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The new integral transform and its applications

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Abstract

We offer a new complex integral transform, the complex SEE transform, in this paper. The properties of this transform are investigated. This complex integral transform is also used to reduce the core problem to a simple algebraic equation. The answer to this primary problem can then be obtained by solving this algebraic equation and using the inverse of this complex integral transform. Finally, the complex integral transform is used to solve higher order ordinary differential equations. Also, we introduce, some important engineering and physics applications.

Keywords: complex integral transform, ordinary differential equations.
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1. Introduction

The complex SEE (complex Sadaq-Emad-Emann) integral transformation is a new complex transformation and It is used to solve differential equations and has applications in domains such as physics. (nuclear physics, engineering (electric circuits, automatic control, mechanical engineering, ...etc.), and bio-medical signal processing. [1, 2, 6, 5, 7, 3, 4].
We analyze functions in the set A defined by a new complex integral transform defined for functions of exponential order.

$$A = \left\{ f(t) : \exists M \ , \ l1, l2 > 0 \ . \ |f(t)| < Me^{-ilj|t|} \ , \ \text{ if } \ t \in (-1)^j \ , \ x \in [0, \infty] \right\}. \tag{1.1}$$

Where i is a complex number, and $i^2 = -1$.

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The constant M must be a finite value for a particular function in the set of A, while $l_1,\ l_2$ may be finite or infinite.
The complex SEE integral transform denoted by the operator $S^c[.]$, the transform formula is as follows:

$$S^c[f(t)] = T(iv) = \frac{1}{v^n}\int_0^\infty f(t)\ e^{-ivt}dt \ \ ,\ t \geq 0 \ \ ,l_1 \leq v \leq l_2 \ \ ,\ n \in \mathbb{Z} \qquad \ldots(2)\ . \qquad (1.2)$$

The variable iv in this complex convert is used to factor the variable t in the argument of the function $f(t)$.

The goal of this research is to demonstrate the utility of this intriguing complex transform in solving ordinary differential equations.

2. A New Complex Integral Transform of Some Famous Functions

We accept that the integral of equation 1.2 exists for any function f(t). The sufficient situations for the being of complex integral transform are that $f(t) \quad for\ t \geq 0$ be piecewise incessant and of exponential order, then the new complex integral transform may or may not exist.

This section, we introduce the complex integral transform of some famous functions:

1. Let $f(t) = C$, where C is a constant number, by the definition we have:

$$S^c[C] = T(iv) = \frac{1}{v^n}\int_0^\infty e^{-ivt}C\ dt = \frac{C}{v^n}\int_0^\infty e^{-ivt}\ dt = -\frac{ic}{v^{n+1}}\ .$$

$$S^c[C] = -\frac{iC}{v^{n+1}}\ .$$

2. Let $f(t) = t$, then: $S^c[t] = \frac{1}{v^n}\int_0^\infty e^{-ivt}t\ dt,,$ integration by parts, we get: $S^c[t] = -\frac{1}{v^{n+2}}$

Similarly, integration by parts

(i) $S^c[t^2] = \frac{(2!)i}{v^{n+3}}$.
(ii) $S^c[t^3] = \frac{3!}{v^{n+4}}$.
(iii) Let $f(t) = e^{bt}$, then $S^c\left[e^{bt}\right] = \frac{1}{v^n}\int_0^\infty e^{-ivt}e^{bt}\ dt$.

After simple computations, we get:

$$S^c\left[e^{bt}\right] = \frac{1}{v^n}\left[\frac{b}{b^2+v^2} + i\frac{v}{b^2+v^2}\right]\ .$$

This outcome will be useful in determining the complicated transform of:

1. $S^c[\sin(bt)\ ] = \frac{-b}{v^n(v^2-b^2)}$.
2. $S^c[\cos(bt)\ ] = \frac{-iv}{v^n(v^2-b^2)}$.
3. $S^c[\sinh(bt)\ ] = \frac{-b}{v^n(v^2+b^2)}$.
4. $S^c[\cosh(bt)\ ] = \frac{-iv}{v^n(v^2+b^2)}$.

