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# Evaluation of longitudinal wave velocity and elasticity modulus of concrete at different ages considering mixing ratios

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**Abstract**

Concrete is a material that the mechanical properties of which change over time. This change is due to chemical reactions within the concrete known as hydration. One of these properties is the modulus of elasticity and longitudinal wave velocity, which has a direct relation with the concrete age and its setting. Just after the materials mixing, the concrete setting is fast, and over time its rate decreases. Here, a series of cubic concrete specimens are prepared and changing in longitudinal wave velocity and modulus of elasticity in different ages is monitored during the process of curing and a relationship has been presented. Materials specifications impact and concrete mixing ratios, including the water to cement ratio and fine to coarse aggregates ratio is studied. Ultrasonic wave velocity has been increased faster at early ages of specimens where the concrete setting process is fast and in last days, rates of increasing the longitudinal wave velocity decreases. An increase in the water to cement ratio leads to increases in the longitudinal wave velocity over time. The empirical equations have been formulated as logarithmic curves. These empirical equations have been developed and a model with more efficiency and precision has been presented. These empirical equations can be used in the analytical and numerical analysis of structures. These models can be used to determine the loading time of concrete structures and to predicting their other physical and mechanical properties, such as strength and stiffness.

*Keywords:* Concrete setting, Concrete age, Non-Destructive Test, Ultrasound waves velocity, Modulus of elasticity.
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## 1. Introduction

### 1.1. Ultrasonic wave velocity

By discovery of the piezoelectric, producing and measuring ultrasound waves have become possible. This discovery is made in 1880 by the Curie brothers. They announced that applying mechanical pressure on small sheets of some crystals causes an electrical Voltage. After a year, reverse piezoelectric phenomenon is described by Lippman. This means that by applying electrical potential shapes of the mentioned crystals is changed. He found that these crystals could fluctuate, or conversely by applying the proper electrical potential and appropriate electrical signals can be obtained by the crystal oscillation. Piezoelectric phenomenon is used to detect ultrasonic waves, and reverse piezoelectric phenomenon is used to generate it(IAEA).

