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Analysis of the structural reliability of dental filling under the effect of mouthwash G.U.M

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Abstract

The application of structural reliability analysis has become very important to ensure the strength of the studied structural design, as it presents results that help in giving evidence about the acceptance of that design according to the materials used under specific operating conditions. In this research an experiment was done, to find out the effect of G.U.M mouthwash on cured and re-cured Visible Light Cured (VLC) composite dental filling material, made according to agreed international standards, where many studies have documented that the surface of restorative materials placed on the tooth may also be affected by the chemical effect of different types of oral health care products, and the experiment was analyzed mathematically by applying structural reliability analysis to know the probability of structural failure when dental filling were exposed to the mouthwash G.U.M (Operational conditions), using analysis technique: the D-vine copula. The results were that the probability of structural failure gave an indication from which to infer the extent to which the experiment was accepted or rejected.

Keywords: Structural reliability, Probability of structural failure, D-vine copula, mouthwash, dental filling, material.

1. Introduction

Structural reliability analysis is a decision-making tool that supports the planning of the studied engineering structure at the design stage to determine its ability to work under certain operating conditions throughout its expected life. It allows the inclusion of relevant information and inference

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from the available data, to estimate the probability of structural failure, which is the main objective of the analysis, that is, it gives safe and more accurate estimates of the design. Structural reliability analysis is often used in the framework of quality controls and results are rejected outside the permissible range, accordingly, it has been used in many engineering fields. The structural reliability analysis is carried out with the help of statistical and probabilistic methods that reflect the engineering reality to evaluate the structural design to ensure satisfactory performance [6, 20, 24, 34].

For the purpose of analysis, uncertain quantities such as loads, material properties, geometric dimensions, environmental factors, etc., are represented by the k-dimensional vector of basic random variables $\mathbf{X} = (X_1,\ \mathrm{X}_2,\ \ldots, X_k)$, then the random variables are formulated through a mathematical model known as the performance function or the limit state function $\mathrm{G}(\mathbf{X})$ that shows the variables included in the design as it plays a key role in the analysis process. The $\mathrm{G}(\mathbf{X})$ is can divide the variables space into two domains: safe domain ($G(\mathbf{X}) > 0$) and failure domain $\mathbf{G(X)} \leq 0$).

Thus, the probability of structural failure p_f is calculated by performing a multidimensional integration of the joint probability density function $f(x_1, x_2, \ldots, x_k)$ of $\mathbf{X}$ within the boundaries provided by the performance function in failure domain $\mathbf{G(X)} \leq 0$.

$$p_f = p\mathbf{G(X)} \leq 0) = \int \ldots \int_{\mathbf{G(X)} \leq 0} f(x_1,x_2, \ldots, x_k)\ dx_1, dx_2, \ldots, dx_k \tag{1.1}$$

but it's difficult to perform the above integration because in most engineering practices there is a multi-correlation between random variables, which makes it difficult to perform the multi-integration in Eq.(1.1) to obtain the probability of structural failure. Accordingly, the researchers presented many techniques, which are an analytical approximation of integration that makes the calculations more flexible to help estimate the probability of structural failure in light of the problem of correlation such as the first-order reliability method (FORM) [2, 16, 25, 30, 31, 39], second-order reliability method (SORM) [36, 42, 44], simulation method [7], neural networks [9, 29], etc.

In this research, a structural reliability analysis by the first-order reliability method (FORM), was used with the aim of evaluate the effect of mouthwash G.U.M on the dental filling made of the light-cured composite material.

The use of mouthwash has become very popular because it prevents and controls some diseases of the teeth and gums because it contains antibacterial agents [17]. Several studies have documented that the surface of restorative materials placed on a tooth may also be affected by the chemical effect of various types of oral health care products [8, 10].

To achieve the aim of the research, an experiment was conducted (Described in detail in Section (3)), for dental filling under agreed laboratory conditions, it was treated with G.U.M mouthwash. After that, a structural reliability analysis was conducted in order to know the probability of structural failure of the dental filling, which shows the extent of the effect resulting from the use of G.U.M mouthwash, the analysis was carried out through the surface hardness and geometric dimensional accuracy (thickness and diameter) of the filling.

The research is organized as follows: In Section (2) the methodological background and techniques used in the structural reliability analysis are clarified, in Section (3) the experience of dental filling and the results obtained based on the application of the reliability analysis methods are explained, while in Section (4) the most important conclusions that have been reached are mentioned.

2. Methodological background and the technique of structural reliability analysis

In this section, the first-order reliability method (FORM) in structural reliability analysis and the analysis techniques based on it are explained.

The FORM is one of the essential methods of structural reliability analysis widely used in its analysis of engineering problems when there are correlations between the studied random variables, it has provided computational procedures to calculating the probability of structural failure [15, 22, 38].

The FORM in structural reliability analysis, in general require that the random variables are independent and have a standard normal distribution, otherwise, the correlated original variables $\mathbf{X} = (X_1, X_2, \ldots, X_k)$ in (X-space) should be transformed into independent standard normal variables $\mathbf{Y} = (Y_1, Y_2, \ldots, Y_k)$ in (Y-space), where transformation vector denoted as $\mathbf{Y} = T(\mathbf{X})$ and the inverse transformation is denoted as $\mathbf{X} = T^{-1}(\mathbf{Y})$, thus the performance function $G(x)$ of correlated original vector $\mathbf{X}$ transform into $g(\mathbf{Y})$ is performance function of independent standard normal vector $\mathbf{Y}$. The researchers have introduced several transformation techniques such as Rosenblatt transformation, Nataf transformation, etc.[6, 32].