Theorem 2.1. *Let $T(iv)$ is the complex new integral transform of $f(t)$ $(T(iv) = S^c[C])$ then:*

(i) $S^c[f'(t)] = \frac{-f(0)}{v^n} + iv\ T(iv)$.

(ii) $S^c\left[f''(t)\right] = \frac{-f'(0)}{v^n} - \frac{if(0)}{v^{n-1}} - v^2\ T\left(iv\right)$.

(iii) In general case:

$$S^c\left[f^{(m)}(t)\right] = \frac{1}{v^n}\left[-f^{(m-1)}\left(0\right) - iv\ f^{(m-2)}\left(0\right) - \left(iv\right)^2\ f^{(m-3)}\left(0\right) - \cdots - \left(iv\right)^{m-1}\ f\left(0\right)\right] + (iv)^m T\left(iv\right)\ .$$

Proof .

(i) *By the definition, we have:* $S^c\left[f'(t)\right] = \frac{1}{v^n}\int_0^\infty\ e^{-ivt}f'(t)\ dt$, *Integration by parts, we get:* $S^c\left[f'(t)\right] = \frac{-f(0)}{v^n} + iv\ T\left(iv\right)$.

(ii) $S^c\left[f''(t)\right] = \frac{1}{v^n}\int_0^\infty\ e^{-ivt}f''(t)\ dt$. *Integration by parts, we get:* $S^c\left[f''(t)\right] = \frac{-f'(0)}{v^n} - \frac{if(0)}{v^{n-1}} - v^2\ T\left(iv\right)$.

(iii) Can be proof by the mathematical induction.

□

Proposition 2.2. *(shifting property):* $S^c\left[e^{at}f\left(t\right)\right] = \frac{1}{v^n}\int_0^\infty\ e^{-ivt}e^{at}f\left(t\right)dt =$
$\frac{1}{v^n}\int_0^\infty f\left(t\right)\ e^{-i(v+ia)t}dt = \frac{\left(v+ia\right)^n}{v^n}.\frac{1}{\left(v+ia\right)^n}\int_0^\infty f\left(t\right)\ e^{-i(v+ia)t}dt = \frac{\left(v+ia\right)^n}{v^n}.T(v+ia)$.

3. Applications of New Complex Integral Transformation in Ordinary Differential Equations

As stated in the introduction to this paper, the innovative complex integral transform may be used as an effect tool for analyzing the basic properties of a liner system governed by the ordinary differential equation in response to initial conditions.
The application of the new complex integral transform to solve initial value issues defined by linear ordinary differential equations is demonstrated in the following problems:

Problem 3.1. *Take the first order differential equation for example:*

$$\acute{y} + y = 0, \qquad y(0) = 1 \tag{3.1}$$

Equation (3.1) can be written as by applying the complex transform to both sides of the equation (3.1) and using the differential property.:

$$S^c\left[y'\right] + S^c\left[y\right] = 0,$$
$$\frac{-y(0)}{v^n} + ivT\left(iv\right) + T\left(iv\right) = 0,$$

when we apply the first condition, we get:

$$T\left(iv\right) = \frac{1}{v^n\left(1+iv\right)},$$
$$T\left(iv\right) = \frac{1}{v^n}\left[\frac{1}{1+iv}.\frac{1-iv}{1+iv}\right], \qquad so,$$
$$T\left(iv\right) = \frac{-1}{v^n}\left[\frac{-1}{1+v^2} + i\frac{v}{1+v^2}\right]$$

Using the inverse of the complex transform, we now have: $y\left(x\right) = e^{-x}$.

