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Designing a multi-period credit portfolio optimization model a nonlinear multi-objective fuzzy mathematical modeling approach (Case study: Ansar banks affiliated to Sepah Bank)

Ali Asghar Tehranipour[a], Ebrahim Abbasi[b,*], Hosein Didehkhani[a], Arash Naderian[c]

[a]*Department of Financial Engineering, Aliabad Katoul Branch, Islamic Azad University, Aliabad Katoul, Iran*
[b]*Department of Financial Management, Alzahra University, Tehran, Iran*
[c]*Department of Accounting, Aliabad Katoul Branch, Islamic Azad University, Aliabad Katoul, Iran*

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Abstract

This study aims to design a multi-period credit portfolio optimization model with a nonlinear multi-objective fuzzy mathematical modeling approach. In terms of data collection, this study is a descriptive-survey research and in terms of the nature and purpose of the research, it is an applied one. The statistical population of the research includes all facility files of the last 10 years as well as the statements of financial position of Ansar Bank branches affiliated with Sepah Bank, selected by census method. The risk criteria used in the models include Average Value at Risk (AVaR), Conditional Value at Risk (CVAR) and Semi-Entropy.Each of the objectives and constraints were specified in a state of uncertainty and ambiguity, based on the principles of fuzzy credibility theory, for a state in which the expected rate of stock return is a triangular fuzzy number. Finally, three multi-objective fuzzy models were designed based on the selected criteria. Research models were implemented using MOPSO algorithm. The software used in conducting the research was MATLAB software. The results indicated that the CVAR model performed better than the other two models, i.e. AVAR and Semi-Entropy, in evaluating optimal portfolios.

Keywords: Portfolio optimization, Fuzzy credit theory, Risk, MOPSO algorithm
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*Corresponding author
Email addresses: ali.tehranippor54@gmail.com (Ali Asghar Tehranipour), abbasiebrahim2000@alzahra.ac.ir (Ibrahim Abbasi), h.didehkhani@gmail.com (Hosein Didehkhani), arashnaderian@yahoo.com (Arash Naderian)

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1. Introduction

Financial and credit institutions such as commercial banks, etc. are among the most important institutions active in the Iranian money market. One of the most important tasks of these institutions, as the name suggests, is on the one hand, receiving people's deposits and on the other hand, granting facilities to customers in the form of Islamic contracts such as Ji'ala (contract of reward), installment sales, Mudharabah (bailment of a capital), etc. [3]. Risk is one of the basic concepts in financial markets and has its own special complexity. Since there is no clear and exact picture of risk realization, financial markets need risk control and management approaches. Risk can be regarded as an uncertain phenomenon [30]. Risks that banks encounter are generally divided into four categories: financial risks, operational risks, business risks and risks incurred by incidents [21]. Currently, the most important reason of bank insolvency is credit risk. If the clients do not repay their liabilities on time, these facilities will turn into non-performing loans and this will disrupt the distribution of bank credits and therefore will disrupt domestic economy [1]. Financial and credit institutions must take into account the credit risk of the clients and satisfy their demands for facilities. Because up to now, loan portfolio management issues and problems have been the most important reason of insolvency or loss of banks and financial and credit institutions [3].

If a model is not designed for how to allocate facilities, or the facilities are paid regardless of the degree of credit risk, in other words, if in the facility portfolio of banks, the allocation of facilities is not done properly, in the long run, the banks face the risk of insolvency. Because the bank may allocate many resources to some of its facilities, leading to an increase in the bank's credit risk, an example of which is increase in the number of non-performing loans. Therefore, the designed model has a vital and key role in the set of lending facility portfolios of banks and financial and credit institutions. Harry Markowitz introduced a portfolio construction theory known as modern portfolio theory, the most significant application of which is to create a portfolio risk-return framework for investors to make decisions. By giving a quantitative definition for investment risk, Markowitz developed a mathematical approach for investors in selecting assets and managing portfolios. Markowitz quantitatively showed that diversification of asset portfolios (bank credits) reduces portfolio risk. In fact, the Markowitz model is used to identify an efficient portfolio collection and to select among an efficient collection.

In fact, risk and return are the basic constituents and factors of a portfolio management system. Therefore, the concept can be applied to the management of loans and facilities in terms of the factors mentioned above. The management of the facility portfolio is a constant process of evaluating and taking advantage of various lending opportunities to achieve maximum return within the framework of macro-management goals with the minimum risk. Therefore, the most important components of a model for facility portfolio management will be return and risk of various lending opportunities in different economic sectors [29]. In recent years, different criteria have been proposed to measure portfolio risks, which are summarized below:

There are generally two sets of criteria for measuring portfolio risk:

1. Dispersion criteria, measured by a tool such as mean deviation-mean absolute deviation, and so on.
2. Downside portfolio risk criteria, measured by a tool such as semi-variance-fuzzy value-at-risk and fuzzy conditional value-at-risk, and so on.

Multi-period portfolio optimization has been one of the interesting topics and challenges for researchers in recent years due to the shortcomings of the single-period portfolio optimization model, some of which are mentioned below:

1. Single-period models can be difficult to use for investors with long-term investment perspectives, such as pension funds, insurance companies, and (natural) persons.
2. The one-period average-variance model is not able to include transaction costs (including taxes) and market impact costs in its model in a natural way. For example, most investors are hesitant about selling securities whose values have increased because of the tax effects. Transaction costs have great importance role for many long-term investors.
3. A single-period model will often offer a fixed asset portfolio, hence, it is not possible to revise the amount of investment in that asset until the end of the period, while the multi-period model offers recommendations that are more rational. This means that in order to maximize wealth at the end of the period, it is possible to assign different weights to an asset in different periods.
4. The last fault of single-period models is related to investment portfolio performance. This is because the single-period mean-variance model does not take advantage of the opportunities that arise during the investment period to increase portfolio returns by adjusting the composition of assets.