After that, the optimal design point or what is known as the most probability point (MPP) for failure in the standard normal space (Y-space) is searched in an iterative manner through one of the optimization algorithms such as HL-RF, iHL-RF and etc.[15], and when the most probable point (MPP) of failure found, the reliability index that symbolizes it has the Greek letter β is calculated at MPP by the following formula [12, 14]

$$\left.\begin{aligned} \beta &= \|\mathbf{Y}\| \\ S.t \quad g(\mathbf{Y}) &= 0 \end{aligned}\right\} \tag{2.1}$$

where $\mathbf{Y} = y_1, y_2, \ldots, y_k$, is the vector of the most probable point (MPP) in the normal space, and $\|.\|$ is the norm of vector:

$$\|\mathbf{Y}\| = \sqrt{\mathbf{Y}^{\mathbf{T}}\mathbf{Y}} = \sqrt{{y_1}^2 + {y_2}^2 + \cdots + {y_k}^2} \tag{2.2}$$

and thus it can calculate the probability of structural failure p_f depending on the reliability index β as [45]

$$p_f = \Phi\left(-\beta\right) = 1 - \Phi\left(\beta\right)$$

where Φ is the cumulative distribution function:

$$\Phi(\beta) = \int_{-\infty}^{\beta} \frac{1}{\sqrt{2\pi}}\, e^{-(\frac{1}{2})y^2} dy \tag{2.3}$$

It should be noted that, there are many techniques adopted by FORM in its general computational procedures, which sometimes leads to those techniques being called: structural reliability analysis techniques based on the idea it presented in the process of transforming correlated variables or providing iterative improvement algorithms and the like.

In this research, FORM used transforming correlated variables technique: Rosenblatt transformation with D-vine model, accordingly, the analysis technique here are named as D-vine copula technique which will be explained below.

2.1. D-vine copula technique

The vine copula was proposed by Joe in 1969 and developed by Bedford and Cook [3, 4], has become one of the important mathematical tools of great importance in statistical analysis, as it has been applied in probability and uncertainty theory, to provide solutions in multidimensional correlation problems. It works on constructing a joint probability distribution function for correlated random variables (more than two), by decomposing the joint probability density function of multidimensional random variables into product of the marginal probability distribution functions and bivariate copula density functions. What is meant by vine is a graphical model structure of a nested set of trees that describes the conditions necessary to build a suitable mathematical model representing the multidimensional joint probability distribution that depends in its construction on the bivariate copula functions, which are selected according to the nature of the data, therefore, the results are more accurate [4, 21, 3, 43].

The vine copula technique was recently developed in the field of structural reliability analysis [11, 40, 41].

There are several types of vine copula each model gives a specific way to represent multidimensional joint density function, in this research, the D-vine was used in structural reliability analysis, this technique is organized as follows: The definition of copula is given in section (2.1.1). The D-vine copula description is introduced in section (2.1.2). The structural reliability analysis based on of D-vine copula is explained in section (2.1.3).

2.1.1. Definition of copula

The copula is defined as the linking function between marginal distribution functions of correlated random variables with each other to form the joint distribution function [28]. According to Sklar's theory [28], consider a vector $\mathbf{X} = X_1, X_2, \cdots, , X_k$ of random variables with a marginal distribution functions $F_{X_1}(x_1), F_{X_2}(x_2), \ldots, F_{X_k}(x_k)$, the joint distribution of $\mathbf{X}$ can be written as [27]

$$\begin{aligned} F_{X_1,X_2,\cdots,X_k}(x_1,x_2,\ldots,x_k) &= C\left(F_{X_1}(x_1)\ ,\ F_{X_2}(x_2),\ldots,F_{X_k}(x_k);\theta\right) \\ &= C(\ u_1,u_2,\ldots,u_K;\theta) \end{aligned} \tag{2.4}$$

where C is denoted copula; θ is parameter of the copula function; $u_i = F_{X_i}(x_i)$, $i = 1, 2, \ldots, k$; all u_i on the interval $[0,\ 1]$ a follow uniform distribution, namely, $C(\ u_1,u_2,\ldots,u_K;\theta)$ is a k-dimensional joint distribution function with marginal distribution on the interval $[0,\ 1]^k \rightarrow [0,\ 1]$ that follow uniform distribution; and C is unique if all marginal distribution $F_{X_i}(x_i)$ are continuous.

Moreover, we get the joint probability density function through the derivation Eq.(2.4) as [35]

$$f_{X_1,X_2,\ldots,X_k}(x_1,x_2,\ldots,x_k)\ = c\left(F_{X_1}(x_1)\ ,\ F_{X_2}(x_2),\ldots\ldots,F_{X_k}(x_k);\theta\right)\prod_{i=1}^{k} f_{X_i}(x_i) \tag{2.5}$$

$f_{X_i}(x_i)$ is marginal probability density function of X_i, c $\left(F_{X_1}(x_1),\ F_{X_2}(x_2),\ldots,F_{X_k}(x_k);\theta\right)$ is the density function of C, which is given by the derivative of the copula function C as

$$c\left(F_{X_1}(x_1)\ ,\ F_{X_2}(x_2),\ldots\ldots,F_{X_k}(x_k);\theta\right) = c\left(u_1,u_2,\ldots,u_k;\theta\right) = \frac{\partial^K C\left(u_1,u_2,\ldots,u_k;\theta\right)}{\partial u_1 \partial u_2 \ldots \partial u_k} \tag{2.6}$$

And the parameter θ is estimated using MLE as

$$\widehat{\theta} = \arg\max\ L\left(c_{12}\right) \tag{2.7}$$

where

$$L(c_{12}) = \sum_{i=1}^{n} lnc_{12}(u_{1i}, u_{2i};\ \theta) \tag{2.8}$$

It is worth noting, there are several types of copula functions to constructing joint distribution; therefore, a suitable copula must be chosen according to the study data. In this paper, according to the study data in the application side, the type of copula that was used is Clayton.

