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# On new classes of neutrosophic continuous and contra mappings in neutrosophic topological spaces

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# Abstract

The aim of this paper is to investigate some new types of neutrosophic continuous mappings like, neutrosophic  $\alpha^*$ -continuous mapping  $(N\alpha^* - CM)$ , neutrosophic irresolute  $\alpha^*$ -continuous mapping  $(NI\alpha^* - CM)$ , and neutrosophic strongly  $\alpha^*$ -continuous mapping  $(NS\alpha^* - CM)$  are given and some of their properties are studied. Moreover, new kind of neutrosophic contra continuous mappings is investigated in this work, it is called neutrosophic contra  $\alpha^*$ -continuous mapping  $(NC\alpha^* - CM)$ .

Keywords: neutrosophic sets, neutrosophic topological space, neutrosophic  $\alpha$ -open sets, neutrosophic  $\alpha^*$ -open set.

# 1. Introduction

In 1998, the connotation of Contra continuity is investigated by Dontchev [6]. Also, the connotation of  $\alpha^*$ -open set ( $\alpha^* - OS$ ) is shown [7]. The idea of neutrosophic sets is presented by Smarandache [35], in 2014, the connotations of "neutrosophic closed set "and" neutrosophic continuous function" are given.

The neutrosophic set is studied in topology, algerbra and other fields. It is one of the non-classical sets, such as soft set, fuzzy sets, nano set, permutation sets and so on, see [1, 3, 4, 6, 8, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 33, 36]. In this research, we introduce a new types of neutrosophic mappings, they are said neutrosophic  $\alpha^*$ -continuous and neutrosophic contra  $\alpha^*$ -continuous mappings. Next, we studied and discussed their basic properties.

# 2. Preliminaries

Here basic definitions and notations, which are used in this section are referred from the references [2, 5, 9, 32, 34].

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**Definition 2.1.** Assume that  $\Psi \neq \emptyset$ . A neutrosophic set (NS)  $\theta$  is defined as

$$\theta = \langle \alpha, \partial_{\varpi}(\alpha), \omega_{\theta}(\alpha), \ell_{\theta}(\alpha) : \alpha \in \Psi \rangle,$$

where  $\partial_{\varpi}(\alpha)$  is the degree of membership,  $\omega_{\theta}(\alpha)$  is the degree of indeterminacy and  $\ell_{\theta}(\alpha)$  is the degree of nonmembership, for all  $\alpha \in \Psi$ .

**Definition 2.2.** We say  $(\Psi, \tau)$  is a neutrosophic topological space (NTS) if and only if  $\tau$  is a collection of (NSs) in  $\Psi$  and it such that:

- (1)  $1_N, 0_N \in \tau$ , where  $0_N = \{ \langle \alpha, (0, 1, 1) \rangle : \alpha \in \Psi \}$  and  $1_N = \{ \langle \alpha, (1, 0, 0) \rangle : \alpha \in \Psi \}$ ,
- (2)  $A \cap \beta \in \tau$  for any  $\theta, \beta \in \tau$ ,
- (3)  $\bigcup_{i \in I} A_i \in \tau$  for any arbitrary family  $\{A_i \mid i \in I\} \subseteq \tau$ .

Moreover, any  $A \in \tau$  is called neutrosophic open set (NOS) and we say neutrosophic closed set (NCS) for its complement.

**Definition 2.3.** Assume A is a neutrosophic set in (NTS) X.

- (i) The neutrosophic closure (resp., neutrosophic  $\alpha$ -closure) of A is the intersection of all neutrosophic closed (resp., neutrosophic  $\alpha$ -closed) sets containing A and is denoted by Ncl(A) (resp.,  $Ncl_{\alpha}(A)$ ).
- (ii) The neutrosophic interior (resp., neutrosophic  $\alpha$ -interior) of A is the union of all neutrosophic open (resp., neutrosophic  $\alpha$ -open) sets are contained in A and is denoted by Nint(A) (resp., Nint<sub> $\alpha$ </sub>(A)), where A is neutrosophic  $\alpha$ -open set (N $\alpha$  – OS) (resp., neutrosophic semi  $\alpha$ -open set (NSe  $\alpha$  – OS), neutrosophic  $\alpha^*$ -open set (N $\alpha^*$  – OS) if  $A \subseteq$  Nint(Ncl(Nint(A))) (resp.,  $A \subseteq$  Ncl(Nint(Ncl(Nint(A)))) or equivalently  $A \subseteq$  Ncl(Nint(A)),  $A \subseteq$  Nint<sub> $\alpha$ </sub> (Ncl (Nint<sub> $\alpha$ </sub>(A))). Also, their complement are called neutrosophic  $\alpha$ -closed set (N $\alpha$  – CS) (resp., neutrosophic semi  $\alpha$ -closed set (NSe $\alpha$  – CS), neutrosophic  $\alpha^*$  – closed set (N $\alpha^*$  – CS).

