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Using Laguerre Polynomials as a Basis for A new Differential Quadrature Methodology to Solve Magneto-Hydrodynamic (MHD) Fourth-Grade Fluid Flow

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Abstract

The addition of an application of a new version of the Differential Quadrature Method is the purpose of this work. The new method, tracing Laguerre polynomials, is applicable to test functions whose purpose is to establish the DQM weighting coefficients, focussing on the use of the DQM in investigating solving nonlinear differential equations numerically for the representation of the steady incompressible flow problem of a fourth-grade non-Newtonian fluid magnetic field between two stationary parallel plates. A series of graphs are used to demonstrate the ways a range of important physical parameters influence the velocity profile. The level of agreement when comparing a small number of grid points in the new technique with analytical solutions is remarkably high.

Keywords: MHD fluid, fourth-grade fluid, Laguerre polynomials, differential quadrature method.

1. Introduction

The considerable implications of non-Newtonian fluids used in technical and industrial fields have given rise to great interest and research undertaken in this field recently. These fluids do not obey Newton's law since their viscosity changes according to the forces that influence them - for example

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hair care products, blood, cooking sauces, or ther substances such as honey, mud, paints, plastics, and polymer melts. These non-Newtonian fluids, which are both elastic and viscous, can be divided into two categories: fluids where the shear stress is only affected by the shear rate, or on both the shear rate and time. Their complexity makes a description of them as a single model impossible. What results is that the behaviour of such fluids under study requires a number of models, including second-grade, third-grade, and fourth-grade fluids. Second-grade fluids are of particular use in predicting differences in normal stress, but do no distinguish between thick and thin shear as the shear viscosity by nature cannot be changed. Thus, certain tests are made using third- or fourth-grade fluids. This research was carried out to solve the problems of fourth-grade fluid flow as a general class of second- and third-grade fluids.

Researchers have used a wide variety of both analytical and numerical methods for such problems. For example, Hayat [1], solved the steady flow of an incompressible fourth-grade fluid in the presence of a magnetic field between parallel plates, with one plate not moving and the other in motion parallel to it at a constant speed with a suction velocity normal to the plates with the finite difference approximation. Numerical solutions for a system of non-linear equations with the use of the spline collocation method based on cubic B-spline functions by Patel [2]. Moakher et al., [3] and [4], used the collocation method (CM) and least square method (LSM) respectively, with the aim ofachieving a solution to the incompressible fully developed flow of fourth-grade fluid in a flat channel under an externally applied magnetic field width, taking slip conditions at the wall of the channel. An alternative method was that used by Al-Saif and Assma [5] where a perturbation-iteration algorithm was utilised to solve nonlinear differential equations that represent the steady incompressible flow problem of a fourth-grade non-Newtonian fluid between two non-moving parallel plates in the presence of a magnetic field. Khan et al.'s work [6], discusses the fundamental governing continuity, momentum, and energy equations for thin film flows of two non-mixing third-grade fluids past a vertical moving belt with slip conditions with a uniform magnetic field present being incorporated and solved with the use of the Adomian decomposition method (ADM) and the homotopy analysis method (HAM). Thus, solving these problems requires finding numerical schemes which offer greater stability, increased accuracy, and improved convergence.

The Differential Quadrature Method (DQM), which Bellman and Casti (1971) [7] first propounded, approximates the partial derivatives in spatial coordinates as linear combinations of the values of the dependent variable at a limited number of grid points, which allow the conversion of the PDE to a series of either algebraic or ordinary differential equations. According to Shu (2000) [8], standard numerical methods can then be used to obtain the solution. The DQM has been widely used as it uses far fewer nodal points and provides a degree of simplicity. Bert and Malik (1996) [9] used it to solve solid mechanics problems. Through the use of a selection of test functions, the development of different types of DQMs has been made possible, such as Bellman et al.'s [10], use of Legendre polynomials and spline functions to obtain weighting coefficients. Meanwhile, Quan and Chang [11] provided an explicit formulation to determine the weighting coefficients utilising Lagrange interpolation polynomials. Shu and Richards (1992) [12] presented explicit formulae featuring both Lagrange interpolation polynomials. Moreover, the explicit determination of the weighting coefficients with the employment of the Lagrange interpolated trigonometric polynomials was carried out by Shu and Xue [13]. Guo and Zhong [14] used a spline function.

Ding et al., (2006) [15] used the radial basis functions as the test functions, rather than utilising high-order polynomials to solve three-dimensional incompressible viscous flows in the primitive variable, employed the local multiquadric Differential Quadrature Method to solve Navier-Stokes equations. Shu and Wu [16] solved a one-dimensional Burger equation and by solving Navier-Stokes equations with integrated radial basis functions were able to simulate natural convection in an aconcentric annulus.

