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On α^*- Continuous and Contra α^*- Continuous Mappings in Topological Spaces with Soft Setting

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Abstract

In this work, some new connotations of continuous mappings such as α^* - continuous mapping $(\alpha^* - CM)$, irresolute α^*- mapping $(I\alpha^* - CM)$, and strongly α^*- continuous mapping $(S\alpha^* - CM)$ are studied and some of their characteristics are discussed. In other side, new some classes of contra continuous mappings are investigated in this work, they are called contra α^* - continuous mappings $(C\alpha^* - CMs)$. Finally they are studied in soft setting.

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1. Introduction

In general the concept of mappings is so useful notion in topology and mathematic. In this work, we study a new classes of mapping that are called α^*- continuous and contra α^*- continuous mappings; the connotation of the $\alpha-$ open set $(\alpha - OS)$ was first introduced by O. Njasted in 1965 [1], where the homely of all $(\alpha - OSs)$ in (X, τ) is also topology on X and it is finer than τ[2]. G. Navalagi defined the connotation of semi $\alpha-$ open sets (Se $\alpha - OSs$)([3], [4]). The connotation of α^*- open set $(\alpha^* - OS)$ was first shown in 2019 by Nadia M. Ali and Shuker M. Khalil for more details see [5] the concept is an extension of $(\alpha - OS)$, it is robustly weaker than $(\alpha - OSs)$ and bigger than (Seα − OSs). The notion of Contra continuity is given in 1998 by Dontchev [6]. The mathematical idea of these notions were studied in non-classical sets, where some types of sets and maps can be extended in non-classical sets like fuzzy sets, nano set, permutation sets and so on and

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applied in topology and algebra see ([7]-[32]). In this research attempt has been considered to apply the concept of $(\alpha^* - OSs)$ to consider some kinds of $(\alpha^* - CMs)$ like; $(I\alpha^* - CMs)$, $(S\alpha^* - CMs)$ and $(C\alpha^* - CMs)$. The interior (resp., closure, α - interior, α - closure) of a subset A of a topological space (TS) will be referred as $\text{int}(A)$ (resp., α int $(A), \text{cl}(A), \text{acl}(A)$).

2. Preliminaries

Here, some definitions and characteristics of $(\alpha^* - OS)$ which we need in our work are recalled.

Definition 2.1. ([1], [3], [5]) *Let A be a set in (TS) X. We say A is α -open set $(\alpha - OS)$ (resp., semi $\alpha-$ open set $(Se\alpha - OS), \alpha^*-$ open set $(\alpha^* - OS)$) if $A \subseteq \text{int}(\text{cl}(\text{int}(A)))$(resp., $A \subseteq \text{cl}(\text{mt}(\text{cl}\,\text{int}(A))))$ or equivalently $A \subseteq cl(\text{int}(A)), A \subseteq \text{int}_\alpha\left(cl\left(\text{int}_\alpha(A)\right)\right))$. Also, their complement are called $\alpha-$ closed set $(\alpha - CS)$ (resp., semi $\alpha-$ closed set $(Se\alpha - CS), \alpha^*-$ closed set $(\alpha^* - CS)$). The symbol of above sets and their complement are referred as $\alpha - O(X)$ (resp., $Se\alpha - O(X), \alpha^* - O(X)$), $\alpha - C(X)$ (resp., $se\alpha - C(X), \alpha^* - C(X)$).*

Proposition 2.2. [5] *(1) Every (OS) (resp., $\alpha-$ open, clopen) set is $(\alpha^* - OS)$, (2) Every $(\alpha^* - OS)$ is $(\text{se}\,\alpha - OS)$.*

Proposition 2.3. [5] *(1) If $\{G_\lambda\}_{\lambda \in \Gamma}$ is a collection of $(\alpha^* - OSs)$, then their union is also $(\alpha^* - OSs)$, (2) If A is $(\alpha^* - OS)$ and B is (OS), then $A \cap B$ is $(\alpha^* - OS)$.*

Proposition 2.4. [5] *Let X_1 and X_2 be two topological spaces (TSs), $A_1 \subseteq X_1$ and $A_2 \subseteq X_1$. Then A_1 and A_2 are $(\alpha^* - OSs)$ (resp., $(\alpha^* - CSs)$) in X_1 and X_2, respectively if and only if $A_1 \times A_2$ is $(\alpha^* - OS)$ (resp., $(\alpha^* - CS)$) in $X_1 \times X_2$.*

