \quad \text{(Eq.13)}$$

It has to be investigated that Hessian matrixes of $E[\pi_{m_1}]$ and $E[\pi_{m_2}]$ functions are negative semi-definite, therefore relation 14 has to be established.

$$\beta > \gamma > \frac{\gamma}{2} > 0 \quad \text{(Eq.14)}$$

With the assumption of condition 14, optimum point has to satisfy equations 15 and 16.

$$\frac{\partial E[\pi_{m_1}(w_1,w_2)]}{\partial w_1} = -\beta w_1 + E[\tilde{d}_1] - \beta S_1 + \gamma S_2 + \frac{1}{2}\gamma w_2 + \frac{1}{2}\beta E[\tilde{c}_1] = 0 \quad \text{(Eq.15)}$$

$$\frac{\partial E[\pi_{m_2}(w_1,w_2)]}{\partial w_2} = -\beta w_2 + E[\tilde{d}_2] - \beta S_2 + \gamma S_1 + \frac{1}{2}\gamma w_1 + \frac{1}{2}\beta E[\tilde{c}_2] = 0 \quad \text{(Eq.16)}$$

$$S_1 = \frac{\beta E[\tilde{d}_1] + \gamma E[\tilde{d}_2] + (\beta^2-\gamma^2)E[\tilde{S}_1]}{2(\beta^2-\gamma^2)}$$

$$S_2 = \frac{\beta E[\tilde{d}_2] + \gamma E[\tilde{d}_1] + (\beta^2-\gamma^2)E[\tilde{S}_2]}{2(\beta^2-\gamma^2)} \quad \text{(Eq.17)}$$

The optimum wholesale prices for producer 1 and 2 (relations 18 and 19) are calculated by solving system of two equations with two unknowns (Eq. 15 and 16).

$$w_1^* = \frac{4\beta E[\tilde{d}_1] + 2\gamma E[\tilde{d}_2] + 2(\gamma^2 - 2\beta^2)S_1 + 2\beta\gamma S_2 + 2\beta^2 E[\tilde{c}_2]}{4\beta^2 - \gamma^2} \quad (18)$$

$$w_2^* = \frac{4\beta E[\tilde{d}_2] + 2\gamma E[\tilde{d}_1] + 2(\gamma^2 - 2\beta^2)S_1 + 2\beta\gamma S_1 + 2\beta^2 E[\tilde{c}_1]}{4\beta^2 - \gamma^2} \quad (19)$$

3.4. The retailer dominant structure

3.4.1. Determining benefit function

This structure is similar to previous one except that retailers like producers determine the prices according to their maximum benefit, which means retailers power is more than producers, since they declare the prices first (relation 20). Firstly optimum wholesale prices have to be distinguished, following that optimum retailers price. Then the equilibrium point could be found.

$$\begin{cases} \max\limits_{r_1,r_2} E[\pi_r] = E[\Sigma_{i=1}^2 (r_i - S_i)(d_i - \beta(r_i + w_i^*) + \gamma(r_{3-i}^* + w_{3-i}^*))] \\ \qquad\qquad Where(w_1^*, w_2^*) solves\ the\ problems: \\ \begin{cases} \max\limits_{w_1} E[\pi_{m_1}] = E[(w_1 - c_1)(d_1 - \beta(r_1 + w_1) + \gamma(r_2 + w_2))] \\ \\ \max\limits_{w_2} E[\pi_{m_2}] = E[(w_2 - c_2)(d_2 - \beta(r_2 + w_2) + \gamma(r_1 + w_1))] \end{cases} \end{cases} \quad (20)$$

3.4.2. Determining optimum wholesale prices

These prices are resulted after solving equations in relation 20. Optimum points would be (w_1^*, w_2^*), if necessary and sufficient conditions are established.

$$\begin{cases} \dfrac{\partial E[\pi_{m_1}(w_1, w_2)]}{\partial w_1} = 0 \\ \dfrac{\partial E[\pi_{m_2}(w_1, w_2)]}{\partial w_2} = 0 \end{cases} \tag{21}$$

Necessary condition:
This point has to be the answer of equation 21:
Sufficient condition: to maximize both $E[\pi_{m_1}(w_1, w_2)]$ and $E[\pi_{m_2}(w_1, w_2)]$ at the optimum point, the second derivative of these functions should be negative. Decisive variables of producer 1 and 2 are and , respectively, and second derivative of benefit functions regarding the decisive variable should be negative (relation 22). Therefore the only sufficient condition is $\beta > 0$ on optimum point, which is established in defining the problem.