Korkmaz and Dag [17] used Chebyshev polynomials as the test function to numerically solve the Schrödinger equation for various initial conditions. Amin et al. [18], developed the Differential Quadrature Method, using harmonic functions rather than polynomials as a test function for harmonics, while Gorgun [19] proposed a Fourier-based Differential Quadrature Method to obtain a solution to the 2D Helmholtz problem. Al-Saif and Al-Saadawi [20] succeeded with their use of Bernstein polynomial to solve the equations relating to the unsteady flow of a polytropic gas. Watson [21] analysed the Radial Basis Function Differential Quadrature (RBFDQ) method, applying the local RBFDQ method to solve boundary value problems in annular Poisson equation domains, a non-homogeneous biharmonic equation, and the non-homogeneous Cauchy-Navier elasticity equations. In order to solve the extended Fisher-Kolmogorov equation, Bashan et al., [22] proposed a modified quartic B-spline Differential Quadrature Method.

The attention all the foregoing authors have dedicated to developing the DQM with the use of different test procedures to compute the DQM weighted coefficients because of the important role it occupies in achieving precise numerical solutions has inspired the authors of this current paper to research into the advantages of polynomials and their suitability for use with the DQM application. The simple definition, and fast, effective calculation on computer systems make Laguerre polynomials especially helpful terms of mathematics. Furthermore, they also represent an extensive range of functions, having frequently been used to solve differential equations. For these reasons and in accordance with our knowledge that the Laguerre polynomials have not yet been used to calculate weighting coefficients, the researchers decided to use them in this work.

In the present research, a modified Laguerre polynomial, called the Laguerre Differential Quadrature Method (the LgDQM), was suggested with the purpose of determining the weighting coefficients of the DQM, and was tested on the issue of the steady flow of fourth-grade fluids between two non-moving parallel plates with a magnetic field effect. Compared with other analytical methods [3-5], this procedure has clearly produced impressive results, despite the number of grid points being small.

2. The Laguerre Differential Quadrature Method (the LgDQM)

The modified Laguerre polynomials are rendered $L_n^k(x)$ and characterised by two parameters, the index n and the other value k , which can be any real number greater than zero. They constitute an important set of orthogonal polynomials over the interval $[0,\infty)$ [23].

Let $\omega_k(x)=e^{-kx}, k>0$, define the weighted space$L^2_{\omega_k}(0,\infty)$ as in normal procedures, with the following inner product and norm:

$$(u,v)_{\omega_k}=\int_0^\infty u(x)v(x)\omega_k(x)dx, \parallel v \parallel_{\omega_k}=(u,v)_{\omega_k} \qquad (1)$$

The general form of these polynomials is defined according to the Rodrigues formula:

$$L_n^k(x)=\frac{1}{n!}e^{kx}\frac{d^n}{dx^n}(x^n e^{-nx}), n\geq 0, k>0, \qquad (2)$$

which satisfies the recurrence relation

$$\frac{d}{dx}L_n^k(x)=\frac{d}{dx}L_{n-1}^k(x)-kL_{n-1}^k(x), n\geq 1, \qquad (3)$$

The set of Laguerre polynomials is a complete $L^2_{\omega_k}(0,\infty)$ orthogonal system, which is to say

$$(L_n^k,L_m^k)_{\omega_k}=\frac{1}{k}\delta_{n,m}, \qquad (4)$$

where $\delta_{n,m}$ is the Kronecker delta symbol. The first few modified Laguerre polynomials are:

$$
\begin{aligned}
&(L_0^k(x) = 1, L_1^k(x) = -k+1, L_2^k(x) = \frac{1}{2}kx^2 - 2x + 1,\\
&L_3^k(x) = -\frac{1}{6}(kx)^3 + \frac{3}{2}(kx)^2 - 3kx + 1,\\
&L_4^k(x) = \frac{1}{24}(kx)^4 - \frac{2}{3}(kx)^3 + 3(kx)^2 - 4kx + 1,\\
&L_5^k(x) = \frac{1}{120}(kx)^5 + \frac{5}{24}(kx)^4 - \frac{5}{3}(kx)^3 + 5(kx)^2 - 5kx + 1..,),, \qquad (5)
\end{aligned}
$$

for $k = 1, L_n^k(x)$ become the normal Laguerre polynomials $L_n(x)$.

Essentially, the DQM is the partial (ordinary) derivative of a function in conjunction with a variable in governing an equation approximated by a weighted linear sum of function values at all discrete points in that direction is:

$$\frac{\partial^m u}{\partial x^m} = \sum_{l=1}^{N} A_{ik}^{(m)} u(x_l, y_j), i = 1,2,...,N, j = 1,2,...,M, m = 1,...,N-1 \qquad (6a)$$

$$\frac{\partial^n u}{\partial y^n} = \sum_{l=1}^{M} A_{jl}^{(n)} u(x_i, y_l), i = 1,2,...,N, j = 1,2,...,M, n = 1,...,M-1 \qquad (6b)$$

where (x_i, y_j) are the discrete points in the variable,$u(x_i, y_j)$ are the function values at these points, and $A_{ik}^{(m)}, A_{jl}^{(n)}$ are the weighting coefficients for the m^{th} andn^{th} order derivatives of the function in conjunction with x and$y$, and N and M denote how many grid points there are. Establishing the weighting coefficients and selecting the sampling points are important factors connected with the accuracy of the DQM solution [24]. In fact, the Laguerre DQM uses two sets of base polynomials, one being the base modified Laguerre polynomial, and the other the terms of the Lagrange interpolating polynomial given by

$$r_l(x) = \frac{M(x)}{(dL_n^k(x) - dL_n^k(x_l))P(x_i)} forl = 1,2,...,N, \qquad (7)$$

