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Maximizing the performance of the Iraqi Armed Forces and determining the optimal path for them using the dynamic programming

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Abstract

The Iraqi Ministry of Defense decided that there would be no escape for the terrorist gang members from the death which it proceeded to it with a false doctrine until it is eradicated from Iraq which can never be an incubator of terrorism, accordingly and with the capabilities available to the ministry, the ministry has developed two strategies: The first is allocating ten regiments from the Iraqi Special Operations Forces (ISOF) to maximize the performance of the Iraqi armed forces (IAF) in defending the homeland and maintaining its security and stability from terrorist gangs in four border regions in Iraq. The second is to reduce the arrival time of the ISOF to the battlefield by determining the optimal possible paths using an efficient scientific approach which is the dynamic programming (DP). The results of this study after solving a real-life problem proved that the proposed approach is an effective mathematical approach for taking a series of related decisions by finding maximization level of performance of ISOF and the shortest time for them to reach the battlefield.

Keywords: Allocating, Maximizing the performance, Optimal path, Reducing arrival time, Dynamic programming; DP.

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1. Introduction

The Iraqi Armed Forces (IAF) consist of the Iraqi Army, the Iraqi Air Force and the Iraqi Navy, the IAF has a particularly long and successful history. It was initially formed in the early 1920s. The

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Armed Forces are administered by the Ministry of Defense, the first units of the IAF were formed in 1921 during the British Mandate of Iraq, where the Musa al-Kadhim regiment was formed and the Armed Forces Command took its headquarters in Baghdad, followed by the formation of the Iraqi Air Force in 1931 and then the Iraqi Naval Force in 1937. The army census reached its peak with the end of the Iraqi war Iranian, to have a population of 1,000,000 individuals. Despite the great capabilities that these terrorist gangs enjoy, represented by the advanced equipment and weapons and the training they receive from some neighbouring countries. One of the important priorities of the Ministry of Defense is to defend Iraq and maintain its security and stability from terrorist gangs. The heroes of the armed forces are in high readiness and ready to confront this terrorist organization and rid the homeland of its evils as soon as possible. On this basis and within the limits of the available capabilities, the Ministry has sought assistance from experts to achieve the desired goals through the use of an effective quantitative approach, which is the dynamic programming approach.

Dynamic programming (DP) approach was improved by Richard Bellman in the 1950s and has found applications in several fields, from aerospace engineering to economics. Dynamic Programming (DP) is a quantitative approach that converts one large problem with several decision variables into a series of small problems with a few decision variables. Thus, a large problem that is difficult to solve can be transformed into a series of small problems, which can be solved easily. As for the meaning of the word "programming" in the dynamic programming approach, it indicates to the mathematical notion of choosing the best allocation of resources, while the word dynamic indicates to decisions taken in several stages.

Many studies addressed this topic, such as Nemhauser and Ullmann (1969) who developed a DP algorithm for optimal capital in allocating the topic of budget constraints. Love (1976) introduced the dynamic programming algorithm to allocate the location of m from the variable means related to n from the current approach that occurred on one path. The algorithm works simultaneously on-site customization. Mathematical experience indicates that relatively large problems may ideally be solved. Mahdavi et al. (2009) proposed a DP approach to obtain the fuzzy shortest chain in a graph with fuzzy distance for every edge using an appropriate ranking approach by using MATLAB. Mafakheri et al. (2011) suggested a two-phase twofold standard DP approach for two of the most standard mission in supply chain management that is the supplier is chosen and demand assignment. In the first phase, to treat several decision criteria in supplier position. In the second phase, supplier positions are fed into a demand assignment model that objects at maximizing an advantage function for the company in addition to minimizing the aggregate supply chain costs. A DP approach is designed to solve the suggested bi-objective model.

In China Yu (2016) used a DP approach to give an active result to decide on the treatment project investment. Additionally, a case study including the Laojuntang coal mine was executed on the investment problem in the processing project using the proposed model. The results showed that the suggested model is active and suitable for making environmental investment decisions at a perfect coal mine in terms of reducing the total losses.

Khalaf and Halim (2018) proposed the fuzzy DP approach to determine the fuzzy maximum flow for Imam Al-Kazim's (AS) visitors on Rajab 25th on the anniversary of his martyrdom, as the research problem emerged through a clear difference in the numbers of visitors during the same day and the clear increase in the numbers visitors during the period of the visit, which culminates in the days 22nd-24th of Rajab.