These issues revealed that multi-period optimization must be paid attention to. Therefore, for long-term investors, a multi-period model will be more effective than single-period mean-variance models. In addition, uncertainty has always been one of the basic challenges in financial markets and investment environments. The first characteristic of a financial asset is the uncertainty the rate of return, so the concept of risk arises from this feature in financial markets. If there is no uncertainty, there will be no risk. To address this uncertainty, several approaches have been taken in portfolio optimization, two of which can be mentioned. While the first approach utilizes stochastic modeling based on probabilistic distribution, the second approach is based on uncertainty modeling and the application of fuzzy concepts. Recently, a lot of research has been done on risk modeling in fuzzy environments. For example, Xu and Kaymak [34] used probabilistic fuzzy systems to estimate value at risk. We will also use a fuzzy approach in this research.

Due to the merger of military banks in Sepah Bank, the officials of this bank definitely try to identify the degree of credit risk of each bank and finally optimize the credit portfolio of the entire bank in compliance with international financial reporting standards. Considering the importance of portfolio optimization in reducing credit risk in banks and financial institutions and increasing credit risk while reviewing the files in the bank archives, we try to identify the components affecting credit risk in this collection and also to provide a good model for optimizing the credit portfolio of Ansar Bank affiliated to Sepah Bank. This will ultimately increase the return on assets and will control the bank's credit risk, and the purpose of the present study will be fulfilled. In fact, in this study, we aim to answer the research question of how to create a model for optimizing the credit portfolio of Ansar Bank affiliated to Sepah Bank, in order to maximize investment returns and minimize credit risk.

No similar research has been conducted that directly addresses the issue in question, and most of the prior research has been on investment companies and stock exchange. In fact, employing this method in a bank regarding the optimization of credit risks may be considered as one of the innovations of the present study. Therefore, we will just provide a few examples of studies on this subject.

Didehkhani et al. [9] wrote an article entitled "Developing a Fuzzy Multi-bjective Model for Multi-period Portfolio Optimization Considering Average Value at Risk". The results of model implementation using MOPSO algorithm showed that the optimal Pareto portfolios resulting from model implementation have better and more desirable performance in terms of achieving predetermined goals compared to portfolios with random weights. The results also indicated that the higher the degree of portfolio diversification, the lower the amount of final wealth. Didehkhani et al. [8] conducted a study entitled "A Comparative Study of Multi-Objective Multi-Period Portfolio Optimization Models in a Fuzzy Credibility Environment Using Different Risk Measures". The results of evaluating the performance of optimal portfolios considering Sharpe and Traynor measures showed that the Mean-AVaR model has a better performance than the other two models, i.e. Mean-Semi Entropy and Mean-VaR. Based on data envelopment analysis (DEA) and taking into account the application of value at risk model in cross efficiencies, Kazemi et al. [17], addressed the problem of stock portfolio optimization. The results showed that Sharpe measures had a better performance in the proposed method than the other methods. Poor Ahmadi and Najafi [26] upgraded single-period optimization to dynamic and multi-period optimization and considering the cost of efficiency transactions concluded that the multi-period model performed better in the long run than the single-period model. Hesabi and Rahmati (2015) conducted a study entitled "Explaining the Optimal Model of Facility Portfolio to Reduce Credit Risk". The results indicated that the implementation of the optimal model of facility portfolio taking an approach of credit risk reduction could reduce the bank's loan portfolio risk by maintaining the minimum expected return. Jafari and Dezfuli Khajehzadeh (2015) conducted a research entitled "Robust Fuzzy Multi Objective Optimization Model for Portfolio Selection". The results confirmed the efficiency of the proposed method in terms of considering both risk, return, the amount of budget and maximum investment per share.Rajabi and Khaloozadeh (2015) compared two important and widely used methods of non-dominated sorting genetic algorithm-II, (NSGA-II) and multi-objective particle swarm optimization (MOPSO). The optimal Pareto fronts obtained, allow the investors to choose the optimal capital portfolio from different risks and values. The value of the capital portfolio and its risk were used as optimization goals and the criterion of conditional value at risk was used as a risk measure, and three practical and applied constraints were considered to solve the problem. The results confirmed the better performance of NSGA2 method versus MOPSO for both criteria, i.e. convergence and the width of Pareto optimal fronts. In addition, in predicting the optimal stock portfolio, the adaptation of the real and projected Pareto optimal fronts shows high efficiency of the applied methods. Vafaei et al. [32] in their research entitled "Using Fuzzy Mathematical Programming to Design Portfolio Optimization Model (Case Study: Melli Bank Investment Co.)" studied the issue of portfolio optimization. They took into account both its significance in the stock market as the main axis of each country's financial system and its role in the optimal allocation of resources, and its importance for investors who want maximum returns and minimum expected risks. They designed a model in fuzzy conditions to bring the maximum expected return to investors while minimizing the risk. To do this, they used multi-objective decision making (MODM) methods. After modeling, the stock portfolio of Bank Melli Investment Company was examined as a case study.