The symbols of the above neutrosophic sets and their complements are referred as  $N\alpha - O(X)$ (resp., NSe  $\alpha - O(X), N\alpha^* - O(X)$ ),  $N\alpha - C(X)$  (resp., SNe  $\alpha - C(X), N\alpha^* - C(X)$ ).

**Proposition 2.4.** (1) If A is  $(N\alpha^* - OS)$  and B is (NOS), then  $A \cap B$  is  $(N\alpha^* - OS)$ .

(2) If  $\{G_{\lambda}\}_{\lambda\in\Gamma}$  is a collection of  $(N\alpha^* - OSs)$ , then their union is also  $(N\alpha^* - OSs)$ .

**Theorem 2.5.** Assume that  $X_1$  and  $X_2$  are two neutrosophic topological spaces (NTSs),  $A_1 \subseteq X_1$ and  $A_2 \subseteq X_1$ . Then  $A_1$  and  $A_2$  are  $(N\alpha^* - OSs)$  (resp.,  $(N\alpha^* - CSs)$ ) in  $X_1$  and  $X_2$ , respectively if and only if  $A_1 \times A_2$  is  $(N\alpha^* - OS)$  (resp.,  $(N\alpha^* - CS)$ ) in  $X_1 \times X_2$ .

**Theorem 2.6.** Assume that W is a subspace of Z satisfies  $G \subseteq W \subseteq Z$ . The following assertions hold.

- (i) If  $G \in N\alpha^* O(Z)$ , then  $G \in N\alpha^* O(W)$ .
- (ii) If  $G \in N\alpha^* O(W)$ , then  $G \in N\alpha^* O(Z)$ , where W is a neutrosophic closed subspace of Z.

**Proposition 2.7.** (1) Every (NOS) (resp.,  $N\alpha$ -open, Ncl-open) set is ( $N\alpha^* - OS$ ).

(2) Every  $(N\alpha^* - OS)$  is  $(NSe\alpha - OS)$ .

**Definition 2.8.** A (NTS) X is called a

- (i) neutrosophic ultra- $T_2$  (N-ultra- $T_2$ ) if for any  $t \neq h \in Z$ , there are two disjoint neutrosophic closed sets (NDCSs) T, H satisfy  $t \in T, h \in H$ .
- (ii) neutrosophic ultra normal, if for all neutrosophic closed sets (NCSs) T, F with  $T \neq \emptyset \neq F$  and  $T \cap F = \emptyset$ , there are two (NCSs) D, H with  $D \cap H = \emptyset$  and  $T \subseteq D, F \subseteq H$ .
- (ii) neutrosophic strongly closed if for any homely of (NCSs) that form a cover of X has a finite sub-homely that form a cover of X, too.

#### 3. The new types of neutrosophic $\alpha^*$ -continuity

The new types of neutrosophic  $\alpha^*$ -continuity like; neutrosophic irresolute  $\alpha^*$ -continuous mapping  $(NI\alpha^* - CM)$ , neutrosophic stronger  $\alpha^*$ -continuous mapping  $(NS\alpha^* - CM)$  and neutrosophic contra  $\alpha^*$ -continuous mapping  $(NC\alpha^* - CM)$  in this work are given. Furthermore, their relationships for these our notions are shown.