Mandal and Venkataraman (2019) improved a model which minimizes perishable inventories costs and hence maximizes the total profit. They suggested modulation of the conventional model by modifying the preferences of the product and hence its price over time. Jenkins et al. (2020) specified the high-quality DPR plans that develop the performance of United States Army MEDEVAC systems and finally raise the fight victim survivability rate. A deducted, unlimited-horizon MDP model of the MEDEVAC DPR problem is designed and solved via an approximate DP approach that uses a propped vector regression value function approximation scheme within an estimated plan iteration algorithmic framework. Eventually, this study notifies the improvement and application of future strategies, techniques, and procedures for military MEDEVAC procedures. Summers et al. (2020) developed the dynamic weapon aim assignment model as a Markov decision procedure and utilized a simulation-based, approximate DP approach to solve models based on a representative scenario. The target for efficient air defence is to find the firing policy for interceptor allocation to incoming missiles that minimize the forecasted entire harm to keep assets over a series of engagements.

Khalaf (2021) used a fuzzy DP method to allocate seven health centers to four of the villages in Baghdad, Iraq, which allows the greatest overall effectiveness of these health centers to be achieved. The results of this study prove after a real-life problem was solved that the proposed method is an effective mathematical model for making a series of related decisions, and it provides us with a systematic procedure to determine the optimal combination of decisions.

In this study, a DP is proposed to apply it in the military field for finding the maximization level of performance of the Iraqi Armed Forces and the shortest time for them to reach the battlefield.

2. Methods

2.1. Recursive nature of DP computation

Dynamic programming (DP) is the general approach to making a series of connected decisions optimally. DP determines the optimal solution to a multivariate problem by dividing the problem into stages, with each stage having a sub-problem that aims to find the optimal value for only one variable. The characteristic feature is due to dealing with only one variable and is much easier mathematically than dealing with all variables at the same time. The model consists of a set of consecutive equations that link the different stages of the original problem in a way that ensures that the best possible solution to the original problem ultimately includes all possible optimal solutions that were obtained when solving sub-problems of different stages. The main idea of DP is to decompose the problem into (more manageable) subproblems. Computations are then carried out recursively where the optimum solution of one subproblem is used as an input to the next subproblem. The optimum solution for the entire problem is at hand when the last subproblem is solved. The manner in which the recursive computations are carried out depends on how the original problem is decomposed. In particular, the subproblems are normally linked by common constraints. The feasibility of these common constraints is maintained at all iterations (Hillier and Lieberman, 2015; Taha, 2017).

2.2. Terminology and formulation of the DP model

The following terms need to be clarified as follows (Hillier and Lieberman, 2015; Taha, 2017):

N = Total number of stages when formulating the dynamic programming model.

n = Label for current stage ($n = 1, 2, ..., N$).

S_n= Current state for stage n, which represents the appropriate choice of "state of the system"

$c_i(x_i)$ = Measurement of contribution or performance.

x_n^* =Optimum decision for the current state S_n (note that s_n is given)

$f_n(s_n,\, x_n)$ = immediate contribution (stage n) + min or max future contribution (stages $n+1$ onward for backward recursion and stages $n-1$ for forward recursion downward)

Formulations for backward recursion as follow:

$$f_n(s_n, x_n) = c_n(x_n) + {f^*}_{n+1}(x_n) \tag{2.1}$$

$$f^*{}_n(s_n) = \max_{x_n=0,1,\ldots,s_n} \{c_n(x_n) + f^*{}_{n+1}(s_n - x_n)\} \tag{2.2}$$

Or

$$f^*{}_n(s_n) = \min_{x_n=0,1,\ldots,s_n} \{c_n(x_n) + f^*{}_{n+1}(s_n - x_n)\} \tag{2.3}$$

Formulations for forward recursion as follow:

$$f_n(s_n, x_n) = c_n(x_n) + f^*{}_{n-1}(x_n) \tag{2.4}$$

$$f^*{}_n(s_n) = \max_{x_n=0,1,\ldots,s_n} \{c_n(x_n) + f^*{}_{n-1}(s_n - x_n)\} \tag{2.5}$$

Or

$$f^*{}_n(s_n) = \min_{x_n=0,1,\ldots,s_n} \{c_n(x_n) + f^*{}_{n-1}(s_n - x_n)\} \tag{2.6}$$