Where the Clayton is a type of copulas that belongs to Archimedes' copulas proposed by Clayton (1978), it has the formula in case bivariate as

$$C(u_1, u_2; \theta) = \left(u_1^{-\theta} + u_2^{-\theta} - 1\right)^{-\frac{1}{\theta}} \tag{2.9}$$

where θ is the copula parameter restricted on interval $(0, \infty)$, the parameter θ is controls the scale of dependence strength, if $\theta \to 0$ the marginal distributions are independent, and when $\theta \to \infty$ the copula reaches the upper bound of the Fre'chet-Hoeffding. But the lower bound of the Fre'chet-Hoeffding cannot be obtained because to the restriction on the copula parameter. This indicates Clayton copula cannot account for negative dependence, the Clayton copula has been used to study correlated risks because it shows a strong dependence on the left tail, while the right-tail is relatively light as shown in Figure.1 [37].

The properties of the Clayton copula as following

(i) The probability density function with two variables is

$$\begin{aligned} C(u_1, u_2; \theta) &= \frac{\partial^2 C(u_1, u_2; \theta)}{\partial u_1 \partial u_2} = \frac{\partial^2}{\partial u_1 \partial u_2}\left(u_1^{-\theta} + u_2^{-\theta} - 1\right)^{-\frac{1}{\theta}} \\ &= (\theta + 1)(u_1 u_2)^{-\theta-1}\left({u_1}^{-\theta} + {u_2}^{-\theta} - 1\right)^{-2-\frac{1}{\theta}} \end{aligned} \tag{2.10}$$

(ii) The conditional function $h(u_1, u_2)$ [39]

$$\begin{aligned} F_{2/1}(x_2/x_1) = C_{2/1}(u_2/u_1) &= \frac{\partial C(u_1, u_2; \theta)}{\partial u_1} = h_{21}(u_1, u_2) \\ &= u_1^{-\theta-1}\left({u_1}^{-\theta} + {u_2}^{-\theta} - 1\right)^{-1-\frac{1}{\theta}} \end{aligned} \tag{2.11}$$

(iii) The tail dependence [36]

- The lower tail dependence is:

$$\lambda_L = 2^{-\frac{1}{\theta}} \tag{2.12}$$

- The upper tail dependence is:

$$\lambda_U = 0 \tag{2.13}$$

Where the tail dependence among the random variables can have great influences in the field of reliability analysis [11].

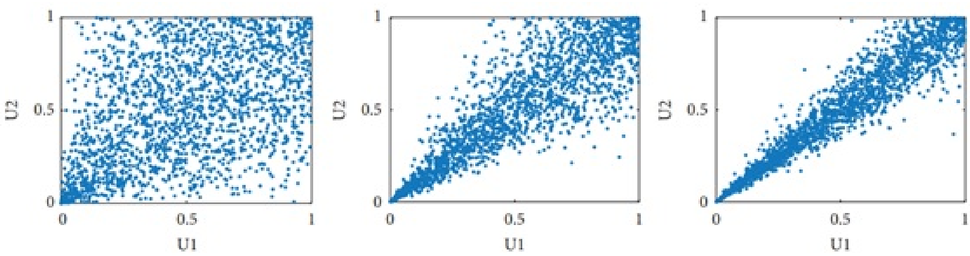

Figure 1: The scatter plots of bivariate Clayton copula function with different parameters ($\theta = 1,\ 5,\ 10$)

(iv) The general formula of clayton copula as [28]

$$C(u_1, u_2, \ldots, u_k; \theta) = \left(u_1^{-\theta} + u_2^{-\theta} + \ldots + u_k^{-\theta} - k + 1\right)^{-\frac{1}{\theta}}, \qquad \theta > 0 \tag{2.14}$$

2.1.2. Description of the D-vine copula model

This section is organized as follows, the graphical model of D-vine is described in section (2.1.2.1), the mathematical model and its construction are explained in sections (2.1.2.2) and (2.1.2.3) respectively

2.1.2.1. Graphical model of D-vine copula

The graphical model of D-vine is structure of a nested set of trees, similar to a bunch of grapes, this trees contains nodes and edges as illustration in Figure.(2). The following is an explanation of the structure and contents of the graphical [18]

(i) Trees T_j , D-vine has tree $\{T_1, T_2, \ldots, T_{k-1}\}$, $j = 1, 2, .., k-1$, k is number of variables.

(ii) Nodes N, for each tree has $(k - j + 1)$ of the nodes, which are important to determining the labels for each edge, and that the edge in the T_j becomes a node in the T_{j+1}, and so on. In tree 1 only the nodes represent the marginal distributions of the random variables.

(iii) Edges E, the nodes are linked to each other by $(k - j)$ of the edges in each tree, the edges represents a copula density functions in the mathematical model, where the total number of edges is $k\ (k-1)\ /\ 2$ which is the copula functions required in the mathematical model, for example in Fig.2 illustrates the graphical model of D-vine copula with three variables, note that edge label 12 represents bivariate copula density function c_{12}, and 13/2 represents the conditional copula density function $c_{13/2}$.