**Definition 3.1.** Assume that  $W_1$  and  $W_2$  are NTSs and  $h: W_1 \to W_2$  is any map from  $W_1$  into  $W_2$ . We say h is a neutrosophic  $\alpha^*$ -continuous mapping  $(N\alpha^* - CM)$  (resp., neutrosophic irresolute  $\alpha^*$ continuous mapping  $(NI\alpha^* - CM)$ , neutrosophic stronger  $\alpha^*$ -continuous mapping  $(NS\alpha^* - CM)$ mapping if for each G (NOS) (resp.  $N\alpha^* - OS$ ) in  $W_2$ , then  $h^{-1}(G)$  is  $N\alpha^* - OS$  (resp., (NOS)) in  $W_1$ .

**Lemma 3.2.** (1) Every  $(N\alpha^* - CM)$  is  $(NI\alpha^* - CM)$ .

(2) Every  $(NI\alpha^* - CM)$  is  $(NS\alpha^* - CM)$ .

**Proof**. It follows from Proposition 2.7.  $\Box$ 

**Theorem 3.3.** Assume that  $W_1$  and  $W_2$  are NTSs and  $h: W_1 \to W_2$ .

- (i) If h is  $(N\alpha^* CM)$ , then  $h|_G : G \to W_2$  is also, where G is (NOS) of  $W_1$ .
- (ii) If h is  $(NI\alpha^* CM)$ , then  $h|_G : G \to W_2$  is also, where G is (NOS) of  $W_1$ .

(iii) If h is  $(NSa^* - CM)$ , then  $h|_G : G \to W_2$  is also, where G is  $(N\alpha^* - OS)$  of  $W_1$ .

**Proof**. (i) Assume B is an (NOS) in  $W_2$ , since h is  $(Na^* - CM)$ ,  $h^{-1}(B)$  is  $(N\alpha^* - OS)$  in  $W_1$ , since G is (NOS) in  $W_1$ . Hence, by Proposition 2.4, we have  $h^{-1}(B) \cap G$  is  $(N\alpha^* - OS)$  in  $W_1$ , but

$$(h|_G)^{-1}(B) = h^{-1}(B) \cap G.$$

Thus by Theorem 2.6,  $(h|_G)^{-1}(B)$  is  $N\alpha^*$  – open in G. (ii) and (iii) are similar to (i).  $\Box$ 

**Theorem 3.4.** Suppose that  $h: W_1 \to W_2$  is any mapping and  $W_1 = T \cup H$ , where T, H are disjoint neutrosophic sets in  $W_1$ . Then,

- (i) h is  $(N\alpha^* CM)$  if and only if  $h|_T$  and  $h|_H$  are also, where T and H are neutrosophic open sets.
- (ii)  $h \text{ is } (NI\alpha^* CM) \text{ if and only if } h|_T \text{ and } h|_H \text{ are also, where } T \text{ and } H \text{ are neutrosophic open sets.}$
- (iii) h is ( $NS\alpha^* CM$ ) if and only if  $h|_T$  and  $h|_H$  are also, where T, H are neutrosophic  $\alpha^*$ -open sets.

**Proof**. (i) Suppose that G is (NOS) in  $W_2$ , since  $h|_T$  and  $h|_H$  are  $(N\alpha^* - CM)$ ,  $(h|_T)^{-1}(G)$  and  $(h|_H)^{-1}(G)$  are  $(N\alpha^* - OS)$  in  $W_1$ . So, their union is also, see Proposition 2.4. However,  $h^{-1}(G) = (h|_T)^{-1}(G) \cup (h|_H)^{-1}(G)$  and hence  $h^{-1}(G)$  is  $(N\alpha^* - OS)$  in  $W_1$ . Thus h is  $(N\alpha^* - CM)$ . Sufficiency, follows by using Theorem 3.3. The proofs of (i) and (iii) are the same way of proof (i).  $\Box$ 

**Theorem 3.5.** Suppose  $h: W_1 \to W_2$  is any mapping and  $h_T: h^{-1}(T) \to T$  is defined as  $h_T(t) = h(t)$ , for any neutrosophic set T in  $W_2$  and  $t \in h^{-1}(T)$ .

- (i) If h is  $(N\alpha^* CM)$ , then  $h_T$  is also, where T is (NOS) in  $W_2$ .
- (ii) If h is  $(NI\alpha^* CM)$  (resp.,  $(NS\alpha^* CM)$ ), then  $h_T$  is also, where T is neutrosophic closed set (NCS) in  $W_2$ .