Problem 3.2. *Solve the following initial value problem:*

$$\acute{y} + 2y = x, \qquad y(0) = 1 \tag{3.2}$$

Equation (3.2) can be expressed as by applying the complex integral transform to both sides of equations (3.2) and exploiting the differential property of this complex transform:

$$S^c[y'] + 2S^c[y] = S^c[x]$$
$$\frac{-y(0)}{v^n} + ivT(iv) + 2T(iv) = \frac{-1}{v^{n+2}}$$

Applying the initial condition, we give: $T(iv) = \frac{1}{v^n}\left[\frac{-1}{v^2(2+iv)} + \frac{1}{(2+iv)}\right]$
Now, take

$$\frac{-1}{v^2(2+iv)} = \frac{A}{v^2} + \frac{B}{v} + \frac{C}{2+iv} \quad .$$

After simple computations, we get: $A = -\frac{1}{2}$, $B = \frac{i}{4}$, $C = \frac{1}{4}$.
Then

$$T(iv) = \frac{-1}{2v^{n+2}} + \frac{i}{4v^{n+1}} + \frac{1}{4v^n(2+iv)} + \frac{1}{v^n(2+iv)} \quad ,$$

$$T(iv) = \frac{-1}{2v^{n+2}} + \frac{i}{4v^{n+1}} + \frac{5}{4}\frac{1}{v^n}\left[\frac{2}{4+v^2} - \frac{iv}{4+v^2}\right] \quad .$$

So,

$$T(iv) = \frac{-1}{2v^{n+2}} + \frac{i}{4v^{n+1}} + \frac{-5}{4v^n}\left[\frac{2}{4+v^2} + i\frac{v}{4+v^2}\right]$$

Take inverse of this complex transform, we get: $y(x) = \frac{1}{2}x - \frac{1}{4} + \frac{5}{4}e^{-2x}$.

Problem 3.3. *Consider the second-order linear differential equation below:*

$$\acute{\acute{y}} + 9y = \cos(2x), \qquad y(0) = 1, \quad y(\pi/2) = -1 \tag{3.3}$$

since $\acute{y}(0)$ is not known, let $\acute{y}(0) = a$.
Take the complex transform of equation (3.3) and using the conditions, we get:

$$\frac{\acute{y}(0)}{v^n} - i\frac{y(0)}{v^{n-1}} - v^2T(iv) + 9T(iv) = -\frac{iv}{v^n(v^2-4)}$$

After simple computations, we have

$$T(iv) = \frac{-iv}{v^n(v^2-4)(9-v^2)} + \frac{-a}{v^n(v^2-9)} + \frac{-iv}{v^n(v^2-9)}$$

Now, take

$$\frac{-iv}{(v^2-4)(9-v^2)} = \frac{Av+B}{v^2-4} + \frac{Cv+D}{9-v^2} \quad .$$

We have, $A = \frac{-i}{5}$, $B = 0$, $C = \frac{-i}{5}$ *and* $D = 0$.
Then

$$T(iv) = \frac{-iv}{5v^n(v^2-4)} - \frac{-iv}{5(v^2-9)} + \frac{-3a}{3v^n(v^2-9)} + \frac{-iv}{v^n(v^2-9)}$$

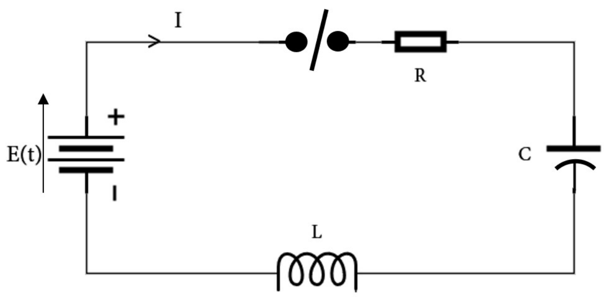
Take inverse of this complex transform, we get:

$$y(x) = \frac{1}{5}\cos(2x) \ + \frac{a}{3}\sin(3x) \ + \frac{4}{5}\cos(3x) \quad .$$

