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Score for some groups $SUT(2, p)$

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Abstract

One of the most important economic goals of countries is fostering more economic growth; this involves the increasing usage of energy sources. Because of the limitations in non-renewable energy sources and environmental pollution caused by burning these sources, renewable energy sources have become a priority. Biomass energy is one of the new and renewable varieties of energy. This energy is more compatible with nature and the environment, and its production and supply cause little environmental pollution; also, since such energies are renewable, there is no near end for their exhaustion. Therefore, biomass energy constitutes a remarkable part of the world energy supply. Because of the importance of energy in economic growth, this study analyzed the relationship between biomass energy consumption and gross domestic production (GDP) during the 1967-2019 time period, using the auto-regressive distributed lag modeling approach (ARDL).

Keywords: Biomass energy consumption, Gross domestic production (GDP), Auto-regressive distributed Lag modeling approach (ARDL), Causality
2010 MSC: 91B62,91B82.

1. Introduction

In 2008, A. B. Mohammed [5] introduced the special upper triangular group $SUT(n, F)$, where $SUT(n, F)$ deduced from $UT(n, F) \cap SL(n, F)$. Authors in [5, 1, 4, 2, 3] compute the Artin character and Artin indicator for the groups $SL(2, p^k)$ where $p \leq 19$ and $p^k = 9, 25$ and 27, $PSL(2, p^k)$ where $p^k = 5, 7, 11, 13$ and 19, $PSL(2, p)$ where $p = 9, 25, 27$ and $PSL(2, F)$, where $F = 9, 25, 49$ respectively. O .M. Salih , N .J. Mohammed, B. M. Alwan [6], they introduced and find the cyclic decomposition of the group $SUT(2, p)$, where $p = 3, 5, 7$.
This research present and find the Artin character and Artin indicator of the group $SUT(2, p)$, where $p = 3, 5, 7$.

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2. Preliminaries

This section will provide some certain results that we will need.

Theorem 2.1. *[6]*
$SUT(2,q) = \left\{ \begin{pmatrix} a & b \\ 0 & a^{-1} \end{pmatrix} : a \in F_q^*, b \in F_q \right\}$. *Therefore* $|SUT(2,q)| = q(q-1)$.

Theorem 2.2. *[6]*
$|SUT(n,q)| = q^{\frac{n(n-1)}{2}}(q-1)^{n-1}$.

3. Main Score

In this section we study Artin character and Artin indicator for $SUT(2,q)$ where $p = 3, 5, 7$.

3.1. A(SUT(2,3))

The group $SUT(2,3)$ has 4 cyclic subgroups which are:

$$< \tau_0^{(1)} >, < -\tau_0^{(1)} >, < \tau_{01}^{(2)} >, < -\tau_{01}^{(2)} >$$

The rational valued character table of $SUT(2,3)$ illustrate in table 1, [6].

Table 1:

$C(g)$	$\tau_0^{(1)}$	$-\tau_0^{(1)}$	$\tau_{01}^{(2)}$	$-\tau_{01}^{(2)}$
$\lvert C_{SUT}(2,3)\rvert$	6	6	6	6
$\lvert C(g)\rvert$	1	1	1	1
$\lvert O(g)\rvert$	1	2	3	6
χ_0	1	1	1	1
χ_1	1	-1	1	-1
$\theta_1 + \theta_2$	2	2	-1	-1
$\chi_1\theta_1 + \chi_1\theta_2$	2	-2	-1	1

The Artin characters given in table 2 by applying the formula

$$\emptyset_H \uparrow^{SUT(2,3)} (\alpha) = \frac{|C_{SUT(2,3)}(X)|}{|C_H(\alpha)|} \sum_{\substack{\alpha \to X \\ H \to SUT(2,3)}} \emptyset(\alpha)$$

where H is the cyclic subgroup

From tables 1 and 2 we get:

$$\begin{aligned}
\chi_0 &= \frac{1}{3}\emptyset_4 + \frac{1}{3}\emptyset_2 - \frac{1}{9}\emptyset_1 \\
\chi_1 &= -\frac{1}{3}\emptyset_4 + \emptyset_3 - \frac{1}{3}\emptyset_2 - \frac{1}{18}\emptyset_1 \\
\theta_1 + \theta_2 &= -\frac{1}{3}\emptyset_4 + \frac{2}{3}\emptyset_2 + \frac{1}{9}\emptyset_1 \\
\chi_1\theta_1 + \chi_1\theta_2 &= \frac{1}{3}\emptyset_4 + \emptyset_3 - \frac{2}{3}\emptyset_2 + \frac{1}{18}\emptyset_1
\end{aligned}$$

Table 2:

$C(g)$	$\tau_0^{(1)}$	$-\tau_0^{(1)}$	$\tau_{01}^{(2)}$	$-\tau_{01}^{(2)}$
$\emptyset_1$	6	0	0	0
$\emptyset_2$	3	3	0	0
$\emptyset_3$	3	0	2	0
$\emptyset_4$	2	0	3	3

Therefore, $6X_i = (Z)\emptyset_i$, $i = 1, \cdot, 4$.
So $A(SUT(2,3)) = 6$.