Where

$$M(x) = \prod_{l=0}^{N} (dL_n^k(x) - dL_n^k(x_l)), \qquad (8)$$

$$P(x_i) = \prod_{l=0, l\neq i}^{N} (dL_n^k(x_i) - dL_n^k(x_l)), \qquad (9)$$

such that, simply, set $\frac{dL_n^k}{dx} = dL_n^k$,

The use of the second set of base vectors became necessary because of the employment of the above two sets of base vectors to extract explicit formulations for the derivation of the weighting coefficients of the first- and second-order derivatives. In the interests of simplicity, we set

$$M(x) = \prod_{l=0}^{N} (dL_n^k(x) - dL_n^k(x_l)) = N(x, x_l)(dL_n^k(x) - dL_n^k(x_l)), \qquad (10)$$

where

$$N(x_i, x_l) = \prod_{l=0, l\neq i}^{N} (dL_n^k(x_i) - dL_n^k(x_l)) = P(x_i), \qquad (11)$$

$$N(x_i, x_j) = N(x_i, x_j)\delta_{ij}, \qquad (12)$$

$$\delta_{ij} = \begin{cases} 1 & if \quad i = j \\ 0 & if \quad i \neq j \end{cases} \qquad (13)$$

is the Kronecker delta operator.

Equation (7) can be reduced to:

$$r_l(x) = \frac{N(x, x_l)}{P(x_l)}, \qquad (14)$$

Let all the base vectors given by equation (14) satisfy equations (6a) and (6b) to obtain:

$$A_{ij}^{(1)}(x) = \frac{N^{'}(x_i, x_j)}{P(x_j)}, \qquad (15)$$

$$A_{ij}^{(2)}(x) = \frac{N^{''}(x_i, x_j)}{P(x_j)}, \qquad (16)$$

From equations (15) and (16), the computation of $A_{ij}^{(1)}$ and $A_{ij}^{(2)}$ is the same as the $N^{'}(x_i, x_j)$ evaluations and is evident, as equation (11) can be used to calculate $P(x_j)$ to obtain

$$M^{'}(x) = N^{'}(x, x_l)(dL_n^k(x) - dL_n^k(x_l) + N(x, x_l d^2 L_n^k(x)), \qquad (17)$$

$$M^{''}(x) = N^{''}(x, x_l)(dL_n^k(x) - dL_n^k(x_l) + 2N^{'}(x, x_l)(d^2 L_n^k(x)) + N(x, x_l)(d^3 L_n^k(x)), \qquad (18)$$

$$M^{'''}(x) = N^{'''}(x, x_l)(dL_n^k(x) - dL_n^k(x_l) + 3N^{''}(x, x_l)(d^2 L_n^k(x)) + 3N^{'}(x, x_l)(d^3 L_n^k(x)) + N(x, x_l)(d^4 L_n^k(x)), \qquad (19)$$

The following are results from the above equations (17, 18, 19):

$$N^{'}(x_i, x_j) = \frac{M^{'}(x_i)}{(dL_n^k(x_i) - dL_n^k(x_j))} fori \neq j, \qquad (20)$$

$$N^{'}(x_i, x_j) = \frac{M^{''}(x_i)}{2(d^2 L_n^k(x))} fori = j, \qquad (21)$$

$$N^{''}(x_i, x_j) = \frac{M^{''}(x_i) - 2(d^2 L_n^k(x_i))N^{'}(x_i, x_j)}{(dL_n^k(x_i) - dL_n^k(x_j))} fori \neq j, \qquad (22)$$

$$N^{''}(x_i, x_i) = \frac{M^{'''}(x_i) - 4(d^3 L_n^k(x_i))N^{'}(x_i, x_i)}{(3d^2 L_n^k(x_i))} fori = j, \qquad (23)$$

When equations (20) and (21) were substituted into equation (15), the following was obtained

$$A_{ij}^{(1)} = \frac{P(x_i)}{(dL_n^k(x_i) - dL_n^k(x_j))P(x_j)}, fori \neq j, \qquad (24)$$

$$A_{ii}^{(1)} = \frac{P(x_i)}{2(d^2 L_n^k(x))P(x_i)}, fori = j, \qquad (25)$$

Equation (25) can be written as [24]

$$A_{ii}^{(1)} = -\sum_{j=1}^{N} A_{ij}^{(1)}, \qquad (26)$$

Equations (22) and (23) were substituted into equation (16), and the following result was obtained

$$A_{ij}^{(2)} = \frac{M''(x_i) - 2(d^2L_n^k(x_i))M'(x_i)}{(dL_n^k(x_i) - dL_n^k(x_j))^2 P(x_j)}, fori \neq j, \qquad (27)$$
$$A_{ii}^{(2)} = \frac{2(d^2L_n^k(x_i))M'''(x_i) - 4(d^3L_n^k(x_i))M''(x_i)}{6(d^2L_n^k(x))P(x_i)}, fori = j, \qquad (28)$$

