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A robust optimization approach for a multi-period location-arc routing problem with time windows: A case study of a bank

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Abstract

A Location-Arc Routing Problem (LARP) is a practical problem, while a few mathematical programming models have been considered for this problem. In this paper, a mixed non-linear programming model is presented for a multi-period LARP with the time windows under demand uncertainty. The time windows modeling in the arc routing problem is rarely. To the best our knowledge, it is the first time that the robust LARP model is verified and an optimal solution is presented for it. For this purpose, the CPLEX solver is used for solving the treasury locate problems of a bank as a case study. These problems are node-based with close nods and can be transformed into arc-based. Therefore, the method LRP and LARP models can be used to solve these problems. The comparing results of the LRP and LARP models prove that the LARP has a better performance regarding timing and optimal solution. Furthermore, comparing the results of deterministic and robust LARP models for this case study shows the validity of the robust optimization approach.

Keywords: Location-arc routing problem; Time windows; Multi-periods; Robust optimization; Demand Uncertainty

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1. Introduction

Routing problems had two major categories: 1) the node routing problem that customers are located on the node, 2) the arc routing problems that customers are located on the arc or edge of a graph [1]. The Vehicle Routing Problem (VRP) and Traveling Salesman Problem (TSP) are node routing problem. The Rural Postman Problem (RPP) and the Chinese Postman Problem (CPP) are arc routing problems. In a CPP, all arcs of a graph are served while in an RPP only a subset of arcs or edges is required to be served [2]. The Capacitated Arc Routing Problem (CARP) was introduced by Golden and Wong [3] and is an expansion of the RPP; however, it is a complex problem because of considering multiple vehicles with capacity constraints. In CARP the customer demands are on arcs (edges) instead of nodes and the goal is to reduce the traveling costs of all links.

A Location-Arc Routing Problem (LARP) is a combination of two well-known problems: 1) CARP and 2) Location-Allocation Problem (LAP) [4]. The LAPR is similar to the CARP. The only major difference between LARP and CARP is that besides routing, the best locations for depots are determined. Therefore, a binary decision variable is added to the mathematical model that has made it even more complicated. As far as we known, a few research is presented seven mathematical models for LARP until now. These models are not verified and are solved with heuristic or meta-heuristic method. For example, in the model of [2] each customer has required one vehicle, and this is not practical. Besides, Huber [5] formulate two objectives functions and don't present constraints. Therefore, this paper presents a new multi-period LARP under uncertain demand. The constraints of time windows, the time limitation for vehicles, delivery or collection from customer, and using the vehicle for some customer are added to the model. Definition of time windows in model reduces the risk of transporting cash. The time windows modeling in arc routing problem has a different structure than the node routing problem, since a required arc with a time window after servicing, can be used as a deadheading arc without a time window.

Çetinkaya et al. [6] claim it is not possible to modeling time windows in arc routing problem and should be transform its to node routing problem. Macedo et al. [7] is not modeled time window and solving with metaheuristic methode. Lystlund et al. [8] proposed a mathematical model for this purpose and we inspiration it. The definition of the time window constraint in a case study is based on the customer requests for the collection or delivery of cash in specified time. Time window modeling in arc routing problems has a different structure rather than node routing problems because an required arc with time window can be selected as an deadheading arc without time window on the other road. The proposed model is linearized and solved with CPLEX solver in GAMS. Beside, Bertsimas and sim [9] method is used for uncertainty condition.

The rest of the paper is structured as follows. Section 2 is reviewed the CARP and LARP. In Section 3, the robust and deterministic linear model is presented. The bank case study problems is described in Section 4. In Section 5, the new model is validated by comparing the results of LARP and LRP for solving a bank case study. The validation of the robust model is also performed in Section Section 6. The final conclusions are presented in Section 7.

2. Literature review

There are a few articles in the LARP and proximity with the CARP, in which this section reviews some studies on the CARP and the LARP.

2.1. CARP review

The review of the CARP from 2000 to 2018 as summarized in Table 1. The most studies have focused on solving basic CARP under deterministic conditions. For an uncertain condition, only

Fleury et al. [10] used a stochastic method for the CARP under uncertainty. Besides, Fleury et al. [11] and Mei et al. [12] applied a robust approach. In most studies, the LARP model has been solved using heuristic or meta-heuristic methods. Other studies proposing solutions based on exact methods have applied either some techniques for transforming the CARP to the CVRP or a heuristic to build an initial solution for decision variables. Furthermore, the CARP with time window is rarely and all of the research are presented a non-linear mathematical model except the model [8]. All of them have not verified proposed model and not presented exact solution.

Table 1: Review on the deterministic CARP

Problem	Exact methods	Heuristics	Meta-heuristics
Capacitated arc routing problem (CARP)	**[13],[14],[15],[16],[17],[18]**	[3],[19],[20],[21],[22],[23],[24],[25],[26],[27],[28],[29],[30],[31],[32],[33],[34]	[35],[36],[37],[38],[39],[40],[41],[42],[43],[44],[45], [46]
CARP with time windows		[8],[6],[47]	[48],[7],[49]
Multi-period CARP		[50]	[51],[52],[53]
Open CARP		[54]	[55],[56]
Split delivery CARP		[57]	
Multi-compartment CARP		[58]	
CARP with profits		[59],[60]	
CARP with refill points		[61]	
CARP with intermediate		[62],[63]	[64]
Time-dependent CARP		[65]	[66],[67]
CARP with mobile depots			[68],[69]
Prize-collecting CARP		[70]	[47]
Rich ARP		[71]	

2.2. LARP review

Although a few studies in the literature review have focused on the LARP, due to the practical applications and complexity of this problem, three review papers can be found on this issue. Ghiani et al. [4] surveyed three main applications of the LARP, namely, postal delivery, garbage collection and road maintenance, and also algorithms used to solve this problem. Liu et al. [72] presented the LARP review and identified the research gap on this problem. Moreover, Laporte et al. [1] surveyed all studies on the LARP and emphasized on developing two presented models [2, 73] and solved this problem with the exact method.