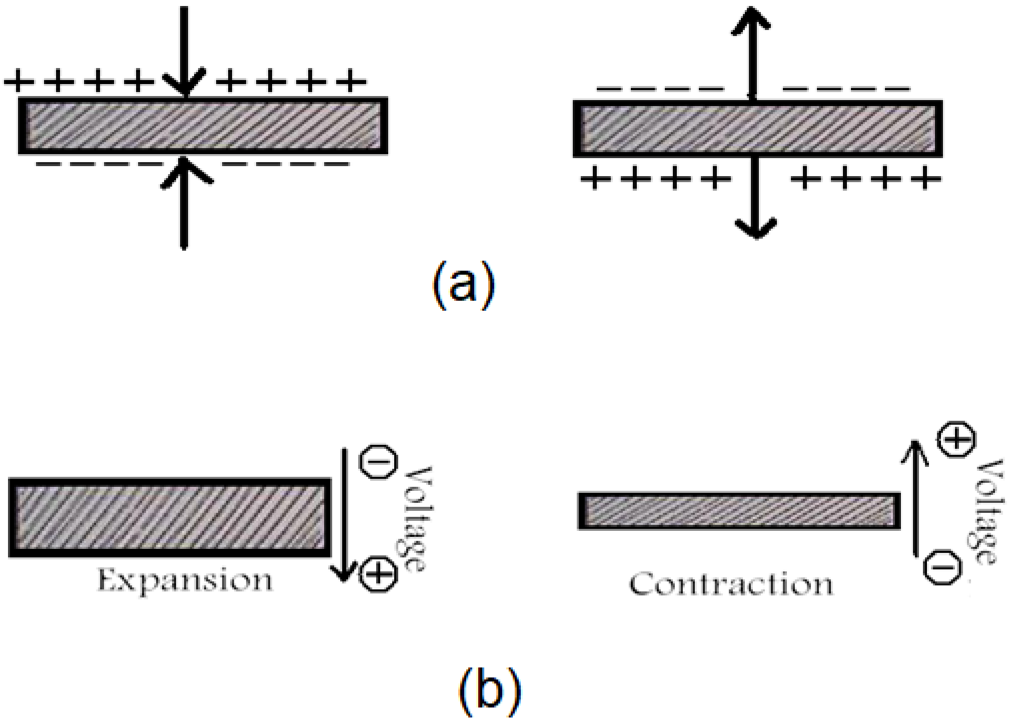


Figure 1: (a) piezoelectric phenomenon, (b) inverse piezoelectric phenomenon

For over 70 years, ultrasound wave velocity technique has been used for assessing concrete (Qasrawi and Marie 2003). The ultrasonic wave velocity technique based on stress wave propagation that measures the transmission time of an ultrasound wave from a given length. The ultrasonic wave is generated by a piezoelectric transducer and after passing through the specified length, it is detected by another piezoelectric transducer. The transmission time between transducers depends on the physical properties of the wave propagation environment. In this regard, many international committees have developed some codes and standards for evaluation of concrete by ultrasonic wave velocity technique(ASTM 2000; BS 1881).

### 1.2. Hypotheses and theory

The constituent materials of concrete are very diverse, and its strength estimation is a difficult and complex problem. Estimation of concrete properties is required in addition to practical and professional issues as well as in physical and numerical modeling(Sabagh and Ghalandarzadeh 2020a, b, c). The main components of concrete are hydrated cement paste, aggregates, water and air. Hydrated cement paste is a complex material consisting of several phases. The aggregates are composite and porous materials that are very different from the surrounding cement paste. The relationship between cement pastes and aggregates is also a complex issue. In this study, concrete is considered as a composite and homogeneous material. In an elastic medium, the longitudinal wave velocity is a function of dynamic elastic modulus ($E_d$), density ($\rho$) and Poisson ratio ($\nu$), and its relationship is expressed as equation (1.1) (Philleo 1955).

$$V_p = \sqrt{\frac{E_d(1-\nu)}{(1+\nu)(1-2\nu)}} \tag{1.1}$$

In analyzing and designing a structure, Strength and Stiffness are the most important properties of materials. Detecting the strength and stiffness of concrete is essential in inspection, evaluating and retrofitting of existing structures. Concrete in different parts of a structure may have different strengths. Concrete strength is important in selection of retrofitting method and effects on structural behavior before and after the retrofitting. Many attempts have been made to obtain the physical and mechanical properties of concrete, in particular strength based on equation (1.1) (Gudra and Stawiski 2000; Krautkrämer and Krautkrämer 2013; Lane 1998; Malhotra and Mehta 1994; Malhotra 1984; Malhotra and Carino 1991; Popovics et al. 1999; Popovics 2001; Popovics et al. 2000; Sabagh et al. 2019).

In UPV a longitudinal wave is made by transmitting transducer and is sent through the concrete block and receives by receiving transducer. The obtained responses were used to develop statistical models, which were then utilized in the optimization of mix design with the help of analysis of variance. Concrete properties such as compressive strength and modulus of elasticity are important in the analysis, design, inspection and retrofitting of concrete structures. These properties change over time as the concrete hardens. Concrete setting time is very important in the construction of a concrete structure, i.e., the appropriate time to remove the framework or to construct a new story, depends on the setting of the concrete. In order to use a concrete structure, concrete setting with time should be considered and start using the structure at an appropriate time.

Just after mixing the concrete, wave velocity is controlled by the liquid phase. In the first hours of hydration, the air contained in the concrete influences the wave velocity. During this time, slight increase is observed at the compressional wave velocity. Most of the previous research have examined the changes in the wave velocity and the modulus of elasticity in the first few hours of concrete setting (Boumiz et al. 1996; Keating et al. 1989; Sayers and Dahlin 1993; Sayers and Grenfell 1993; Trtnik and Gams 2015; Zhu et al. 2011). Here, the longitudinal wave velocity is considered as stiffness criterion of concrete and variations of longitudinal wave velocity are monitored over a period of two months. Also, the effect of mixing ratios on these variations is evaluated.