$$\begin{cases} \dfrac{\partial^2 E[\pi_{m_1}(w_1, w_2)]}{\partial w_1^2} = -2\beta \\ \dfrac{\partial^2 E[\pi_{m_2}(w_1, w_2)]}{\partial w_2^2} = -2\beta \end{cases} \tag{22}$$

By solving equation system 21, optimum values of $w_1^*(r_1, r_2)$ and $w_2^*(r_1, r_2)$ are calculated according to relation 23.

$$w_1^*(r_1, r_2) = \frac{(-2\beta^2 + \gamma^2)r_1 + \beta\gamma r_2 + 2\beta(E[d_1] + \beta E[c_1]) + \gamma(E[d_2] + \beta E[c_2])}{4\beta^2 - \gamma^2}$$
$$w_2^*(r_1, r_2) = \frac{(-2\beta^2 + \gamma^2)r_2 + \beta\gamma r_1 + 2\beta(E[d_2] + \beta E[c_2]) + \gamma(E[d_1] + \beta E[c_1])}{4\beta^2 - \gamma^2}$$

3.4.3. Determining optimum retail prices

As retailer benefit is a two variables function (decisive variables,r_2, r_1), to obtain the optimum point for each of them to maximize the benefit, both necessary and sufficient conditions have to be established. Same as earlier, necessary condition is that the first derivative function equalizes to zero, and sufficient condition is negative semi- definite Hessian matrix of related benefit function.
Necessary condition:

$$\frac{\partial E[\pi_r(r_1, r_2)]}{\partial r_1} = 0$$
$$\frac{\partial E[\pi_r(r_1, r_2)]}{\partial r_2} = 0 \tag{24}$$

Sufficient condition:

$$H_3 = \begin{vmatrix} \dfrac{\partial^2 E[\pi_r(r_1, r_2)]}{\partial r_1^2} & \dfrac{\partial^2 E[\pi_r(r_1, r_2)]}{\partial r_1 \partial r_2} \\ \dfrac{\partial^2 E[\pi_r(r_1, r_2)]}{\partial r_2^2} & \dfrac{\partial^2 E[\pi_r(r_1, r_2)]}{\partial r_2 \partial r_1} \end{vmatrix} = \begin{vmatrix} \dfrac{-2\beta(2\beta^2 - \gamma^2)}{4\beta^2 - \gamma^2} & \dfrac{\beta^2\gamma}{4\beta^2 - \gamma^2} \\ \dfrac{-2\beta(2\beta^2 - \gamma^2)}{4\beta^2 - \gamma^2} & \dfrac{\beta^2\gamma}{4\beta^2 - \gamma^2} \end{vmatrix} \tag{25}$$

Negative semi- definite Hessian matrix of a function is the sufficient condition for being a local maximum point of a two variables function (relation 25).

If $\beta > \gamma > 0$, would be a semi- definite matrix. As this condition is established, by solving equation 24 system, optimum retail prices would be determined (relation 26).

$$\begin{aligned}
&r_1^* = \frac{A(C_1 + E[d_1]) + (A^2 - B^2)E[s_1] + B(C_2 + E[d_2])}{2(A^2 - B^2)} \\
&r_2^* = \frac{A(C_2 + E[d_2]) + (A^2 - B^2)E[s_2] + B(C_1 + E[d_1])}{2(A^2 - B^2)} \\
&A = \frac{2\beta^3 - \beta\gamma^2}{4\beta^2 - \gamma^2}, B = \frac{\beta^2\gamma}{4\beta^2 - \gamma^2} \\
&C_1 = \frac{(-2\beta^2 + \gamma^2)(E[d_1] + \beta E[c_1]) + \beta\gamma(E[d_2] + \beta E[c_2])}{4\beta^2 - \gamma^2} \\
&C_2 = \frac{(-2\beta^2 + \gamma^2)(E[d_2] + \beta E[c_2]) + \beta\gamma(E[d_1] + \beta E[c_1])}{4\beta^2 - \gamma^2}
\end{aligned} \tag{26}$$

3.5. Members identical power

3.5.1. Determining benefit function

In this structure, all producers and retailers have equal power for pricing. In other words, none of them is superior to others. Each member determines the price to maximize its benefit; therefore competitive pricing takes place between members. This model is called Nash-Bertrand game and it is solved by relation 27 (Nagorni and wolf, 2014).