The first research on the LARP was carried out by Levy et al. [74] for solving the routing problem at the post office in the USA. They used a Location-Allocation-Routing (L-A-P) method to solve the model. Based on this method, firstly, the depot location is determined and then required arcs be allocated depots. Finally, required arcs can be solved a Vehicle Routing Problem (VRP). In another method, called Allocate-Routing-Location, required arcs are first allocated to the depots, and then the depot location is determined based on the selected routes. According to the reports, this method has been more efficient than the previous methods [4]. Ghiani et al. [75] presented a linear multi-depot model for the LARP. Then, they transformed this model into the RPP and used a branch-and-cut algorithm for solving their model.

A few research studies have proposed a mathematical model for the LARP, and all have used a meta-heuristic algorithm for solving the deterministic models Table 2. Doulabi et al. [73] presented two Mixed-Integer Linear Programming (MILP) models considering the flow variables for single and multi-depot problems. They proposed a Simulated Annealing (SA) algorithm, which used an allocation-routing-location method at each iteration. It first builds a routing solution and then improves the depot locations.

Lopes et al. [2] proposed a mathematical model and solved it by several heuristics. They tested different constructive heuristics combining Variable Neighbourhood Search (VNS), Greedy Randomized Adaptive Search Procedure (GRASP) and Tabu Search (TS). According to their results, the

Table 2: Features LARP in this paper and other paper

Refrence	Depot				Customer			Vehicle				Material		Model Features						
																Data		Solving Method		
	Multi	Capacity	Heterogeneous	Inventory	Capacity	Splitting	Time Windows	Determine number	Capacity	Heterogeneous	Time available	Multi Produce	Risk	Multi-objective	Multi period	Linear	Uncertain	Exact	heuristic	Metaheuristic
Doulabi et al. [73]	×	×			×															×
Lopes et al. [2]	×				×															×
Essink et al. [76]	×				×							×			×					×
Riquelme-Rodríguez et al. [77]	×			×																×
Huber et al. [5]	×			×										×						×
Amini et al. [78]	×	×															×		×	
Tavakkoli-Moghaddam et al. [79]	×	×										×							×	
This paper	×	×	×		×		×	×	×	×	×	×			×		×	×		

combination of TS and GRASP is the best case. The disadvantage of this model is that Constraint (7) in this paper implies that each customer is required an individual vehicle and this is not practical. The mathematical model in [76] is the same as [2]; however, they are used hybrid TS-GRASP. Riquelme-Rodríguez et al. [77] compared two methods for locating depots in the network. They proposed a non-linear model for a periodic LARP and used a heuristic method for solving this model. Huber et al. [5] presented a model with two objectives functions without formulating constraints. He using heuristic for solving benchmark instances. Amini et al. [78] addressed an uncertain LARP and employing two scenario-based approaches. Performance of scenario-based models is evaluated with results of the numerical example.

The review on LARP shows that the time window has not been modeled for it, until now. However, according to Table 1, in CARP have several studies on this asspect. Vansteenwegen et al. [49] used a meta-heuristic method to solve the ARP problem with a soft time window for a case study of mobile mapping van problem. But, they did not present mathematical model. Lystlund et al. [8] presented two linear models for the CARP problem with soft and hard time window and solved with heuristic method. Black et al. [47] defined a non-linear model for the prize-collection CARP. Proposed mathematical model was non-linear and solved with heuristic methode. Çetinkaya et al. [6] presented a hybrid non-linear integer model for a two-step ARP problem. Also, Vincent and Lin [70] did not present a mathematical model for prize-collection ARP and solved with meta-heuristic method. A review of related studies to the time window on arc routing shows that limited studies focus on presenting a mathematical model and except Lystlund et al. [8] model all models are non-linear. Because of the similarities between LARP and CARP, in this paper has been used the model presented in [8].

3. Mathematical formulations

The LARP is defined as a graph $G = (V, E \cup R)$, which V is a set of vertex, E is a set of edges, and R is a set of arcs, which can be directed, undirected, or a combination of both. Here, the LARP problem is a directional graph and the connection of two vertexes is called an arc. The vertex set V contains a non-empty subset J of n potential depot locations . Every arc has a non-negative traversal cost and a non-negative demand for service. The arcs with positive demand form the subset A of the arcs required to be serviced, only once, by a vehicle with capacity. Vehicles start and end their route in the same depot, and each new vehicle. The sets, parameters, uncertain parameters, decision variables for multi-period robust LARP model are presented in the following Table 3. The main assumptions in proposed LARP problem are as follows:

- Every required arc is traversed by one vehicle
- The vehicle is returned to the depot that was started
- The total demand for the arcs selected on tour is less than the vehicle capacity
- The deadheading arc is movement from the end vertex of one required arc to the start vertex of another required arc without servicing the traversed arcs
- Each required arc is serviced once that can be used as a deadheading without time window
- The number of deadheading arc selected in every tour is minimized
- Waiting time is allowed for each required arc at the start of service
- The distribution of products is delivery and collection
- The number of deadheaded traverses in each arc is not limited

Table 3: The sets, parameters, uncertain parameters, decision variables

Nomenclature	
Sets	
I	Set of all vertices $I = \{1, ..., i\}$, which includes customers and depots
J	Set of depots $J = \{1, , j'\}$ that j' is the maximum number of depots
K	Set of vehicle $K = \{1, , k\}$
P	Set of period $P = \{1, , p\}$
S	Set of steps that every vehicle travels $S = \{1, , s\}$
A	Set of required arcs that has to be visited
V_A	Set of vertices that are extremities of the arcs in set A
$\hat{V}$	Union of the set J and V_A ($\hat{V} = V_A \cup J$)
$\hat{A}$	Set of arcs forming a complete graph with $\hat{V}$
$\delta^+(J)(\delta^-(J))$	Set of arcs leaving (entering) on the set of vertices J. When S contains a single vertex v, $\delta^+(v)$ is a simplification for $\delta^+(v)$.
Parameters	
$f_{(j'p)}$	Costs of creating depot j' in period p
c_{ijp}	Costs of traversing arc (i,j) in period p, if arc $(i,j) \in A$ cost of servicing is equal to $\hat{c}_{ij}$ and if the arc is deadheading (i.e. arc $(i,j) \notin A$) is equal c'_{ij}
$b_{(j'p)}$	Capacity of depot j in period p
t_{ij}	Time of traversing arc (i,j), if arc $(i,j) \in A$ time of traveling is equal to $\hat{t}_{ij}$ and if the arc is deadheading (i.e. arc $(i,j) \notin A$)) is equal arcs t'_{ij}
T_{kp}	The maximum time allowable for vehicle k in period p
F_k	Fixed costs of using vehicle k
N_p	Number of customers in period p
Q_{kp}	Capacity of vehicle k in period p
M	A very large number
Uncertain Parameters	
$\tilde{d}_{ijp}$	Demand for service of arc $(i,j) \in A$ in period p
Decision variables	
x_{ijksp}	1 if arc $(i,j) \in \hat{A}$ is traveled by vehicle k in period p at step s; and 0, otherwise
Decision variables	
x_{ijksp}	1 if arc $(i,j) \in \hat{A}$ is traveled by vehicle k in period p at step s; and 0, otherwise
y_{ijksp}	1 if arc $(i,j) \in A$ is served by vehicle k in period p at step s; and 0, otherwise
$w_{ijj'p}$	1 if arc $(i,j) \in A$ is allocated to depot j' in period p
ω_{ijp}	Time that service of the arc $(i,j) \in \hat{A}$starts in period p
φ_{iksp}	Time that vehicle k arrives in node i at step s in period p
ρ_{ijksp}	Time that vehicle k starts traversing arc $(i,j) \in \hat{A}$ at step s
u_{ikp}	Slack variable for eliminating sub-tours