To determine the value of a, note that $y\left(\frac{\pi}{2}\right) = -1$ then we find $a = \frac{12}{5}$, then,

$$y(x) = \frac{1}{5}\cos(2x) \ + \frac{4}{5}\cos(3x) \ + \frac{4}{5}\sin(3x) \quad .$$

The New Complex Integral Transform in a Simple Electric Circuit Examine an electric circuit with a series of resistance R, inductance L, a capacitive condenser C, and electromotive power of voltage E. In the circuit, there is also a switch,
Then by Kirchhoff's law, we have: $L\frac{dI}{dt} + RI + \frac{Q}{C} = E$.



Problem 3.4. *An inductance of* 3 *henry, a resistor of* 16 *ohms and a capacitor of* 0.02 *farad are linked in series with an emf of* 300 *volts. At* $t = 0$*, the charge on The circuit's capacitor and current are both 0. At any time, you can check the charge and current.*$t > 0$.

let Q and I be prompt charge and current correspondingly at time t.
Then by Kirchhoff's law: $L\frac{dI}{dt} + RI + \frac{Q}{C} = E$

$$2\frac{d^2Q}{dt^2} + 16\frac{dQ}{dt} + 50Q = E \qquad \ldots\left(since\ I = \frac{dQ}{dt}\right)$$

Then $\frac{d^2Q}{dt^2} + 8\frac{dQ}{dt} + 25Q = 150$,
For simplify

$$\acute{\acute{y}} + 8\acute{y} + 25y = 150$$

Applying the new complex integral transform on both sides, we get

$$\frac{-y'(0)}{v^n} - \frac{iy(0)}{v^{n-1}} - v^2T(iv) + 8\left[\frac{-y(0)}{v^n} + ivT(iv)\right] + 25T(iv) = \frac{-150i}{v^{n+1}}$$

Applying the initial condition, we give:

$$T(iv) = \frac{-150i}{v^{n+1}\left(-v^2 + 8iv + 25\right)} \quad ,$$

$$T(iv) = \frac{150i}{v^n}\left[\frac{A}{v} + \frac{Bv + C}{v^2 - 8iv - 25}\right] \quad .$$

After simple computations, we get: $A = \frac{-1}{25}$, $B = \frac{1}{25}$ *and* $C = \frac{-8}{25}\, i$

$$T\left(iv\right) = \frac{-6i}{v^{n+1}} + \frac{-6iv}{v^{n}\left(-v^{2}+8iv+25\right)} + \frac{-48}{v^{n}\left(-v^{2}+8iv+25\right)} \quad .$$

Then

$$T\left(iv\right) = \frac{-6i}{v^{n+1}} - \frac{6\left(iv+4\right)-24}{v^{n}\left[\left(iv+4\right)^{2}+9\right]} - \frac{24}{v^{n}\left[\left(iv+4\right)^{2}+9\right]} \quad ,$$

$$T\left(iv\right) = \frac{-6i}{v^{n+1}} - \left[\frac{-6i\left(v-4i\right)}{v^{n}\left[\left(v-4i\right)^{2}-9\right]}\right] - [\frac{-\left(8\right)\left(3\right)}{v^{n}\left[\left(v-4i\right)^{2}-9\right]}$$

Take the inverse of the complex transform and shifting property, we get:

$$y\left(x\right) = 6 - 6e^{-4x}\cos\left(3x\right) - 8e^{-4x}\sin\left(3x\right)\ , \quad \text{or}$$

$$Q(t) = 6 - 6e^{-4t}\cos\left(3t\right) - 8e^{-4t}\sin\left(3t\right)$$

, And $I = \frac{dQ}{dt} = 50e^{-4t}\sin\left(3t\right)$.
This is required expression for charge and current at any time $t > 0$.

