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Analysis and control of a 4D hyperchaotic system with passive control

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Abstract

In this study, the dynamical behavior of the four-dimensional (4D) hyperchaotic system is analyzed. Its chaotic dynamical behaviors and basic dynamical properties are presented by Lyapunov exponents, stability analysis, and Kaplan-Yorke dimension. Then, the control of 4D hyperchaotic system is implemented by using passive control. The global asymptotic stability of the system is guaranteed by using Lyapunov function. Simulation results are shown to validate all theoretical analysis and demonstrate the effectiveness of the proposed control method. By applying the passive controllers, the system under chaotic behavior converges to the equilibrium point at origin asymptotically.

Keywords: 4D hyperchaotic system, passive control, chaos control, complex dynamic behavior

1. Introduction

Recently, chaos has been extensively studied in a number of fields such as engineering, communication, mathematics, etc. Chaos and hyperchaos can occur in any nonlinear system. The chaotic systems are defined by great sensitiviy to initial conditions [1]. The system must be at least third order autonomus or second order nonautonomous to see chaotic behavior [2]. The chaotic system must also have at least one positive Lyapunov exponent. Moreover, if the system has more than one positive Lyapunov exponent, the system is called as hyperchaotic system [3, 4].

It is believed that high dimensional chaotic systems will provide much more benefits in applications. Chaotic systems with higher dimensional have been proposed to improve the security of the communication due to their more-complex dynamics rather than chaotic systems [5]. Although Rossler firstly reported the hyperchaos in 1979 [6], the hyperchaos is firstly observed in a fourth

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order electrical circuit among the real physical systems [7]. Therefore, a lot of scientists have been proposed and generated many hyperchaotic systems recently [6, 7, 8, 9, 10, 11, 12, 13, 14, 15]. Different methods have been proposed in recent years to obtain hyperchaos or hyperchaotic system from chaotic systems such as linear feedback control method [9, 10, 11, 12], nonlinear feedback [13], time delay state feedback [14] and state feedback [15]. Using these methods, hyperchaotic attractors and tri-chaos attractors can be generated with two positive Lyapunov exponents and three positive Lyapunov exponents respectively [2, 16]. A new 4D hyperchaotic system generated from three dimensional autonomous Chen-Lee chaotic system by adding a nonlinear controller is reported by Cheng [13]. It is showed that the obtained system has two lyapunov exponents and has more complex than 3D autonomous Chen-Lee chaotic system.

However, chaos in systems can lead the undesirable behavior such as harmful oscillations for the system. So, if it is undesirable, this is needed to be diminished or control to avoid the undesirable behavior. After Ott, Grebogi and Yorke have firstly proposed OGY method for control of chaos [17], the control of chaotic behavior in the systems has received intensive attention. Recently, a number of control methods have been applied to control the chaos and undesirable behavior in the chaotic systems such as sliding mode control [18], time-delayed feedback control [19], linear control [20], state feedback control [21], adaptive control [22, 23], passive control [24, 25, 26, 27, 28] and so on.

Since chaos can have a negative effect on the stable operation of the system, the studies about the chaos control of the system have great significance. Several studies have shown that the passive controller can asymptotically provide stability, suppressing chaos and stabilize the system effectively. Also, the passive control technique is applied for the synchronization of the chaotic systems. Therefore, the researchers are interested in the passive control as an effective method. Using the effectiveness of the passive control technique, Kocamaz proposed a new controller designed based on combining the passive and feedback controllers [29]. While classical passive control does not applied for the hyperchaotic finance system, it has been shown that the new passivity-based feedback controller can stabilize the hyperchaotic system [29]. Takhi proposed passivity based sliding mode control for chaos control and synchronization of chaotic systems using the combination of sliding model control and passivity theory [30]. Zhang showed that the passivity based controllers can stabilize hyperchaotic Chen system by controlling the hyperchaos and also achieve synchronization between different hyperchaotic systems by ensuring the asymptotic stability [31].