3.1. LARP model

The mixed-integer non-linear Robust multi-period LARP model is presented in this subsection.

$$Min \sum_{j' \in J} \sum_{p \in P} f_{j'p} w_{ijj'p} + \sum_{(i,j) \in \hat{A}} \sum_{k \in K} \sum_{s \in S} \sum_{p \in P} c_{ijp} x_{ijksp} + \sum_{k \in K} \sum_{(i,j) \in \delta^+(J)} \sum_{s \in S} \sum_{p \in P} F_k x_{ijksp} \tag{1}$$

$s.t.$

$$\sum_{k \in K} y_{ijksp} = 1 \qquad \forall (i,j) \in A, s \in S, p \in P \tag{2}$$

$$\sum_{(i,j) \in \delta^+(i)} x_{ijksp} - \sum_{(i,j) \in \delta^-(i)} x_{ijksp} = 0 \qquad \forall i \in V_A, k \in K, s \in S, p \in P \tag{3}$$

$$x_{ijksp} \geq y_{ijksp} \qquad \forall (i,j) \in A, k \in K, s \in S, p \in P \tag{4}$$

$$u_{ikp} - u_{jkp} + N_p x_{ijksp} \leq N_p - 1 \qquad \forall i \in V_A, i \neq j, k \in K, s \in S, p \in P \tag{5}$$

$$\sum_{(i,j) \in A} \tilde{d}_{ijp} y_{ijksp} \leq Q_{kp} \qquad \forall k \in K, s \in S, p \in P \tag{6}$$

$$\sum_{m \in V_A} x_{j'mksp} y_{ijksp} \leq M w_{ijj'p} \qquad \forall j' \in J, (i,j) \in A, k \in K, s \in S, p \in P \tag{7}$$

$$\sum_{(i,j) \in A} \tilde{d}_{ijp} w_{ijj'p} \leq b_{j'p} \qquad \forall j' \in J, k \in K, p \in P \tag{8}$$

$$\sum_{(i,j) \in \hat{A}} \sum_{s \in S} t_{ij} x_{ijksp} \leq T_{kp} \qquad \forall k \in K, p \in P \tag{9}$$

$$\varphi_{iksp} - \rho_{ijksp} \leq M(1 - x_{ijksp}) \qquad \forall (i,j) \in \hat{A}, k \in K, s \in S, p \in P \tag{10}$$

$$\rho_{ijksp} + \hat{t}_{ij} - \varphi_{jks+1p} \leq M(1 - y_{ijksp}) \qquad \forall (i,j) \in A, k \in K, s \in S, p \in P \tag{11}$$

$$\varphi_{jks+1p} - (\rho_{ijksp} + \hat{t}_{ij}) \leq M(1 - y_{ijksp}) \qquad \forall (i,j) \in A, k \in K, s \in S, p \in P \tag{12}$$

$$\rho_{ijksp} + t'_{ij} - \varphi_{jks+1p} \leq M(1 - (x_{ijksp} - y_{ijksp})) \qquad \forall (i,j) \in \hat{A}, k \in K, s \in S, p \in P \tag{13}$$

$$\varphi_{jks+1p} - (\rho_{ijksp} + t'_{ij}) \leq M(1 - (x_{ijksp} - y_{ijksp})) \qquad \forall (i,j) \in \hat{A}, k \in K, s \in S, p \in P \tag{14}$$

$$\rho_{ijksp} - \omega_{ijp} \leq M(1 - y_{ijksp}) \qquad \forall (i,j) \in A, k \in K, s \in S, p \in P \tag{15}$$

$$\omega_{ijp} + \hat{t}_{ij} - \rho_{ijksp} \leq M(1 - (x_{ijksp} - y_{ijksp})) \qquad \forall (i,j) \in \hat{A}, k \in K, s \in S, p \in P \tag{16}$$

$$\varphi_{0k1p} = 0 \qquad \forall k \in K, p \in P \tag{17}$$

$$x_{ijksp}, y_{ijksp}, w_{ijj'p}, u_{ikp} \in \{0,1\}, \omega_{ijp}, \rho_{ijksp}, \varphi_{iksp} \geq 0 \tag{18}$$

The objective function (1) minimizes the sum of the fixed costs of creating the depots, the costs of all traversed arcs, and the cost of selected vehicles, respectively. Constraint (2) ensures that each required arc is served once by exactly one vehicle. Constraint (3) guarantees that the flow is preserved (i.e., it ensures the number of arrivals at any vertex is equal to the number of departures). Constraint (4) implies that an arc is served by a particular vehicle only if it traverses the same vehicle. Constraint (5) is a sub-tour elimination constraint. In fact, a tour ends at the same depot, from which it was started. Constraint (6) ensures that the vehicle capacity is not exceeded. Constraint (7) makes sure that the required arcs are correctly allocated to a depot. Therefore, if the route that the vehicle takes, starts from and ends at depot j, the route is devoted to depot j. This restriction is necessary to establish Constraint (8). Constraint (8) ensures that the capacity of the depots is not violated. Constraint (9) ensures that the time taking the vehicle to finish the tour does not exceed the maximum available time.

