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Modelling covid-19 data using double geometric stochastic process

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Abstract

Some properties of the geometric stochastic process (GSP) are studied along with those of a related process which we propose to call the Double geometric stochastic process (DGSP), under certain conditions. This process also has the same advantages of tractability as the geometric stochastic process; it exhibits some properties which may make it a useful complement to the multiple Trends geometric stochastic process. Also, it may be fit to observed data as easily as the geometric stochastic process. As a first attempt, the proposed model was applied to model the data and the Coronavirus epidemic in Iraq to reach the best model that represents the data under study. A chicken swarm optimization algorithm is proposed to choose the best model representing the data, in addition to estimating the parameters a, b, μ, and σ^2 of the double geometric stochastic process, where μ and σ^2 are the mean and variance of X_1, respectively.

Keywords: double geometric stochastic process, geometric stochastic process, parameter estimation, chicken swarm optimization algorithm, multiple monotone trends, root mean square criteria.

Nomenclature

X_1 The distribution of the first inter-arrival time.
h(k) Function of k with $\{h(1) = 1\}$ for $k = 1, 2, \ldots$.
DGSP Double Geometric Stochastic Process.
GSP Geometric Stochastic Process.
CSO Chicken Swarm Optimization Algorithm

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RMSE Root Mean Square Criteria .
RSP Renewal Stochastic Process.
Greek Symbols
a Ratio of Geometric Stochastic Process.
b Parameter of shape impact factor.
μ & σ^2 **The mean and variance of** X_1 **respectively.**

1. Introduction

During the outbreak of a particular epidemic disease, for example, the severe acute respiratory syndrome Coronavirus (SARS-CoV-2), the number of daily cases of a particular epidemic often shows multiple trends: a monotonic increase during the growing stage or the outbreak of the epidemic, stationary in the number of daily cases called in Some sources are the stabilization stage, that is, controlling the epidemic to eliminate it, and then decreasing during the declining stage. In recent years, The Double Geometric Stochastic Process (DGSP) has begun to be used as a model to fit data from a series of repetitive events, Since it provides a more flexible application model than the geometric stochastic process (GSP) [3].

In practice, however, the data of successive inter-arrival times will almost always indicate a pattern. In the study of a repairable device, for example, the system will deteriorate due to aging and accumulated wear, so the successive running times will decrease. Trend data is general. The Nonhomogeneous Poisson process (NHPP) with a monotone trend was used to model trend data. Examples include the Cox–Lewis model (CLM) and the Weibull process model (WPM). That was demonstrated in [1]. A more straightforward approach is to model the data using a monotone method known as the geometric stochastic process [4, 5].

For the first time, authors in [4, 5] looked at the (GSP) as a generalization of the (RSP). Since then, authors from a broad variety of scientific disciplines have shown interest in the (GSP). For example, the (GSP) was used as a basis in infectious disease outbreak modeling and software reliability analysis by the authors in [9]. The GSP is a random process described as follows [3]: a sequence $\{X_k,\ k = 1,\ 2,\ ...\}$ of nonnegative random variables which represent the inter-arrival times of the process. The GSP $\{X_k,\ k = 1,\ 2,\ ...\}$ is said to be a GSP with the ratio (a) if there exists a real number $(a > 0)$ such that $a^{k-1}X_k$ for $k = 1,\ 2,\ \ldots$ from a (RSP) with a common distribution function F where the distribution F is the distribution function of the first inter-arrival time X_1.

As can be seen, the distinction between the GSP and the RSP lies in the fact that the inter-arrival times of the RSP have the same distribution F(t) over k's and the inter-arrival times of the GSP have a cumulative distribution function (CDF) $F(a^{k-1}t)$, which changes over $k's$ [1, 2].

Although this GSP is an important model that has been widely used in various research fields to solve various problems, its scope is restricted and does not meet the needs of various empirical studies [13]. First, for a stochastic process in which the inter-arrival times must be interpreted by distributions with varying shape parameters, this model is inappropriate. Second, it can only describe stochastic processes that are stochastically increasing and decreasing. These limitations can limit the GSP's application in the real world [10, 11].