In this article, firstly, the dynamical behavior of the 4D hyperchaotic system presented by Cheng [13] is analyzed. The chaotic behavior of the 4D system is investigated by analyzing the Lyapunov exponents, Lyapunov or Kaplan-Yorke dimension and eigenvalues. The complex dynamics behavior is also shown by phase portraits and time series. Moreover, the passive control technique is applied for control of hyperchaotic system presented by Cheng [13]. The asymptotic stability of the system is ensured by using the Lyapunov function with passive control.

The structure of this paper is organized as follows. In Section 2, the hyperchaotic system obtained from chaotic system is introduced and the dynamical behavior of the hyperchaotic system is investigated by using Lyapunov exponents, Kaplan-Yorke dimension, eigenvalues, and phase diagrams. In Section 3, the control of the chaos in the hyperchaotic system is implemented via passive control technique. In Section 4, the numerical simulations are given by showing very good control achievement. Finally, conclusions are given in Section 5.

2. The System Description and Dynamical Analysis of 4D Hyperchaotic System

The hyperchaotic 4D system generated from 3D autonomous system by adding nonlinear controller is given by [13],

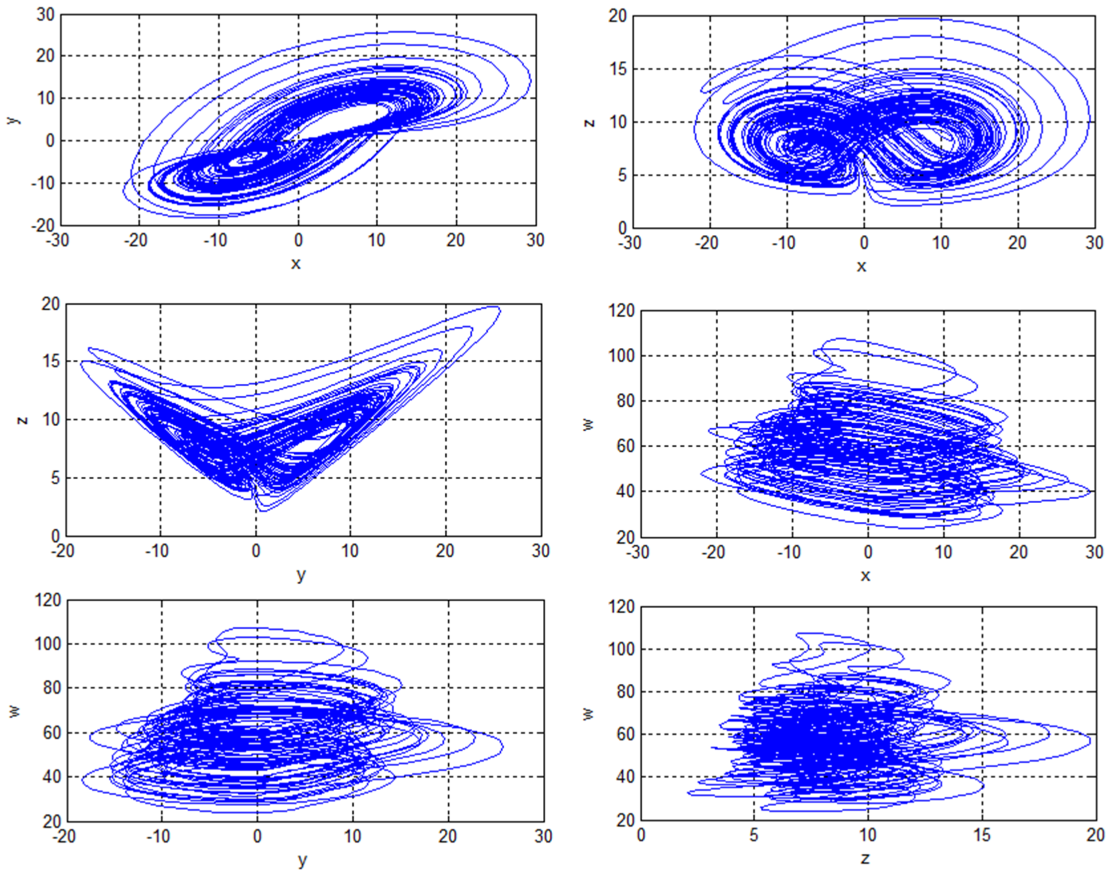


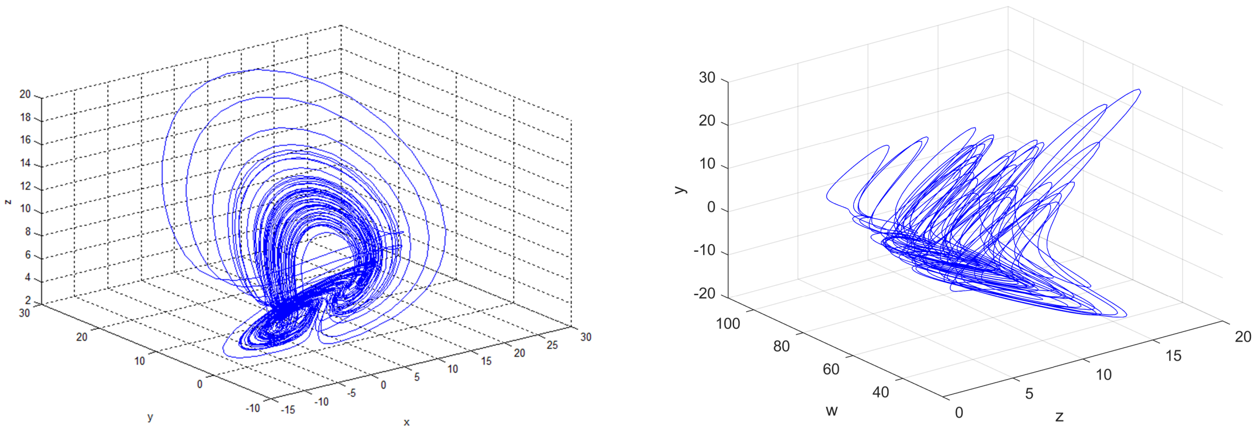
Figure 1: Phase diagrams of the system

$$
\begin{aligned}
\dot{x} &= -yz + ax \\
\dot{y} &= xz + by \\
\dot{z} &= (1/3)\,xy + cz + 0.2w \\
\dot{w} &= dx + 0.5yz + 0.05w
\end{aligned}
\tag{2.1}
$$