$$\begin{aligned}
&\max_{w_1} E[\pi_{m_1}] = E[(w_1 - c_1)(d_1 - \beta(r_1^* + w_1) + \gamma(r_2^* + w_2))] \\
&\max_{w_2} E[\pi_{m_2}] = E[(w_2 - c_2)(d_2 - \beta(r_2^* + w_2) + \gamma(r_1^* + w_1))] \\
&\max_{r_1, r_2} E[\pi_r] = E[\Sigma_{i=1}^2 (r_i - s_i)(\tilde{d}_i - \beta(r_i + w_i) + \gamma(r_{3-i} + w_{3-i}))]
\end{aligned} \tag{27}$$

By optimizing expected benefit function of retailer ($E[\pi_r]$) optimum retail price (r_1^* and r_2^*), by optimizing expected benefit function of producer 1 ($E[\pi_{m_1}]$) optimum wholesale price of product 1 (w_1^*) and by optimizing expected benefit function of producer 2 ($E[\pi_{m_2}]$) optimum wholesale price of product 2 (w_2^*) would be calculated.

These optimum prices are shown in relation 28.

$$\begin{aligned}
&r_1^* = S_1 - \frac{(-3\beta^2 + \gamma^2)S_1 + 2\beta\gamma S_2 + 3\beta(E[d_1] + \beta E[c_1]) + \gamma(E[d_2] + \beta E[c_2])}{9\beta^2 - \gamma^2} \\
&r_2^* = S_2 - \frac{(-3\beta^2 + \gamma^2)S_2 + 2\beta\gamma S_1 + 3\beta(E[d_2] + \beta E[c_2]) + \gamma(E[d_1] + \beta E[c_1])}{9\beta^2 - \gamma^2} \\
&w_1^* = \frac{(-6\beta^2 - 2\gamma^2)S_1 + 4\beta\gamma S_2 + 6\beta(E[d_1] + \beta E[c_1]) + 2\gamma(E[d_2] + \beta E[c_2])}{9\beta^2 - \gamma^2} \\
&w_2^* = \frac{(-6\beta^2 - 2\gamma^2)S_2 + 4\beta\gamma S_1 + 6\beta(E[d_2] + \beta E[c_2]) + 2\gamma(E[d_1] + \beta E[c_1])}{9\beta^2 - \gamma^2} \\
&S_1 = \frac{\beta E[d_1] + \gamma E[d_2] + (\beta^2 - \gamma^2)E[s_1]}{2(\beta^2 - \gamma^2)}
\end{aligned}$$

$$S_2 = \frac{\beta E[d_2] + \gamma E[d_1] + (\beta^2 - \gamma^2)E[s_2]}{2(\beta^2 - \gamma^2)}$$

3.5.2. Expected benefit function of producers and retailers

Referring to relation 3, expected benefit function of producer 1, producer 2 and retailer are indicated in relations 29, 30 and 31, respectively.

$$E[\pi_{m_1}] = -\beta w_1^2 + \gamma w_1 w_2 + \left(-\beta r_1 + \gamma r_2 + E[d_1] + \beta E[c_1]\right) w_1 + \beta r_1 E[c_1] - \gamma(r_2 + w_2)E[c_1] - E[c_1 d_1] \tag{29}$$

$$E[\pi_{m_2}] = -\beta w_2^2 + \gamma w_2 w_2 + \left(-\beta r_2 + \gamma r_1 + E[d_2] + \beta E[c_2]\right) w_2 + \beta r_2 E[c_2] - \gamma(r_1 + w_1)E[c_2] - E[c_2 d_2] \tag{30}$$

$$E[\pi_r] = \Sigma_{i=1}^{2} \left\{-\beta r_i^2 + \gamma r_i r_{3-i} + \left(-\beta w_i + \gamma w_{3-i} + E[d_i] + \beta E[s_i]\right) r_i + \beta w_i E[s_i] - \gamma(r_{3-i} + w_{3-i})E[s_i] - E[s_i d_i]\right. \tag{31}$$

4. Numerical results

Numerical examples are used to verify presented models, besides the effect of power structure on balanced prices and expected benefit of producers and retailers would be determined. In all examples two producers and one retailer play in a non- cooperative game and in three different scenarios,$\beta = 200$,$\gamma = 150$ and producers and retailer information are indicated in table 1 (all fuzzy variables with triangle probability distribution). Examples are taken from literature, but all exact numbers are the fuzzy triangle, instead of uncertain.