Constraints (10) to (17) related to time windows Constrait. The time windows modeling in arc routing problem has a different structure than the node routing problem, since a required arc with a time window after servicing, can be used as a deadheading arc without a time windows. Constraint (10) denotes that the start time of the traversal arc $(i,j)\in\hat{A}$ cannot be before the arrival time in node i at step s. Constraints (11) to (14) implies that the arrival time in node j equals the start time of the service or traversal plus the service time or the travel time for required arc or deadheads arc, respectively. Constraint (15) fixes the service time variable. Constraint (16) ensures that any required arc cannot be deadheads arc earlier than the finish time of the service of this arc. Constraints (10) to (16) are restrictive if the arc $(i,j)\in A$ selecting in optimum routes. Index s that shows steps travels every vehicle does not effect on the model before Constraint (9). Therefore, the definition of this index is not required if the problem is without time windows. However, this index is necessary for Constraints (10) to (17) because for calculate arrival time in node j is used arrival time in node i in previous step vehicle. Constraint (17) ensures that at the first step the vehicles start in the depot at time zero. Finally, Constraints (18) shows the binary and positive decision variables.

3.2. Robust-based Bertsimas model

A robust approach is a method for solving problems of linear optimization under an uncertain condition. When the data change to ensure that the solution remains near-optimal and feasible, this approach has accepted a sub-optimal solution for the nominal values of the data [9]. Optimal solutions that are less sensitive to uncertainty is called a robust solution. This method is alternative for the stochastic programming and sensitivity analysis. This approach is used in discrete problems. To describe [9], the reader may consider the following linear programming:

$$Min \quad cx$$
$$s.t.$$
$$\sum_j \tilde{a}_{ij} x_j \le b_i \qquad \forall i \quad (19)$$
$$x\in X$$

where coefficients $\tilde{a}_{ij}$ is uncertainty and random variable in $[a_{ij}-\hat{a}_{ij}, a_{ij}+\hat{a}_{ij}]$ that a_{ij} and $\hat{a}_{ij}$ are the nominal value and the variation magnitude of the uncertain parameter, respectively. J_i denote the set of uncertain coefficients of row i. For the $i-th$ constraint, a control parameter Γ_i, called price of robustness is introduced. Parameters $\Gamma_i\in[0,|j_i|]$ and $|j_i|$ are the cardinality of set j_i. In the proposed LARP model, $\tilde{d}_{ijp}\in[d_{ijp}-\hat{d}_{ijp}, d_{ijp}+\hat{d}_{ijp}]$ in Constraints (6) and (8) is uncertainty. Accordingly, based on [9], Constraint (6) can be shown in the following non-linear form:

$$\sum_{(i,j)\in A}\sum_{s\in S} d_{ijp}y_{ijksp} + \max_{S_{kp}\cup\{t_{kp}\}|S_{kp}\subseteq J_{kp},|S_{kp}|=\lfloor\Gamma_{kp}\rfloor,t_{kp}\in J_{kp}\setminus S_{kp}} \{\sum_{(i,j)\in A}\sum_{s\in S}\hat{d}_{ijp}y_{ijksp} + (\Gamma_{kp}-\lfloor\Gamma_{kp}\rfloor)\hat{d}_{ijp}y_{ijksp}\}$$
$$\le Q_{kp} \qquad \forall k\in K, p\in P \quad (20)$$

Given a vector y^*, the protection function of Constraint (20) is as follows:

$$B_{kp}(y^*,\Gamma_{kp}) = \max_{S_{kp}\cup\{t_{kp}\}|S_{kp}\subseteq J_{kp},|S_{kp}|=\lfloor\Gamma_{kp}\rfloor,t_{kp}\in J_{kp}\setminus S_{kp}} \{\sum_{(i,j)\in A}\sum_{s\in S}\hat{d}_{ijp}|y^*_{ijksp}| + (\Gamma_{kp}-\lfloor\Gamma_{kp}\rfloor)\hat{d}_{ijp}|y^*_{ijksp}|\}$$
$$\le Q_{kp} \qquad \forall k\in K, p\in P \quad (21)$$

It is equal to the following linear optimization problem in (22):

$$\begin{aligned}
&B_{kp}(y^*, \Gamma_{kp}) = \max\{\sum_{j''\in J_{ksp}} \hat{d}_{ijp}|y^*_{j''}|z_{ijj''}\} \\
&s.t. \\
&\sum_{j''\in J_{ksp}} z_{ijj''} \leq \Gamma_{kp} \qquad (22)\\
&0 \leq z_{ijj''} \leq 1 \qquad j''\in J_{ksp}
\end{aligned}$$

By introducing variables q^1_{kp} and r^1_{ijksp} for linear optimization problem (22), the dual model is as follows:

$$\begin{aligned}
&Min \qquad q^1_{kp}\Gamma^1_{kp} + \sum_{(j,j)\in A} r^1_{ijksp} \\
&s.t. \\
&q^1_{kp} + r^1_{ijksp} \geq \hat{d}_{ijp}|y_{ijksp}| \qquad (23)\\
&q^1_{kp}, r^1_{ijksp} \geq 0
\end{aligned}$$

By interleaving the dual model Constraint (6) will be replaced with the following constraints:

$$\sum_{(i,j)\in A} d_{ijp}y_{ijksp} + q^1_{kp}\Gamma^1_{kp} + \sum_{(j,j)\in A} r^1_{ijksp} \leq Q_{kp} \qquad \forall k\in K, p\in P \qquad (24)$$

$$q^1_{kp} + r^1_{ijksp} \geq \hat{d}_{ijp}y_{ijksp} \qquad \forall k\in K, p\in P \qquad (25)$$

$$q^1_{kp}, r^1_{ijksp} \geq 0 \qquad (26)$$

Similarly, (8) will be replaced with the following constraints:

$$\sum_{(i,j)\in A} d_{ijp}w_{ijj'p} + q^2_{j'p}\Gamma^2_{j'p} + \sum_{(j,j)\in A} r^2_{ijj'p} \leq b_{j'p} \qquad \forall j', p\in P \qquad (27)$$

$$q^2_{j'p} + r^2_{ijj'p} \geq \hat{d}_{ijp}w_{ijj'p} \qquad \forall j', p\in P \qquad (28)$$