Equation (28) can also be written as [24]

$$A_{ii}^{(2)} = -\sum_{j=1}^{N} A_{ij}^{(2)}, \qquad (26)$$

The second-order derivative weighted coefficients can be obtained thus:

$$[A_{ii}^{(2)}] = [A_{ii}^{(1)}][A_{ii}^{(1)}] = [A_{ii}^{(1)}]^2, \qquad (30)$$

Weighting coefficients $A_{ii}^{(m)}$ can be obtained employing the same technique:

3. Governing equations

It is possible to express the basic equations which govern the flow of an incompressible fluid as follows:

$$\nabla . V = 0,$$
$$\rho \frac{Dv}{Dt} = \nabla . \tau + J \times B, \qquad (31)$$

Here, V denotes the velocity vector, ρ is the constant density, ∇ is the Nabla operator,τ is the stress tensor, J is the electric current density and B is the total magnetic field such that $J \times B = \sigma(V \times B) \times B, \sigma$ is the electrical conductivity of the fluid,$B = (0, B_0, 0)$, and $\frac{D()}{Dt}$ signifies the material derivative. The stress tensor τ defining an-grade fluid is the result of

$$\tau = -pI + \sum_{i=1}^{n} S_i \qquad (32)$$

where p is the pressure, I is the identity tensor, $\mu, \alpha_1, \alpha_2, \beta_1, \beta_2, \beta_3, \gamma_1, \gamma_2, \gamma_3, \gamma_4, \gamma_5, \gamma_6, \gamma_7$ and γ_8 are material constants, and

$$A_0 = I,$$
$$A_n = \frac{dA_{n-1}}{dt} + A_{n-1}(\nabla V) + (\nabla V)^t A_{n-1}, n \geq 1 \qquad (33)$$

The flow of a fourth-grade fluid between two non-moving parallel plates is considered by the researchers to be towards x at distance 2d in a magnetic field, where the constant pressure gradient drives, as illustrated in Figure 1 below.