Table 1: Parameters and their numerical values

Parameter	Linguistic value	Fuzzy value that corresponds to linguistic value
$\tilde{c_1}$	Approximately 6	$(7,6,5)$
$\tilde{c_2}$	Approximately 6	$(7,6,5)$
$\tilde{s_1}$	Approximately4	$(5,4,3)$
$\tilde{s_2}$	Approximately4	$(5,4,3)$
$\tilde{d_1}$	Approximately 3.5	$(3,3/4,5)$
$\tilde{d_2}$	Approximately 3.5	$(3,3/4,5)$

Table 1 indicates expected values of fuzzy triangle parameters, and table 2 represents multiplication of fuzzy triangle parameters.

Table 2: Expected value of fuzzy parameters

Parameter	$E[\tilde{c_1}]$ - $E[\tilde{c_2}]$	$E[\tilde{d_1}]$-$E[\tilde{d_2}]$	$E[\tilde{c_1}\tilde{d_1}]$	$E[\tilde{c_2}\tilde{d_2}]$	$E[\tilde{s_1}]$	$E[\tilde{s_2}]$	$E[\tilde{s_1}\tilde{d_1}]$	$E[\tilde{s_2}\tilde{d_2}]$
Value	6-6	3.5 - 3.5	21166/67	21166/67	4	4	14166/67	14166/67

MS, MR and N games are used for 1, 2 and 3 scenarios, respectively. The results are shown in table 3.

According to table 3, as one of the members in the Stackelberg game becomes leader; it tries to increase selling price and expected benefit as high as possible. At MS game, producers increase the wholesale price to maximize their expected benefit, since they are rulers in pricing. At RS game same trend is established but for retailers. At N game all members have identical pricing power and expected benefit is almost similar compared with two other games. It has to be noted that selling

Table 3: Results

Variable	MS game	RS game	N game
w_1^*	16/69	12	12/67
$E[\pi_{m_1}]$	13012/31	7033/33	8722/22
w_2^*	16/69	12	12/67
$E[\pi_{m_2}]$	13012/31	7033/33	8722/22
(r_1^*, r_2^*)	(28/65,28/65)	(34,34)	(30/67, 30/67)
$E[\pi_r]$	60451/32	71666/66	70777/77
(p_1^*, p_2^*)	(45/34, 45/34)	(46,46)	(43/34 , 43/34)
$E[\pi_t]$	86475/93	85733/32	88222/21

cost (advertising, …), an important factor in sale, is assigned to retailers, which affect retail price and increase retailers benefit much higher than two other producers. Retailers can obtain 24 to 30 of currency by paying about 4 units of cost per unit of product. If sales power for all chain members is equal, customer receive the product with the lowest price. From customers viewpoint this state is the best, on the other hand as the retailer rules pricing it would be the worst state.

It has to be investigated that if taking sale cost (advertising, transfer,..) into account for the producer and exempt it for the retailer, is the best state for customers from the final price point of view and expected benefits for producers.

In the following another numerical problem would be proceeded. Table 4 parameters of the problem.

Table 4: Parameters and their numerical values

Parameter	Linguistic value	Fuzzy value that corresponds to linguistic value
$\tilde{c_1}$	Approximately 8	$(9, 8, 7)$
$\tilde{c_2}$	Approximately 8	$(9, 8, 7)$
$\tilde{s_1}$	Approximately 13	$(7, 6, 5)$
$\tilde{s_2}$	Approximately 13	$(7, 6, 5))$
$\tilde{d_1}$	Approximately 4	$(5, 4, 3)$
$\tilde{d_2}$	Approximately 4	$(5, 4, 3)$

Supposing MS, MR and N games for scenario 1, 2 and 3, respectively, results are presented in table5.

5. Sensitivity analysis

As production costs, attainable market share for each producer and retail costs (advertising, marketing,…) are of great importance; sensitivity analysis of these factors in each scenario would be measured. Effects of these factors on the wholesale price and benefit of producers would be distinguished.

5.1. Scenario 1

In this scenario, the producer is dominant over the retailer on pricing. In the case of production cost fluctuations for producer 1, referring to relation 18, it can be said that optimum products wholesale price for producer 1 is not dependent to production cost, thus no changes in price would happen.