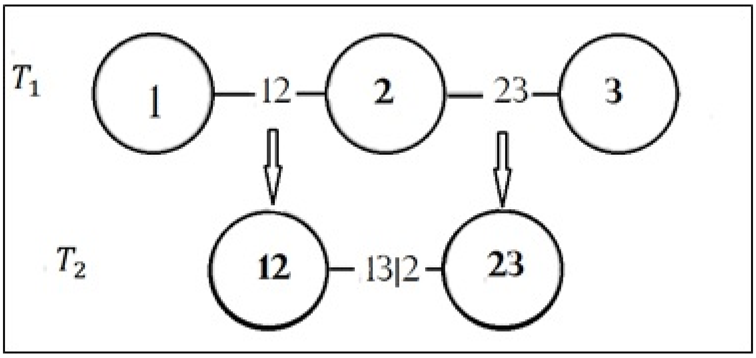

Figure 2: D-vine with three variable

2.1.2.2. Mathematical model of D-vine copula

To explant the mathematical model for D-vine copula, let the multidimensional joint probability density function

$f\,(x_1, x_2, \ldots, x_k)$ of random vector $\mathbf{X} = (X_1, X_2, \ldots,\ X_k)$, can be decomposed as [40]

$$f\,(x_1, x_2, \ldots, x_k) = f_1\,(x_1)\, f_{2/1}\,(x_2/x_1)\, f_{3/1,2}\,(x_3/x_1, x_2) \ldots\ldots f_{k/1,2,\ldots,k-1}\,(x_k/x_1, x_2, \ldots, x_{k-1}) \tag{2.15}$$

the conditional densities functions of correlated variables in Eq.(2.15) are obtained through copula function, to illustrate this, let's take a case of three random variables X_1, X_2, X_3, the PDF decompose as follows [1]

$$f\,(x_1, x_2, x_3) = f_1\,(x_1)\, f_{2/1}\,(x_2/x_1)\, f_{3/1,2}\,(x_3/x_1, x_2) \tag{2.16}$$

where $\mathrm{f}_1(x_1)$ is marginal density function of X_1, and $f_{2/1}(x_2/x_1)$ is conditional density function can be obtained using copula function as

$$\begin{aligned} f_{2/1}(x_2/x_1) &= \frac{f(x_1,x_2)}{f(x_1)} = \frac{c_{12}\{F(x_1),F(x_2);\theta_{12}\}f(x_1)f(x_2)}{f(x_1)} \\ &= C_{12}\{F(x_1),F(x_2);\theta_{12}\}f(x_2) \end{aligned} \tag{2.17}$$

where c_{12} is bivariate copula density function.

and by same way the conditional density function $\mathrm{f}_{3/1,2}(x_3/x_1,x_2)$ can be obtained as

$$\begin{aligned} f_{3/1,2}(x_3/x_1,x_2) &= \frac{f_{13/2}(x_1,x_3/x_2)}{f_{2/1}(x_2/x_1)} \\ &= c_{13/2}\left\{F_{1/2}(x_1/x_2),F_{\frac{3}{2}}(x_3/x_2);\theta_{13/2}\right\}.\mathrm{f}_{3/2}\left(\frac{x_3}{x_2}\right) \end{aligned} \tag{2.18}$$

$$\begin{aligned} f_{3/2}\left(\frac{x_3}{x_2}\right) &= \frac{f(x_2,x_3)}{f(x_2)} = \frac{c_{23}\{F_2(x_2),F_3(x_3);\theta_{23}\}.f(x_2).\mathrm{f}_3(x_3)}{f(x_2)} \\ &= c_{23}\{F_2(x_2),F_3(x_3);\theta_{23}\}.f_3(x_3) \end{aligned} \tag{2.19}$$

then, substituting Eq.(2.19) in (2.18)

$$f_{3/1,2}\left(\frac{x_3}{x_1},x_2\right) = c_{13/2}\{F_{1/2}(x_1/x_2),F_{3/2}(x_3/x_2);\theta_{13/2}\}\,c_{23}\{F_2(x_2),F_3(x_3);\theta_{23}\}.f_3(x_3) \tag{2.20}$$

where $\mathrm{c}_{13/2}\left\{F_{\frac{1}{2}}(x_1/x_2),F_{\frac{3}{2}}(x_3/x_2);\theta_{13/2}\right\}$ is conditional copula density function, $F(x_1/x_2)$ and $F(x_3/x_2)$ are the conditional cumulative distribution functions, we obtain it by the derivation of the copula function with respect to the conditional variable as

$$F_{1/2}(x_1/x_2) = \frac{\partial C_{12}(u_1,u_2)}{\partial u_2} = h_{12}(u_1,u_2) \tag{2.21}$$

$$F_{3/2}(x_3/x_2) = \frac{\partial C_{23}(u_2,u_3)}{\partial u_2} = h_{32}(u_3,u_2) \tag{2.22}$$

where $h(.)$ represents the conditional distribution. and the function $c_{13/2}(u_3/u_2,u_1/u_2)$ is corresponding to the conditional distribution function [26] $C_{3/1,2}(u_3/u_2,u_1/u_2)$. thus, substituting the Eq. (2.17) and Eq.(2.20) in Eq. (2.16), the PDF became as

$$\begin{aligned} f(x_1,x_2,x_3) = f_1(x_1).f_2(x_2).f_3(x_3).c_{12}\{F_1(x_1),F_2(x_2);\theta_{12}\} \\ .c_{23}\{F_2(x_2),F_3(x_3);\theta_{23}\}\ .c_{13/2}\left\{F(x_1/x_2),F(x_3/x_2);\theta_{13/2}\right\} \end{aligned} \tag{2.23}$$

the joint probability density function in Eq.(2.23) is the mathematical model of three-dimensional D-vine.