Theorem 2.5. *Assume W is a subspace of Z satisfy $G \subseteq W \subseteq Z$. Then; (i) if $G \in \alpha^* - O(Z)$, then $G \in \alpha^* - O(W)$ (ii) if $G \in \alpha^* - O(W)$, then $G \in \alpha^* - O(Z)$, where W is a closed subspace of Z.*

Definition 2.6. ([34]) *A (TS) X is called: (i) Ultra $-T_2$ if for any $t \neq h \in Z$, there exist two disjoint closed sets (DCSs) T, H satisfy $t \in T, h \in H$ (ii) Ultra Normal if for each T, F closed sets (CSs) with $T \neq \phi \neq F$ and $T \cap F = \phi$. Then, $\exists D, H$ two (CSs) with $D \cap H = \phi$ and $T \subseteq D, F \subseteq H$, (iii) Strongly closed if for any homely of (CSs) that form a cover of x has a finite sub-homely that form a cover of X too.*

Definition 2.7. ([36]) *Assume $\left(\int, A\right)$ is soft subset in soft topological space (STS) (Ψ, E, ℓ). We say $\left(\int, A\right)$ is a soft $\alpha-$ open set$(S\alpha - OS)$ if $\left(\int, A\right) \cong \text{int}^S\left(cl^S\left(\text{int}^S\left(\int, A\right)\right)\right)$ and its complement is called soft $\alpha-$ closed set $(S\alpha - CS)$. The intersection of all soft closed sets (SCSs) [res. (Sα-CSs)] which containing $\left(\int, A\right)$ is called soft closure [resp., soft $\alpha-$ closure] of $\left(\int, A\right)$ and referred as $cl^S\left(\int, A\right)$ $\left(cl^S_\alpha\left(\int, A\right)\right)$. Moreover, the union of all soft open sets (SOSs) [resp., $(S\alpha - OSs)$] is contained in $\left(\int, A\right)$ is called soft interior [res. soft α -interior] of $\left(\int, A\right)$ and referred as int $^S\left(\int, A\right)$ (int $\alpha^S\left(\int, A\right)$)*

Definition 2.8. ([35]) *Assume (Ψ, E) and (Ψ', E') are two soft spaces and $u : \Psi \to \Psi', p : E \to E'$ are two maps. We say $L : (\Psi, E) \to (\Psi', E')$ is a soft mapping (SM) and recognized as:, $\left(L\left(\int, A\right), B\right)$ in (Ψ', E') whenever $\left(\int, A\right) \text{in}(\Psi, E), B = P(A) \subseteq K$ and $L\left(\int, A\right)(\omega) = u\left(\cup_{\alpha \in p^{-1}(\omega) \cap A} \int(\alpha)\right)$, for $\omega \in K$ $\left(L\left(\int, A\right), K\right)$ is referred as $\left(L\left(\int, A\right)\right.$, when $B = K$.*

Definition 2.9. ([33]) *Let (Ψ, E, ℓ) be a (STS) has soft subset $\left(\int, A\right)$. Then $\left(\int, A\right)$ is said to be a soft α^*- open* set *$(S\alpha^*-$ OS) if $(f, A) \subseteq \operatorname{int}_\alpha^s\left(cl^s\left(\operatorname{int}_\alpha^s(I, A)\right)\right)$ and its complement is called soft α^* -closed set $(S\alpha^*-CS)$.*

Definition 2.10. ([33]) Let(Ψ, E, ℓ) *and* (Ψ', E', ℓ') *be (STSs) and* $L:(\Psi, E) \rightarrow (\Psi', E')$ *be a* (SM), *then L is said to be a soft contra α^*- continuous mapping $(SC\alpha^*-CM)$, if for each $\left(\int, A\right) \widetilde{\in} \ell', L^{-1}\left(\left(\int, A\right)\right)$ is $(S\alpha^*-$ CS)* in(Ψ, E, ℓ)

3. Some New Classes of α^*- Continuity

Some new classes of α^*- continuity such as; irresolute α^*- continuous mapping $(I\alpha^*-CM)$, stronger α^*- continuhgous mapping $(S\alpha^*-CM)$ and contra α^* - continuous mapping $(C\alpha^*-CM)$ in this section are investigated. Also, their relationships among them are given.