The outline of the paper is as follows. In the next section, we define and discuss the properties of the DGSP and GSP models. In Section 3, the steps of the CSO algorithm are illustrated. In Section 4, we estimate the parameters a, b, μ, and $\sigma^2$of the DGSP by using the CSO algorithm. In Section 5, three examples are analyzed by using the methodology developed in this paper. Finally, we present the conclusions and future work in Section 6.

2. The two models

In recent years, several authors have worked hard to establish new methods for modeling data from a series of events [4, 9]. One of them is the geometric stochastic process is a stochastic process that is defined as Lam,1988 in [4]: a sequence $\{X_k,\ k = 1,\ 2,\ \dots\}$ of nonnegative random variables which represent the inter-arrival times of the process. The GSP can be constructed as follows.

Let

$$X_k = \frac{Y_k}{a^{k-1}} \qquad k = 1, 2, 3, \dots \tag{2.1}$$

For some $(a > 0)$. If $S_0^G = 0 \quad and \quad S_n^G = \sum_{k=1}^{n} X_k$

Then $\{X_k\ ,\ k = 1,\ 2,\ \dots\}$ is said to be a GSP with the ratio (a) if there exists a real number $(a > 0)$ such that $a^{k-1}X_k$ for $k = 1,\ 2,\ \dots$ form a (RSP) with a common distribution function F .where the distribution F is the distribution function of the first inter-arrival time X_1 [2].

The difference between the GSP and the RSP is that the RSP's inter-arrival times have the same distribution F(t) over k's, while the GSP's inter-arrival times have a cumulative distribution function (CDF) $F(a^{k-1}t)$ that varies over k's. This distinction makes the GSP more attractive in certain scenarios, such as reliability mathematics, since it can model the failure process of aging or failing systems, which may have declining working times between failures [5].

Although the GSP is a useful model that has been applied to a variety of problems in various fields of study, its scope is still limited and does not meet the needs of various empirical studies. First, this model is inappropriate for a stochastic process in which the inter-arrival times must be modeled using distributions with varying shape parameters. Second, it can only characterize stochastic processes that are increasing or decreasing stochastically. Due to these constraints, the GSP may be unable to find a broader application in the real world. For these limitations, [12] provides detailed examples.

Due to the GSP's limitations, several authors have been working hard in recent years to develop new methods for modeling data from a series of events; see Wu and Pham [10], Wu and Wang in [12] and Wu in [13] for more information. The doubly geometric stochastic process (DGSP), suggested by Wu in [13] as an extension of the GSP, is considered in this paper. The following is a description of this process as well as some theoretical properties [12].

Definition 2.1. *Given a sequence of non-negative random variables $\{X_k,\ k = 1,\ 2,\ \dots\}$, If they are independent and the cumulative distribution function of X_k is given by $\{F\left(a^{k-1}X_k^{h(k)}\right)$ for $k = 1, 2, \dots\}$ where a is a positive constant, $h(k)$ is a function of k and the probability of the parameters in $h(k)$ has a known closed form, and $\{h(k) > 0\}$ for $k \in \mathbb{N}$, then $\{X_k,\ k = 1,\ 2,\ \dots\}$ is called a DGSP [13].*

Since the process can involve two geometric processes, we refer to it as a DGSP. As an example from [8], consider the following:

- The geometric series $\{a^{k-1},\ k = 1,\ 2,\ \dots\}$ is refer as the scale impact factor.
- The geometric series $\{h(k),\ k = 1,\ 2,\ \dots\}$ is refer as the shape impact factor.

From definition, we can find the following results:

1. If h(k) = 1, then $\{X_k,\ k = 1,\ 2,\ \cdots\}$ reduces to the geometric stochastic process (GSP).

2. The expectation and variance of the doubly geometric stochastic process can be calculated as follows:

$$\mu_{1k} = \mathbb{E}\left[{X_1}^{h^{-1}(k)}\right] = \int_0^\infty {X_1}^{h^{-1}(k)} f\left(x\right) \mathrm{dx} \tag{2.2}$$

$$\mu_{2k} = \mathbb{E}\left[{X_1}^{2h^{-1}(k)}\right] = \int_0^\infty {X_1}^{2h^{-1}(k)} f\left(x\right) \mathrm{dx} \tag{2.3}$$

$$h^{-1}\left(k\right) = \frac{1}{h(k)} \tag{2.4}$$

Since then, the probability density function has been defined and is given as:

$$f\left(x\right) = \frac{\partial F(x)}{\partial x} \tag{2.5}$$

as well as assuming:

$$\mathbb{E}\left[{X_1}^{h^{-1}(k)}\right] < \infty \quad \& \quad \mathbb{E}\left[{X_1}^{2h^{-1}(k)}\right] < \infty. \tag{2.6}$$