**Proof**. We shall prove the second case. The first case is similar to (ii). Suppose that B is  $(N\alpha^* - OS)$  in T. Since T is (NCS) in  $W_2$ , B is  $(N\alpha^* - OS)$  in  $W_2$ , see Theorem 2.6(ii). Also, since h is  $(NI\alpha^* - CM)$  (resp.,  $(NS\alpha^* - CM)$ ),  $h^{-1}(B)$  is  $(N\alpha^* - OS)$  (resp., (NOS)) in  $W_1$ . Therefore,  $h^{-1}(B)$  is  $(N\alpha^* - OS)$  (resp., (NOS)) in  $h^{-1}(T)$ , see Theorem 2.6(i).  $\Box$ 

**Theorem 3.6.** Suppose that  $X_1, X_2, X_3$  are three (NTSs)  $L : X_1 \to X_2$  and  $X_2 \subseteq X_3$ . If  $L : X_1 \to X_2$  is  $(N\alpha^* - CM)$  (resp., (NIa"-CM),  $(NS\alpha^* - CM)$ ), then  $L : X_1 \to X_3$  is also.

**Proof**. Assume that A is(NOS)  $(resp., (N\alpha^* - OS))$  in  $X_3$ , then A is (NOS)  $(resp., (N\alpha^* - OS))$  in  $X_2$ , see Theorem 2.6(i) and hence  $L^{-1}(A)$  is a neutrosophic  $\alpha^*$ -open set  $(N\alpha^* - OS, neutrosophicopen)$  in  $X_1$ , Now, we recall that the set  $\{(x, L(x)), x \in X\} \subseteq X \times Y$  is called the neutrosophic graph of the mapping  $L: X \to Y$  and is denoted by NG(L).  $\Box$ 

**Theorem 3.7.** Suppose that  $W_1$  and  $W_2$  are two (NTSs),  $h : W_1 \to W_2$  is any mapping and  $L : W_1 \to W_1 \times W_2$  is a neutrosophic graph mapping of h defined by L(t) = (t, h(t)), for all  $t \in W_1$ . If L is  $(N\alpha^* - CM)$  (resp., (NI  $\alpha^* - CM$ ),  $(NS\alpha^* - CM)$ ), then h is also.

**Proof**. Assume that K is (NOS) (resp.,  $(N\alpha^* - OS)$ ) in  $W_2$ . Since  $W_1$  is (NOS) (resp.,  $(Na^* - OS)$ ) in any NTS),  $W_1 \times K$  is (NOS) (resp.,  $(Na^* - oS)$ ) in  $W_1 \times W_2$ , see Theorem 2.5. Therefore,  $L^{-1}(W_1 \times K) = h^{-1}(K)$  is a neutrosophic  $a^*$ -open (resp.,  $(N\alpha^* - OS)$ , (NOS)) in  $W_1$ . Hence, the proof is complete.  $\Box$ 

#### 4. Neutrosophic contra $\alpha^*$ -continuity:

In this section, we define a new type of neutrosophic  $\alpha^*$ -continuity that we call it a neutrosophic contra  $\alpha^*$ -continuous mapping ( $NC\alpha^*$ -CM) and several propositions related to this new notion are investigated.

**Definition 4.1.** Assume that  $W_1$  and  $W_2$  are two (NTSs) and  $h: W_1 \to W_2$  is a mapping, then h is called a neutrosophic contra  $\alpha^*$ -continuous mapping (NC $\alpha^* - CM$ ). If  $h^{-1}(K)$  is (N $\alpha^* - CS$ ) in  $W_1$ , for any (NOS) K in  $W_2$ .

**Theorem 4.2.** Let  $h: W_1 \to W_2$  be a mapping. The following statements are equivalent:

- (i) h is  $(NC\alpha^* CM)$ ,
- (ii) for each  $t \in W_1$  and each (NCS) K in  $W_2$  containing h(t), there exists  $(N\alpha^* OS) B$  in  $W_1$ , such that  $\in B, h(B) \subseteq K$ ,
- (iii) for every (NCS) K of  $W_2$ ,  $h^{-1}(K)$  is  $(N\alpha^* OS)$  of  $W_1$ .