Definition 3.1. *Assume W_1 and W_2 are (TSs) and $h: W_1 \rightarrow W_2$ is any map from W_1 into W_2. We say h is α^*- continuous mapping (α^*-CM) (resp., irresolute α^* - continuous mapping $(I\alpha^*-CM)$, stronger α^*- continuous mapping $(S\alpha^*-CM)$) mapping if each G(OS) (resp., α^*-OS) in W_2, then $h^{-1}(G)$ is $\alpha^*-OS($ resp. ,(OS)) in W_1*

Lemma 3.2. *(1) Every (α^*-CM) is $(I\alpha^*-CM)$, (2) Every $(I\alpha^*-CM)$ is $(S\alpha^*-CM)$.*

Proof . It follows from [Proposition (2.2)]. □

Theorem 3.3. *Assume W_1 and W_2 are (TSs) and $h: W_1 \rightarrow W_2$. Then, (i) If h is (α^*-CM), then $\left.h\right|_G: G \rightarrow W_2$ is (ii) If h is $(I\alpha^*-CM)$, then $\left.h\right|_G: G \rightarrow W_2$ is also, where G is (OS) also, where G is (OS) of W_1, of W_1, (iii) If h is $(S\alpha^*-CM)$, then $\left.h\right|_G: G \rightarrow W_2$ is also, where G is (α^*-OS) of W_1.*

Proof . (i) Assume B is an (OS) in W_2, since h is (α^*-CM), then $h^{-1}(B)$ is (α^*-OS) in W_1, since G is (OS) in W_1. Hence, by Proposition (2.3) we have $h^{-1}(B) \cap G$ is (α^*-OS) in W_1, but $\left(\left.h\right|_G\right)^{-1}(B) = h^{-1}(B) \cap G$, thus by Theorem (2.5)$\left(\left.h\right|_G\right)^{-1}(B)$ is α^*- open in G.
(ii) and (iii) are similar to (i). □

Theorem 3.4. *Suppose $h: W_1 \rightarrow W_2$ is any mapping and $W_1 = T \cup H$, where T, H are disjoint sets in W_1. Then, (i) h is (α^*-CM) iff $\left.h\right|_T$ and $\left.h\right|_H$ are also, where T, H are open sets, (ii) h is $(I\alpha^*-CM)$ iff $\left.h\right|_T$ and $\left.h\right|_H$ are also, where T, H are open sets, (iii) h is $(S\alpha^*-CM)$ iff $\left.h\right|_T$ and $\left.h\right|_H$ are also, where T, H are α^*- open sets.*

Proof . (i) Necessity: Suppose that G is (OS) in W_2, since $\left.h\right|_T$ and $\left.h\right|_H$ are (α^*-CM), then $\left(\left.h\right|_T\right)^{-1}(G)$ and $\left(\left.h\right|_H\right)^{-1}(G)$ are (α^*-OS) in W_1. So, their union is also, see Proposition (2.3) However, $h^{-1}(G) = \left(\left.h\right|_T\right)^{-1}(G) \cup \left(\left.h\right|_H\right)^{-1}(G)$ and hence $h^{-1}(G)$ is (α^*-OS) in W_1. Thus h is (α^*-CM). Sufficiency: Follows by using Theorem (3.3). The proofs of (i) and (iii) are the same way of proof (i). □

Theorem 3.5. *Suppose $h: W_1 \rightarrow W_2$ is any mapping and $h_T: h^{-1}(T) \rightarrow T$ is defined as $h_T(t) = h(t)$, for any set T in W_2 and $t \in h^{-1}(T)$. Then, (i) If h is (α^*-CM), then h_T is also, where T is (OS) in W_2, (ii) If h is $(I\alpha^*-CM)$ (resp., $(S\alpha^*-CM)$), then h_T is also, where T is closed set* (CS) *in W_2.*