where x, y, z, w are the state variables and a, b, c, d are real constants. When $a = 5, b = -10, c = -3.8, d = 0.4$, the system exhibits hyperchaotic behavior. The phase portraits of the hyperchaotic attractor of system with certain parameters and initial points $[0.2, 0.1, 0.1, 0.2]^T$ are shown in Fig. 1 and Fig. 2.

Lyapunov exponents of the system are computed as $L_1 = 0.3265$, $L_2 = 0.0946$, $L_3 = 0$ and $L_4 = -9.1711$ with $T = 10000\ s$ for parameter $a = 5, b = -10, c = -3.8, d = 0.4$ and the initial state values $(x_0, y_0, z_0, w_0) = (0.2, 0.1, 0.1, 0.2)$ using the Wolf's algorithm [32]. The system is chaotic due to the Lyapunov characteristic exponents. If any system has one or more positive Lyapunov exponent, it is described as chaotic.[32] Also, any system containing more than one positive Lyapunov exponent can be defined as a hyperchaotic system [2].

The Lyapunov or Kaplan–Yorke dimension [33, 34] of the corresponding chaotic system is calculated as,



(a) (b)

Figure 2: 3D phase spaces of the system (a) x-y-z plane, (b)z-w-y plane.

$$D_L = j + \frac{1}{|L_{j+1}|}\sum_{i=1}^{j} L_i = 3 + \frac{L_1 + L_2 + L_3}{|L_4|} = 3.0459. \tag{2.2}$$

The dissipation of the system (2.1) is given as follow [35] :

$$\nabla V = \frac{\partial \dot{x}}{\partial x} + \frac{\partial \dot{y}}{\partial y} + \frac{\partial \dot{z}}{\partial z} + \frac{\partial \dot{w}}{\partial w} = a + b + c + 0.05 = -8,75 < 0 \tag{2.3}$$

As the divergence flow and sum of Lyapunov exponents are negative, the system (2.1) is obviously dissipative with an exponential contraction rate:

$$\frac{dV}{dt} = e^{-8.75t}$$

The volume of the trajectories of the dynamical system shrinks to zero exponentially as $t \to \infty$ with -8.75 rate in the dynamical system (2.1).