In general, the probability density function in Eq.(2.15) has D-vine model as follows

$$\begin{aligned} &f(x_1,x_2,\ldots,x_k) = \prod_{p=1}^{k} f_p(x_p) \times \prod_{j=1}^{k-1}\prod_{i=1}^{k-j} c_{i,i+j/i+1,\ldots,i+j-1} \\ &\left\{F_{i/i+1,\ldots,i+j-1}\left(x_i/x_{i+1},\ldots.,x_{i+j-1}\right)\ ,F_{i+j/i+1,\ldots,i+j-1}\left(x_{i+j}/x_{i+1},\ldots.,x_{i+j-1}\right),\theta\ \right\} \end{aligned} \tag{2.24}$$

where $f_p(x_p)$, $p=1,2,\ldots,k$ is the marginal probability density function of X_p, $c_{i,i+j/i+1,\ldots,i+j-1}$ is copula density function, subscript j represents tree T_j, subscript i represents edge in tree T_j.

2.1.2.3. Construction the D-vine model

To construction the functions of D-Vine model in Section (2.1.2) illustration as follows [19].
We take the D-vine model with three-dimensional as Eq.(2.23) and Figure. 2 :

$$.c_{23}\left\{F_2\left(x_2\right), F_3(x_3); \theta_{23}\right\}.c_{13/2}\left\{F\left(x_1/x_2\right), F(x_3/x_2); \theta_{13/2}\right\}$$

(i) Characterize the marginal density functions $f_1\left(x_1\right), f_2\left(x_2\right), f_3\left(x_3\right)$ of random variables X_1, X_2 and X_3 , respectively, based on the study sample data (x_{1i}, x_{2i}, x_{3i}), $i = 1, 2, \ldots, n$ where n is the total number of samples.

(ii) Determined cumulative distribution functions, $F_{X_1}\left(x_{1i}\right) = u_{1i}$, $F_{X_2}\left(x_{2i}\right) = u_{2i}$, $F_{X_3}\left(x_{3i}\right) = u_{3i}$, $i = 1, 2, \ldots, n$.

(iii) Determined which bivariate copula type in tree 1, for each the $c_{12}\left\{F_1\left(x_1\right), F_2\left(x_2\right); \theta_{12}\right\}$ and $c_{23}\left\{F_2\left(x_2\right), F_3\left(x_3\right); \theta_{23}\right\}$, by plotting the original data, then choosing the appropriate copulas.

(iv) Estimate the parameters of the selected copula using the original data by using MLE.

(v) Construction the bivariate conditional copula density function $c_{13/2}\left\{F\left(x_1/x_2\right), F(x_3/x_2); \theta_{13/2}\right\}$ in tree 2 as follows,

based on the conditional function $h(.)$ of copulas in tree 1:

$$h12\left(u_1, u_2\right) = \; F\left(x_1/x_2\right) = \frac{\partial C\left(u_1, u_2; \theta_{12}\right)}{\partial u_2} = v_1 \tag{2.25}$$

and

$$h_{32}\left(u_3, u_2\right) = \; F\left(x_3/x_2\right) = \frac{\partial C\left(u_2, u_3; \theta_{23}\right)}{\partial u_2} = \; v_2 \tag{2.26}$$

Then, $c_{13/2}\left\{F\left(x_1/x_2\right), F(x_3/x_2); \theta_{13/2}\right\}$ is transform as Eq.(2.27) through transform observations :

$$c_{13/2}\left(v_1, v_2; \; \theta_{13/2}\right) \tag{2.27}$$

and estimate the parameter $\theta_{13/2}$ by using MLE method, and then appropriate copula is chosen as mentioned in above. That is, the estimation of copula functions for the vine model depended on the sequential estimation of the successive tree on the previous tree, etc.

2.1.3. Structural Reliability Analysis Adoptive D-vine Copula technique

The D-vine Copula technique helped to analysis the structural reliability according to the FORM approach, where first the Rosenblatt transform method with D-vine model was used to transform random variables with multi-dimensional correlations from (x-space) to independent standard normal random variables (y-space), and then the analysis is carried out [11], according to this, the transformation procedure, and the reliability analysis is carried out will be explained in sections (2.1.3.1) and (2.1.3.2) respectively.

2.1.3.1. Transformation variables

For the structural reliability analysis, the correlated random vector $X = (x_1,\ x_2, \ldots x_k)$ are transformed into independent standard normal vector $Y = (y_1, y_2\ , \ldots, y_k)$, here, by adopting the Rosenblatt transformation with the D-vine model [11, 39, 41].
The Rosenblatt transformation as

$$\begin{aligned} \Phi(y_1) &= F_1\,(x_1) \\ \Phi\,(y_2) &= F_{2/1}\,(x_2/x_1) \\ \Phi(y_3) &= F_{3/1,2}\,(x_3/x_1,x_2) \\ \Phi\,(y_k) &= F_{k/1,2,k-1}(x_k/x_1, x_2, \ldots, x_{k-1}) \end{aligned} \tag{2.28}$$

where Φ is standard normal distribution function of Y_i, and $F_{k/1,2,k-1}(x_k/x_1, x_2, \ldots, x_{k-1}\ \ )$ is conditional distribution function of original variables X_i. But the Rosenblatt transform requires knowledge of the joint distribution function, and since the variables $(X_1, X_2, \ldots, X_k)$ are correlated it is difficult to obtain the joint functions, therefore, the D-vine model was used, for its ability to provide the joint distribution functions as well as the conditional functions required in the transformation, as explained in Sections (2.1.2.2) and (2.1.2.3), then the transformation process will be as follows
In a three-dimensional case,
let $F_1\,(x_1) = u_1,\quad F_2\,(x_2) = u_2\ ,\ F_3\,(x_3) = u_3$; then, obtained $\mathbf{Y} = T(\mathbf{X})$ as:

(i)

$$\Phi\,(y_1) = F_1\,(x_1) =\ u_1;\ \ then,\ \ y_1 = \Phi^{-1}\ \,(\ u_1) \tag{2.29}$$

(ii)

$$\Phi\,(y_2) = F_{2/1}\,(x_2/x_1) = \frac{\partial C_{12}\,(F_1\,(x_1)\,, F_2\,(x_2))}{\partial F_1\,(x_1)}\ \ = \frac{\partial C_{12}\,(u_1, u_2;\theta_{12})}{\partial u_1} = h_{21}\,(u_1, u_2)\,;\ then,$$

$$y_2 =\ \ \Phi^{-1}\ \ (h_{21}\,(u_1, u_2)\ ) \tag{2.30}$$

(iii)

$$\begin{aligned} \Phi\,(y_3) = F_{\frac{3}{1,2}}(x_3/x_1,x_2) &= \frac{\partial C_{13/2}\left(F_{3/2}\,(x_3/x_2)\,, F_{1/2}\,(x_1/x_2)\right)}{\partial F_{1/2}\,(x_1/x_2)} \\ &= h_{3/2,1/2}((h_{32}\,(u_3\ , u_2)\,, (h_{12}\,(u_1\ , u_2)), \end{aligned}$$

then

$$y_3 = \Phi^{-1}\ \ \left(\ \ \ h_{3/2,1/2}((h_{32}\,(u_3\ , u_2)\,, (h_{12}\,(u_1\ , u_2))\ \right) \tag{2.31}$$

where $\Phi^{-1}\,(.)$ is inverse CDF of standard normal variable Y_i, $h_{12}\,(u_1, u_2)$ and
$h_{3/2,1/2}((h_{32}\,(u_3\ , u_2)\,, (h_{12}\,(u_1\ , u_2))$ are conditional copula function.
Also, the inverse transformation is performed to obtain vector X corresponding to vector Y based on the above , $\mathbf{X} = T^{-1}\mathbf{X}$ as

(i) description From Eq.(2.29); then,

$$x_1 = {F_1}^{-1}(\Phi\,(y_1)) \tag{2.32}$$

(ii) description From Eq.(2.30); then, $u_2 = h_{21}^{-1}\left(\Phi\left(y_2\right), u_1\right); and\ u_2 = F_2\left(x_2\right)$ then,

$$x_2 = F_2^{-1}(h_{21}^{-1}\left(\Phi\left(y_2\right), u_1\right) \tag{2.33}$$

(iii) description From Eq.(2.31); then, $u_3 = h_{32}^{-1}(h_{3/2,1/2}^{-1}(\ \Phi\left(y_3\right), h_{12}\left(u_1\ , u_2\right)),\ u_2), and\ \mathrm{u}_3 = F_3\left(x_3\right)$;then,

$$X_3 = F_3^{-1}(h_{32}^{-1}(h_{3/2,1/2}^{-1}(\ \Phi\left(y_3\right), h_{12}\left(u_1\ , u_2\right)),\ u_2)) \tag{2.34}$$

where $F_{X_i}^{-1}(.)$ is invers cdf of X_i , $h^{-1}(.,.)$ invers function of $h(.,.)$.

2.1.3.2. Structural reliability analysis

In this section, the procedure for reliability analysis is explained adopting algorithm improved HL-RF (IHL-RF) with D-vine copula [11], where the IHL-RF is one of the common iterative algorithm applied in structural reliability analysis to obtain the most probable point (MPP) in standard normal space (Y-space) according to the technique used in the reliability analysis process, as there are techniques due to of their computational complex furcation require convergence and efficiency in iterative steps, in IHL-RF this is done by controlling the step size α when search of the MPP for calculating the reliability index β and then obtain the probability of structural failure p_f [12]. Accordingly, the working steps of the analysis algorithm are explained in Section (2.1.3.2.2).

But before that, the gradient vector of the performance function $\nabla g(\mathbf{Y})$ that is fundamental to the analysis process, will be explained in the section (2.1.3.2.1).

2.1.3.2.1. Computing the gradient vector

The gradient vector of the performance function $\nabla g(\mathbf{Y})$ of standard normal vector $\mathbf{Y}$ calculate as [11]

$$\nabla g\left(\mathbf{Y}\right) = \mathbf{J}_{\mathbf{XY}}^T \nabla G(\mathbf{X}) \tag{2.35}$$

where $\nabla G(\mathbf{X})$ is gradient vector of the performance function of original vector $\mathbf{X}$:

$$\nabla G(\mathbf{X}) = \left\{\frac{\partial G}{\partial x_1}\ ,\ \frac{\partial G}{\partial x_2}\ ,\ \ldots\ldots, \frac{\partial G}{\partial x_k}\right\} \tag{2.36}$$