3.2. A (SUT(2,5))

The group $SUT(2,5)$ has 5 cyclic subgroups which are:

$$< \tau_0^{(1)} >, < -\tau_0^{(1)} >, < \tau_{01}^{(2)} >, < -\tau_{01}^{(2)} >, < \tau_{2,-2}^{(3)} >$$

The rational valued character table of $SUT(2,3)$ illustrate in table 3, [6].

Table 3:

$C(g)$	$\tau_0^{(1)}$	$-\tau_0^{(1)}$	$\tau_{01}^{(2)}$	$-\tau_{01}^{(2)}$	$-\tau_{2,-2}^{(3)}$
$\lvert C_{SUT}(2,5)(g)\rvert$	20	20	10	10	4
$\lvert C(g)\rvert$	1	1	2	2	5
χ_0	1	1	1	1	1
$\chi_1 + \chi_3$	2	-2	2	-2	0
χ_2	1	1	1	1	-1
$\theta_1 + \theta_2$	4	-4	-1	-1	0
$\chi_1\theta_1 + \chi_1\theta_2$	4	4	-1	1	0

The Artin characters given in table 4 by applying the formula

$$\emptyset_H \uparrow^{SUT(2,5)} (\alpha) = \frac{|C_{SUT(2,5)}(X)|}{|C_H(\alpha)|} \sum_{\substack{\alpha \to X \\ H \to SUT(2,5)}} \emptyset(\alpha)$$

where H is the cyclic subgroup

From tables 3 and 4 we get:

Table 4:

$C(g)$	$\tau_0^{(1)}$	$-\tau_0^{(1)}$	$\tau_{01}^{(2)}$	$-\tau_{01}^{(2)}$	$\tau_{2,-2}^{(3)}$
$\emptyset_1$	20	0	0	0	0
$\emptyset_2$	10	10	0	0	0
$\emptyset_3$	5	0	2	0	0
$\emptyset_4$	2	0	0	5	0
$\emptyset_5$	2	2	0	5	5

$$\begin{aligned}
\chi_0 &= \frac{1}{5}\emptyset_5 + \frac{1}{2}\emptyset_3 + \frac{3}{50}\emptyset_3 - \frac{1}{8}\emptyset_1 \\
\chi_1 + \chi_3 &= -\frac{2}{5}\emptyset_4 + \emptyset_3 - \frac{1}{5}\emptyset_2 - \frac{1}{10}\emptyset_1 \\
\chi_2 &= -\frac{1}{5}\emptyset_5 + \frac{2}{5}\emptyset_4 + \frac{1}{2}\emptyset_3 + \frac{7}{5}\emptyset_2 - \frac{33}{10}\emptyset_1 \\
\theta_1 + \theta_2 &= -\frac{1}{5}\emptyset_4 - \frac{1}{2}\emptyset_3 - \frac{4}{10}\emptyset_2 - \frac{109}{10}\emptyset_1 \\
\chi_1\theta_1 + \chi_1\theta_2 &= \frac{1}{5}\emptyset_4 + -\frac{1}{2}\emptyset_3 + \frac{4}{10}\emptyset_2 + \frac{21}{200}\emptyset_1
\end{aligned}$$

Therefore, $20X_i = (Z)\emptyset_i, i = 1, ..., 5$. So we obtain $A(SUT(2,5)) = 20$.

3.3. *A(SUT(2,7))*

The group SUT(2,7) has 6 cyclic subgroups which are:

$$< \tau_0^{(1)} >, < -\tau_0^{(1)} >, < \tau_{01}^{(2)} >, < -\tau_{01}^{(2)} >, < \tau_{2,-2}^{(3)} >, < \tau_{5,-5}^{(3)} > .$$

The rational valued character table of $SUT(2,7)$ illustrate in table 5, [6].