The system (2.1) is linearized for the zero equilibirum point, the jacobian matrix is,

$$J_{(0,0,0)} = \begin{bmatrix} a & 0 & 0 & 0 \\ 0 & b & 0 & 0 \\ 0 & 0 & c & 0.2 \\ d & 0 & 0 & 0.05 \end{bmatrix} = \begin{bmatrix} 5 & 0 & 0 & 0 \\ 0 & -10 & 0 & 0 \\ 0 & 0 & -3.8 & 0.2 \\ 0.4 & 0 & 0 & 0.05 \end{bmatrix} \tag{2.4}$$

By using $| J - \lambda I |= 0$, the characteristic equation is obtained as

$$\lambda^4 + 8.75\lambda^3 - 31.44\lambda^2 - 188.45\lambda^1 + 9.5 = 0 \tag{2.5}$$

From the characteristic equation, eigenvalues are calculated as $\lambda_1 = -3.8, \lambda_2 = 0.05, \lambda_3 = 5$ and $\lambda_4 = -10$ Obviously, zero equilibrium point (0, 0, 0, 0) is unstable due to the positive eigenvalues.

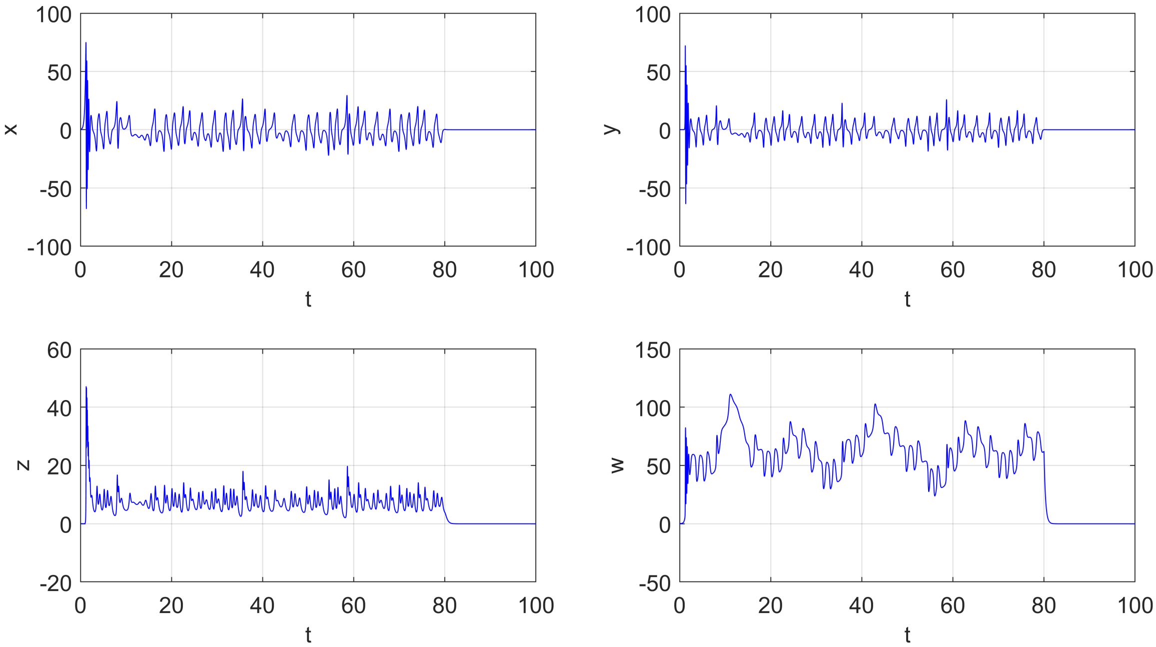


Figure 3: The times series of the system with controller activated at 80 s with $\alpha = 1$ and $v = 0$.