As a result, finding both the expectation and variance of the geometric stochastic process of the random variable (X_k) is simple as:

$$\mathbb{E}\left[X_k\ \right] = \frac{\mu_{1k}}{a^{(k-1)h(k)}} \qquad for\ all\ k = 1, 2, \ldots \tag{2.7}$$

$$\mathbb{V}\left[X_k\right] = \frac{\mu_{2k} - \left[\mu_{1k}\right]^2}{a^{2(k-1)h(k)}} \qquad for\ all\ k = 1, 2, \ldots \tag{2.8}$$

Wu in [13] applies the Doubly Geometric Stochastic Process (DGSP) to some reliability problems to detect the best form of $h\left(k\right)$. It is obtained that the DGSP with $h\left(k\right) = \left(1 + log10\left(k\right)\right)^b$ performs better than the processes with the other $h\left(k\right)$'s.

3. Chicken swarm optimization algorithm

The chicken swarm optimization (CSO) algorithm is a common intelligence optimization algorithm that performs well in solving global optimization problems (GOPs).The genetic algorithm (GA), particle swarm optimization (PSO), bat algorithm (BA), artificial bee colony (ABC) algorithm, and others are swarm intelligent optimization algorithms that simulate the swarm behavior of natural organisms. These algorithms simulate the physical laws of natural phenomena, the living patterns, and behavioral characteristics of different biological populations in nature to find the best solution to an optimization problem [6]. In the fields of computer science, mathematical science, and other fields, swarm optimization intelligent algorithms provide a new way to solve global optimization problems. Swarm intelligent optimization algorithms have become a hotbed of research and are especially significant. Meng, et al. [6] introduced the CSO algorithm in 2014. It is a stochastic search approach based on chicken swarm search behavior. The entire chicken swarm is divided into several groups in CSO, with each group consisting of a rooster, a couple of hens, and several chicks. Different laws of motion apply to different chickens. Mutual learning and rivalry occur among different chickens, and the chicken group's hierarchy is updated after several generations of evolution. Because of its high convergence speed and precision, the CSO algorithm has a lot of research potential. However, the simple chicken swarm optimization algorithm, like other swarm intelligent optimization algorithms, suffers from premature convergence, with iteration easily falling into a local minimum, when solving large-scale optimization problems of greater complexity. In addition, by [14] the algorithm for CSO is as follows:

The unconstrained continuous optimization problems can be expressed as follows.

$$\begin{gathered} \min f(X) \qquad X \in R^D \\ \text{if} \quad \mathrm{X}^* \in R^D \quad satisfies\ that: f(X^*) \le f(X) \qquad \forall X \in R^D \end{gathered} \tag{3.1}$$

X^* is called the global minimum point of $f(X)$ in the whole space R^D .

The CSO algorithm is inspired by the hierarchical order and behaviors of a swarm of hungry chickens. Depending on their hierarchical status, different chickens follow different laws of motion. The CSO algorithm employs the following four rules to idealize chicken action:[7]

1. The chicken swarm is divided into many subgroups. A dominant rooster, a couple of hens, and chicks make up each subgroup.
2. The fitness values of the chickens themselves determine how the entire chicken swarm is divided into several groups and the species of chicken is determined. In the whole chicken swarm, a few individuals with the best fitness values are labelled as roosters; the chickens with the worst fitness values are acted as chicks, and the rest are hens. The hen, like the mother-child relationship between the hen and the chick, chooses its subgroup at random.
3. The hierarchical order, dominance relationship, and mother-child relationship in a group remain intact for several years unless the roles are reassigned.
4. In their quest for food, the hens chase their rooster-mate, while the chicks seek food in the vicinity of their mothers. The dominant individuals have an edge when it comes to finding food.