Proof . We shall prove the second case. The first case is similar to (ii). Suppose that B is $(\alpha^* - OS)$ in T, since T is (CS) in W_2, then B is $(\alpha^* - OS)$ in W_2, see Theorem (2.5-(ii)). Also, since h is $(I\alpha^* - CM)$ (resp., $(S\alpha^* - CM)$), then $h^{-1}(B)$ is $(\alpha^* - OS)$ (resp., (OS)) in W_1. Therefore, $h^{-1}(B)$ is $(\alpha^* - OS)$ (resp., (OS)) in $h^{-1}(T)$, see Theorem (2.5-(i)). □

Theorem 3.6. *Suppose X_1, X_2, X_3 are three (TSs) $L : X_1 \to X_2$ and $X_2 \subseteq X_3$. If $L : X_1 \to X_2$ is $(\alpha^* - CM)$ (resp. $(I\alpha^* - CM), (S\alpha^* - CM)$), then $L : X_1 \to X_3$ is also.*

Proof . Assume A is (OS) (resp., $(\alpha^* - OS)$) in X_3, then A is (OS) (resp., $(\alpha^* - OS)$) in X_2, see Theorem $(2.5-(\mathrm{i}))$ and hence $L^{-1}(A)$ is $\boldsymbol{\alpha}^*-$ open set $(resp., (\alpha^* - OS, open))$ in X_1. Now, we recall that the set $\{(x, L(x)), x \in X\} \subseteq X \times Y$ is called the graph of the mapping $L : X \to Y$ and is denoted by $G(L)$. □

Theorem 3.7. *Suppose W_1 and W_2 are two (TSs), $h : W_1 \to W_2$ is any mapping and $L : W_1 \to W_1 \times W_2$ be a graph mapping of h defined by $L(t) = (t, h(t)), \forall t \in W_1$. If L is $(\alpha^* - CM)$(resp. ,$(I\alpha^* - CM), (S\alpha^* - .\ CM\ )$), then h is also.*

Proof . Assume that K is (OS) (resp., $(\alpha^* - OS)$) in W_2, since W_1 is (OS) (resp., $(\alpha^* - OS)$) in any (TS). Hence, $W_1 \times K$ is (OS) (resp., $(\alpha^* - OS)$) in $W_1 \times W_2$, see Theorem (2.4). Therefore, $L^{-1}(W_1 \times K) = h^{-1}(K)$ is α^*- open (resp., $(\alpha^* - OS)$, (OS)) in W_1. Hence, the proof is complete. □

4. Contra $\boldsymbol{\alpha}^*-$ continuity

New class of $\boldsymbol{\alpha}^*$ - continuity is called contra $\boldsymbol{\alpha}^*-$ continuous mapping $(\mathrm{C}\alpha^* - \mathrm{CM})$. and some theorems are shown in this section.

Definition 4.1. *Assume W_1 and W_2 are two (TSs) and $h : W_1 \to W_2$ is a mapping, then h is called contra $\boldsymbol{\alpha}^*$ - continuous mapping $(C\alpha^* - CM)$. If $h^{-1}(K)$ is $(\alpha^* - CS)$ in W_1, for any* (OS)K *in W_2.*

Theorem 4.2. *The following statements are equivalent, if $h : W_1 \to W_2$ is a mapping:*
(i) h is $(C\alpha^* - CM)$
(ii) for each $t \in W_1$ and each (CS) K in W_2 containing $h(t)$, there exists $(\alpha^* - OS)\, B$ in W_1, such that $t \in B, h(B) \subseteq K$,
(iii) for every (CS)K *of W_2, then $h^{-1}(K)$ is $(\alpha^* - OS)$ of W_1.*

Proof . (i) $\to$ (ii): Assume$t \in W_1$, and K is any (CS) in W_2, then K^c is (OS) in W_2, thus $h^{-1}(K^c)$ is $(\alpha^* - CS)$ in W_1, but $h^{-1}(K^c) = [h^{-1}(K)]^c$, hence $h^{-1}(K)$ is $(\alpha^* - OS)$ in W_1, and $t \in h^{-1}(K)$. Put $B = h^{-1}(K)$, thus $h(B) \subseteq K$. (ii) $\to$ (iii): Assume that K is a closed set in W_2 and $t \in h^{-1}(K)$, then $h(t) \in K$ and hence there exists $(\alpha^* - OS)\, B$ containing $t, h(B) \subseteq K$, thus $t \in B = h^{-1}(K)$. So $h^{-1}(K) = \cup\{B_t \mid t \in h^{-1}(K)\}$. Hence by Theorem $(2.3-(1))$ we get $h^{-1}(K)$ is $(\alpha^* - OS)$ in W_1. (iii) $\to$ (i): Obvious. □