Forward and backward recursion relationship for the last stage will always be of the form:

$$f^*{}_n(s_n) = \max_{x_n=0,1,\ldots,s_n} f_n(s_n, x_n) \tag{2.7}$$

Or

$$f^*{}_n(s_n) = \min_{x_n=0,1,\ldots,s_n} f_n(s_n, x_n) \tag{2.8}$$

3. A Real Practical Example

3.1. The first strategy: Maximizing the Performance of the Iraqi Armed Forces (IAF).

As a result of the exposure of four border areas in Iraq to organized attacks by terrorist gangs, the Iraqi Ministry of Defense decided to send ten regiments of the Iraqi Special Operations Forces (ISOF) to defend these areas, provided that at least one regiment be allocated to each of those four areas, the ministry aspires after the arrival of the required reinforcements to those areas are to be allocated in a way that achieves the maximum performance of the IAF to defend Iraq and maintain its security and stability from those terrorist gangs on the battlefield. Table 1 shows the points added to maximize the performance of the IAF after the arrival of the required reinforcements (regiments of ISOF) to those areas and according to the assessment of specialized military experts:

Table 1: Points added to maximize the performance of the IAF after the arrival regiments of the ISOF to those areas

Attacked areas	Points added to maximize the performance of the IAF after the arrival of the ISOF to those areas						
	Regiments						
	1	2	3	4	5	6	7
1	250	500	600	800	1000	1000	1000
2	200	700	900	1000	1000	1000	1000
3	400	600	800	1000	1000	1000	1000
4	100	200	300	400	5000	600	700

3.2. The second strategy: Determining the Shortest Path for the Regiments to Reach the Enemy at the Appropriate Time.

The time factor is important for deciding the fate of the battle, therefore the time for reinforcements from the ISOF to arrive on the battlefield at an appropriate time is considered one of the important factors for deciding the battle and victory over the enemy, figure 1, shows the possible paths among territories, the time required between one area and another and the precedence between them.