The whole swarm of chickens is made up of three different types of chickens. Each chicken represents a possible solution when the CSO algorithm solves the optimization problem and different chickens use different optimization strategies. In the CSO algorithm, N chickens are assumed, and the chickens are ordered in ascending order based on their fitness values. The roosters in the front are NR chickens, the chicks in the back are NC chickens, and the hens in the center are $(NH = N - NR - NC)$ chickens. Let us look at the Chicken Movement [7].

The Roosters with higher fitness values have preference over those with lower fitness values when it comes to food. For the sake of convenience, this condition may be simulated by cocks with higher fitness values being able to forage in a wider variety of locations than cocks with lower fitness values. This can be expressed as follows.

$$x_{i,j}^{t+1} = x_{i,j}^{t} * \left(1 + Randn(0, \sigma^2)\right) \tag{3.2}$$

$$\sigma^2 = \begin{cases} 1 & if\ f_i \le f_k \\ \exp\left(\frac{(f_k - f_i)}{|f_i| + \varepsilon}\right) & otherwise \end{cases} \qquad k \in [1, N], k \ne i, \tag{3.3}$$

where Randn $(0, \sigma^2)$ represents a Normal distribution with a zero mean and variance (σ^2). (ε), which is used to avoid zero-division error, is the smallest constant in the computer. A rooster's index (k) is chosen at random from the rooster's group, and the fitness value of the corresponding x is (f). The hens, on the other hand, will join their roosters in hunting for food. Furthermore, while being repressed by the other chickens, they would steal good food found by other chickens at random. When competing for food, dominant hens will have an advantage over submissive hens. These phenomena can be expressed mathematically as follows [7].

$$x_{i,j}^{t+1} = x_{i,j}^{t} + \ S_1 * Rand * \left(x_{r1,j}^{t} - x_{i,j}^{t}\right) + S_2 * Rand * \left(x_{r2,j}^{t} - x_{i,j}^{t}\right) \tag{3.4}$$

$$S_1 = \frac{\exp\left(f_i - f_{r1}\right)}{\left(\text{abs}\left(f_i\right) + \varepsilon\right)} \tag{3.5}$$

$$S_2 = \exp\left(\left(f_{r2} - f_i\right)\right) \tag{3.6}$$

Rand is a uniform random number between zero and one. $\{r1 \in [1, \ldots,\ N]\}$ is the index of the rooster, which is the i[th] hen's group-mate, while $\{r2 \in [1, \ldots,\ N]\}$ is the index of the chicken (rooster or hen), which is selected at random from the swarm. $(r1 \neq r2)$.

Obviously, $(f_i > f_{r1})$, $(f_i > f_{r2})$, resulting in S_2<1< S_1. If $S_1 = 0$, then the i[th] hen will be the first to forage for food, followed by other chickens. The greater the disparity in fitness values between the two chickens', the smaller S_2, and the greater the distance between their positions. As a result, the hens will be less likely to snatch food from other chickens. The reason that the formula form of S_1 differs from that of S_2 is that there exist competitions in a group. the fitness values of the chickens about the rooster's fitness value are simulated as the competitions between chickens in a group. If $S_2 = 0$, then the i[th] hen will look for food within its territory. the fitness rooster's value is unique and it's special in this category. As a result, the smaller the i[th] hen's fitness value, the closer S_1 approximates to 1, and the smaller the difference between the ith hen's location and that of its group-mate rooster. Hence the more dominant hens would be more likely than the more submissive ones to eat the food.

To forage for food, the chicks move around their mother. This is how it's put together [7].
To forage for food, the chicks move around their mother. This is how it's put together:

$$x_{i,j}^{t+1} = x_{i,j}^{t} + FL * \left(x_{m,j}^{t} - x_{i,j}^{t}\right) \tag{3.7}$$

The location of the i^{th} chick's mother ($m \in [1,\ N]$) is defined by $x_{m,j}^{t}$. FL $(FL \in (0, 2))$ is a parameter, which means that the chick would follow its mother to forage for food. Taking into account individual variations, each chick's FL will fluctuate between zero and two.