3. Control of 4D Hyperchaotic System

In this section, the passive control technique is used to control the hyperchaotic system around the unstable equilibrium point at origin. The controlled hyperchaotic system is given by,

$$\begin{aligned} \dot{x} &= -yz + ax + u_1 \\ \dot{y} &= xz + by \\ \dot{z} &= (1/3)\, xy + cz + 0.2w \\ \dot{w} &= dx + 0.5yz + 0.05w + u_2 \end{aligned} \tag{3.1}$$

where u_1 and u_2 are the control input vectors which drive the hyperchaotic system to zero equilibirum point.

Supposing the variables,

$$\begin{aligned} z &= \begin{pmatrix} z_1 \\ z_2 \end{pmatrix} = \begin{pmatrix} y \\ z \end{pmatrix} \\ y &= \begin{pmatrix} y_1 \\ y_2 \end{pmatrix} = \begin{pmatrix} x \\ w \end{pmatrix} \end{aligned} \tag{3.2}$$

and then, the controlled system can be described by generalized form of passive control theory.

$$\begin{aligned} \dot{z_1} &= y_1 z_2 + b z_1 \\ \dot{z_2} &= (1/3)\, y_1 z_1 + c z_2 + 0.2 y_2 \\ \dot{y_1} &= -z_1 z_2 + a y_1 + u_1 \\ \dot{y_2} &= d y_1 + 0.5 z_1 z_2 + 0.05 y_2 + u_2 \end{aligned} \tag{3.3}$$

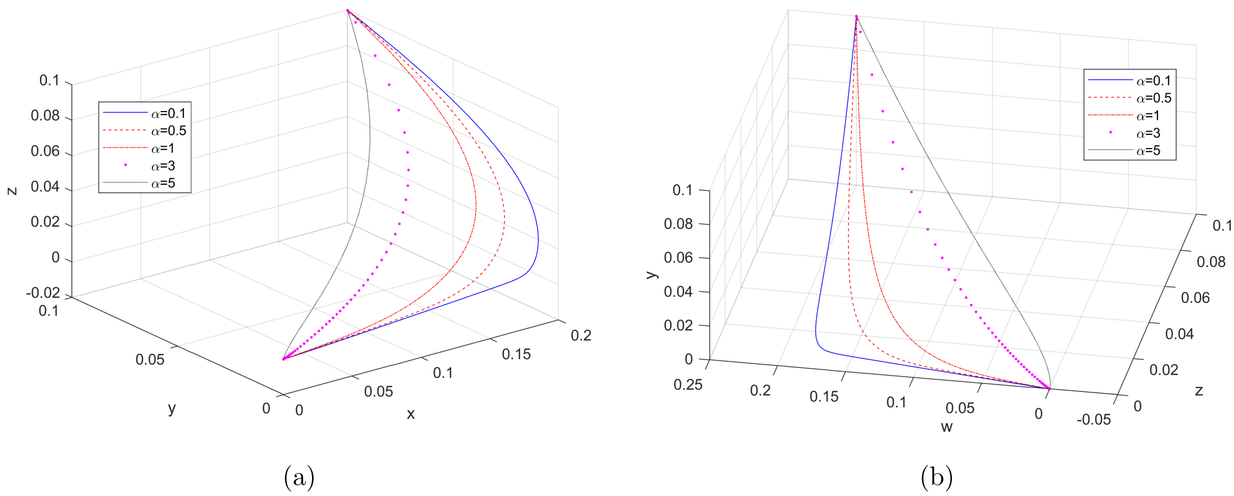


Figure 4: The phase planes of the controlled system with different α values.