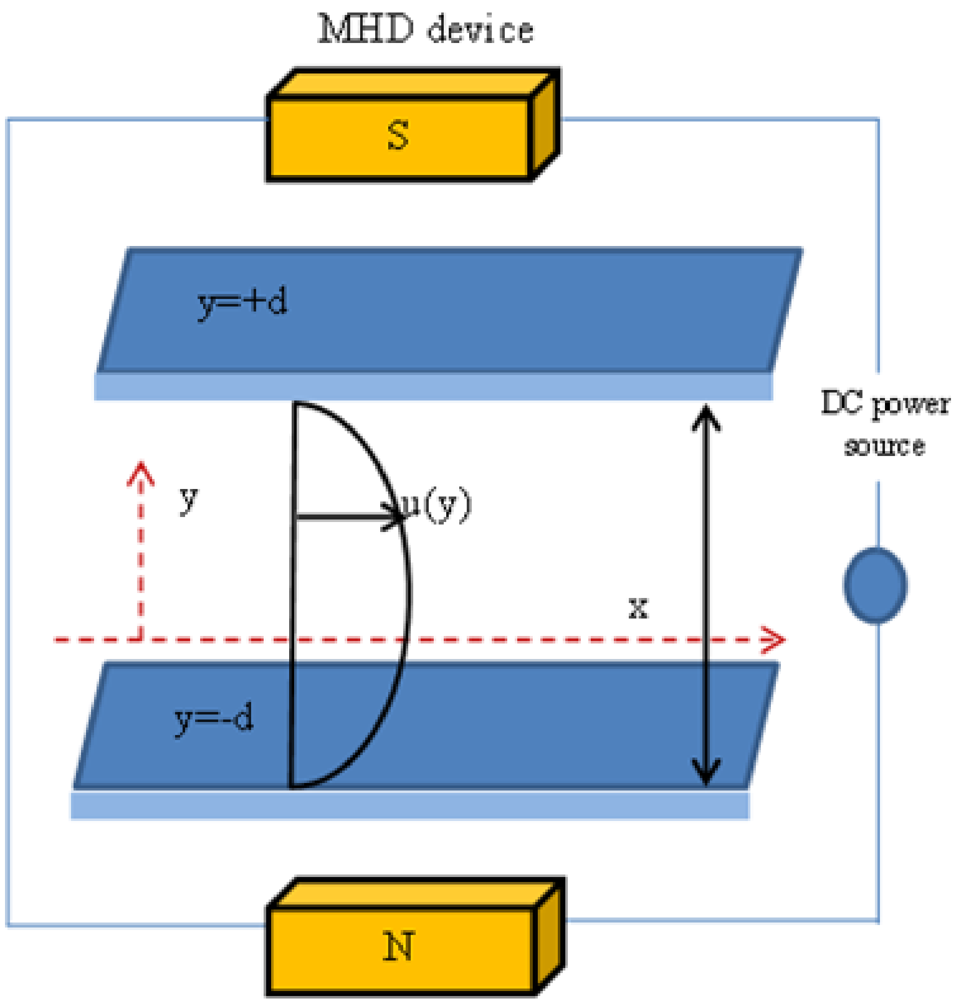

Figure 1: The physical system of the flow problem

It was noted that when $n = 4$ in Eq. (2) the fourth-grade fluid can be inferred. Then, the definition of the components of the stress tensor τ is

$$
\begin{aligned}
S_1 =& \mu A_1,\\
S_2 =& \alpha_1 A_2 + \alpha_2 A_1^2,\\
S_3 =& \beta_1 A_3 + \beta_2 (A_1 A_2 + A_2 A_1) + \beta_3 (tr(A_1)^2) A_1,\\
S_3 =& \beta_1 A_3 + \beta_2 (A_1 A_2 + A_2 A_1) + \beta_3 (tr(A_1)^2) A_1,\\
S_4 =& \gamma_1 A_4 + \gamma_2 (A_3 A_1 + A_1 A_3) + \gamma_3 (A_2)^2 + \gamma_4 (A_2 A_1^2 + A_1^2 A_2) + \gamma_5 (tr A_2) A_2 + \gamma_6 (tr A_2) A_1^2\\
&+ (\gamma_7 (tr A_3) + \gamma_8 (tr(A_2 A_1))) A_1, \qquad (34)
\end{aligned}
$$

The component forms of the momentum Equation (31) can be rendered as

$$x - component: \quad -\frac{dp}{dx} + \mu \frac{d^2u}{dy^2} + 6(\beta_2 + \beta_3)(\frac{du}{dy})^2 \frac{d^2u}{dy^2} - \sigma B_0^2 u(y) = 0, \qquad (35)$$

$$y - component: \quad -\frac{dp}{dy} + (2\alpha_1 + \alpha_2)\frac{d}{dy}(\frac{du}{dy})^2 + 4(\gamma_3 + \gamma_4 + \gamma_5 + 0.5\gamma_6)\frac{d}{dy}(\frac{du}{dy})^4 = 0, \qquad (36)$$

$$z - component: \quad -\frac{dp}{dz} = 0, \qquad (37)$$

By integrating Equation (7) with respect to , the following is obtained:

$$p^* = -p + (2\alpha_1 + \alpha_2)\frac{d}{dy}(\frac{du}{dy})^2 + 4(\gamma_3 + \gamma_4 + \gamma_5 + 0.5\gamma_6)\frac{d}{dy}(\frac{du}{dy})^4 = 0, \qquad (38)$$

Since $\frac{dp^*}{dy} = 0$, and $\frac{dp^*}{dz} = 0$, then $p^* = p^*(x)$. Equations (35), (36), and (37) can then be reduced to a single equation:

$$\mu(\frac{d^2u}{dy^2}) + 6\beta(\frac{du}{dy})^2(\frac{d^2u}{dy^2}) - \sigma B_0^2 u(y) = A, \qquad (39)$$

where $\beta = \beta_2 + \beta_3$ and $A = \frac{dp^*}{dx}$, reducing the problem of solving the second-order nonlinear ordinary differential Eq. (39) with these boundary conditions as follows:

$$\frac{du}{dy}|_{y=0} = 0 \& \frac{du}{dy}|_{y=d} = -\lambda u(d), \qquad (40)$$

The equations above are non-dimensional with scales

$$\eta = \frac{y}{d}, U(\eta) = \frac{\mu u(y)}{Ad^2}, N_f = \frac{A^2 d^2 \beta}{\mu^3}, Ha = B_0 d\sqrt{\frac{\sigma}{\mu}}, \qquad (41)$$

Eqs. (39) and (40) in a non-dimensional form become

$$\frac{d^2U}{d\eta^2} + 6N_f(\frac{dU}{d\eta})^2 \frac{d^2U}{d\eta^2} - Ha^2 U - 1 = 0, \qquad (42)$$

with

$$\frac{dU}{d\eta}|_{\eta=0} = 0 \& \frac{dU}{d\eta}|_{\eta=1} = -\lambda U(1), \qquad (43)$$

4. Application of the LgDQM to the problem, and results

In this problem, a range of values of equally-spaced grid points was used. Equation (42) can be approximated by using the LgDQM as follows:

$$\sum_{j=1}^{N} A_{ii}^{(2)} u_j + 6N_f(\sum_{j=1}^{N} A_{ii}^{(1)} u_j)^2(\sum_{j=1}^{N} A_{ii}^{(2)} u_j) - Ha^2 u_i - 10,$$

where $A_{ii}^{(2)}$ and $A_{ii}^{(1)}$ are the weighted coefficients of the first- and second-order derivatives defined in equations (24), (26), and (30). We approximate the boundary conditions for the above problem by means of the centre finite difference method. The resulting set of $N \times N$ nonlinear systems for u are solved by Newton-Raphson.