The algorithm can be described in steps that reflect the iterative processes used to arrive at the problem's solution[14]:

The CSO Algorithm is Set Up

Build a population of N chickens and specify the relevant parameters;
compute the fitness values of the N chickens, t=0;
While (t < Max Generation)
If($t\%G == 0$)
Rank the chicken's fitness values and create a hierarchal order in the swarm;
Determine the relationship between the chicks and mother hens in each group by dividing the swarm into separate groups.
If you want to quit, you should.
For $i = 1 : N$ If $i ==$ rooster Update its solution/location using equation (3.2); End if
If $i ==$hen Update its solution/location using equation (3.4); End if
If $i ==$ chick Update its solution/location using equation (3.7); End if Test the new solution;
If the new solution is superior to the previous one, it should be modified;
End for
End while

4. Covid-19 virus infection case

Corona virus disease (Covid-19) is one of the most serious global health threats due to the ease of transmission and the long incubation period of the epidemic, making it one of the most widespread epidemics, as the first case of the epidemic was recorded in Wuhan, China, on December 31,2019, and began the spread of a global epidemic. Beginning on February 24,2020, in the city of Najaf, Iraq, other cases infected with an epidemic were detected. Covid-19 in various regions of Iraq. As of 12/31/2020, the total number of confirmed total cases in Iraq was (595291), whereas the total number of deaths in Iraq was (12813).

Our investigation focuses on modeling data on daily infection numbers in Iraq as a result of the Corona epidemic, as well as for the three governorates (Baghdad, Erbil, and Basra). The covid-19 data for the three regions include information on daily infected cases, daily death cases, daily recovery cases, and daily cases in care. Our research focuses on modeling the number of regular infected cases for each of the three regions separately. Table 1. contains a description of the data.

Table 1: The basic details for the three region's covid-19 data.

Governorate	*Baghdad*	*Erbil*	*Basra*
Start date	*15/3/2020*	*15/3/2020*	*15/3/2020*
End date	*31/12/2020*	*31/12/2020*	*31/12/2020*
n *= number of data*	*214*	*214*	*214*
Number of cases in total S_n	*38980*	*35987*	*177628*
Number of Cases Per Day (S_n/n)	*182.149*	*168.163*	*830.037*

5. Choosing the best model for modeling covid-19 data

In this section, many functions of both the shape impact factor $h(k)$and the scale impact factor $g(k)$ were assumed to arrive at the best model representing the data under study. For this purpose, a CSO algorithm was used to choose the best model representing the data. The following table represents the different functions of each of the shape impact factor and the scale impact factor.

Table 2: The different functions of the shape impact factor and scale impact factor of the double geometric stochastic process models.

Number	***Function code***	***Scale Impact Factor***	***Shape Impact Factor***
1	$\boldsymbol{DGSP}_{\mathbf{log1}}$	$g(k)=a^{k-1}$	$h(k)=(1+\log(k))^b$
2	$\boldsymbol{DGSP}_{\mathbf{log2}}$	$g(k)=a^{k-1}$	$h(k)=1+b\log(k)$
3	$\boldsymbol{DGSP}_{\mathbf{log3}}$	$g(k)=a^{k-1}$	$h(k)=b^{\log(k)}$
4	$\boldsymbol{DGSP}_{\boldsymbol{GP}}$	$g(k)=a^{k-1}$	$h(k)=b^{k-1}$
5	$^*\boldsymbol{DGSP}_{\exp}$	$g(k)=a^{k-1}$	$h(k)=\log k^b$
6	$^*\boldsymbol{DGSP}_{\boldsymbol{Mixed.P}}$	$g(k)=a^{k-1}$	$h(k)=\ k^b$
7	$^*\boldsymbol{DGSP}_{\boldsymbol{Q.R.P}}$	$g(k)=a^{1-k}$	$h(k)=(1+\log(k))^b$

Where (log) is the natural logarithm based on (10) and that (b) is the shape impact factor parameter, which has many characteristics, including, Estimable, Closed And Bounded Form.