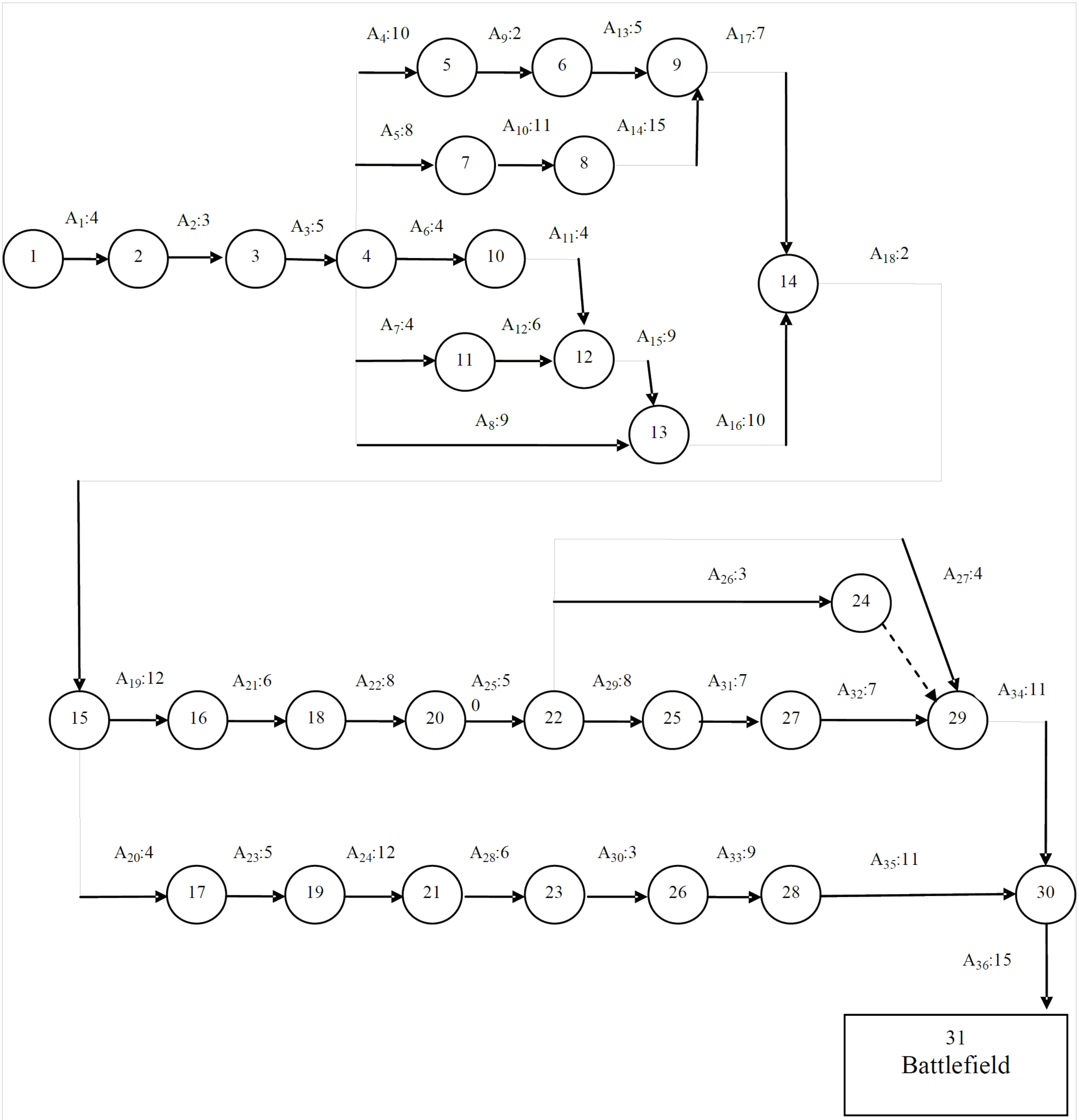

Figure 1: The possible paths among territories and the time required between one area and another and the precedence between them

3.2.1. Solution procedures for the first strategy

Equation (2.7) was applied to obtain the results of stage 4, while Eq.(2.1) and (2.2) were applied to obtain the results of the other stages.

Stage 4

Table 2: Points added to maximize the performance of the IAF and the optimum number of the ISOF regiments that can be allocated to stage 4.

s_4	$f^*{}_4\ (s_4)$	x_4
1	100	1
2	200	2
3	300	3
4	400	4
5	500	5
6	600	6
7	700	7

Stage 3

Table 3: Points added to maximize the performance of the IAF and the optimum number of the ISOF regiments that can be allocated to stage 3.

s_3 \ x_3	$f_3(s_3, x_3) = c_3(x_3) + f^*{}_4(s_3 - x_3)$							$f_3(s_3)$	x_3*
	1	2	3	4	5	6	7		
2	500							500	1
7	600	700						700	2
4	700	800	900					900	3
5	800	900	1000	1100				1100	4
6	900	1000	1100	1200	1100			1200	4
7	1000	1100	1200	1300	1200	1100		1300	4
8	1100	1200	1300	1400	1300	1200	1100	1400	4

Stage 2

Table 4: Points added to maximize the performance of the IAF and the optimum number of the ISOF regiments that can be allocated to stage 2.

s_2 \ x_2	$f_2(s_2, x_2) = c_2(x_2) + f^*{}_2(s_2 - x_2)$							$f_2(s_2)$	x_2*
	1	2	3	4	5	6	7		
3	700							700	1
4	900	1200						1200	2
5	1100	1400	1400					1400	2 or 3
6	1300	1600	1600	1500				1600	2 or 3
7	1400	1800	1800	1700	1500			1800	2 or 3
8	1500	1900	2000	1900	1700	1500		2000	3
9	1600	2000	2100	2100	1900	1700	1500	2100	3 or 4

Stage 1

Table 5: Points added to maximize the performance of the IAF and the optimum number of the ISOF regiments that can be allocated to stage 1.

s_1 \ x_1	$f_1(s_1, x_1) = c_1(x_1) + f^*{}_1(s_1 - x_1)$							$f_1(s_1)$	x_1*
	1	2	3	4	5	6	7		
10	2350	2500	2400	2400	2400	2200	1700	2500	2

3.2.2. Results of the first strategy

The DP approach was applied to find the optimal solution for the first strategy of the research problem, which is to achieve the maximum performance of the Iraqi Armed Forces on the battlefield. After using this approach to find the optimal solution, it was found that allocating the ten regiments of the Iraqi Special Operations Forces to the military units located in the four border areas, as shown in tables (3-6) gave the maximum level of performance (2500) points.

3.3. Solution procedures for the second strategy

Equation (2.8) was applied to obtain the results of stage 1, while Eq.s (2.4) and (2.5) were applied to obtain the results of the other stages.

Stage 1

Table 6: The path between (territory 1) and (territory 2) and the shortest time between them for stage 1

s_1 \ x_1	$f^*{}_n(s_n) = \min\limits_{x_n=0,1,\ldots,s_n} f_n(s_n, x_n)$	$f_1^*(s_1)$	x_1*
	2		
1	4	4	2

The shortest time from territory 1 to territory 2 is 4 hours.