Theorem 4.3. *The restriction L_A of $(C\alpha^* - CM)\, L : X \to Y$ to $(\alpha^* - CS)\, A \subseteq X$ is also $(C\alpha^* - CM)$.*

Proof . Assume B is (OS) in Y, thus $L^{-1}(B)$ is $(\alpha^* - CS)$ in X, since A is $(\alpha^* - CS)$ in X. Then $L^{-1}(B) \cap$ Ais also $(\alpha^* - CS)$ inXand hence it is also $(\alpha^* - CS)$ in A, see [Theorem (2.5-(i))], but $(L|_A)^{-1}(B) = L^{-1}(B) \cap A$ hence the proof is complete. □

Theorem 4.4. *If $L : X \to Y$ is $(C\alpha^* - CM)$, then $L_A : L^{-1}(A) \to A$ is also, where A is (CS) in Y.*

Proof . Assume B is (CS) in A, since A is (CS) in Y, thus B is (CS) in Y, then $L^{-1}(B)$ is $(\alpha^* - OS)$ in X, since $L^{-1}(B) \subseteq L^{-1}(A) \subseteq X$, then $L^{-1}(B)$ is $(\alpha^* - OS)$ in $L^{-1}(A)$, see [Theorem (2.5 – (i))]. □

Theorem 4.5. *Assume X and Y are two (TSs), $L : X \to Y$ is a mapping and $X = A \cup B$, where A, B are disjoint $(\alpha^*-$ $CSs)$ in X. Then $L|_A$ and $L|_B$ are $(C\alpha^* - CMs)$ iff L is $(C\alpha^* - CM)$*

Proof . Necessity: Follows by using Theorem (4.3). Sufficiency: Assume that G is (CS) in Y, since $L|_A$ and $L|_B$ are $(C\alpha^* - CMs)$, thus $(L|_A)^{-1}(G)$ and $(L|_B)^{-1}(G)$ are $(\alpha^* - OS)$ in X. So, their union is also, see [Proposition (2.3)]. But $L^{-1}(G) = (L|_A)^{-1}(G) \cup (L|_B)^{-1}(G)$ and hence the proof is complete. □

Definition 4.6. *A (TS) W is called: (i) $\alpha^* T_2$ (resp., Ultra- $_{\alpha^*}T_2$) space if for each $t \neq d \in W$, there (ii) α^*- Ultra Normal exist two disjoint $(\alpha^* - OSs)$ (resp., $(\alpha^* - CSs)$) T, D satisfy$t \in T, d \in D$ space if for each pair nonempty (DCSs) can be separated by disjoint $(\alpha^*-$ clopen), (iii) α^*- Compact space $(\alpha^* C-$ space) if for each α^*- open cover of W has a finite subcover.*

Theorem 4.7. *Suppose $h : W_1 \to W_2$ is injective $(C\alpha^* - CM)$ and W_2 is T_2- space. Then W_1 is Ultra $-_{\alpha^*}T_2$ space.*

Proof . Assume$t \neq d \in W_1$, since h is injective, then $h(t) \neq h(d)$ in W_2, since W_2 is T_2- space, then there exist two (DOSs) T, D satisfy $h(t) \in T, h(d) \in D$. Since$h$ is $(C\alpha^* - CM)$, then $h^{-1}(T), h^{-1}(D)$ are $(\alpha^* - CS)$ in W_1 containing t, d and $h^{-1}(T) \cap h^{-1}(D) = \varphi = h^{-1}(T \cap D)$.
Hence W_1 is Ultra- $_{\alpha^*}T_2$ space. □

Theorem 4.8. *Suppose $L : X \to Y$ is injective $(C\alpha^* - CM)$ and Y is Ultra T_2- space. Then X is $\alpha^* T_2$ space.*