Katagiri et al. [16] considered bi-level linear programming through possibility-based value at risk model. Mussa et al. [22] also estimated the value at risk (VAR) and the expected shortfall (ES) using fuzzy random variables. Peng [25] discussed the calculation of value at risk based on fuzzy credit theory and found relations for value at risk and the average value at risk under fuzzy credit theory. Wang and Watada [33] also estimated the value-at-risk measure using the credit theory in their multi-criteria investment portfolio selection model. Hung [14] used fuzzy variable variance under credit theory to measure risk in the investment portfolio selection model.

2. Theoretical foundations of research

2.1. Optimization

Mathematical optimization or mathematical programming in mathematics, economics, and management refers to selecting the best member of a set of attainable members. In the simplest form, it is attempted to find the maximum and minimum value of a real function through a systematic selection of the data from an attainable set and calculation of the value of the real function.

According to the related literature, if the model parameters vary of the nominal value, the optimal response that is obtained based on the nominal value of the parameters may no longer satisfy the optimality conditions or violate some constraints, hence will be out of the range of reasonable responses [6]. Optimization in the conditions of uncertainty was introduced by Dantzig [7] by offering stochastic optimization and Charnes and Cooper [4] by offering optimization in possible constraints. These two approaches (stochastic optimization and optimization in conditions of possible constraints) are based on assumptions. In other words, the information regarding "probability distribution of the random parameters of the model" which is available for these parameters is used to convert the stochastic model into a deterministic model. The final deterministic model may be in the form of linear programming, nonlinear programming, and otherwise. In mathematical programming, normally the problems are solved in advance with the assumption that the data are certain, whereas in the real world most data are uncertain. The main default in the mathematical programming of model development is based on explicitly defined data and equivalent to nominal value. Whereas, data uncertainty has no effect on the quality and feasibility of the answers. In real-world problems, if an item of the data changes, a large number of constraints will be violated and the obtained response may be non-optimal or even impossible. As a result, the main question of providing a solution to the problem arises which can withstand the uncertainty of the data. This solution is called robust solution and this type of optimization is termed as robust optimization.

The approach studied in the literature related to this subject classifies the assets in groups based on the similarity between assets and selects a representative from each group [6]. The criterion used to measure the similarity between assets is the correlation coefficient between the assets. In this model, it is assumed that a portfolio manager intends to select n shares of q shares constituting the index, in order to form a portfolio. The following model divides the shares into q groups and selects a representative from each group. According to Markowitz's model, risk is associated with efficiency fluctuations and fluctuations are measured by the efficiency variance. The rate of return on a portfolio, consisting of different assets, is derived from the weighted average return on individual assets forming that portfolio:

$$r_p = \sum_{i=1}^{N} x_i r_i \tag{2.1}$$

In the above relation, r_p is the portfolio rate of return, r_i is the rate of return on asset i, x_i is the asset weight of asset i in the portfolio (ratio of the net value of asset i to the net value of the total portfolio) and N is the number of assets available in the portfolio.
The risk of the portfolio is obtained from the following relationship:

$$\sigma^2_{p=} \sum_{i=1}^{N} \sum_{j=1}^{N} x_i x_j \rho_{ij} \sigma_i \sigma_j = \sum_{i=1}^{N} \sum_{j=1}^{N} x_i x_j \rho_{ij} COV_{ij}. \tag{2.2}$$

In the above relation, σ_p^2 is portfolio variance; σ_i and σ_j are standard deviations of assets i and j, respectively; ρ_{ij} is the correlation coefficient between assets i and j; x_i and x_j are weights of the assets i and j in the portfolio, respectively; and N is the number of assets in the portfolio.

To optimize a portfolio based on the Markowitz model of risk management, we use the following nonlinear programming model:

$$\begin{aligned}
&\text{Min } Z{=}\delta_P^2\\
&S.T{:}\overline{r}_p{=}\sum_{j=1}^{M} x_j\overline{r}_j\\
&\sum_{j=1}^{M} x_j{=}\ 1\\
&x_j = 0.
\end{aligned} \tag{2.3}$$

Some of the advantages of the Markowitz model, leading to its wide application, include being bi-parametric and the various ratios that have been suggested to measure its efficiency. This Model also has some shortcomings, including high complexity in solving the nonlinear model, the logical problem of increasing the risk resulting from a quantitative rise in numbers, lack of a precise and correct understanding of variance compared with other risk measures, and that the variance to considers positive and negative variations in the same way [12].

2.2. Fuzzy credibility theory

As mentioned, generally a fuzzy event occurs if its credibility is equal to 1 and it does not occur if its credibility is equal to zero. Suppose (Θ) is a nonempty set that represents the sample space and $P(\Theta)$ is a power set of (Θ) consisting of all possible subsets, and each element in $P(\Theta)$ is called an event. In order to provide an evident definition of credibility, it is necessary to assign a value of *Cr(A)* to the event A, which indicates the credibility of the occurrence of event A. In addition, to ensure that *Cr(A)* has particular mathematical properties, it must obey the following 4 principles:

Normality: $\mathrm{Cr}\{\Theta\}{=}\ 1$.

Uniformity: $A{\subset}B \to Cr\{A\}{\leq}Cr\{B\}$.

Self-Duality: $Cr\{A\} + Cr\{A^c\} = 1$, for all $A{\in}\mathrm{P}(\Theta)$.

Maximizing: (3.1) $Cr\{U_iA_i\}{=}\sum \sup_i Cr\{A_i\}$, for all $\{A_i\}, \sup_i Cr\{A_i\} < 0.5$.