Stage 2

Table 7: The path between (territory 2) and (territory 3) and the shortest time between them for stage 2.

S_2 \ X_2	$f_2(s_2, x_2) = c_2(x_2) + f^*{}_1(x_2)$	$F_2^*(s_2)$	x_2*
	3		
2	3+4=7	7	3

The shortest time from territory 2 to territory 3 is 7 hours

Stage 3

Table 8: The path between (territory 3) and (territory 4) and the shortest time between them for stage 3.

X_3 \ S_3	$f_3(s_3, x_3) = c_3(x_3) + f^*{}_2(x_3)$	$F_3^*(s_3)$	$x_3{}^*$
	4		
3	5+7=12	12	4

The shortest time from territory 3 to territory 4 is 12 hours

Stage 4

Table 9: The possible paths between (territory 4) and (territories 5, 7, 10, 11 and 13) and the shortest time to reach them.

X_4 \ S_4	$f_4(s_4, x_4) = c_4(x_4) + f^*{}_3(x_4)$					$F_4^*(s_4)$	$x_4{}^*$
	5	7	10	11	13		
4	10+12=22	8+120	4+12=16	4+12=16	9+12=21	16	10 or 11

The shortest time from territory 4 to territory 10 or 11 is 16 hours

Stage 5

Table 10: The possible paths between (territories 5, 7, 10 and 11) and (territories 6, 8 and 12) and the shortest time to reach them.

X_5 \ S_5	$f_5(s_5, x_5) = c_5(x_5) + f^*{}_5(x_5)$			$F_5^*(s_5)$	$x_5{}^*$
	6	8	12		
5	2+22=24	—	—	24	6
7	—	11+20=31	—	31	8
10	—	—	4+16=20	20	12
11			6+16=22	22	12

The shortest time from territory 10 to territory 12 is 20 hours

Stage 6

Table 11: The possible paths between (territories 6, 8, 12 and 4) and (territories 9 and 13) and the shortest time to reach them.

X_6 \ S_6	$f_6(s_6, x_6) = c_6(x_6) + f^*{}_6(x_5)$		$F_6^*(s_6)$	$x_6{}^*$
	9	13		
6	5+24=29	—	29	9
8	15+31=46	—	46	9
12	—	9+20=29	29	13
4	—	9+12=21	21	13

The shortest time from territory 4 to territory 13 is 21 hours

Stage 7

Table 12: The possible paths between (territories 9 and 13) and (territory 14) and the shortest time to reach it.

S_7 \ X_7	$f_7(s_7, x_7) = c_7(x_7) + f^*{}_7(x_6)$	$F_7^*(s_7)$	$x_7{}^*$
	14		
9	7+29=36	36	14
13	10+21=31	31	14

The shortest time from territories 9 and 13 to territory 14 is 31 hours

Stage 8

Table 13: The possible paths between (territory 14) and (territory 15) and the shortest time to reach it.

S_8 \ X_8	$f_8(s_8, x_8) = c_8(x_8) + f^*{}_8(x_7)$	$F_8^*(s_8)$	$x_8{}^*$
	15		
14	2+31=33	33	15

The Shortest time from territory 14 to territory 15 is 33 hours

Stage 9

Table 14: The possible paths between (territory 15) and (territories 16 and 17) and the shortest time to reach it.

S_9 \ X_9	$f_9(s_9, x_9) = c_9(x_9) + f^*{}_9(x_8)$		$F_9^*(s_9)$	$x_9{}^*$
	16	17		
15	12+33=45		45	16
15		4+33=37	37	17

The shortest time from territory 15 to territory 16 or 17 is 37 hours

Stage 10

Table 15: The possible paths between (territories 16 and 17) and (territories 18 and 19) and the shortest time to reach them.

S_{10} \ X_{10}	$f_{10}(s_{10}, x_{10}) = c_{10}(x_{10}) + f^*{}_{10}(x_9)$		$F_{10}^*(s_{10})$	x_{10}^*
	18	19		
16	6+45=51	—	51	18
17	—	5+37=42	42	19

The Shortest time from territory 17 to territory 19 is 42 hours

Stage 11

Table 16: The possible paths between (territories 18 and 19) and (territories 20 and 21) and the shortest time to reach them.