$$q^2_{j'p}, r^2_{ijj'p} \geq 0 \qquad (29)$$

3.3. Linear Programming

Because of the multiplication of two binary variables in Constraint (7), the model is nonlinear. Therefore, instead of $x_{j'mksp} \times y_{ijksp}$, we introduce a new variable, namely, $h_{j'mijksp}$. Then, Constraint (7) is replaced with the following constraints:

$$\sum_{(i,j)\in A}\sum_{s\in S} h_{j'mijksp} \leq Mw_{ijj'p} \qquad \forall j'\in J, (i,j)\in A, k\in K, p\in P \qquad (30)$$

$$x_{j'mksp} + y_{ijksp} \leq 1 + h_{j'mijksp} \qquad \forall j'\in J, m\in V_A, (i,j)\in A, k\in K, p\in P \qquad (31)$$

$$x_{j'mksp} + y_{ijksp} \geq 2h_{j'mijksp} \qquad \forall j'\in J, m\in V_A, (i,j)\in A, k\in K, p\in P \qquad (32)$$

$$h_{j'mijksp}\in\{0,1\} \qquad (33)$$

3.4. Linear robust model

Robust model based on Bertsimas and sim method is as follow:

$$Min \sum_{(j'\in J)}\sum_{p\in P} f_{j'p}w_{ijj'p} + \sum_{((i,j)\in \hat{A}}\sum_{k\in K}\sum_{s\in S}\sum_{p\in P} c_{ijp}x_{ijksp} + \sum_{k\in K}\sum_{\delta^+(J)}\sum_{s\in S}\sum_{p\in P} F_k x_{ijksp} \tag{34}$$

s.t.

$$\sum_{k\in K} y_{ijksp} = 1 \qquad \forall (i,j)\in A, s\in S, p\in P \tag{35}$$

$$\sum_{(i,j)\in\delta^+(i)} x_{ijksp} - \sum_{(i,j)\in\delta^-(i)} x_{ijksp} = 0 \qquad \forall i\in V_A, k\in K, s\in S, p\in P \tag{36}$$

$$x_{ijksp} \geq y_{ijksp} \qquad \forall (i,j)\in A, k\in K, s\in S, p\in P \tag{37}$$

$$u_{ikp} - u_{jkp} + N_p x_{ijksp} \leq N_p - 1 \qquad \forall i\in V_A, i\neq j, k\in K, s\in S, p\in P \tag{38}$$

$$\sum_{(i,j)\in A} d_{ijp}y_{ijksp} + q^1_{kp}\Gamma^1_{kp} + \sum_{(j,j)\in A} r^1_{ijksp} \leq Q_{kp} \qquad \forall k\in K, p\in P \tag{39}$$

$$q^1_{kp} + r^1_{ijksp} \geq \hat{d}_{ijp}y_{ijksp} \qquad \forall k\in K, p\in P \tag{40}$$

$$\sum_{(i,j)\in A}\sum_{s\in S} h_{j'mijksp} \leq M w_{ijj'p} \qquad \forall j'\in J, (i,j)\in A, k\in K, p\in P \tag{41}$$

$$x_{j'mksp} + y_{ijksp} \leq 1 + h_{j'mijksp} \qquad \forall j'\in J, m\in V_A, (i,j)\in A, k\in K, p\in P \tag{42}$$

$$x_{j'mksp} + y_{ijksp} \geq 2h_{j'mijksp} \qquad \forall j'\in J, m\in V_A, (i,j)\in A, k\in K, p\in P \tag{43}$$

$$\sum_{(i,j)\in A} d_{ijp}w_{ijj'p} + q^2_{j'p}\Gamma^2_{j'p} + \sum_{(j,j)\in A} r^2_{ijj'p} \leq b_{j'p} \qquad \forall j'\in J, p\in P \tag{44}$$

$$q^2_{j'p} + r^2_{ijj'p} \geq \hat{d}_{ijp}w_{ijj'p} \qquad \forall j'\in J, p\in P \tag{45}$$

$$\sum_{(i,j)\in \hat{A}}\sum_{s\in S} t_{ij}x_{ijksp} \leq T_{kp} \qquad \forall k\in K, p\in P \tag{46}$$

$$\varphi_{iksp} - \rho_{ijksp} \leq M(1 - x_{ijksp}) \qquad \forall (i,j)\in \hat{A}, k\in K, s\in S, p\in P \tag{47}$$

$$\rho_{ijksp} + \hat{t}_{ij} - \varphi_{jks+1p} \leq M(1 - y_{ijksp}) \qquad \forall (i,j)\in A, k\in K, s\in S, p\in P \tag{48}$$

$$\varphi_{jks+1p} - (\rho_{ijksp} + \hat{t}_{ij}) \leq M(1 - y_{ijksp}) \qquad \forall (i,j)\in A, k\in K, s\in S, p\in P \tag{49}$$

$$\rho_{ijksp} + t'_{ij} - \varphi_{jks+1p} \leq M(1 - (x_{ijksp} - y_{ijksp})) \qquad \forall (i,j)\in \hat{A}, k\in K, s\in S, p\in P \tag{50}$$

$$\varphi_{jks+1p} - (\rho_{ijksp} + t'_{ij}) \leq M(1 - (x_{ijksp} - y_{ijksp})) \qquad \forall (i,j)\in \hat{A}, k\in K, s\in S, p\in P \tag{51}$$

$$\rho_{ijksp} - \omega_{ijp} \leq M(1 - y_{ijksp}) \qquad \forall (i,j)\in A, k\in K, s\in S, p\in P \tag{52}$$

$$\omega_{ijp} + \hat{t}_{ij} - \rho_{ijksp} \leq M(1 - (x_{ijksp} - y_{ijksp})) \qquad \forall (i,j)\in \hat{A}, k\in K, s\in S, p\in P \tag{53}$$

$$x_{ijksp}, y_{ijksp}, w_{ijj'p}, u_{ikp}, h_{j'mijksp}\in\{0,1\}, \omega_{ijp}, \rho_{ijksp}, \varphi_{iksp}, q^1_{kp}, r^1_{ijksp}, q^2_{j'p}, r^2_{ijj'p} \geq 0, \varphi_{0k1p} = 0 \tag{54}$$

4. Case study

In this paper, proposed model is used for modelling bank case study problem. This bank has 1200 branches and 30 treasury centers in a special city for providing services to the branches (Figure 1). Treasury centers and their branches have been divided into 10 clusters based on the bank strategy. The two main criteria considered by the bank for clustering include:

- There have to be safe roads between cluster members,

- The cash exchanges between clusters has to be minimized.