The first three principles are self-evident. The fourth principle is that if the credibility of a fuzzy event is equal to 1 or zero, there is no uncertainty as to the outcome of that event, because it is believed that the event occurs (or does not occur). On the other hand, an event has a high degree of uncertainty, if its credibility is equal to 5% because in this case, possibility of occurrence or non-occurrence of the event is the same. In addition, if no information is available about the credibility measure of an event, its value should be assumed 5%. Accordingly, Liu [20] proposed the principle of maximum uncertainty, which states that if for each event, there are different logical values for its credibility measure; the nearest value to 5% will be assigned to it. The set of Cr functions is called the credibility measure if it satisfies the principles of normality, uniformity, self-duality, and maximization.

Now suppose ξ is a fuzzy variable with function μ. For each set B of the set of real numbers, we can write the Equation (2.4):

$$Cr\left(\xi \in B\right){=}\frac{1}{2}\left(\sup_{x\in B}\mu(x) + \sup_{x\in B^c}\mu(x)\right) \tag{2.4}$$

Now in a more special case it can be proved (for each $r \in R$)

$$Cr\left(\xi{\leq}r\right){=}\frac{1}{2}\left(\sup_{x\leq r}\mu\left(x\right){+}1{-}\sup_{x>r}\mu\left(x\right)\right). \tag{2.5}$$

The expected value of the fuzzy variable ξ was also defined by Liu and Liu [19] as Equation (2.6):

$$E\left[\xi\right]=\int_{0}^{+\infty}Cr\left\{\xi\geq r\right\}dr-\int_{-\infty}^{0}Cr\left\{\xi\leq r\right\}dr. \tag{2.6}$$

Of course, equation (2.6) is conditional upon at least one of the two integrals being finite. Now if ξ is a fuzzy variable with finite expected value e, its variance is defined as Equation (2.7):

$$V\left[\xi\right]=E\left[\left(\xi-e\right)^{2}\right] \tag{2.7}$$

Now if $\left(\xi-e\right)^{2}$ is an indeterminate non-negative variable, we can write Equation (2.8):

$$V\left[\xi\right]=\int_{0}^{+\infty}Cr\left\{\left(\xi-e\right)^{2}\geq r\right\}dr. \tag{2.8}$$

The credibility distribution Φ: $\mathrm{R}\rightarrow[0, 1]$ for a fuzzy variable ξ is defined by Liu [20] as Equation (2.9):

$$\Phi\left(x\right)=Cr\left\{x\in\Theta\,|\xi\left(x\right)\leq x\right\} \tag{2.9}$$

Liu and Gao [21] also introduced the concept of independence of fuzzy variables. Accordingly, the variables $\xi_1, \xi_2, \ldots, \xi_m$ are termed as independent variables, if and only if for each set $B_1, B_2, \ldots, B_m \in R$, there is the following relationship:

$$Cr\left\{\bigcap_{i=1}^{m}\left\{\xi_i\in B_i\right\}\right\}=\min_{1\leq i\leq m}Cr\left\{\xi_i\in B_i\right\}. \tag{2.10}$$

Now, based on the definitions and relations expressed, the fuzzy credibility theory can be used to obtain the value at risk in the fuzzy environment and under the credibility theory. It should be noted that this estimate is also referred to as credibility value at risk.

2.3. Fuzzy average value at risk

Value at risk, one of the indicators of downside risk, is a measure of the maximum potential loss of portfolio introduced by Weatherstone in 1994. A value at risk measures risk quantitatively and it is considered as one of the key and widely used tools in risk management. By definition, value at risk is the maximum loss that the reduction in the value of the portfolio will not exceed in a given period in the future with a certain confidence level. In other words, VaR measures the worst expected loss under normal market conditions over a period of time and at a certain level of confidence [2]. For example, if $A = (a.b.c)$ is a triangular fuzzy number for any confidence level $0 < \alpha\leq 1$, the fuzzy average value at risk using the theory of credibility can be expressed as follows:

(1)	$AVaR_{\alpha}\left(\phi\right)=\begin{cases}(a-b)\alpha-a, & \text{if } \alpha=0.5,\\ c-2b-\frac{1}{4\alpha}(a-2b+c)+(b-c)\alpha, & \text{if } \quad \alpha>0.5.\end{cases}$

2.4. Conditional value at risk

Although the concept of value at risk seems very simple and understandable, its calculation is very difficult. Calculating the value at risk statistically means finding the critical value for the desired probability level. Since the probability distribution of return over time is not constant, there are problems in calculating the value at risk. One of the main problems of value at risk is the incoherence of this criterion. Therefore, conditional value at risk has been introduced in order to improve the value at risk [13] and it is considered as an interesting measure of risk (coherent risk measure) that has gained acceptance in recent years. Conditional value at risk, abbreviated as CVaR, gradually emerged as a useful tool for risk measurement and management [14]. This criterion estimates the expected loss equal to or higher than the value at risk, at a certain level of confidence. Therefore, this view is more conservative than the previous view [13].

Ever since Rockafellar and Uryasev introduced the CVaR model, the theory and application of this model has expanded rapidly. VaR and CVaR have been frequently used as risk criteria in risk management. VaR measures the maximum expected loss over a specified time perspective at a certain level of confidence, and the equivalent CVaR, is defined as a conditional expectation of portfolio loss that is greater than or equal to VaR [11], however VaR has nothing to say about the losses beyond itself [8].

CVaR is a more coherent measure of risk and has interesting features:

1. First, the CVaR performs better than the mean-variance analysis in the face of an asymmetric distribution of return on capital.
2. Second, CVaR minimization is usually the result of solving a convergent programming problem, such as linear programming problems, that allows the decision maker to handle a large-scale portfolio problem more efficiently [11].