Table 5: Results

Variable	MS game	RS game	N game
w_1^*	19/23	15,6	16.3/67
$E[\pi_{m_1}]$	12811/06	6824/12	8202/14
w_2^*	19/23	15,6	16.3/67
$E[\pi_{m_2}]$	12811/06	6824/12	8202/14
(r_1^*, r_2^*)	(31/24,31/24)	(36.3,36.3)	(33/54, 33/54)
$E[\pi_r]$	60085/11	70124/12	69923/77
(p_1^*, p_2^*)	(48/12, 48/12)	(49.26,,49.26)	(46/17 , 46/17)
$E[\pi_t]$	85956/11	84988/06	87956/11

But it's clear that production affects the wholesale price of the competent producer (producer 2) as well as benefits of both producers and retailer (figures 2 and 3).

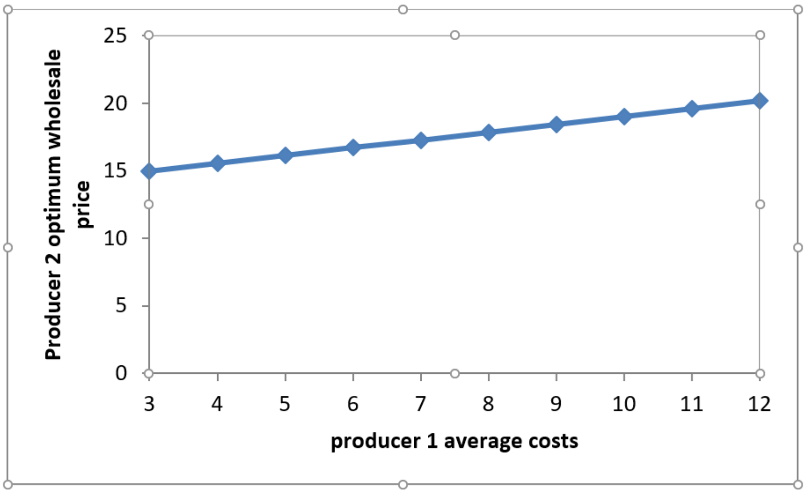

Figure 2: Producer 2 optimum wholesale price vs. producer 1 average costs

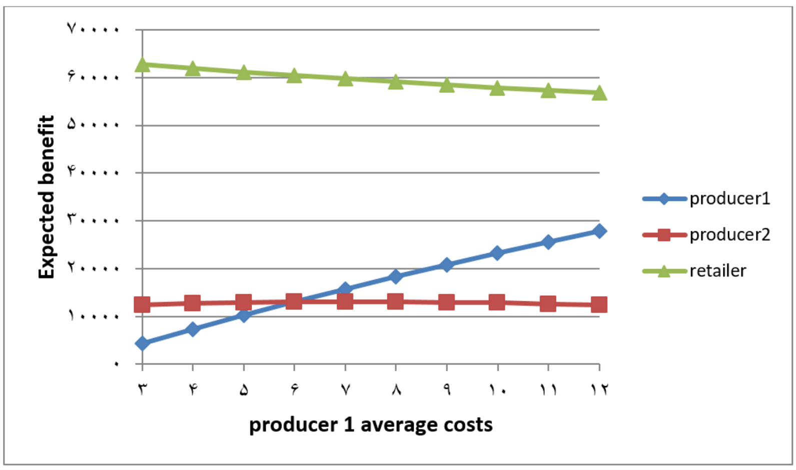

Figure 3: Expected benefit of chain members vs. producer 1 average costs

It is obvious that changing attainable market share for producer 1, has undeniable effects on wholesale price and benefits of both producers' as well as retailer's benefit (according to relations 18,19 and 33-35) (figures 4 and 5)