X_{11} \ S_{11}	$f_{11}(s_{11}, x_{11}) = c_{11}(x_{11}) + f^*{}_{11}(x_{10})$		$F^*_{11}(s_{11})$	x^*_{11}
	20	21		
18	8+51=59	—	59	20
19	—	12+42=54	54	21

Shortest time from territory 19 to territory 21 is 54 hours

Stage 12

Table 17: The possible paths between (territories 20 and 21) and (territories 22 and 23) and the shortest time to reach them.

X_{12} \ S_{12}	$f_{12}(s_{12}, x_{12}) = c_{12}(x_{12}) + f^*{}_{12}(x_{11})$		$F^*_{12}(s_{12})$	x^*_{12}
	22	23		
20	5+59=64	—	64	22
21	—	6+54=60	60	23

The shortest time from territory 21 to territory 23 is 60 hours

Stage 13

Table 18: The possible paths between (territories 22 and 23) and (territories 24, 25, 26 and 29) and the shortest time to reach them.

X_{13} \ S_{13}	$f_{13}(s_{13}, x_{13}) = c_{13}(x_{13}) + f^*{}_{13}(x_{12})$				$F^*_{13}(s_{13})$	x^*_{13}
	24	25	26	29		
22	3+64=67	8+64=72		4+64=68	67	24
23	—	—	3+60=63		63	25

The shortest time from territory 23 to territory 26 is 63 hours

Stage 14

Table 19: The possible paths between (territories 25 and 26) and (territories 27 and 28) and the shortest time to reach them.

X_{14} \ S_{14}	$f_{14}(s_{14}, x_{14}) = c_{14}(x_{14}) + f^*{}_{14}(x_{13})$		$F^*_{14}(s_{14})$	x^*_{14}
	27	28		
25	7+72=79	—	79	27
26		9+63=72	72	28

The shortest time from territory 26 to territory 28 is 72 hours

Stage 15

Table 20: The possible paths between (territories 22, 24, 27, 28 and 29) and (territories 29 and 30) and the shortest time to reach them.

X_{15} / S_{15}	$f_{15}(s_{15}, x_{15}) = c_{15}(x_{15}) + f^*{}_{15}(x_{14})$		$F^*_{15}(s_{15})$	x^*_{15}
	29	30		
22	4+64=68		68	29
24	0+67=67		67	29
27	7+79=86	—	86	29
28	—	11+72=83	83	30
29	—	11+67=78	78	30

The shortest time from territory 24 to territory 29 is 67 hours

Stage 16

Table 21: The possible paths between (territory 30) and (territory 31) and the shortest time to reach it.

X_{16} / S_{16}	$f_{16}(s_{16}, x_{16}) = c_{16}(x_{16}) + f^*{}_{16}(x_{15})$	$F^*_{16}(s_{16})$	x^*_{16}
	31		
30	15+78=93	93	31

The shortest time from territory 30 to territory 31 is 93 hours.

3.3.1. Results of the second strategy

Maximizing the performance of the Iraqi Armed Forces is related to the arrival of the Iraqi Special Operations Forces regiments in the shortest time to the battlefield, and the shortest time for the arrival of the Iraqi Special Operations Forces regiments to the battlefield is 93 hours, and it is the optimal path that starts from the territory (1) and ends in the territory (31) and as shown in the tables (7-22).

4. Discussion of results

The DP approach was used to assist the decision-maker in finding optimal solutions for two important strategies in deciding the battle and achieving victory over the enemy. The first strategy is to achieve the maximum performance of the Iraqi Armed Forces on the battlefield to defend the land of Iraq and preserve its security and stability from those terrorist gangs. After applying the dynamic programming approach, the optimal solution was to allocate two regiments of the Iraqi Special Operations Forces for the first territory (X_1*= 2), three regiments for the second territory (X_2*= 3), four regiments for the third territory (X_3*= 4) and one regiment for the fourth territory (X_4*= 1), As the maximum level of performance of the Iraqi Armed Forces was (2500) points, as shown in table 6, and this is what the ministry aspires to after the arrival of the required reinforcements of the Iraqi Special Operations Forces. As for the second strategy, which focused on the time factor because of its great importance in determining the fate of the battle and an important element of victory, the goal of this strategy is to reduce the arrival time of the Iraqi Special Operations Forces to the battlefield where the dynamic programming approach was used to determine the optimal path that guarantees the arrival of these regiments to the battlefield in the shortest time, whereas the arrival time was 93 hours and figure 2 shows the optimal path for the arrival of the regiments in the shortest time.