Conditional Value at Risk (CVaR) has all the characteristics of Value at Risk (VaR) and it also has more advantages than VaR, including simplicity of calculations, more accurate measurement of risk and consideration of different probabilities for different situations [14]. For example, if $A = (a.b.c)$ is a triangular fuzzy number for any confidence level $0 < \alpha \leq 1$, the value of the fuzzy conditional risk can be expressed as follows using credibility theory:

*(1)	$\xi CVaR(\alpha) = \begin{cases} \alpha a + (1+\alpha)b & \text{if } \alpha = 0.5, \\ (a-1)b - \alpha c & \text{if } \alpha > 0.5. \end{cases}$

2.5. Entropy and financial issues

Entropy measures the degree of difficulty in predicting the specific value that a variable will take.

Philippatos and Wilson (1972) were the first researchers who applied the concept of entropy to portfolio selection. In their thesis, a mean-entropy approach was proposed and compared to traditional methods. Since then, many researchers have refined the theory of portfolio selection with the concept of entropy, proposing different types of entropy and using it in financial matters. One of the most important issues in capital markets is choosing the right investment strategy; hence, many models have been developed for this purpose. Most of these models are based on two factors: risk and return. As a result, a lot of research has been done on risk measurement. One of the applications of entropy in portfolio optimization is the use of entropy as a measure of risk [31]. Entropy considers both high and low returns in expressing return uncertainty. Uncertainty of return that is undesirable for the investor refers to returns lower than expected. Therefore, Zhou et al. [35] introduced fuzzy semi-entropy and used it as a risk measure in portfolio optimization. Entropy has been used as an accepted measure of portfolio diversification. The degree of portfolio diversification is measured using the entropy of the ratio. Entropy has also been used in asset pricing, option pricing and other financial derivatives [36].

3. Research method

In terms of data collection, this study is a descriptive-survey research and in terms of the nature and purpose of the research, it is an applied one. The data collection in this study was based on the credit files in Ansar Bank affiliated to Sepah Bank and the reports taken from the credit dashboard in the bank. The risk criteria used in the models include Average Value at Risk (AVaR), Conditional Value at Risk (CVAR) and Semi-Entropy. The statistical population of this research includes all facility files of the last 10 years as well as the statements of financial position of Ansar Bank branches affiliated to Sepah Bank, which were selected by census method. In this research, first, having reviewed the research literature, the objectives and indices of the portfolio optimization issue were investigated based on the practical character of this issue and the main criteria were selected. Then, each of the objectives and constraints were specified in a state of uncertainty and ambiguity, based on the principles of fuzzy credibility theory, for a state in which the expected rate of stock return is a triangular fuzzy number. Finally, three multi-objective fuzzy models were designed based on the selected criteria. Research models were implemented using MOPSO algorithm. The software used in conducting the research was MATLAB software.

3.1. Method of measuring research objectives, based on credibility theory

In this section, the objectives used in the research and the way they are measured will be introduced.

Criterion	How to measure goals in fuzzy and portfolio optimization
Expected fuzzy value Liu (2008)	$E\left[\xi\right]=\frac{a+2b+c}{4}$
Fuzzy Value at Risk Peng (2011)	$\xi V_a R\left(\alpha\right)=\begin{cases} 2\left(a-b\right)\alpha-a & \text{if } \alpha\leq 0.5 \\ 2\left(b-c\right)\alpha+c-2b & \text{if } \alpha>0.5 \end{cases}$
Average Fuzzy Value at Risk Peng (2008)	$\xi AV_a R\left(\alpha\right)=\begin{cases} \left(a-b\right)\alpha-a & \text{if } \alpha\leq 0.5 \\ c-2b-\frac{1}{4\alpha}\left(a-2b+c\right)+\left(b-c\right)\alpha & \text{if } \alpha>0.5 \end{cases}$
Fuzzy semi-entropy Zhou et al. (2016)	$S_h\left(\xi\right)=\begin{cases} \left(b-a\right)\rho-\left(b-a\right)\zeta\left(\rho\right) & \text{if } \frac{\alpha+2^{*}b+c}{4}\leq b \\ \frac{b-a}{2}+\left(c-b\right)\zeta\left(\tau\right) & \text{if } \frac{\alpha+2^{*}b+c}{4}>b \end{cases}$ $\zeta\left(x\right)=x^2\ln x-\left(1-x\right)^2\ln\left(1-x\right)$ $\rho=\left(2b+c-3a\right)/8\left(b-a\right)$ and $t=\left(3c-2b-a\right)/8\left(c-b\right)$

$$\max W_{T+1}=\text{Min } Avar(\alpha) \quad \text{s.t.} \quad \sum_i w_i=1, \quad w_i\geq 0.$$

3.2. Objective functions

Objective 1: Maximizing the rate of return on assets at the end of the period.
Objective 2: Minimizing credit risk.

3.3. Solving the model using MOPSO algorithm

One of the problems we face in stock portfolio optimization applications is how to solve the developed models. Since the use of different risk criteria and higher torques in stock portfolio optimization leads to nonlinearity and NP Hard problem, meta-heuristic methods should be used in applied problems. A wide range of these methods and different algorithms have been used in this field. One of the best methods is the MOPSO algorithm due to the multi-objective nature of the developed model. Coello et al. (2002) developed this algorithm for multi-objective problems by modifying the particle swarm optimization algorithm. The main difference between MOPSO and PSO is in determining the best particle in the population as well as determining the best personal memory of each particle. In the MOPSO algorithm, a new concept called archive is introduced to the single-objective mode, which is actually the storage of unsuccessful responses. By defining the archive in this algorithm, the concept of the best particle in the population has also changed. The main steps of the MOPSO algorithm are summarized as follows:

1. Creating the initial population and initialization of the speed and location vectors of each particle (in the initialization of the speed vector the particles equal to the zero vector and the location vector are created randomly).
2. Calculating cost functions for particles
3. Finding the non-dominated members of the population and storing these particles in the archive
4. Generating hypercubes in the target space and place particles in these hypercubes.