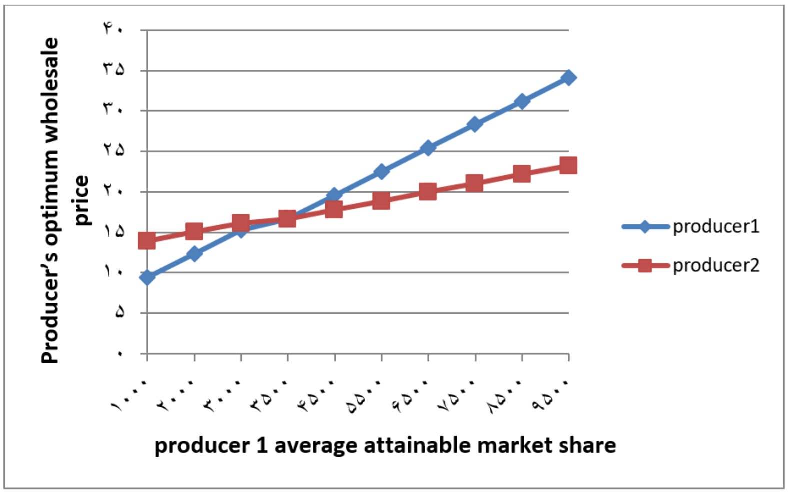

Figure 4: Producer's optimum wholesale prices vs. producer 1 average attainable market share

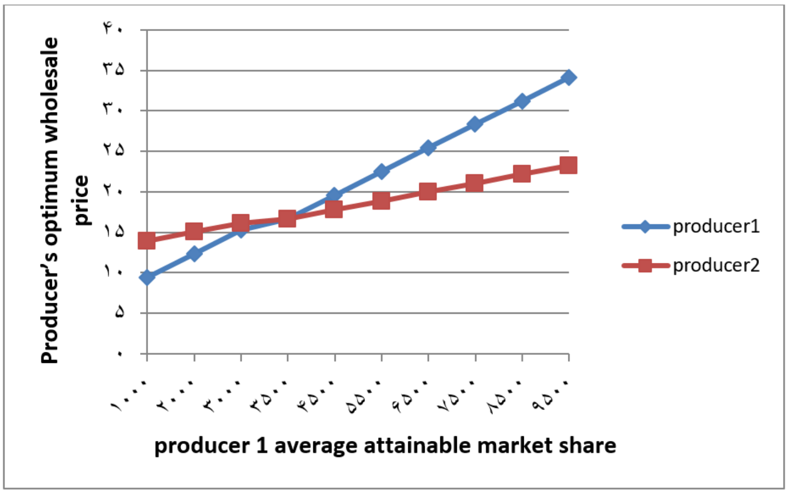

Figure 5: Expected benefits of chain members vs. producer 1 average attainable market share

5.2. Scenario 2

In this scenario, the retailer is dominant over producers on pricing. According to relation 23, production cost fluctuations for producer 1 affects wholesale price and benefits of both producers' as well as retailer's benefit (relations 33-35) (figures 6 and 7).