4. The New Complex Integral Transform in Nuclear Physics

The following problem is based on nuclear physics fundamentals.
Consider the linear ordinary differential equation of first order.: $\frac{dN}{dt} = -\beta N$
The essential relationship describing radioactive decay is given above., where $N = N\left(t\right)$ During time t, denotes the number of undecayed atoms left in a sample of radioactive isotope, and β is the decay constant. we can use the complex integral transform $S^{c}\left[.\right]$, to have

$$S^{c}\left[N'\right] + \beta S^{c}\left[N\right] = 0, \quad \text{therefore}$$

$$-\frac{N\left(0\right)}{v^{n}} + iv\overline{N} + \beta\overline{N} = 0$$

Here $S^{c}\left[N\right] = \overline{N}$ and $N\left(0\right) = N_{0}$
So,

$$\overline{N} = \frac{N_{0}}{v^{n}(iv+\beta)} = \frac{N_{0}}{v^{n}}\left[\frac{1}{iv+\beta}\ .\frac{-iv+\beta}{-iv+\beta}\right]$$

$$\overline{N} = \frac{N_{0}}{v^{n}}\left[\frac{-iv+\beta}{v^{2}+\beta^{2}}\right]$$

$$\overline{N} = \frac{-N_{0}}{v^{n}}\left[\frac{-\beta}{v^{2}+\beta^{2}} + \frac{iv}{v^{2}+\beta^{2}}\right]$$

Now, taking inverse the complex integral transform on both sides, we get: $N\left(t\right) = N_{0}e^{-\beta t}$.
This is, in fact, the proper type of radioactive decay.

5. Problem of Pharmacokinetics

Solution of the problem pharmacokinetics

$$\left.\begin{array}{c} \frac{d}{dt}C(t)+\lambda C(t)=\frac{\gamma}{VOL} \quad , \quad t>0 \\ with\ C(0)=0 \end{array}\right\} \tag{5.1}$$

Here $C(t)$: at any time, the medication concentration in the blood t
λ: constant velocity of elimination.
γ: the proportion of infusion (in mg/min).
VOL: volume inn which drug is distributed.
Now, take complex the integral transformation of both sides of eq.(5.1) gives

$$S^c[C'(t)]+\lambda S^c[C(t)]=\frac{\gamma}{VOL}S^c[1]$$

Applying, complex Sadaq-Emad-Emann integral transform, we get:

$$\left[\frac{C(0)}{v^n}+iv\ T(iv)\right]+\lambda T(iv)=\frac{\gamma}{VOL}\left(\frac{-i}{v^{n+1}}\right) ,$$

$$iv\ T(iv)+\lambda T(iv)=\frac{\gamma}{VOL}\left(\frac{-i}{v^{n+1}}\right) ,$$

$$(iv\ +\lambda)T(iv)=\frac{\gamma}{VOL}\left(\frac{-i}{v^{n+1}}\right) ,$$

$$T(iv)=\frac{\gamma}{VOL}\frac{1}{v^n}\left[\frac{1}{i(iv\ +\lambda)v}\right] ,$$

$$T(iv)=\frac{\gamma}{VOL}\frac{1}{v^n}\left[\frac{A}{iv}+\frac{B}{iv\ +\lambda}\right]$$

$$A=\frac{1}{\lambda} \quad , \quad B=\frac{-1}{\lambda}$$

$$T(iv)=\frac{\gamma}{VOL}\frac{1}{v^n}\left[\frac{\frac{1}{\lambda}}{iv}+\frac{\frac{-1}{\lambda}}{iv}\right]$$

$$T(iv)=\frac{\gamma}{\lambda\ VOL}\left[\frac{-i}{v^{n+1}}+\frac{1}{v^n}\left(\frac{-\lambda}{\lambda^2+v^2}+\frac{iv}{\lambda^2+v^2}\right)\right]$$

Take inverse, we get

$$C(t)=\frac{\gamma}{\lambda\ VOL}\left[1-e^{-\lambda t}\right] .$$

6. Conclusion

The definition and application of the new complex SEE integral transform to solution of ordinary differential equations has been demonstrated.

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