Proof . Take $x \neq y$ in X, since L is injective, then $f(x) \neq f(y)$ in Y, since Y is Ultra T_2- space, then there exist two (DCSs) A, B satisfy $L(x) \in A, L(y) \in B$. Since L is $(C\alpha^* - CM)$, then $L^{-1}(A), L^{-1}(B)$ are $(\alpha^* - OSs)$ in X containing x, y and $L^{-1}(A) \cap L^{-1}(B) = {}_L^{-1}(A \cap B) = L^{-1}(\varphi) = \varphi$. Then X is $_{\alpha^*}T_2$ space. □

Theorem 4.9. *Suppose $h : W_1 \to W_2$ is closed injective $(C\alpha^* - CM)$ and W_2 is Ultra Normal space. Then W_1 is $\boldsymbol{\alpha}^*$ -Ultra Normal space.*

Proof . Assume A_1, A_2 are two (CSs) in W_1 with $A_1 \cap A_2 = \varphi$, since h is closed mapping, then $h(A_1), h(A_2)$ are (CSs) in W_2, since W_2 is Ultra Normal space, then there exist two disjoint clopen sets B_1, B_2 in W_2 satisfy $h(A_1) \subseteq B_1, h(A_2) \subseteq B_2$. Hence $A_1 \subseteq h^{-1}(B_1), A_2 \subseteq h^{-1}(B_2)$. Since h is injective $(C\alpha^* - CM)$, then $h^{-1}(B_1), h^{-1}(B_2)$ are disjoint α^*- clopen sets. Thus W_1 is $\boldsymbol{\alpha}^*$ -Ultra Normal space. □

Theorem 4.10. *Supposeh: $W_1 \to W_2$ is closed surjective $(C\alpha^* - CM)$ and W_1 is $(\alpha^* C-$ space) Then W_2 is strongly closed space.*

Proof . Assume $\{V_i \mid i \in I\}$ is any closed cover of W_2, since h is $(C\alpha^* - CM)$, then $\{h^{-1}(V_i) \mid i \in I$ $\}$is $\boldsymbol{\alpha}^*$ -open cover of W_1, but W_1 is (α^* C- space) , thus W_1 has finite subcover. That means $W_1 = \mathrm{U}_{i\in \mathrm{I}} \quad h^{-1}(V_i)$ and hence $\left(h(W_1) = h\left(\bigcup_{i\in\mathrm{Io}} h^{-1}(V_i)\right) = \bigcup_{i\in\mathrm{Io}} \quad hh^{-1}(V_i) \Rightarrow W_2 = \bigcup_{i\in\mathrm{Io}} \quad V_i\right($ since h is surjective). Thus W_2 is strongly closed space. □

Theorem 4.11. *If* $L : (\Psi, E) \rightarrow (\Psi', E')$ *is* $(SC\alpha^* - CM)$, *then* $L_{\left(\int, A\right)} : L^{-1}\left(\left(\int, A\right)\right) \rightarrow \left(\int, A\right)$ *is also, where* $\left(\int, A\right)$ *is* (SCS) *in* (Ψ', E').

Proof . Suppose that (Θ, B) is a (SCS) in $\left(\int, A\right)$, thus (Θ, B) is (SCS) in (Ψ', E') (since $\left(\int, A\right)$ is (SCS) in (Ψ', E')). Then $L^{-1}((\Theta, B))$ is $(S\alpha^* - OS)$ in (Ψ, E), since $L^{-1}((\Theta, \mathrm{B}))\tilde{\subseteq} L^{-1}\left(\left(\int, A\right)\right)\tilde{\subseteq}(\Psi, E)$, then $L^{-1}((\Theta, B))$ is $(S\alpha^* - OS)$ in $L^{-1}\left(\left(\int, A\right)\right)$. □

Theorem 4.12. *Assume* (Ψ, E) *and* (Ψ', E') *are two* (STSs), $L : (\Psi, E) \rightarrow (\Psi', E')$ *be any* (SM) *and* $(\Psi, E) = \left(\int, A\right) \mathrm{L}(\Theta, B)$, *where* $\left(\int, A\right), (\Theta, B)$ *are disjoint* $(S\alpha^* - CSs)$ in(Ψ, E). *Then* $L|_{\left(\int, A\right)}$ *and* $L|_{(\theta, B)}$ *are* $(SC\alpha^* - CMs)$ *iff* L *is* $(SC\alpha^* - CM)$