Each particle randomly selects a leader from the archive and moves towards it. Like the single-purpose PSO, here the motion of each particle requires updating the speed and location of the particle. The difference is that the concepts of the best particle in the whole population and the best personal memory of each particle are different from the single-purpose state. The movement of each particle towards the selected leader from the archive can be considered as the following steps.
The speed of each particle is updated using the following equation.

$$V^i\left(t-1\right)=W\times V^i\left(t\right)+c_1r_1\left(x^i_{p\ best}-x^i\left(t\right)\right)+c_2r_2\left(x_{g\quad best}-x^i\left(t\right)\right). \tag{3.1}$$

This is similar to the PSO algorithm. The best personal memory is the p_i particle (the concept and how to determine the best personal memory is explained below) which is the value selected from the archive as the leader. To select the argument h, first, the target space is separated by hypercubes in the bi-objective tabulation mode with square cells, then, according to the number of particles in each hypercube, the probability of selection is calculated. In other words, a merit coefficient is assigned to that hypercube. A hypercube with fewer particles will have a higher merit coefficient. In this research, the inverse proportion method has been proposed to assign these probabilities to hypercubes. In this method, the following equation is used to assign probabilities to hypercubes.

$$p_i=\left|\frac{\left(1/n_i\right)^\beta}{\sum\left(1/n_i\right)^\beta}\right|. \tag{3.2}$$

Here, the number of particles in the above cube is p_i and the pressure is selective. In this case, all hypercubes are selected with equal probability, otherwise only hypercubes containing one particle will be selected. The location of each particle is updated using the following equation:

$$x^i\left(t+1\right)=x^i\left(t\right)+v^i\left(t+1\right). \tag{3.3}$$

After updating the particle location, the speed vector becomes symmetric if these particles are out of the search space. Dominated members of the current population are added to the archive, and previous members of the archive dominated by new members are removed from the archive. Moreover, due to the hardware limitations in the implementation it is necessary to consider a capacity for archiving. If the number of archive members exceeds this capacity, additional members will be removed using a secondary criterion. In this algorithm, the secondary criterion is considered to improve the order of dispersion of non-dominated particles in the target space. Hence, particles that are in denser regions are more likely to be removed and particles in more vacuous regions less likely to be removed. To do this, like the selection of the hypercubes, a set of deletion probabilities is specified and the members to be deleted are selected using the roulette wheel.

5. Updating the best personal memory of each particle, given that in multi-objective optimization, the responses can have three different modes in relation to the each other.

If the new position of the particle is compared with its best personal memory, the following three situations occur:

1. If the new situation dominates the best personal memory, the new situation replaces the best personal memory.
2. If the new situation is dominated by the best personal memory, no change will occur.
3. If the new situation and the best personal memory do not affect each other, one of them is randomly selected as the best memory.
4. If the termination condition is not met, go to step four, otherwise the algorithm terminates.

3.4. Mathematical modeling of credit portfolio based on fuzzy credit theory

In this section, we introduce the symbols and constraints of the model and among the introduced target functions, we develop the model based on the fuzzy objective functions of expected value, variance, skewness, semi-elongation, value at risk and so on.

According to the research literature on credibility portfolio optimization, it can be said that so far, the meta-heuristic algorithm of multi-objective particle swarm has not been used. Accordingly, this study has the following characteristics:

1. Using fuzzy credibility theory.
2. Using criteria of average, conditional and semi-entropy value at risk, for example coherent risk criterion.
3. Considering the costs of general and specific reserves corresponding to the creation of new assets and changing the class of assets in the credit portfolio that is required to be disclosed in compliance with international reporting standards.
4. Considering risk-free income as the bank allocates part of its assets to risk-free assets for deposit with the central bank and providing short-term liquidity.
5. Considering the maximum and minimum amounts allocated to each facility based on the policies of the Central Bank in the form of policy and regulatory packages communicated to commercial banks.
6. Considering the minimum and maximum variety of facilities that can be present in the portfolio (taking into account the degree of diversification).
7. Using a multi-objective particle swarm algorithm to solve the proposed mathematical model.
8. Using ratio entropy to achieve the minimum degree of credibility portfolio diversification.

3.5. The structure of the proposed model

The fuzzy multi-objective optimization model based on credibility theory for the problem of facility portfolio selection is formulated as follows:

Problem

Min $AVaR\left(x_1\xi_1+x_2\xi_2+\cdots+x_n\xi_n\right)$

Min $CVaR\left(x_1\xi_1+x_2\xi_2+\cdots+x_n\xi_n\right)$

Max $SEMI\ ENT\left(x_1\xi_1+x_2\xi_2+\cdots+x_n\xi_n\right)$

Subject to

max $W_T=W_1\prod_{t=1}^{T}\left(1+\sum_{i=1}^{n}\left(\frac{a_{it}+2*b_{it}+c_{it}}{4}\right)x_{it}+r_f(0.132\sum_{i=1}^{n}x_{it})-\ \sum_{i=1}^{n}\mathrm{COST}_{it}\left(|x_{it}|\right)\right)$