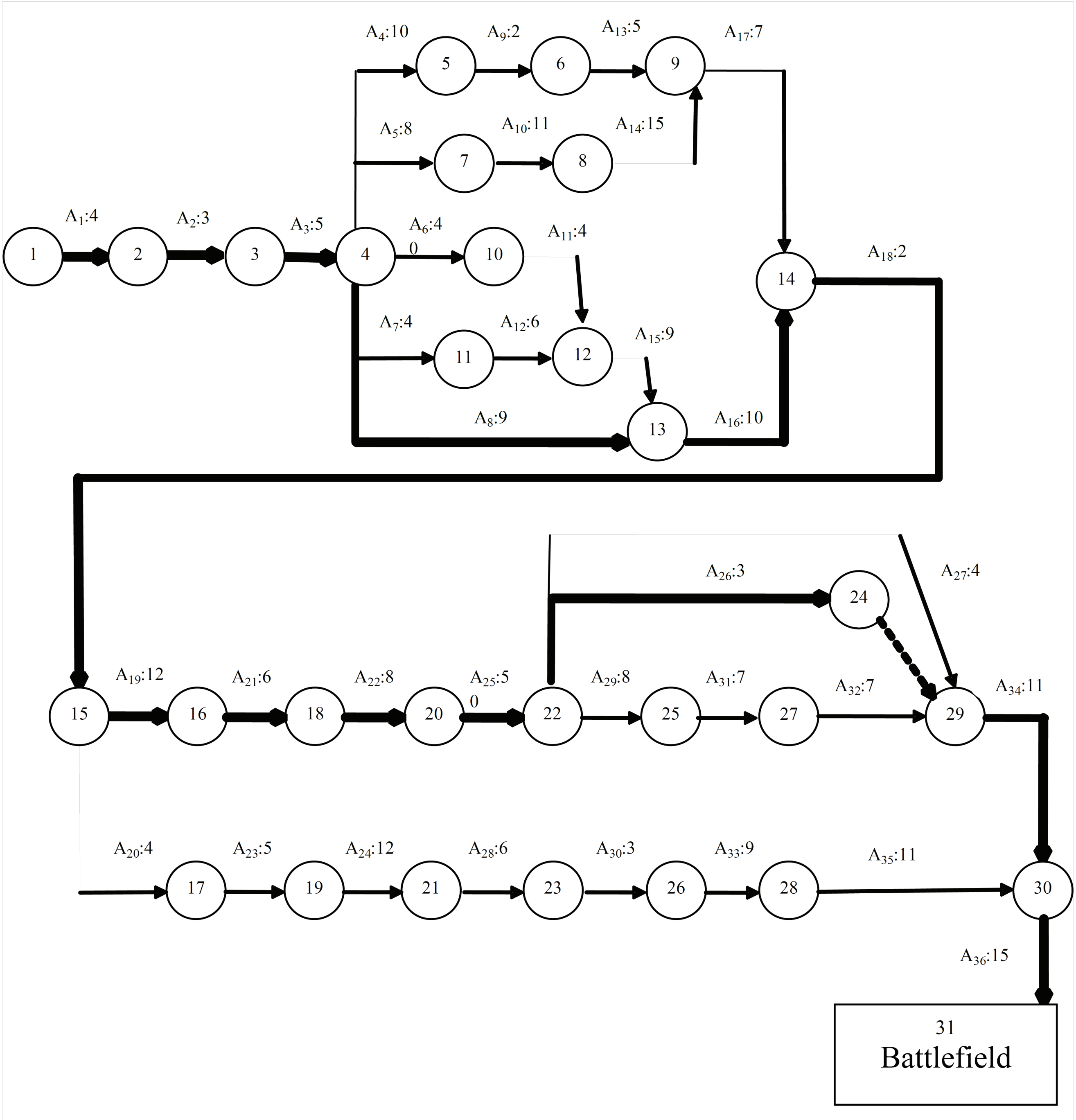

Figure 2: The optimal path for the arrival of the Iraqi Special Operations Forces regiments to the battlefield in the shortest possible time.