In the present study, comparing the results the new technique's numerical solution with those of other studies for this problem is focused upon, with the purpose of examining the accuracy of the current solution process and methodology.

In the following computations, the parameters $\kappa = 0.625$ and $n = 3$ are used to compute the weighting coefficient in Eqs. (24, 25, 27, 28) by using the LgDQM. The velocity distribution equation can be established as in Tables (1-2) for different grid points ($N = 7, 9, 11$). Table 3 shows the comparison of results to compute velocity $U(\eta)$ for $N = 7, 9, 11, \lambda = 0.4, Ha = 1$ and $N_f = 1$ between the applied method, the LgDQM, with the results of other studies obtained by a perturbation iteration (PIA) [5] and [3, 4] utilising the collocation method and least square method, respectively. From Tables (1-3), it becomes evident that the LgDQM results more closely approximate to the results [5], [3], and [4]. All the results are calculated using Maple language programs.

The physical meaning of the governing parameters is as follows: Ha signifies the Lorentz force, which acts perpendicularly to the direction of the applied magnetic field, i.e. in the present geometry it acts against the u-velocity component. This is to say, the Ha number suppresses the fluid velocity in the x-direction. The non-Newtonian parameter N_f accounts for the effect of the viscosity variation with the shear rate. In effect, by increasing N_f, the viscosity of the fluid increases with the increasing shear rate. With the roles of these two parameters in mind, the following graphical results can be explained.

Figures 2 and 3 provide the results obtained from making use of the LgDQM and are for different active parameter values. Figure 4 shows the inverse relationship between the velocity and the magnetic parameter Ha, where the value of velocity is reduced as the magnetic parameter value is increased, since the applied magnetic field affects the Lorentz force form, so diminishing the velocity value. Figure 5 demonstrates the non-Newtonian parameter Nf variation on the velocity at $\lambda = 1$ and $Ha = 1$. Meanwhile, Figure 6 shows the effect of the slip parameter λ on the velocity$U(\eta)$, at $Nf = 0.5$ and $Ha = 1.5$. In this figure the decrease in velocity with an increase in the slip parameter can be seen. The principal reason for this is the increase of the slip parameter in some parts of fluid molecules striking a solid surface and then being reflected diffusely, which increases and then decreases the velocity. It was recorded that the LgDQM gives results close to other results for this problem.

Table 1: The present results of the LgDQM for$N = 7, \lambda = 0.4, Ha = 1\ and\ N_f = 1$

η	$U(\eta)$
0	-0.7556566416
0.16667	-0.7556313323
0.33333	-0.7462780727
0.50000	-0.7283021259
0.66666	-0.7021499925
0.83333	-0.6680972818
1.0000	-0.6263189611

Table 2: The present results of the LgDQM for $N = 7, \lambda = 0.4, Ha = 1\ and\ N_f = 1$

η	$U(\eta)$
0	-0.7567633194
0.1250	-0.7567657026
0.2500	-0.7516878018
0.3750	-0.7418421284
0.5000	-0.7274546013
0.6250	-0.7086856546
0.7500	-0.6856480982
0.8750	-0.6270702929
1.0000	-0.6270702929

Table 3: Comparison between the present result using the LgDQM and other results using PIA, CM and LSM for $U(\eta)$ at $N = 11, \lambda = 0.4, Ha = 1\ and\ N_f = 1$

η	LgDQM	PIA [5]	CM [3]	LSM [4]
0	-0.7564385546	-0.7541296359	-0.755435424	-0.756148161
0.10	-0.7564200582	-0.7528998171	-0.754233549	-0.754928597
0.20	-0.7532155367	-0.7492048374	-0.750610552	-0.751266500
0.30	-0.7469888395	-0.7430287564	-0.744540374	-0.745151678
0.40	-0.7378675118	-0.7343471653	-0.735996956	-0.736567863
0.50	-0.7259497463	-0.7231306774	-0.724954241	-0.725493618
0.60	-0.7113104257	-0.7093503270	-0.711386168	-0.711903466
0.70	-0.6940064395	-0.6929854610	-0.695266681	-0.704156173
0.80	-0.6740813991	-0.6740349439	-0.676569720	-0.677060964
0.90	-0.6515698593	-0.6525328333	-0.6552692277	-0.655748735
1.00	-0.6265011272	-0.6285701164	-0.631339143	-0.631802927