Each treasury center is responsible for delivering or collecting cash from its branches. These centers need to exchange money among themselves. Therefore, the goal is to determine at least one main center (depot) in each cluster based on a demand of treasury centers.

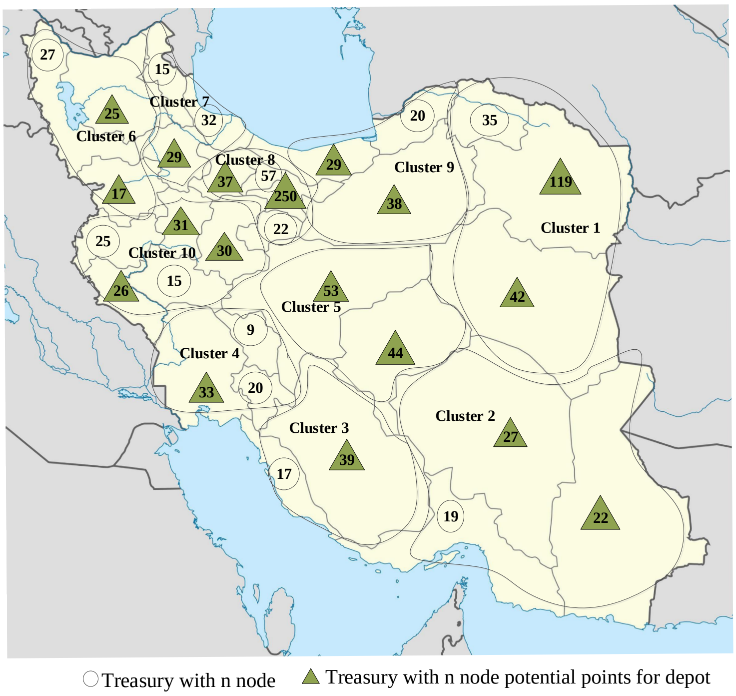


Figure 1: Clusters in case study. (number of branches is noted on the image)

The case study problems have a large number of branches in each cluster and large distance dimension, therefore, it is assumed in each cluster, the nodes are the close and node-based problem converting into the arc-based problem. Therefore, the branches of each city have been replaced by the ignorance of the distance and location with the required arc. The arc demand value of each city is equal to the sum demands branches of that city. But arc demand value of non-candidate treasury center is equal to demand of same treasury center. Service to the branches of these treasury is not responsible for central treasury. Selecting deadheading arc that are used as paths should be done before solving problems. Due to the case study conditions, these routes should have adequate and acceptable security. Therefore, all roads are evaluated by the AHP method based on indicators of road safety, traffic, accident history and road type (highway or subway). The time of travel on arcs are equal averaging the traveling times under different conditions. This case study is a two-period problem because the changes in cash demand in the first and second half of the year. The dimensions of the 10 problem of bank case study are presented in Table 4.

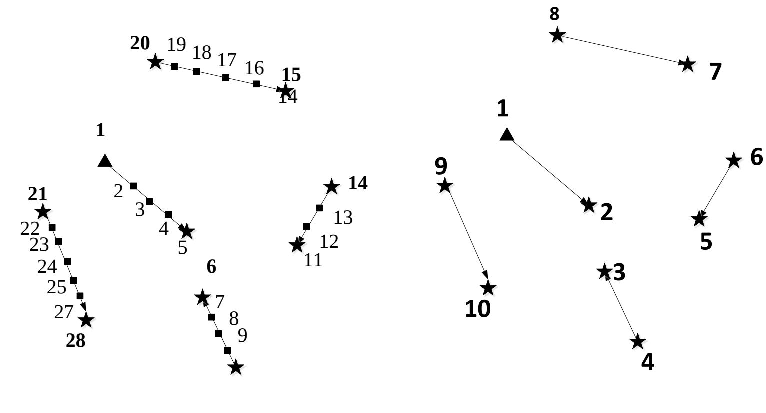
Table 4: Dimensions of case study problem

	Number of Cluster	1	2	3	4	5	6	7	8	9	10
period 1	Number of nodes	8	12	14	18	23	25	27	27	32	36
	Number of depot	1	1	1	2	2	2	2	2	2	3
	Number of vehicles	5	6	6	6	7	7	7	8	10	12
	Number of required arcs	8	10	14	15	20	22	28	30	34	36
period 2	Number of nodes	8	12	14	18	23	25	27	27	32	36
	Number of depot	1	1	1	2	2	2	2	2	2	3
	Number of vehicles	2	3	4	4	5	5	6	6	7	8
	Number of required arcs	10	14	12	17	18	24	24	28	33	38

5. Verification and validation of the model

The proposed mathematical model is solved by GAMS software, and all the numerical examples are tested on a PC with Intel Pentium 4, core i5, 2.4 GHz processor with 4 GB memory. In all of the LARP studies, it is noted that because this problem is NP-hard with a large number of binary variables, any exact method cannot be used to solve the model. Therefore, in this paper, we use the CPLEX solver in GAMS software for the bank case study. Selecting depot(s) in each cluster is possible by using two different models: 1) Since this problem is discrete and the demand of a treasury is dependent on the demands for its branches, this problem can be solved by the LRP model. 2) Since this problem is large-size (Figure 1), the problem can be solved by the LARP model considering the assumption of using arcs instead of nodes. This strategy reduced the dimension of clusters.