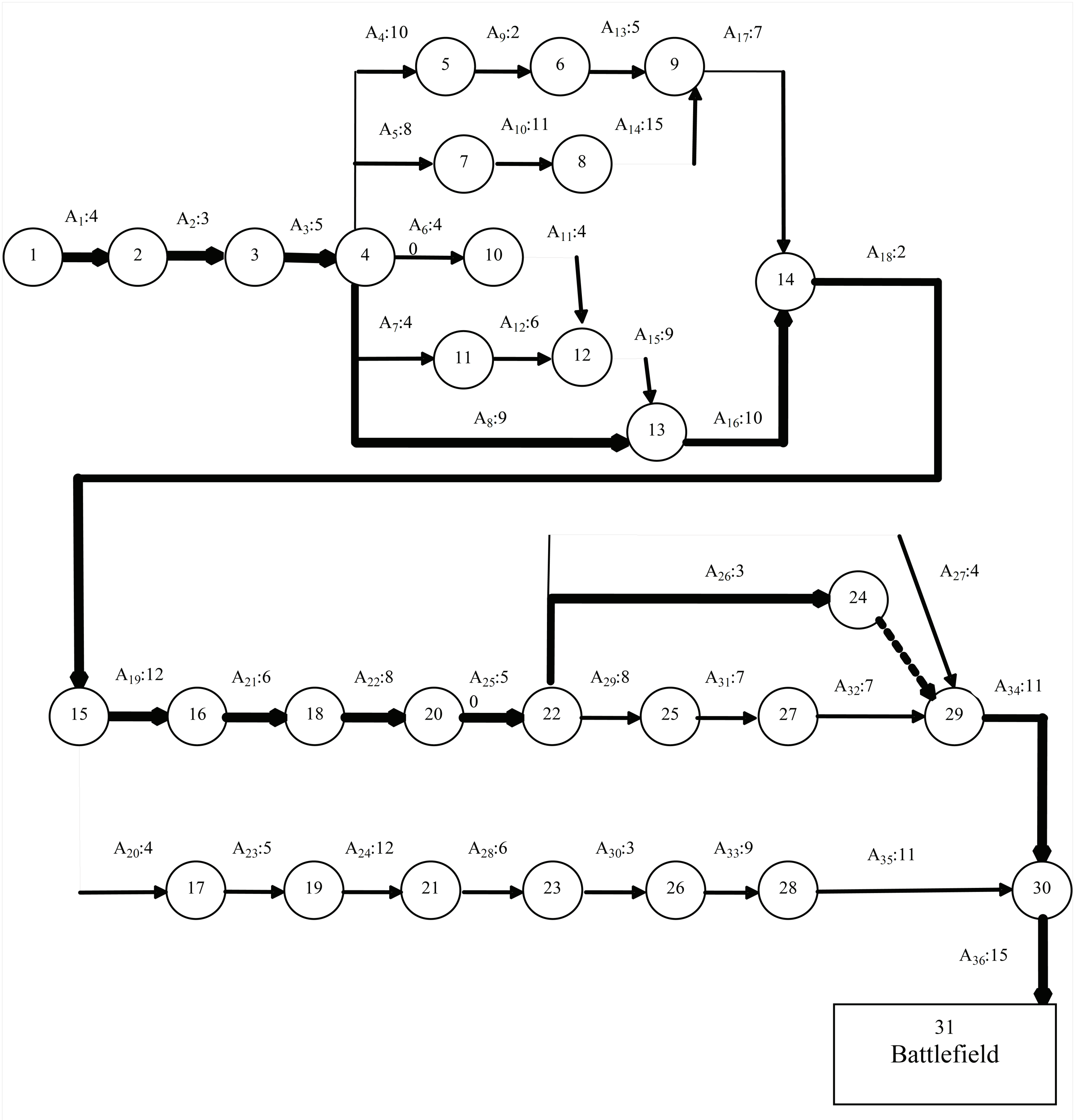

Figure 3: The optimal path for the arrival of the Iraqi Special Operations Forces regiments to the battlefield in the shortest possible time.

5. Conclusions

1. After solving a real-life problem, the results of this study proved that the proposed approach (dynamic programming) is an effective mathematical model for making a series of related decisions.

2. DP is a helpful mathematical approach for creating a series of interrelated decisions. It offers a systematic process for finding the optimum combination of decisions.
3. DP is a common type of approach to problem-solving and the specific equations used must be developed to appropriate each state as is evident from the formulation of mathematical equations for the first and second strategies of the research problem.
4. The use of the DP approach helped the decision-maker in implementing the first strategy, which is to maximize the performance of the Iraqi Armed Forces through the arrival of the required reinforcements of the Iraqi Special Operations Forces to the battlefield to maintain the security and stability of the Iraqi's territories from the terrorist gangs and that was through finding the optimal allocation of the ten Iraqi Special Operations Forces regiments in four border areas, it also helped him in implementing the second strategy which is to reduce the time for the Iraqi Special Operations Forces regiments of to arrive on the battlefield by choosing the optimal path.

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