Min $AVaR\left(\alpha\right)\ =\ \sum_{t=1}^{T}\left[\sum_{i=1}^{n}\left((c-2b)-\frac{1}{4a}\left(a-2b+c\right)+\left(\mathrm{b}-\mathrm{c}\right)a\right)x_{it}\right]\qquad \alpha\geq\ 0.5$

Min $CVaR\left(\alpha\right)\ =\ \sum_{t=1}^{T}\left[\sum_{i=1}^{n}\left((a-1)\mathrm{b}-ac\right)x_{it}\right]\qquad \alpha\geq\ 0.5$

$$\min\ S_h\left[\xi\right]=\begin{cases}\sum_{t=1}^{T}\left[\sum_{i=1}^{n}\left(\left(b_{it}-a_{it}\right)\rho_{it}-\left(b_{it}-a_{it}\right)\zeta\left(\rho_{it}\right)\right)x_{it}\right], & \text{if } \frac{a_{it}+2*b_{it}+c_{it}}{4}\leq b_{it}\\ \sum_{t=1}^{T}\left[\sum_{i=1}^{n}\left(\frac{b_{it}-a_{it}}{2}+\left(c_{it}-b_{it}\right)\zeta\left(\tau_{it}\right)\right)x_{it}\right] & \text{if } \frac{a_{it}+2*b_{it}+c_{it}}{4}>b_{it}\end{cases}$$

$\rho_{it}=\frac{(2b_{it}+c_{it}-3a_{it})}{8(b_{it}-a_{it})}, i=1\ldots,n;\ t=1,\ldots,T.$

$\tau_{it}=\frac{(3c_{it}-2b_{it}-a_{it})}{8(c_{it}-b_{it})}, i=1\ldots,n;\ t=1,\ldots,T.$

s.t

1	$\sum_{i=1}^{n}x_{it}+x_{r_ft}=1, i=1\ldots,n, t=1,\ldots,T.$
2	$*R_t\geq \min\_r_t, t=1,\ldots,T.$
3	$-\sum_{i=1}^{n}x_{it}Lnx_{it}\geq e, t=1,\ldots,T,\ i=1,2\ldots,n, 0\ \leq e\leq \ln\ n.$
4	$x_{it}\geq 0, i=1\ldots,n, t=1,\ldots,T.$
5	$L_{it}\leq x_{it}\leq \mathcal{U}_{it}, i=1\ldots,n, t=1,\ldots,T.$
6	$h_t\leq\sum_{i=}^{n}\mathcal{Y}_{it}\leq K_t, i=1\ldots,n, t=1,2,\ldots,T.$
7	$\mathcal{Y}_{it}\epsilon\ \left[0,1\right], i=1\ldots n, t=1,\ldots,T.$
8	$R_t=\sum_{i=1}^{n}\left(\frac{a_{it}+2*b_{it}+c_{it}}{4}\right)x_{it}-\sum_{i=1}^{n}COST_{it}\left(\lvert x_{it}-x_{it-1}\rvert\right); i=1\ldots,n, t=1,\ldots,T.$

W_t : Expected income at the beginning of the tth period $t=1,2,\ldots,T$.

x_{it} : The amount (ratio of total funds) of investment in the ist asset in the period $i=1,2\ldots n$, $t=1,2,\ldots,T$.

x_{r_ft}: The amount (ratio of total funds) of investment in risk-free assets (central bank deposit and liquidity) in the period $t=1,2,\ldots,T$.

ξ_{it}: Fuzzy rate of return on assets i in the period $i=1,\ 2\ldots,n$ and $t=1,2,\ldots,T$.

$\mathrm{r_f}$: Risk-free rate of return.

R_t: Net return rate of portfolio x_t in the period t, $t=1,\ 2,\ldots,T$.

$\mathrm{min\_r_t}$: The minimum acceptable rate of return of the portfolio in the period $t=1,2,\ldots,T$.

(α): confidence coefficient of average and conditional value at risk.

e: Degree of expected portfolio diversification.

U_{it}: The maximum asset ratio that can be assigned to asset i in the period t. $i=1,2\ldots,n, t=1,2,\ldots,T$.

L_{it}: The minimum asset ratio that can be assigned to asset i in the period t, $i=1,2\ldots,n$ and $t=1,2,\ldots,T$.

COST_{it}: Cost of creating each asset unit i (cost of non-current assets created) in the portfolio $i=1,2\ldots,n$ and $t=1,2,\ldots,T$.

K_t: The maximum number of assets that can be in the portfolio.

h_t : The minimum number of assets that can be in the portfolio.

$\mathcal{Y}_{it}$: A binary variable that indicates whether the asset i is present in portfolio t or not.

$$\mathrm{Y}_{it}=\begin{cases}1 & \text{if asset } i \text{ exists in portfolio } t\\ 0 & \text{otherwise}\end{cases}, t=1,2,\ldots,T, i=1,2\ldots,n.$$

It should be noted that the figures are in million Rials.

4. Data analysis

A. Identifying the main indicators of portfolio optimization based on the research background and information available in the bank facility files

The main indicators of portfolio optimization
Debt
Mudarabeh (profit-sharing contract)
Predecessor
Civil partnership
Ji'ala (contract of reward)
Installment Sale
Lease on condition of ownership
Debt Purchase
capital-plus-interest
Exchange account

B. Implementing Mean-AVAR model

Running the model by MOPSO algorithm, Figure 1 shows the optimal Pareto fronts with 1000 repetitions. Taking into account the degree of portfolio diversification equal to 1.5, the number of failed portfolios obtained is 100 portfolios.