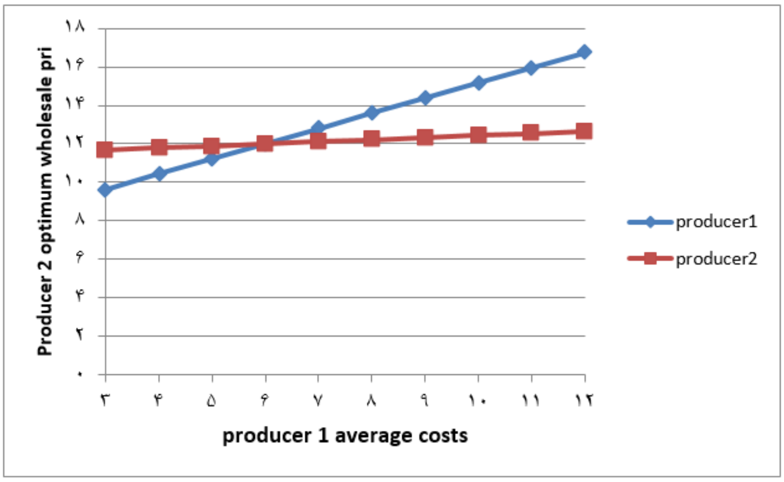

Figure 6: Producer's optimum wholesale prices vs. producer 1 average cost

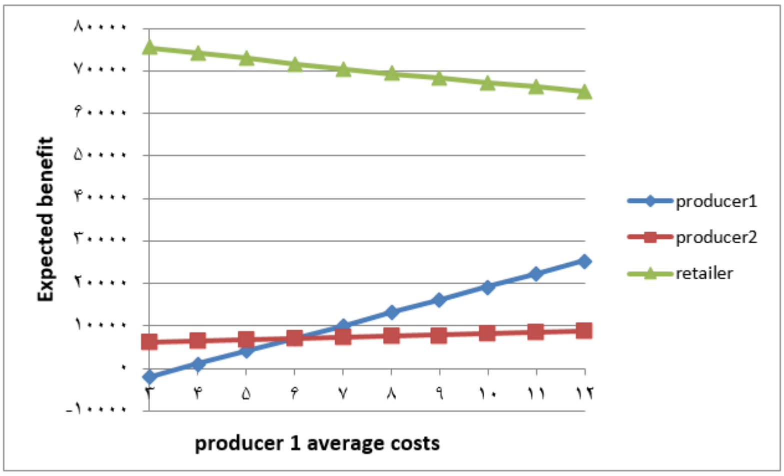

Figure 7: Expected benefits of chain members vs. producer 1 average cost

It is clear that changing attainable market share for producer 1, has impressions on wholesale price and benefits of both producers' as well as retailer's benefit (according to relations 23 and 33-35) (figures 8 and 9).

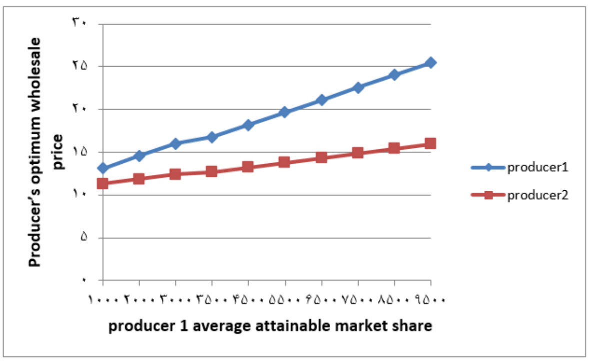

Figure 8: Producer's optimum wholesale prices vs. producer 1 average attainable market share

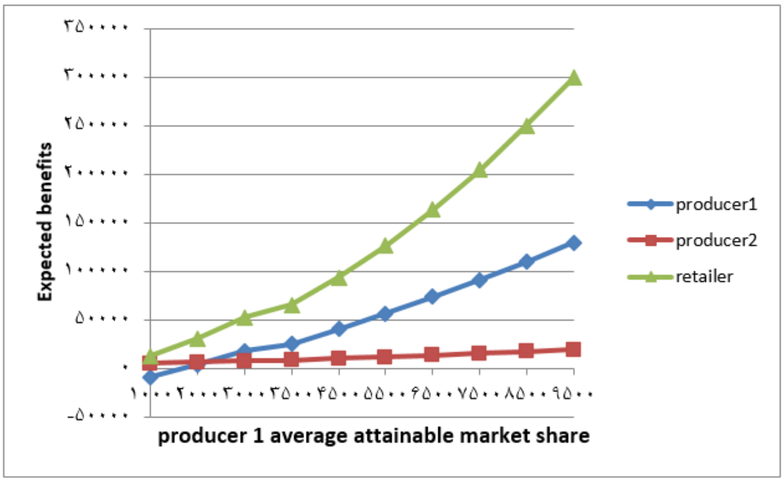

Figure 9: Expected benefits of chain members vs. producer 1 average attainable market share

5.3. Scenario 3

In this case none of chain members are dominant over others. Referring to relation 28, production cost fluctuations for producer 1 affects both producers wholesale price, as well as benefits of both producers and retailer (relations 33-35) (figures 10 and 11).

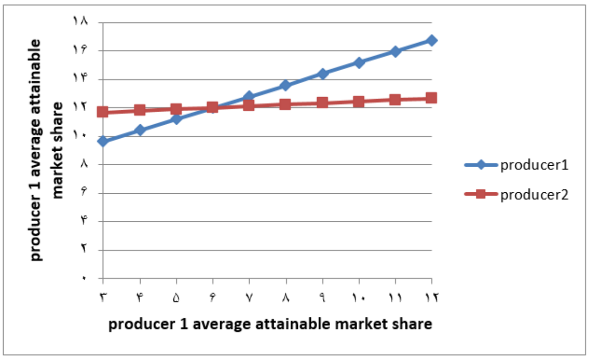

Figure 10: Producer's optimum wholesale prices vs. producer 1 average attainable market share