So, writing the general form as in passive control as follows. [24, 25, 26],

$$\begin{aligned}\dot{z} &= f_0(z) + p(z,y)y,\\ \dot{y} &= b(z,y) + a(z,y)u\end{aligned} \tag{3.4}$$

where

$$f_0(z) = \begin{bmatrix} bz_1 \\ cz_2 \end{bmatrix}$$
$$p(z,y) = \begin{bmatrix} z_2 & 0 \\ 1/3z_1 & 0.2 \end{bmatrix}$$
$$b(z,y) = \begin{bmatrix} -z_1z_2 + ay_1 \\ dy_1 + 0.5z_1z_2 + 0.05y_2 \end{bmatrix}$$
$$a(z,y) = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

Selecting the following storage function,

$$W(z) = \frac{1}{2}({z_1}^2 + {z_2}^2) \tag{3.5}$$

is the Lyapunov function of $f_0(z)$ with $W(0) = 0$.

$$\dot{W} = \frac{d}{dt}W(z) = \frac{\partial W(z)}{\partial z}f_0(z) = \begin{bmatrix} z_1 & z_2 \end{bmatrix}\begin{bmatrix} bz_1 \\ cz_2 \end{bmatrix} = bz_1^2 + cz_2^2 \tag{3.6}$$

where b and c are negative, so

$$bz_1^2 + cz_2^2 \leq 0 \tag{3.7}$$

$f_0(z)$ is globally asymptotically stable, as $W(z) \geq 0$ and $\dot{W}(z) \leq 0$, it can be achieved that the zero dynamics of the controlled system (3.1) is Lyapunov stable. The system with controller can be said to be equivalent to a passive system and it becomes minimum phase.

Using passive control theory, the control signal u is achieved by

$$u = a(z,y)^{-1}\left[-b^T(z,y) - \frac{\partial}{\partial z}W(z)p(z,y) - \alpha y + v\right] \tag{3.8}$$

According to the passive control theory [25], the system (3.3) is a passive system. u control signal from the property of passive control theory can be defined as,

$$\begin{aligned} u_1 &= yz - (a+\alpha)x - xy - 1/3xz + v_1 \\ u_2 &= -dx - 0.5yz - 0.2z - (0.05+\alpha)w + v_2 \end{aligned} \tag{3.9}$$

where α is a positive parameter, $u = (u_1, u_2)^T$ and $v = (v_1, v_2)^T$ are the control and an external input vector respectively.

4. Numerical simulations

In this section, numerical simulations have been performed to show the effectiveness and feasibility of the proposed control method on 4D hyperchaotic system. The parameters used in simulations are $a = 5, b = -10, c = -3.8, d = 0.4$ and the initial state values $x(0) = 0.2, y(0) = 0.1, z(0) = 0.1, w(0) = 0.2$.

The passivity-based control signal with external input $v = 0$ is applied to the 4D hyper chaotic system at $t = 80$ s. The trajectories of the controlled system are shown in Fig. 3. Also, the phase diagrams of the controlled chaotic system with passivity-based controller activated in the beginning of the simulations are shown in Fig. 4. Moreover, the effect of the α parameter of the controller is investigated. As can be seen in Fig. 4, the controlled system converges faster to the zero equilibrium point, as α parameter increases. As we can see from these Figs. 3 and 4, the passivity based controller (3.9) can effectively stabilize and derive the chaotic system to the equilibrium point $(0,0,0,0)$.

5. Conclusion

In this paper, dynamical analyses of the 4D hyperchaotic system are presented by Lyapunov exponents, Kaplan-Yorke dimension, eigenvalues, times series and phase spaces. Moreover, passive control scheme is investigated for 4D hyperchaotic system which was generated from 3D autonomous system by adding nonlinear controller. We have proposed a passive controller to asymptotically stabilize a 4D hyperchaotic system to zero equilibrium point based on the stability features of a passive system. The proposed controller can be easily applied in practical applications due to its simplicity and efficiency. Simulation results have demonstrated that the proposed controller can effectively suppress the hyperchaos in the system and stabilize the hyperchaotic system at zero equilibrium point.

Conflict of interest. On behalf of all authors, the corresponding author states that there is no conflict of interest.

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