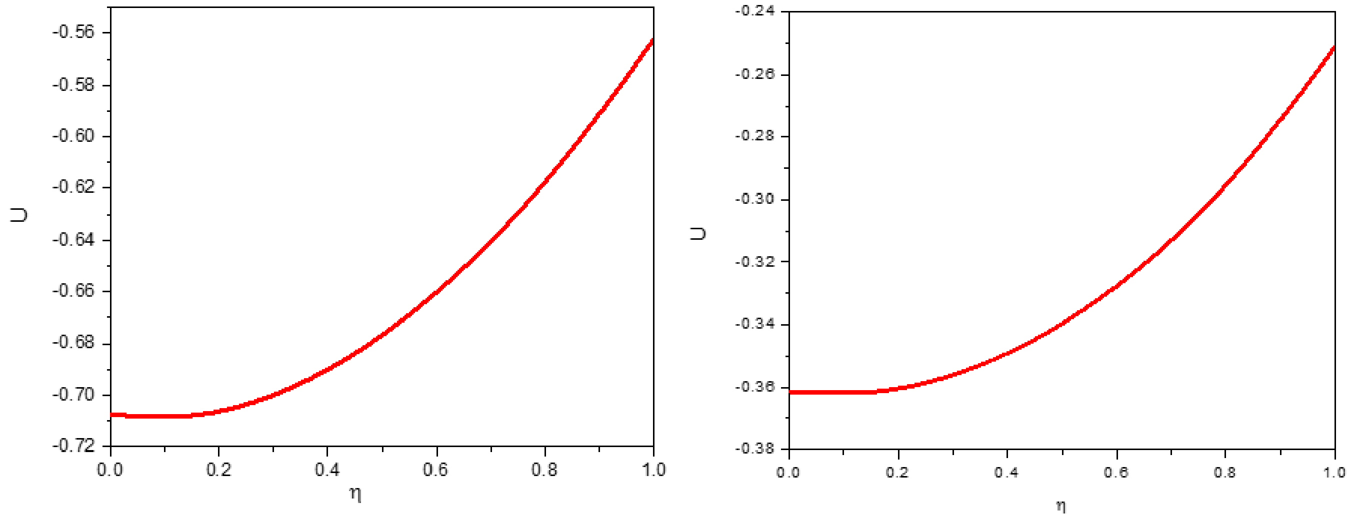

Figure 2: Numerical soution of velocities in $Ha = 1, N_f = 1, \lambda = 0.5$

Figure 3: Numerical soution of velocities in $Ha = 1.5, N_f = 0.3, \lambda = 0.9$

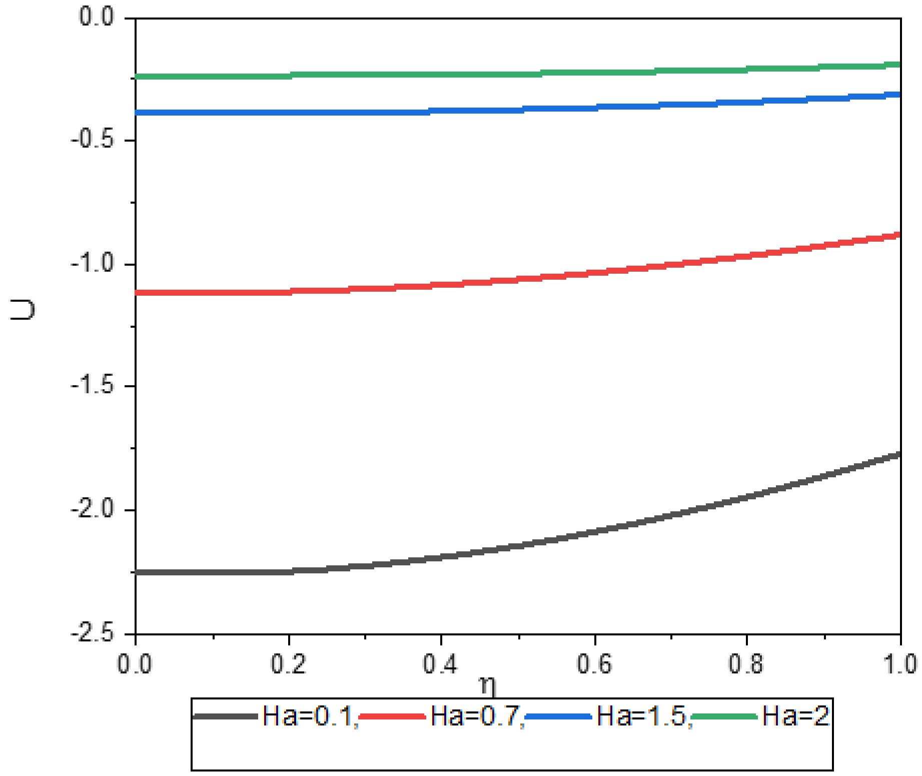

Figure 4: Numerical soution of velocities in $\lambda = 0.5, N_f = 0.1$ for different Ha