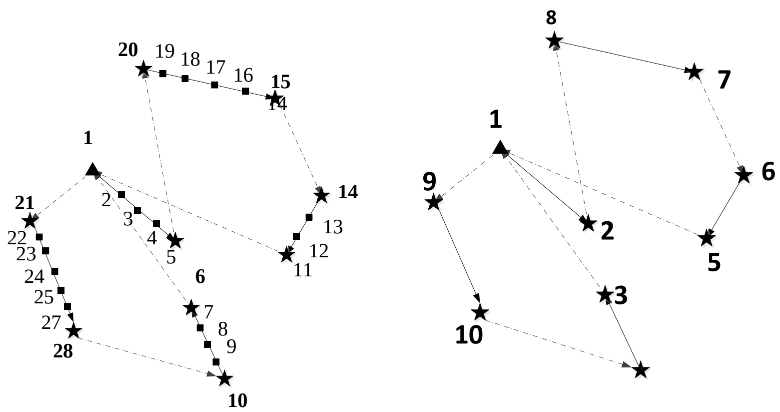
To check the proposed model, cluster 1 is solved with both the LRP model (Figure 2a) and deterministic LARP model (Figure 2b) and the solutions are analysed. To solve the model, the following points are considered: In Figure 2a, 1) All the branches of a city assumed to be located on a straight line and 2) the treasury in each city are assumed as a branch that has cash demand. In Figure 2b, branches located on a straight line is replaced with an arc, whose demand is equal to the total demands of those branches. Although the results of the LRP (Figure 3a) and LARP (Figure 3b) show that the selected path is the same in both cases, it takes GAMS software 50 hours to solve the LRP model while the solution for LARP model is obtained in only 1 second. Certainly, using the LRP in other clusters with larger sizes is not reasonable, even if it results in a better solution.



(a) LRP (b) LARP

■ Branch ▲ Depot ★ Branches begin and end of each city

Figure 2: Cluster 1 for the LRP and LARP models



(a) LRP (b) LARP

Figure 3: Experimental results

6. Experimental results

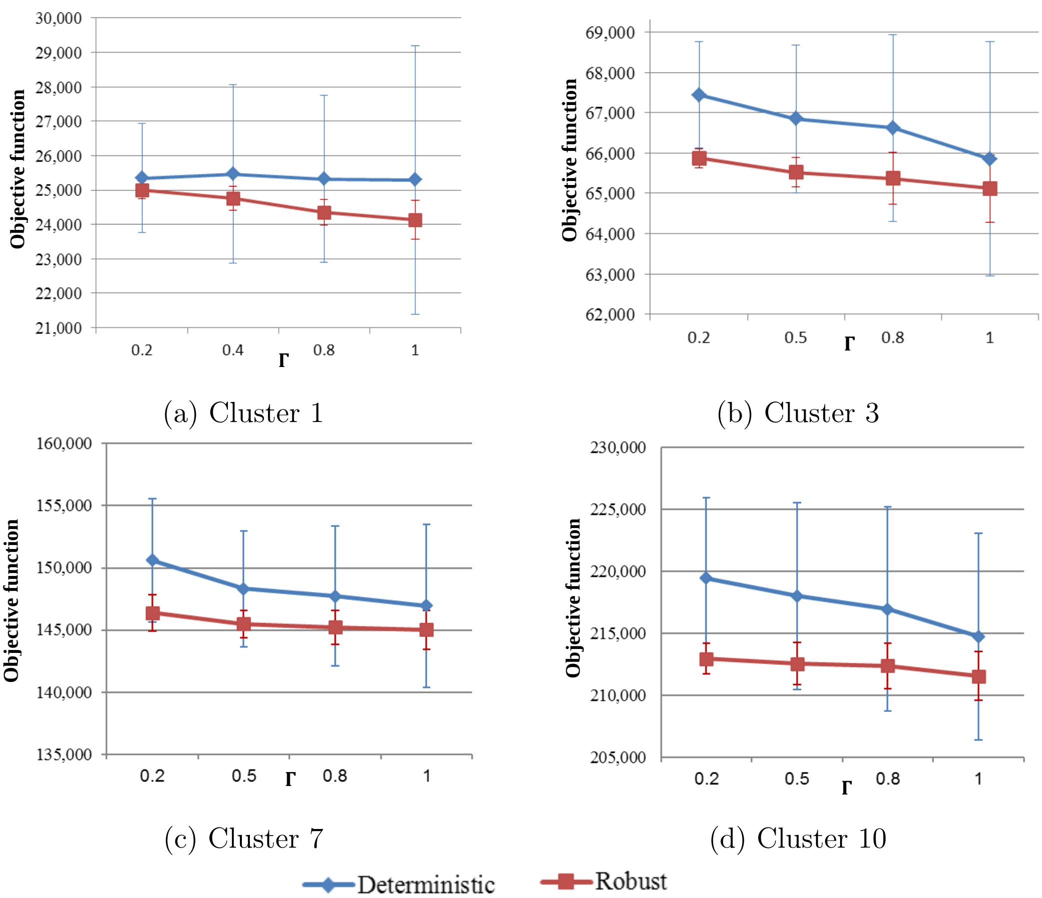
The proposed model has high complexity because of a large number of binary variables. To evaluate the performance of the proposed robust models, 10 clusters of the case study (Figure 1) are selected. The deterministic and robust model is solved for each cluster under nominal data. It should be noted that in the robust model are solved with $\Gamma = 0.2, 0.5, 0.8, 1$. Then for realization, 5 problems are solved in each cluster under each Γ. The performance measures of both the deterministic and robust models are the mean, standard deviation of objective function values under problems of case study realizations. Objective function value and computational time of solving each model are reported in Table 5 and Figure 4.