In this section of the analysis, the CSO algorithm was used to find the best function of the shape impact factor and the scale impact factor from which to find the best random model representing the data under study. The following are the results of using the CSO algorithm to compare the different functions using the root mean square error (RMSE) criteria see ([3] for more details) to comparison for the three Iraqi governorates (Baghdad, Erbil, and Basra).

The parameters of the various random models of the double geometric stochastic process model was estimated using programs written for this purpose in the programming language MATLAB v.R2016a.

Now, we use the methodology developed to modelling the covid-19 virus infection data in the three governorates of Iraq (Baghdad, Erbil, and Basra).

5.1. The covid-19 virus infection data in Baghdad

we test the monotone trend in the data set. This test is the first process in analyzing monotone-trend data, and to test the monotonic trend in the data set, the graphical technique (GT) was used as follows:

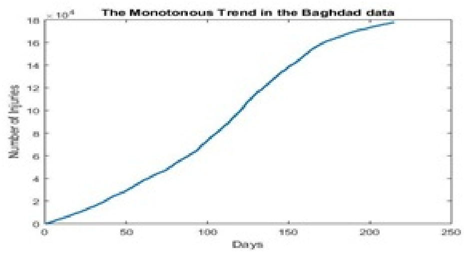

Figure 1: monotone trend in the Baghdad data

Figure 1 represent the plotting of the number of cases versus the number of days may be convex, and the plotting of $\ln X_k$ against (k) may be represented by the linear form. this means that the data understudy agreement with DGSP.

While Table 3 represent the parameters of the double geometric stochastic process estimated using the chicken swarm algorithm under the assumption of related parameter value equal to $\{G = 10, RN = 0.2N, HN = 0.7N, MN = 0.5HN, CN = N - RN - HNandFL \in [0.4, 1]\}$ based on the RMSE criteria and rank for each model, with the following results:

Table 3: Estimates of the parameters of the double geometric stochastic process using CSO algorithm for data from the Baghdad governorate.

Function	***Lower Bound*** $[\hat{a}, \ddot{b}, \hat{\mu}]$	***Upper Bound*** $[\hat{a}, \ddot{b}, \hat{\mu}]$	***Estimate of Parameters*** $[\hat{a}, \ddot{b}, \hat{\mu}]$	***RMSE***	***Rank***
$DGSP_{\log 1}$	$[0.8, -1, 5]$	$[0.8, -1, 5]$	$[1.0036, -0.795, 130.9377]$	***335.3904***	***6***
$DGSP_{\log 2}$	$[0.8, -1, 5]$	$[1.2, 1, 1000]$	$[1.0079, -0.2136, 120.3224]$	***305.1617***	***3***
$DGSP_{\log 3}$	$[0.8, 0.1, 5]$	$[1.2, 0.9, 1000]$	$[1.0062, 0, 6875, 48.4936]$	***306.7368***	***4***
$DGSP_{GP}$	$[0.8, 0.5, 5]$	$[1.2, 2, 1000]$	$[1.0215, 0.9932, 305.2056]$	***211.3443***	***1***
$DGSP_{\exp}$	$[0.8, -0.9, 5]$	$[1.2, 0.1, 1000]$	$[1, 0078, -0.6578, 185.525]$	***306,9911***	***5***
$DGSP_{MixedP}$	$[0.8, -0.9, 5]$	$[1.2, 1.9, 1000]$	$[1.0054, -0.1575, 50.1148]$	***297.5833***	***2***
$DGSP_{Q.R.P}$	$[0.8, -1, 5]$	$[1.2, 1, 1000]$	$[0.9973, -0.2705, 207.8432]$	***345.0587***	***7***

5.2. The covid-19 virus infection data in Erbil

use Fig. 2 to test if the Erbil data agree with DGSP.

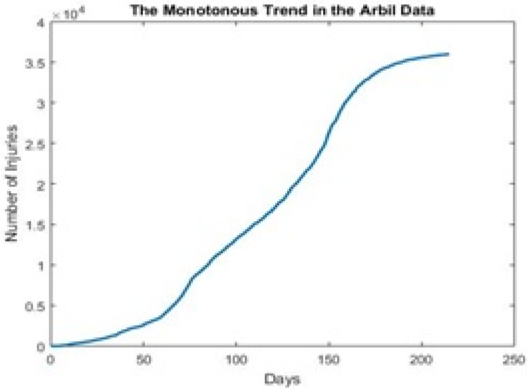

Figure 2: monotone trend in the Erbil data

Whereas the Table 4 represent the RMSE and the Rank to estimates for the Erbil data for each model.