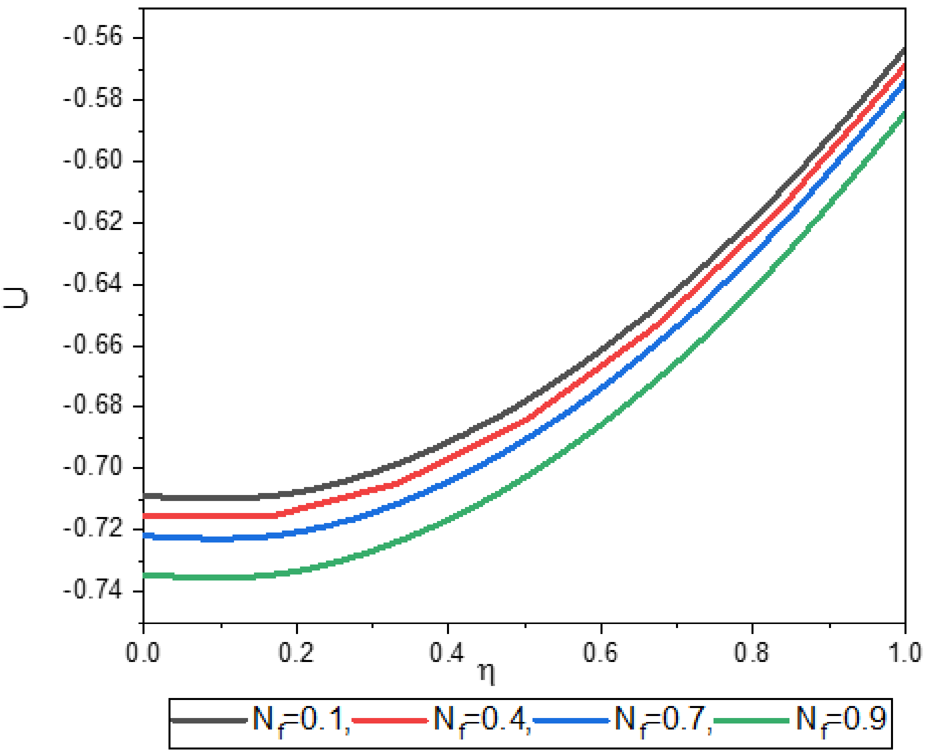

Figure 5: Numerical soution of velocities in $\lambda = 0.5, Ha = 1$ for different N_f

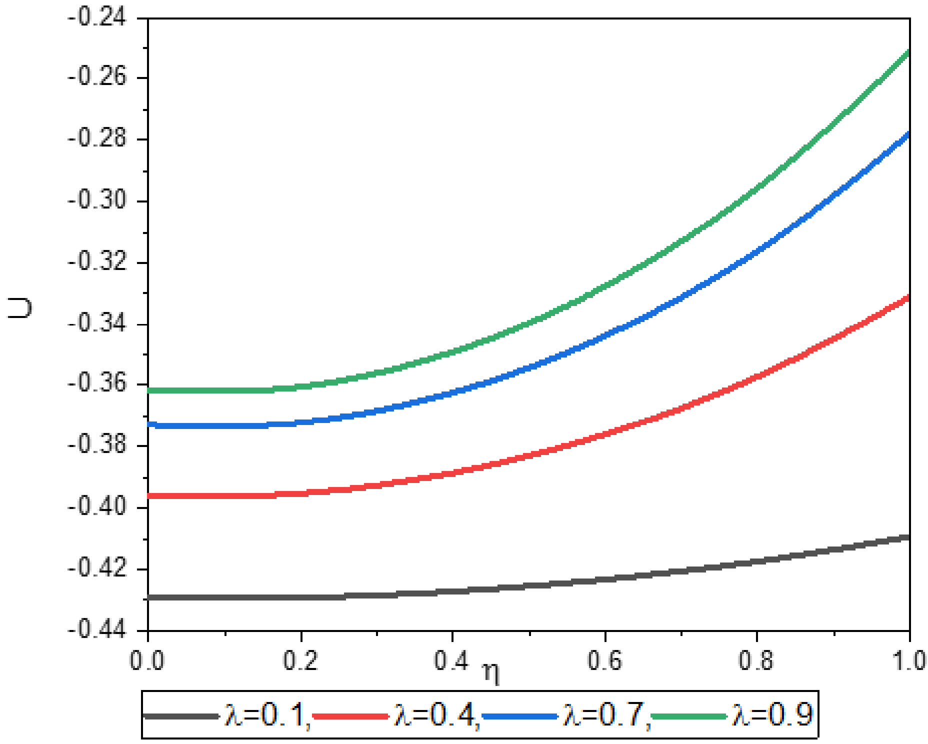

Figure 6: Numerical soution of velocities in $N_f = 0.5, Ha = 1.5$ for different λ

5. Conclusions

The problem of steady incompressible flow of a fourth-grade fluid between two non-moving parallel plates where a magnetic field is present has been solved with the successful application of the new LgDQM. Calculations and comparisons of the numerical results of this method were carried out with the results of previous studies [3], [4], [5]. Accompanying the research were investigations of the effects of the slip parameter, the non-Newtonian parameter, and the magnetic field parameter on the velocity. The numerical results evidence the fact that the LgDQM is advantageous, including its greater accuracy resulting from a diminished number of grid points for all the values of the physical parameters utilised in this problem.

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