**Proof**. (i)  $\rightarrow$  (ii) Assume that  $\in W_1$ , and K is any (NCS) in  $W_2$ , then  $K^c$  is (NOS) in  $W_2$ . Thus  $h^{-1}(K^c)$  is  $(N\alpha^* - CS)$  in  $W_1$ , but  $h^{-1}(K^c) = [h^{-1}(K)]^c$ . Hence  $h^{-1}(K)$  is  $(N\alpha^* - OS)$  in  $W_1$ , and  $t \in h^{-1}(K)$ . Put  $B = h^{-1}(K)$ , thus  $h(B) \subseteq K$ .

(ii)  $\rightarrow$  (iii) Assume that K is a neutrosophic closed set in  $W_2$  and  $t \in h^{-1}(K)$ , then  $h(t) \in K$ and hence there exists  $(N\alpha^* - OS) B$  containing  $t, h(B) \subseteq K$ , thus  $t \in B = h^{-1}(K)$ . So  $h^{-1}(K) = \bigcup \{B_t \mid t \in h^{-1}(K)\}$ . Hence by Proposition 2.4(1), we get  $h^{-1}(K)$  is  $(N\alpha^* - OS)$  in  $W_1$ .

(iii)  $\rightarrow$  (i) Obviously holds.  $\Box$ 

**Theorem 4.3.** The restriction  $L_A$  of  $(NC\alpha^* - CM)L : X \to Y$  to  $(N\alpha^* - CS)A \subseteq X$  is also  $(NC\alpha^* - CM)$ .

**Proof**. Assume that B is (NOS) in Y, thus  $L^{-1}(B)$  is  $(N\alpha^* - CS)$  in X. Since A is  $(N\alpha^* - CS)$  in X,  $L^{-1}(B) \cap A$  is also  $(N\alpha^* - CS)$  in X and hence it is also  $(Na^* - CS)$  in A, see Theorem 2.6(i), but  $(L|_A)^{-1}(B) = L^{-1}(B) \cap A$ , hence the proof is complete.  $\Box$ 

**Theorem 4.4.** If  $L: X \to Y$  is  $(NCa^* - CM)$ , then  $L_A: L^{-1}(A) \to A$  is also, where A is (NCS) in Y.

**Proof**. Assume that B is (NCS) in A. Since A is (NCS) in Y, B is (NCS) in Y. Then  $L^{-1}(B)$  is  $(N\alpha^* - OS)$  in X. Since  $L^{-1}(B) \subseteq L^{-1}(A) \subseteq X$ ,  $L^{-1}(B)$  is  $(N\alpha^* - OS)$  in  $L^{-1}(A)$ , see Theorem 2.6(i).  $\Box$ 

**Theorem 4.5.** Assume that X and Y are two (NTSs),  $L: X \to Y$  is a mapping and  $X = A \cup B$ , where A, B are disjoint  $(N\alpha^* - CSs)$  in X. Then  $L|_A$  and  $L|_B$  are  $(NC\alpha^* - CMs)$  if and only if L is  $(NC\alpha^* - CM)$ .

**Proof**. Necessity follows by using Theorem 4.3. Assume that G is (NCS) in Y. Since  $L|_A$  and  $L|_B$  are  $(NC\alpha^* - CMs)$ ,  $(L|_A)^{-1}(G)$  and  $(L|_B)^{-1}(G)$  are  $(N\alpha^* - OS)$  in X. So, their union is also, see Proposition 2.4. But  $L^{-1}(G) = (L|_A)^{-1}(G) \cup (L|_B)^{-1}(G)$  and hence the proof is complete.  $\Box$ 

**Definition 4.6.** An (NTS) W is called:

- (i) an  $N -_{a^*} T_2$  (resp., N-ultra- $_{a^*} T_2$ ) space if, for each  $t \neq d \in W$ , there exist two disjoint  $(N\alpha^* OSs)$  (resp.,  $(N\alpha^* CSs)$ ) T, D satisfy  $t \in T, d \in D$ .
- (ii)  $anN \alpha^*$ -ultra normal space if for each pair nonempty (NDCSs) can be separated by disjoint  $N\alpha^*$ -clopen).

• (iii) a neutrosophic  $\alpha^*$ -compact space (Na\*C-space) if for each N $\alpha^*$ -open cover of W has a finite subcover.