$\mathbf{J}_{\mathbf{XY}} = \left[\frac{\partial x_i}{\partial y_i}\right]_{k\times k}$ is the Jacobean matrix obtained through the inverse of the$\mathbf{J}_{\mathbf{YX}}$, where the $\mathbf{J}_{\mathbf{YX}}$ can be obtained by differentiating both sides of the Eq.(2.28) as [11, 31]

$$\mathbf{J}_{\mathbf{YX}} = \left[\frac{\partial y_i}{\partial x_i}\right]_{k\times k} = \begin{bmatrix} 0 & i<j \\ \frac{1}{\varphi(y_i)} f_{i/1,\ldots,i-1}\left(x_{\mathrm{i}}\ /x_1, x_2, .., x_{i-1}\right) & i=j \\ \frac{1}{\varphi(y_j)}\ \frac{\partial F_{i/1,..i-1}(x_{\mathrm{i}}\ /x_1,x_2,..,x_{i-1})}{\partial x_i} & i>j \end{bmatrix}, \tag{2.37}$$

the $\mathbf{J}_{\mathbf{YX}}$ can be illustration with three variables as [11]

$$\mathbf{J}_{\mathbf{YX}} = \begin{bmatrix} \frac{f_1(x_1)}{\varphi(y_1)} & 0 & 0 \\ \frac{1}{\varphi(y_2)}\frac{\partial F_{2/1}(x_2/x_1)}{\partial x_1} & \frac{f_{2/1}(x_2/x_1)}{\varphi(y_2)} & 0 \\ \frac{1}{\varphi(y_3)}\frac{\partial F_{3/1,2}(x_3/x_1,x_2)}{\partial x_1} & \frac{1}{\varphi(y_3)}\frac{\partial F_{3/1,2}(x_3/x_1,x_2)}{\partial x_2} & \frac{f_{3/1,2}(x_3/x_1,x_2)}{\varphi(y_3)} \end{bmatrix} \tag{2.38}$$

where the conditional functions of the $\mathbf{J}_{\mathbf{YX}}$ are obtained according to the D-vine copula model in section (2.1.2.2).

After obtaining $\mathbf{J}_{\mathbf{YX}}$, the $\mathbf{J}_{\mathbf{XY}}$ is obtained as

$$\mathbf{J}_{\mathbf{XY}}\ = \mathbf{J}_{\mathbf{YX}}^{-1} \tag{2.39}$$

2.1.3.2.2. Algorithm of Structural reliability analysis

The algorithm of Structural reliability analysis to calculate the probability of structural failure as

(i) Definition of the performance function $textbfG(X)$.

(ii) transformation the correlated random variables from (X- space) into the independent standard (Y-space) according to Eqs.(2.29), (2.30) and (2.31).

(iii) Based on inverse transformation, can be obtain vector $\mathbf{X}$ corresponding to vector $\mathbf{Y}$ by Eqs.(2.32), (2.33) and (2.34) and then, substituting these equations into the performance function $\mathbf{G(X)}$ for each random variableX_i, thus, $\mathbf{G(X)}$ transformed into $g\left(\mathbf{Y}\right)$ of independent standard normal vector $\mathbf{Y}$:

$$\mathbf{G(X)} \xrightarrow{g(T^{-1}(\mathbf{X})} g(\mathbf{Y}) \tag{2.40}$$

(iv) Set $s=0$, where s is iteration.

(v) Select an initial point of vector $\mathbf{X}_s$, set $\mathbf{X}_s = \mu_{\mathbf{X}}$,where $\mu_{\mathbf{X}}$ is vector mean of original variables.

(vi) Compute points of vector $\mathbf{Y}_s$ in standard normal space, by Eq. (2.29), (2.30) and (2.31).

(vii) Compute value of Performance function $g(\mathbf{Y})$ at $\mathbf{Y}_s$.

(viii) Compute gradient vector $\nabla g\left(\mathbf{Y}\right)$ by Eq.(2.35) at $\mathbf{X}_s$ and $\mathbf{Y}_s$

(ix) Obtain most probable point MPP as:

$$\mathbf{Y}_{s+1} = \mathbf{Y}_s + \alpha\ d_s \tag{2.41}$$

where d_s is search direction, calculate as:

$$d_s = \frac{(\nabla g\left(\mathbf{Y}_s\ \right))^T \mathbf{Y}_s\ - g\left(\mathbf{Y}_s\right)}{(\nabla g\left(\mathbf{Y}_s\ \right))^T \nabla g\left(\mathbf{Y}_s\ \right)}\ \nabla g\left(\mathbf{Y}_s\ \right) - \mathbf{Y}_s \tag{2.42}$$

α is step size ,as

$$\alpha = \frac{1}{2}\left\|\mathbf{Y}_s\right\| + c\left|g\left(\mathbf{Y}_s\right)\right| \tag{2.43}$$

and

$$c = \frac{2\left\|\mathbf{Y}_s\right\|}{\left\|\nabla g\left(\mathbf{Y}_s\right)\right\|} + 10 \tag{2.44}$$

(x) Compute vector $\mathbf{X}_{s+1}$ corresponding to vector $\mathbf{Y}_{s+1}$.

(xi) If $\left\|\mathbf{X}_{s+1} - \mathbf{X}_s\right\| \leq \varepsilon$ where ε is the permissible error($\varepsilon = 10^{-6}$); then, go to step (xii). otherwise, $s = s+1$, go to step (v)

(xii) Calculated reliability index as :

$$\beta = \parallel \mathbf{Y}_{M+1} \parallel \tag{2.45}$$

and then probability of structural failure obtained as:

$$p_f = \Phi(-\beta) \tag{2.46}$$

(xiii) Stop

3. Case study

3.1. Introduction

This research aims to find out the effect of G.U.M mouthwash on cured and re-cured Visible Light Cured (VLC) composite filling material (Smile USA, shade A2) through the extent to which their surface hardness and dimensional geometric (thickness and diameter) are affected when using G.U.M mouthwash. The study established in the dental lab, Department of Prosthetic Dental Technology, College of Health and Medical Technology, Middle Technical University, Baghdad, Iraq

For the purpose of achieving the aim of the research, the structural reliability analysis was performed to obtain the probability of a structural failure p_f for the dental filling by using the analysis technique D-Vin copula referred to in Section (2).