Proof . Necessity: Follows by using Theorem (3.4) in [33]. Sufficiency: Assume that (H, M) is (SCS) in (Ψ', E'), since $L|_{\left(\int, A\right)}$ and $L|_{(\theta, B)}$ are $(SC\alpha^* - CMs)$, thus $\left(L|_{\left(\int, A\right)}\right)^{-1}(H, M)$ and $\left(L|_{(\theta, B)}\right)^{-1}((H, M))$ are $(S\alpha^* - OSs)$ in (Ψ, E). So, their union is also. But $L^{-1}((H, M)) = \left(h|_{\left(\int, A\right)}\right)^{-1}((H, M)) \coprod \left(h|_{(\theta, B)}\right)^{-1}((H, M))$ and hence the proof is complete. □

References

[1] O. Njstad, *On some classes of nearly open sets*, Pacific Journal Mathematics, 15(3), (1965), 961-970.
[2] N. M. Abbas, *On Some Types of Weakly Open Sets*, M. SC.Thesis, Baghdad University, (2004).
[3] G.B.Navalagi , *Definition Bank in general topology*, Topogy Atlas, (2002).
[4] H.A. Othman, *New Types of* α *-continuous mapping*, Mustansiriya University, Thesis, (2004).
[5] N. M. Ali and S. M. Khalil, *On* α^* *- Open Sets in Topological Spaces*, IOP Conference Series: Materials Science and Engineering, 5 7 1(2019) 012021, doi: 10.1088 / 1757-899 X / 571 / 1 / 012021.
[6] J. Dontchev, *Survey on preopen sets*, Japan, August,(1998),1-8.
[7] A. R. Nivetha, M. Vigneshwaran, N. M. Ali Abbas and S. M. Khalil, *On* $N_{*g\alpha}$ *- continuous in topological spaces of neutrosophy*, Journal of Interdisciplinary Mathematics, 24(3),(2021), 677-685
[8] S. M. Khalil, & F. Hameed, *An algorithm for the generating permutation algebras using soft spaces.* Journal of Taibah University for Science, 12(3),(2018), 299-308.
[9] S. M. Khalil and M. H. Hasab, *Decision making using new distances of intuitionistic fuzzy sets and study their application in the universities*, INFUS, Adv. Intel. Syst. Comput. 1197 (2020), 390-396.
[10] S. M. Khalil and A. Rajah, *Solving the Class Equation* $x^d = \beta$ *in an Alternating Group for each* $\beta \in H \cap C^\alpha$ *and* $n \notin \theta$, Journal of the Association of Arab Universities for Basic and Applied Sciences, 10,(2011), 42-50
[11] S. M. Khalil and A. Rajah, *Solving Class Equation* $x^d = \beta$ *in an Alternating Group for all* $n \in \theta \& \beta \in H_n \cap C^\alpha$, journal of the Association of Arab Universities for Basic and Applied Sciences, 16(2014), 38-45.
[12] S. M. Khalil, Enoch Suleiman and Modhar M. Torki, *Generated New Classes of Permutation I/B-Algebras*, Journal of Discrete Mathematical Sciences and Cryptography, (2021), to appear
[13] S. M. Khalil, *The Permutation Topological Spaces and their Bases*, Basrah Journal of Science (A), 32(1), (2014), 28-42.
[14] S. M. Khalil and N. M. A. Abbas, *On Nano with Their Applications in Medical Field*, AIP Conference Proceedings 2290, 040002 (2020).
[15] S. M. Khalil, *New category of the fuzzy d-algebras*, Journal of Taibah University for Science, 12(2), (2018),143-149.
[16] N. M. Ali Abbas, S. M. Khalil and M. Vigneshwaran, *The Neutrosophic Strongly Open Maps in Neutrosophic Bi-Topological Spaces*, Journal of Interdisciplinary Mathematics, 24(3), (2021), 667-675.