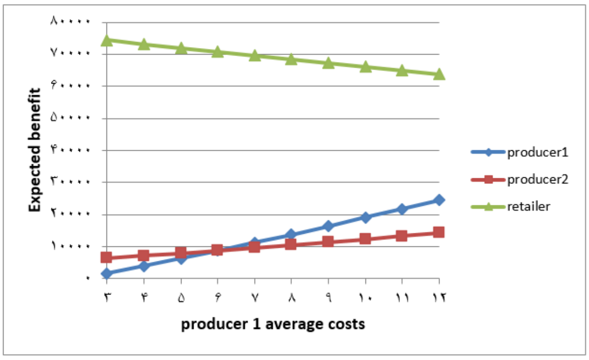

Figure 11: Expected benefits of chain members vs. producer 1 average cost

It is distinguishable that changing attainable market share for producer 1, has influence on producers 1 and 2 wholesale prices (according to relations 28) as well as benefits of both producers and retailer (according to relations 33-35) (figures 12 and 13).

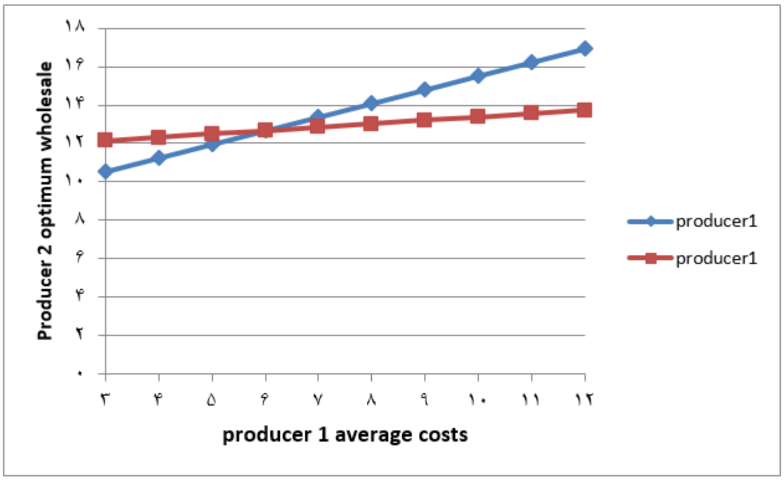

Figure 12: Producer's optimum wholesale prices vs. producer 1 average attainable market share

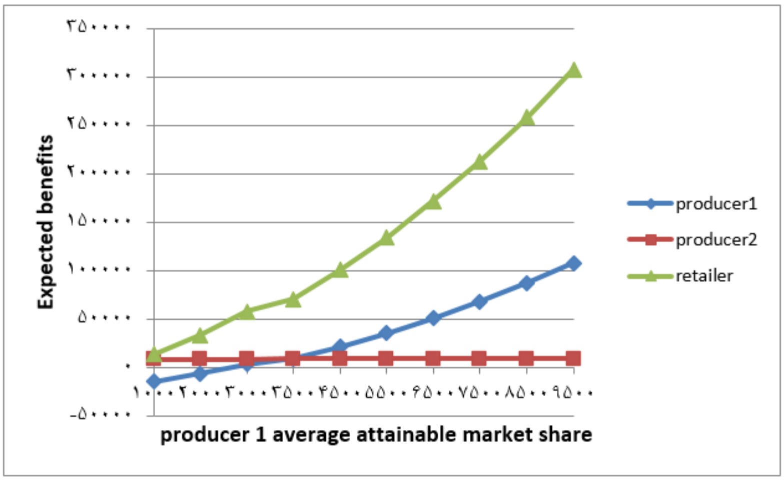

Figure 13: Expected benefits of chain members vs. producer 1 average attainable market share

6. Conclusion

In this study a game theory based model for pricing a two level supply chain in conditions of uncertainty and advertising is presented. The problem includes optimum pricing of two substitutable products that are manufactured by two different producers. A product which is most adjustable (servicing aspect) to the targeted one is defined substitution. Market is considered highly competitive, means producers selling power is not equal (brands, after sale services,..). The model was used to solve problems of different strategies, and optimum strategy was determined for each of chain members. In this regard, three main strategies including producer dominance, retailer dominance and members' equality were inspected. In each strategy benefit function, optimum wholesale and retail prices were investigated. Finally, numerical problems from various angels were presented, solved and discussed. Sensitivity analysis for producers pricing, attainable market share for each producer and retail costs have been done and effects of each parameter fluctuation on model variables were discovered.

This research can be developed to distinguish optimum supply chain power structure for customers and to investigate inequality of them. Besides, developments can move further to consider horizontal competition between chain members.

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