3.2. Experiment description

To evaluate the effect of mouthwash on the light- cured composite filling material, this is through the extent to which the surface hardness and dimensional accuracy (thickness and diameter) are affected by the mouthwash.

Accordingly, the Nano hybrid light-cured composite resin dental filling material were prepared according to ISO/ASTM D2240 standardization [5], with the dimension of 12($\pm$0.02) mm in diameter and 3$(\pm 0.02) mm$ thickness, for a sample ($n = 50$). Each filling cured according to the manufacturer's instruction using the light-cure unit (Ivoclar-Vivadent, Germany) for 40 seconds on each filling surface side and kept it without finishing in distilled water for 24 hours before starting the treatment with mouthwash treatment. The G.U.M (Alcohol-free) mouthwash (Ivohealth, South Africa) was selected for this study. The prepared fillings that were kept in distilled water for 24h immersed in G.U.M mouthwash of 1ml for 2min/day for 4 weeks (28 days$\pm$2h), then the treated fillings re-cured for 40 seconds.

3.3. Analysis of the experiment data

In this part, the data obtained for the surface hardness and the accuracy of the geometric dimensions (thickness and diameter) of the filling are analyzed after conducting the experiment. To analysis the experiment data, represent the hardness and geometric dimensions of dental filling with the following random variables:

Diameter is X_1, Hardness is X_2 , Thickness is X_3.

The data is analyzed to learn the general features of the data which include descriptive statistics, and the type of distribution that the data follow

where the results of the analysis of sample (n=50) for three variables, that the three variables have a Weibull distribution with shape parameter α_i and scale parameter β_i , designed as

$X_i \sim Weibull(\alpha_i, \ \beta_i)$, $i = 1, 2, 3$, as shown in Table 1, which also shows the information of the random variables. In addition, the results of the analysis using the Pearson correlation coefficient showed that there are strong positive correlation between the variables as shown in Table 2.

Table 1: Probability distribution and information of the random variables to experience dental filling

Variables	***Distribution***	***Mean***	***Standard deviation***	***Skewness***	***Kurtosis***
X_1	***Weibull (152.5824,11.6386)***	***11.5966***	***0.08905***	***-0.427***	***-0.263***
X_2	***Weibull (21.5805, 91.6015)***	***89.3296***	***5.08127***	***-0.554***	***-0.289***
X_3	***Weibull (11.9043, 2.4480)***	***2.350***	***0.21719***	***-0.060***	***-0.349***

Table 2: The value of correlations coefficient ρ_{ij} between random variables

ρ_{ij}	X_1	X_2	X_3
X_1	***1***	***0.815***	***0.919***
X_2	***0.815***	***1***	***0.829***
X_3	***0.919***	***0.829***	***1***

3.4. Structural reliability analysis

In this section, the structural reliability is analyzed to obtain the probability of a structural failure p_f for the dental filling. In the structural reliability analysis, the performance function G($\mathbf{X}$) must first be determined for the basic design variables, in this research, the basic random variables (Diameter X_1, Hardness X_2, Thickness X_3) are formulated by using the following performance function

$$G(\mathbf{X}) = \sum_{i=1}^{3} X_i - \sum_{i=1}^{3} M_{X_i} \tag{3.1}$$

then, to obtain the probability of a structural failure p_f , the technique referred to in Section (2) were used, the Table (2) showed that there is a multidimensional correlation between the variables involved in the design studied, where that techniques work to remove the correlation statistically and standardized the space of variables, then the analysis performed. The computational work was carried out by using the MATLAB program.

The analysis results listed in Table (3) were obtained, and interpreted as follows:

Initially in D-vine copula technique, for this experiment a three-dimensional D-vine model of was built as a section (1.2.2.), since all variables follow a Weibull distribution, and after studying the types of copulas, the copula suitable is the Clayton for building the joint distributions, accordingly, for all the copulas required in construction the D-vine model the Clayton was adopted. The parameters of the copula functions were estimated and their values were ($\theta_{12} = 1.052$, $\theta_{13} = 1.0995$, $\theta_{13/2} = 0.9640$).

The analysis was carried out according to the algorithm in section (2.1.3.2.2), the results were that the reliability index is $\beta = 0.2744$ and the probability of structural failure is p_f =0.3919.

Table 3: The results of the analysis techniques

Technique	***Most probability point (MPP)*** $(y_1,\ y_2, y_3)$	***Reliability index*** β	***Probability of structural failure*** p_f
D-vine copula	***0.0278, -0.0323, -0.2711***	***0.2744***	***0.3919***

4. Conclusions

1. The concept of structural reliability analysis has become pervasive in all applied fields as long as it is included in the quality of the work presented, and therefore when applying structural reliability by using D-Vine copula technique in this research and calculating the probability of structural failure, it gave an indicator based on it, was the clarify the extent of the impact of dental filling with MGU mouthwash in laboratory experiments.

2. Within the limits of the current study, it can be concluded that mouthwashes have shown an effect on dental fillings according to the material from which they are made and the laboratory operating conditions through the indicator of the probability of structural failure, but despite this it is considered an acceptable percentage and it can be worked on clinically in the future and resume the research on that

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