[17] K . Damodharan, M. Vigneshwaran and S . M. Khalil, $N_{\delta^*\ g\alpha}$ *-Continuous and Irresolute Functions in Neutrosophic Topological Spaces*, Neutrosophic Sets and Systems, 38(1)(2020), 439-452.

[18] M. M. Torki and S. M. Khalil, *New Types of Finite Groups and Generated Algorithm to Determine the Integer Factorization by Excel*, AIP Conference Proceedings 2290, 040020 (2020).

[19] S. M. Khalil and N. M. Abbas, *Applications on New Category of the Symmetric Groups*, AIP Conference Proceedings 2290, 040004 (2020).

[20] S. M. Khalil and F. Hameed, *Applications on Cyclic Soft Symmetric Groups*, IOP Conf. Series: Journal of Physics, 1530 (2020) 012046.

[21] S. M. Saied and S. M. Khalil, *Gamma Ideal Extension in Gamma Systems*, Journal of Discrete Mathematical Sciences and Cryptography, (2021), to appear.

[22] S. M. Khalil, *Dissimilarity Fuzzy Soft Points and their Applications*, Fuzzy Information and Engineering, 8(3), (2016), 281-294.

[23] S. M. Khalil, F. Hameed, *Applications of Fuzzy-Ideals in-Algebras*, Soft Computing, 24(18), (2020), 13997- 14004.

[24] S. A. Abdul-Ghani, S. M. Khalil, M. Abd Ulrazaq, and A. F. Al-Musawi, *New Branch of Intuitionistic Fuzzification in Algebras with Their Applications*, International Journal of Mathematics and Mathematical Sciences,Volume 2018, Article ID 5712676, 6 pages.

[25] S. M. Khalil, Decision making using algebraic operations on soft effect matrix as new category of similarity measures and study their application in medical diagnosis problems, Journal of Intelligent & Fuzzy Systems, 37, (2019), 1865-1877. doi: 10.3233/JIFS-179249.

[26] S. M. Khalil, S. A. Abdul-Ghani, *Soft M-Ideals and Soft S-Ideals in Soft S-Algebras*, IOP Conf. Series: Journal of Physics, 1234 (2019) 012100.

[27] S. M. Khalil and F. Hameed, *An algorithm for generating permutations in symmetric groups using soft spaces with general study and basic properties of permutations space*s. J Theor Appl Inform Technol, 96(9), (2018), 2445-2457.

[28] S. M. Khalil, M. Ulrazaq, S. Abdul-Ghani, Abu Firas Al-Musawi, σ *-Algebra and* σ *-Baire in Fuzzy Soft Setting*, Advances in Fuzzy Systems, Volume 2018 , Article ID 5731682,10 pages.

[29] M. A. Hasan, S. M. Khalil, and N. M. A. Abbas, *Characteristics of the soft-(1, 2)-gprw closed sets in soft bi-topological spaces*, Conference, IT-ELA 2020, 9253110, (2020), 103-108.

[30] S. M. Khalil and A. Hassan, *Applications of Fuzzy Soft* ρ *-Ideals in* ρ *-Algebras*, Fuzzy Information and Engineering, 10(4),(2018), 467-475.

[31] S. M. Khalil, *Decision Making Using New Category of Similarity Measures and Study Their Applications in Medical Diagnosis Problems*, Afrika Matematika,(2021), https://doi.org/10.1007/s13370-020-00866-2.

[32] N. M. Ali Abbas and S. M. Khalil, *On new classes of neutrosophic continuous and contra mappings in neutrosophic topological spaces*, Int. J. Nonlinear Anal. Appl. 12(1), (2021), 719-725.

[33] M. A. Hasan, N. M. Ali Abbas and S. M. Khalil, *On Soft* α^* *-Open Sets and Soft Contra* α^* *-Continuous Mappings in Soft Topological Spaces*, Journal of Interdisciplinary Mathematics, 24(3),(2021), 729-734.

[34] R. Stam, *The algebra of bounded continuous functions into a non archimedean field*, Pacific J Math, 50(1974), 169-185.

[35] A. Kharal, & B. Ahmad, *Mappings on soft classes*. New Math. Nat. Comput., 7(3), (2011), 471481.

[36] M. Shabir and M. Naz, *On Soft topological spaces*, Comp. And Math. with applications, 61(7), (2011), 1786-1799.