Table 4: Estimates of the parameters of the double geometric stochastic process using CSO algorithm for data from the Erbil governorate.

Function	***Lower Bound*** $[\hat{a}, \ddot{b}, \hat{\mu}]$	***Upper Bound*** $[\hat{a}, \ddot{b}, \hat{\mu}]$	***Estimate of Parameters*** $[\hat{a}, \ddot{b}, \hat{\mu}]$	***RMSE***	***Rank***
$DGSP_{\log 1}$	$[0.8, -1, 5]$	$[1.2, 1, 1200]$	$[1.0027, -0.7468, 13.7074]$	***114.9962***	***4***
$DGSP_{\log 2}$	$[0.8, -1, 5]$	$[1.2, 1, 1200]$	$[1.0021, -0.1162, 68.7288]$	***122.0383***	***6***
$DGSP_{\log 3}$	$[0.8, 0.1, 5]$	$[1.2, 1.9, 1200]$	$[1.003, 0.5562, 6.7974]$	***111.6268***	***3***
$DGSP_{GP}$	$[0.8, -1, 5]$	$[1.2, 1, 1200]$	$[1.0159, 0.9925, 64.6180]$	***932.3533***	***1***
$DGSP_{\exp}$	$[0.8, -1, 5]$	$[1.2, 1, 1200]$	$[1, 006, -0.9807, 29.1034]$	***105.5272***	***2***
$DGSP_{MixedP}$	$[0.8, -1, 5]$	$[1.2, 1, 1200]$	$[1.009, -0.0607, 54.8099]$	***122.8223***	***7***
$DGSP_{Q.R.P}$	$[0.8, -1, 5]$	$[1.2, 1, 1200]$	$[0.9981, -0.3855, 38.7985]$	***121.3974***	***5***

5.3. The covid-19 virus infection data in Basra

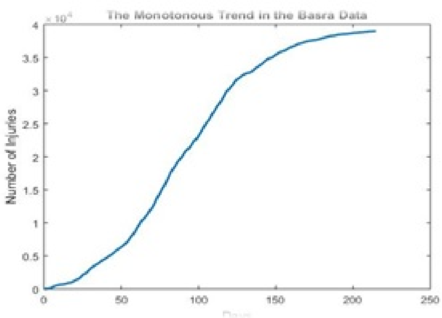

Figure 3: monotone trend in the Basra data

Whereas Figure 3 use to test if the Basra data agree with DGSP. The Table 5 represent the CSO algorithm to estimate the Basra data for each model.

Table 5: Estimates of the parameters of the double geometric stochastic process using chicken swarm algorithm for data from the Basra governorate.

Function	***Lower Bound*** $[\hat{a}, \ddot{b}, \hat{\mu}]$	***Upper Bound*** $[\hat{a}, \ddot{b}, \hat{\mu}]$	***Estimate of Parameters*** $[\hat{a}, \ddot{b}, \hat{\mu}]$	***RMSE***	***Rank***
$DGSP_{\log 1}$	$[0.8, -1, 5]$	$[1.2, 1, 1200]$	$[1.0065, -0.9499, 12.7148]$	***103.2879***	***6***
$DGSP_{\log 2}$	$[0.8, -1, 5]$	$[1.2, 1, 1200]$	$[1.0099, -0.2234, 59.5223]$	***102.5052***	***5***
$DGSP_{\log 3}$	$[0.8, 0.1, 5]$	$[1.2, 1.9, 1200]$	$[1.0078, 0.6798, 28.7072]$	***94.8592***	***3***
$DGSP_{GP}$	$[0.8, -1, 5]$	$[1.2, 1, 1200]$	$[1.0228, 0.9930, 163.7898]$	***82.0157***	***1***
$DGSP_{\exp}$	$[0.8, -1, 5]$	$[1.2, 1, 1200]$	$[1, 0106, -1, 48.4671]$	***86.5737***	***2***
$DGSP_{MixedP}$	$[0.8, -1, 5]$	$[1.2, 1, 1200]$	$[1.0071, -0.1196, 48.3096]$	***103.5211***	***7***
$DGSP_{Q.R.P}$	$[0.8, -1, 5]$	$[1.2, 1, 1200]$	$[0.9950, -0.9866, 10.8812]$	***94.6077***	***4***