Table 5:

$C(g)$	$\tau_0^{(1)}$	$-\tau_0^{(1)}$	$\tau_{01}^{(2)}$	$-\tau_{01}^{(2)}$	$\tau_{2,-2}^{(3)}$	$\tau_{5,-5}^{(3)}$
$\lvert C_{SUT}(2,7)(g)\rvert$	42	42	14	14	6	6
$\lvert C(g)\rvert$	1	1	3	3	7	7
χ_0	1	1	1	1	1	1
$\chi_1 + \chi_3$	2	-2	2	-2	-2	1
$\chi_2 + \chi_4$	2	2	2	2	2	-1
χ_3	1	-1	1	-1	-2	-1
$\theta_1 + \theta_2$	6	6	-1	-1	0	0
$\chi_1\theta_1 + \chi_1\theta_2$	6	-6	-1	1	0	0

The Artin characters given in table 6 by applying the formula

$$\emptyset_H \uparrow^{SUT(2,7)} (\alpha) = \frac{|C_{SUT(2,7)}(X)|}{|C_H(\alpha)|} \sum_{\substack{\alpha\to X \\ H\to SUT(2,7)}} \emptyset(\alpha)$$

where H is the cyclic subgroup

Table 6:

$C(g)$	$\tau_0^{(1)}$	$-\tau_0^{(1)}$	$\tau_{01}^{(2)}$	$-\tau_{01}^{(2)}$	$\tau_{2,-2}^{(3)}$	$\tau_{5,-5}^{(3)}$
$\emptyset_1$	42	0	0	0	0	0
$\emptyset_2$	21	21	0	0	0	0
$\emptyset_3$	7	0	3	0	0	0
$\emptyset_4$	3	0	0	7	0	0
$\emptyset_5$	3	3	0	0	7	0
$\emptyset_6$	3	0	3	0	7	7

From tables 5 and 6 we get:

$$\begin{aligned}
\chi_0 &= \frac{1}{7}\emptyset_6 + \frac{1}{7}\emptyset_4 + \frac{4}{21}\emptyset_3 + \frac{1}{21}\emptyset_2 - \frac{23}{441}\emptyset_1 \\
\chi_1 + \chi_3 &= \frac{1}{7}\emptyset_6 - \frac{3}{7}\emptyset_5 - \frac{2}{7}\emptyset_4 + \frac{11}{21}\emptyset_3 - \frac{5}{147}\emptyset_2 + \frac{8}{441}\emptyset_1 \\
\chi_2 + \chi_4 &= -\frac{1}{7}\emptyset_6 + \frac{3}{7}\emptyset_5 + \frac{2}{7}\emptyset_4 + \frac{17}{21}\emptyset_3 + \frac{5}{147}\emptyset_2 - \frac{64}{441}\emptyset_1 \\
\chi_3 &= -\frac{1}{7}\emptyset_6 - \frac{1}{7}\emptyset_5 - \frac{1}{7}\emptyset_4 + \frac{10}{21}\emptyset_3 - \frac{4}{147}\emptyset_2 - \frac{4}{126}\emptyset_1 \\
\theta_1 + \theta_2 &= -\frac{1}{7}\emptyset_4 - \frac{1}{3}\emptyset_3 + \frac{2}{7}\emptyset_2 + \frac{29}{441}\emptyset_1 \\
\chi_1\theta_1 + \chi_1\theta_2 &= \frac{1}{7}\emptyset_4 - \frac{1}{3}\emptyset_3 - \frac{2}{7}\emptyset_2 + \frac{2}{7}\emptyset_1
\end{aligned}$$

Therefore, $42\chi_i = (Z)\emptyset_i$, $i = 1, \cdot, 6$. So we obtain $A(SUT(2,7)) = 42$.

4. Conclusion

From this work we deduced these results:

1. $A(SUT(2,q) = q(q-1$ the order of $SUT(2,q)$
2. For every prime number q, $<\tau_{2,-2}^{(3)}>, <\tau_{5,-5}^{(3)}>$
   i The order of cyclic subgroup $<\tau_0^{(1)}>, <-\tau_0^{(1)}>= q(q-1)$
   ii The order of cyclic subgroup $<\tau_{0,1}^{(2)}>, <-\tau_{0,1}^{(2)}>= 2q$
   iii The order of remaining cyclic subgroup equal to $q-1$.

5. Suggestions for future works

Based on the present work, the following topics are put forward for future works

1. Study the same idea for the groups $SUT(2, p^k)$, where p is an odd prime number and $k = 2$.
2. Study the same idea for the groups $PSL(3, p^k)$, where p is even number and $k = 3$.

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