**Theorem 4.7.** Suppose that  $h: W_1 \to W_2$  is injective  $(NC\alpha^* - CM)$  and  $W_2$  is  $N - T_2$ - space. Then  $W_1$  is N-ultra- $\alpha \cdot T_2$  space.

**Proof**. Assume that  $t \neq d \in W_1$ . Since h is injective,  $h(t) \neq h(d)$  in  $W_2$  and since  $W_2$  is  $N - T_2$ - space, there exist two (NDOSs) T, D satisfy  $h(t) \in T, h(d) \in D$ . Since h is  $(NC\alpha^* - CM)$ ,  $h^{-1}(T), h^{-1}(D)$  are  $(N\alpha^* - CS)$  in  $W_1$  containing t, d and  $h^{-1}(T) \cap h^{-1}(D) = \varphi = h^{-1}(T \cap D)$ . Hence  $W_1$  is N-ultra- $\alpha \cdot T_2$  space.  $\Box$ 

**Theorem 4.8.** Suppose that  $L: X \to Y$  is injective  $(NC\alpha^* - CM)$  and Y is an N-ultra  $T_2$ -space. Then X is an  $N - _{\alpha^*} T_2$  space.

**Proof**. Take  $x \neq y$  in X. Since L is injective,  $f(x) \neq f(y)$  in Y. Since Y is an N-ultra  $T_2$ - space, there exist two (NDCSs) A, B satisfy  $L(x) \in A$ ,  $L(y) \in B$ . Moreover, from L is  $(NCa^* - CM)$ , we have  $L^{-1}(A), L^{-1}(B)$  are  $(N\alpha^* - OSs)$  in X containing x, y and  $L^{-1}(A) \cap L^{-1}(B) = \emptyset$ . Then X is an  $N - a^*T_2$  space.  $\Box$ 

**Theorem 4.9.** Suppose that  $h: W_1 \to W_2$  is a neutrosophic closed injective  $(NC\alpha^* - CM)$  and  $W_2$  is a neutrosophic ultra normal space. Then  $W_1$  is  $N - \alpha^* - is$  an ultra normal space.

**Proof**. Assume that  $A_1, A_2$  are two (NCSs) in  $W_1$  with  $A_1 \cap A_2 = \varphi$ . Since h is a neutrosophic closed mapping,  $h(A_1), h(A_2)$  are (NCSs) in  $W_2$ . Since,  $W_2$  is a neutrosophic ultra normal space, there exist two disjoint neutrosophic clopen sets  $B_1, B_2$  in  $W_2$  satisfy  $h(A_1) \subseteq B_1, h(A_2) \subseteq B_2$ . Hence  $A_1 \subseteq h^{-1}(B_1), A_2 \subseteq h^{-1}(B_2)$ . From injectivity of h, we get  $h^{-1}(B_1), h^{-1}(B_2)$  are disjoint neutrosophic  $\alpha^*$ -clopen sets. Thus  $W_1$  is a neutrosophic  $\alpha^*$ -ultra normal space.  $\Box$ 

**Theorem 4.10.** Suppose that  $h: W_1 \to W_2$  is a neutrosophic closed surjective  $(NC\alpha^* - CM)$  and  $W_1$  is  $(N\alpha^*C - space)$ . Then  $W_2$  is a neutrosophic strongly closed space.

**Proof**. Assume that  $\{V_i \mid i \in I\}$  is any neutrosophic closed cover of  $W_2$ . Since h is  $(NC\alpha^* - CM)$ ,  $\{h^{-1}(V_i) \mid i \in I\}$  is a neutrosophic  $\alpha^*$ -open cover of  $W_1$ , but  $W_1$  is  $(N\alpha^*C - \text{space})$ , thus  $W_1$  has finite subcover. This means that  $W_1 = \bigcup_{i \in I_0} h^{-1}(V_i)$ , where  $I_0 = \{1, \ldots, n\}$ . Since h is neutrosophic surjective, we have

$$h(W_1) = h\left(\bigcup_{j=1}^n h^{-1}(V_i)\right) = \bigcup_{j=1}^n hh^{-1}(V_i)$$

Hence,  $W_2 = \bigcup_{i \in I_0} V_i$ . Thus  $W_2$  is a neutrosophic strongly closed space.  $\Box$ 

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