According to the results in Table 3-5, the estimation using function DGSP_{GP} is better than estimation using the other functions. Despite the fact that they seem to be very similar. also Table 6 displays the estimates of the geometric stochastic process parameters, When the proposed model was compared to the geometric stochastic process model and RSP model, the results revealed that the proposed model had a higher prevalence.

The Fitted values for the observations X_k are:

$$\widehat{X}_k = \begin{cases} \overline{X}_k & \textit{by a RSP} \\ \widehat{\mu} a^{1-k} & \textit{by a GSP} \\ \left(\widehat{\mu}_D a^{1-k}\right)^{b^{1-k}} & \textit{by a DGSP} \end{cases} \tag{5.1}$$

Table 6: The different functions of the shape impact factor and scale impact factor of the double geometric stochastic process models.

Cities	***Baghdad***	***Erbil***	***Basra***
$\hat{a}$	***0.9999***	***0.9983***	***1.0028***
$\hat{\mu}$	***819.6147***	***139.4078***	***244.3201***
RMSE	***389.0015***	***130.2086***	***134.7548***

In Figures 4-6, We plot the Corona epidemic in the three region's covid-19 data and fitted times against the number of days by an RSP, GSP, and a DGSP, respectively. One concludes that fitting of a DGSP is better than fitting of an RSP and GSP.

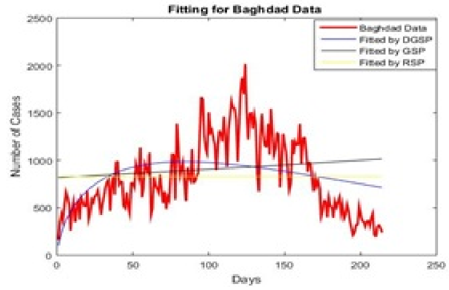

Figure 4: The Baghdad data and their fitted values.

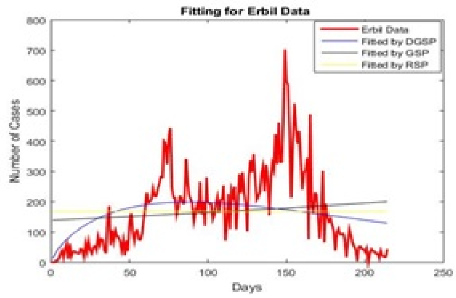

Figure 5: The Erbil data and their fitted values

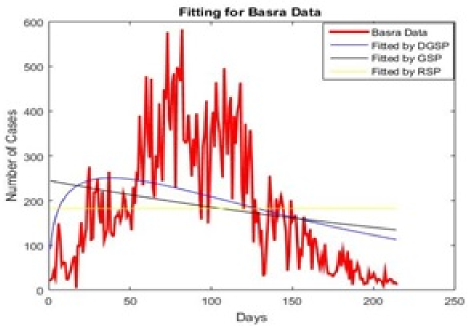

Figure 6: The Basra data and their fitted values.

6. Conclusion and future work

In this paper, the covid-19 data was modeled using the chicken swarm optimization algorithm. a novel model of stochastic processes. We aim to generalize the model in terms of modeling methodologies, methods of estimation, and fields of application. The estimators of the model parameters a, b, and μ are obtained by using the CSO algorithm. Then, we compare various models for modeling data sets and conclude that the $\mathrm{DGSP_{GP}}$ model is preferable., To achieve this result, we suggest the following strategy. To begin, we assume that the data sets conform to a DGSP by employing graphical techniques. Secondly, calculate the parameters for each model (RSP, GSP, and DGSP). Third, we make comparisons between them. Since DGSP is a new concept, many questions must be discussed. For example, what are the differences in DGSP application reliability between one DGSP and the other models? How do we know if a dataset is in agreement with DGSP before fitting it with DGSP?. We will work to answer these questions in the future.

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