Table 5: Result of the deterministic and robust LARP models for case study

Cluster	Γ	Results with nominal data				Results under realizations			
		Cost		Time(s)		Mean		Standard deviation	
		Deterministic	Robust	Deterministic	Robust	Deterministic	Robust	Deterministic	Robust
1	0.2	25,698	25,949	1	1	25,346	25,000	1,590	251
	0.5		26,106		1.2	25,469	24,757	2,592	346
	0.8		26,430		1.45	25,326	24,348	2,415	367
	1		27,957		1.6	25,298	24,130	3,902	560
2	0.2	32,478	34,246	2,720	2,680	34,951	33,538	1,478	183
	0.5		34,884		2,825	34,997	32,942	2,346	209
	0.8		35,435		3,078	34,423	32,439	2,629	558
	1		38,103		3,234	34,136	32,178	3,762	630
3	0.2	65,421	67,514	5,806	6,080	67,443	65,870	1,325	236
	0.5		68,926		6,825	66,847	65,524	1,839	364
	0.8		71,583		7,634	66,623	65,373	2,323	647
	1		73,047		7,978	65,856	65,117	2,905	832
4	0.2	99,736	105,714	11,650	12,812	102,547	100,740	1,578	368
	0.5		106,978		13,354	101,940	100,273	2,476	509
	0.8		109,000		13,862	101,621	99,624	3,449	958
	1		112,546		14,200	100,941	99,480	3,862	1,260
5	0.2	123,654	124,937	16,320	17,031	129,347	125,382	1,979	718
	0.4		126,034		17,524	127,607	124,809	3,146	969
	0.8		127,745		17,860	126,402	124,373	4,749	1,128
	1		130,602		18,432	124,846	123,270	5,576	1,360
6	0.2	129,349	131,345	26,524	27,012	133,623	129,950	2,478	1,318
	0.5		132,489		27,854	133,321	129,573	4,546	1,579
	0.8		135,243		27,462	132,740	129,324	5,249	1,958
	1		137,030		28,900	130,247	128,974	4,762	2,260
7	0.2	145,654	149,364	33,047	35,321	150,623	146,370	4,972	1,435
	0.5		151,314		35,647	148,321	145,473	4,654	1,099
	0.8		157,974		36,250	147,734	145,224	5,650	1,365
	1		163,546		36,834	146,950	145,007	6,546	1,567
8	0.2	179,421	183,039	52,391	54,321	183,706	180,870	5,478	945
	0.5		184,986		55,734	182,422	179,773	6,546	1,309
	0.8		186,634		56,568	181,824	179,124	6,249	1,558
	1		188,912		56,630	181,132	179,007	8,762	1,660
9	0.2	192,345	200,378	87,034	89,107	204,313	193,587	6,478	1,225
	0.5		203,236		91,348	200,124	193,324	7,546	1,509
	0.8		206,879		92,420	198,740	192,873	8,249	1,658
	1		215,647		93,312	196,547	192,270	8,762	1,960
10	0.2	212,340	224,127	111,456	123,110	219,464	212,934	6,478	1,234
	0.5		227,923		124,265	218,010	212,562	7,546	1,709
	0.8		229,647		125,732	216,965	212,373	8,249	1,858
	1		236394		126,974	214,735	211,550	8,362	1,960

Computational results show that the robust model outperforms the deterministic model in both quality and standard deviation of the solutions. For results under realizations in the robust model, some demand values is in the worst case instead of the nominal value. Therefore, Bertsimas objective value is larger than the deterministic objective value.

The results Figure 4 imply that the robust strategy has a better performance on the large-sized problems and also higher Γ versus the deterministic one. The gap between (mean and Standard deviation) the two approaches increases with problem size and Γ. The results emphasize disregarding uncertainty can lead some losses in uncertain occurrences. Therefore, deterministic model is not optimum under realizations data. Time-solving has increased in the large-size problem. Time-solving in the robust model versus deterministic model is increased, because of the increasing number of constraints and complexity of model.



(a) Cluster 1

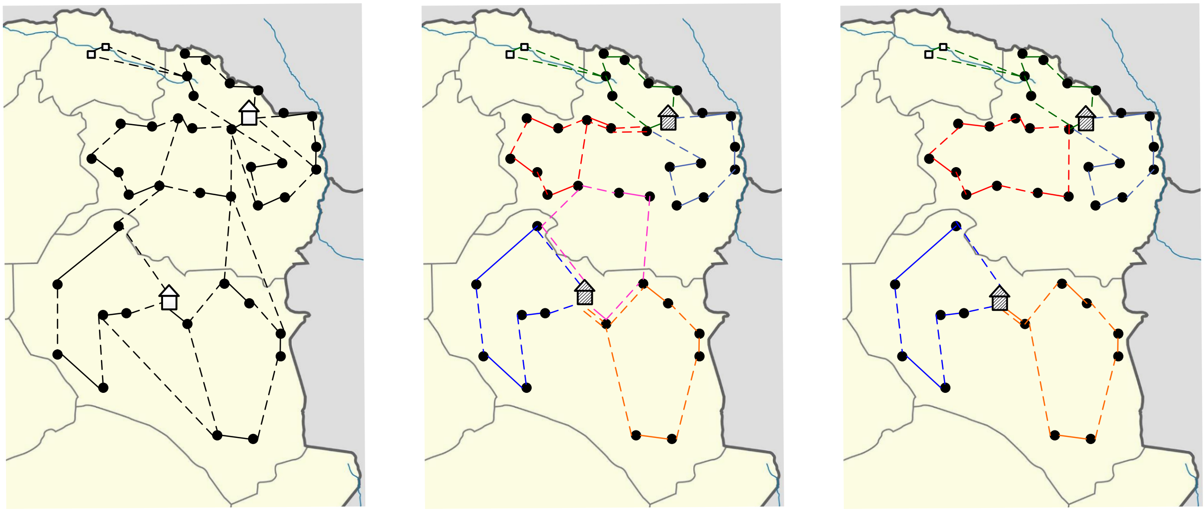
(b) Cluster 3

(c) Cluster 7

(d) Cluster 10

Figure 4: Results of case study solving and selected depot

To further explain of results Table 5, Cluster 1 is simplified and is shown in (Figure 5a). The solving results of deterministic and robust models under nominal data (Figure 5b) and realizations data (Figure 5c) are illustrated in Figure 5. Although, in both methods, the number of selected treasury is the same and all the required arcs are served, but in the deterministic method the selected routes have higher cost. In addition, according to the transported material(cash), choice of routes within the province is more desirable, which has happened in the robust method. Therefore, the robust method performs better than the results of the deterministic method.



(a) Cluster 1 (b) Deterministic result (c) Robust result

Depot potential point Selected depot Requeired arc Deadheading Arc

Figure 5: Results of case study solving and selected depot

Results of solving clusters case study with robust models in Figure 6 shows that 13 treasury from 18 potential point has selected. Selected this treasury for transforming cash between treasury central (in Tehran) and other treasury reduce bank costs and risk transforming cash.

Treasury with n node Selected Treasury with n node as depot

Figure 6: Results of case study solving and selected depot

7. Conclusion and future research

This paper presented a new mixed non-linear programming model for a multi-period LARP under uncertainty with time windows, time limitation for vehicles and using the vehicle for multiple customers. The mathematical model for time windows in the arc routing problem is rarely. The Bertsimas method was used for uncertainty in demand. The proposed model was linearized and solved with CPLEX solver in GAMS. The LARP model is used for reduced dimension problem size for a bank cast study and has better performance than LRP model. The realization of model demonstrates that the robust strategy versus deterministic has a better performance on the large-size problem and high uncertainty level. For future research, the model can be expanded with other real constraints under multiple objectives, and also different meta-heuristic algorithms can be proposed to solve the presented model.

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