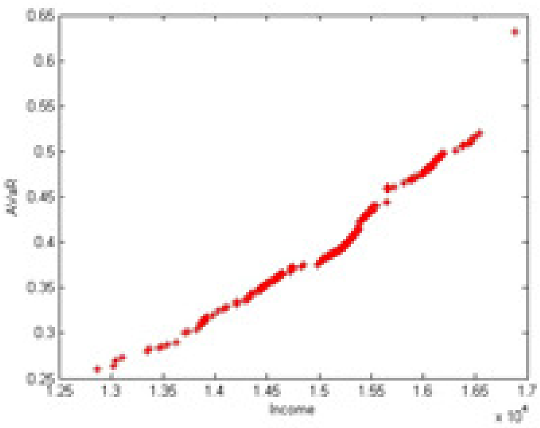

Figure 1. Pareto optimal fronts resulting from the implementation of the Mean-AVAR model.

The table below shows the maximum, minimum, mean and standard deviation of the optimal portfolios in the Mean-AVAR model.

Table 1. Maximum, minimum, mean and standard deviation of optimal portfolios in Mean-AVAR model

	Final income	Risk
Minimum	15359	2602%
Maximum	16185	6314%
Mean	15572	4024%
Standard Deviation	711	0618%

Among the optimal portfolios created in the Mean-AVAR model, the minimum amount of income generated is 15359 million Rials with a risk of 2% and the maximum final income is 16185 million Rials with a risk of 6%. The mean of total portfolios is 15572 with a mean risk of 4%.

C. Implementing Mean-CVAR model

By implementing this model, the number of nondominated portfolios obtained is 75 portfolios. Figure 2 shows the optimal Pareto portfolios.

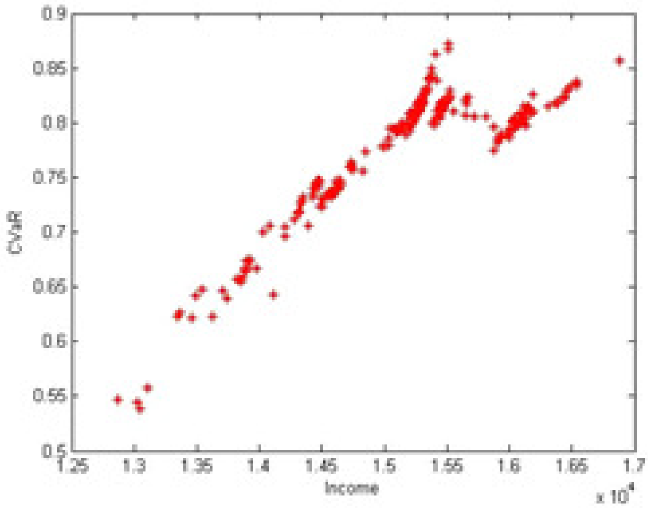

Figure 2. Pareto optimal fronts resulting from the implementation of the Mean-CVAR model.

	Final income	Risk
Minimum	15479	5381%
Maximum	16878	8718%
Mean	16187	7748%
Standard Deviation	758.9	0622%

Among the optimal portfolios created in the Mean-CAVAR model, the minimum amount of income generated is 15479 million Rials with a risk of 5% and the maximum final income is 16878 million Rials with a risk of 8%. The average of total portfolios is 16187 with an average risk of 7%.

D. Implementing Mean-Semi-Entropy model

By implementing this model, the number of nondominated portfolios is 200 portfolios. Figure 3 shows the optimal Pareto portfolios.

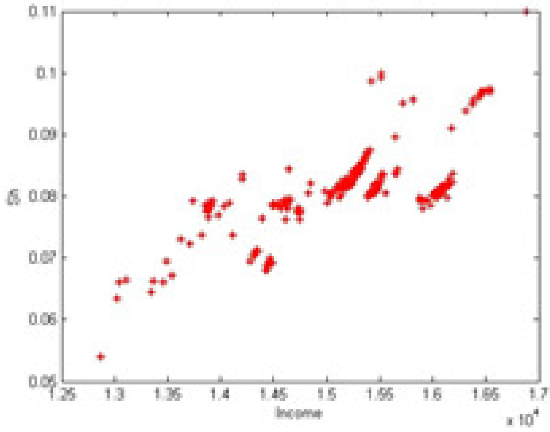

Figure 3. Pareto optimal fronts resulting from the implementation of the mean-semi-entropy model.

	Final income	Risk
Minimum	15339	0541%
Maximum	16086	1099%
Mean	15712	0810%
Standard Deviation	394	0055%

Among the optimal portfolios created in the mean-semi-entropy model, the minimum amount of income generated is 15339 million Rials with a risk of 5% and the maximum final income is 16086 million Rials with a risk of 81%. The average of total portfolios is 15712 with an average risk of 8%.

5. Discussion and conclusion

In this study, multi-period portfolio optimization with different risk criteria was measured by three models of average value at risk (AVaR), conditional value at risk (CVAR) and semi-entropy. All research models were implemented using MOPSO algorithm. The results show that the performance of CVAR model is better than the other two models, AVAR and semi-entropy, in evaluating optimal portfolios.

Many financial institutions seek to identify sources of risk and try to control and manage it. Considering the research results and considering the capabilities of the value-at-risk criterion, it is proposed as a coherent risk criterion. Due to the advantages of the multi-period model, portfolio managers can use this risk criterion to measure portfolio risk well, and finally identify the financial assets that increase the risk and take action to minimize the portfolio risk. To optimally and reallocate assets.

Other future researchers who intend to conduct research in this field are advised to use other multi-objective algorithms as well as other criteria and risk measures.

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