## 2. Experimental activities

### *2.1. Specimens preparation*

For experimental studies, a series of 42 cubic specimens (10cm x 10cm x 10cm) are prepared. According to previous studies, the water to cement ratio is the most important factor affecting mechanical properties of concrete, including stiffness and strength (Felekoğlu et al. 2007; Popovics 1990; Sahmaran et al. 2007; Sahu et al. 2004). Concrete water-cement ratio and concrete strength have an inverse relationship. In these specimens, water to cement ratio is 0.3, 0.4, 0.5, 0.6, 0.7 and 0.8 and the ratio of fine aggregates to coarse aggregates in different specimens are equal to $\frac{0}{1}$, $\frac{1}{5}$, $\frac{1}{2}$,

$\frac{1}{1}$, $\frac{2}{1}$, $\frac{5}{1}$ and $\frac{1}{0}$. The molds is removed after 24 hours and then, immersed in a water pond. During 60 days the ultrasound wave transmission time is measured and according to the dimensions of the specimens, the ultrasound wave velocity is calculated at each test.

Figure 2: Preparation of specimens for ultrasound wave velocity monitoring test

*2.2. Testing procedure*

An ultrasonic test device (Concrete Tester) made by Toni Thechnik with precision of one microsecond is used to measure the wave transmission time and calculate longitudinal wave velocity (Figure 3). At different ages of concrete specimens, ultrasonic transducers is placed on specimens opposite sides and wave transmission time is measured. Petroleum jelly is used as an intermediate medium for better performance of the ultrasonic test device.

The ultrasonic velocity is monitored continuously in specimens that are under curing. On a daily basis, the ultrasonic wave transmission time in every specimen is measured in three directions. The ultrasonic wave velocity is obtained by dividing specimen dimensions over transmission time as equation (2.1).

$$V = \frac{L}{T} \tag{2.1}$$

Where $L$ is the specimen dimension, $T$ is the wave transmission time and $V$ is the wave velocity.

## 3. Results

Microsoft Excel and IBM SPSS Statistics are used to analyze the results of the tests. wave velocity at different ages of the concrete for different water to cement and different sand to gravel ratios are


Figure 3: Ultrasonic test device

shown in Figures 4 to 10. Based on these diagrams the ultrasonic wave velocity is increased faster at early ages of specimens where concrete setting process is fast and in last days, rates of increasing the longitudinal wave velocity decreases. In all aggregate ratios, by increasing the water to cement ratio, the longitudinal wave velocity increases over time.

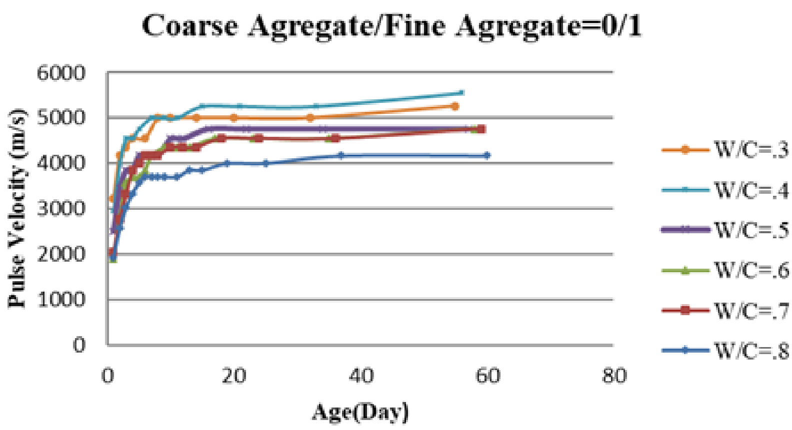


Figure 4: Ultrasound wave velocity versus concrete age in specimens with a coarse aggregate to fine aggregates ratio equal to $\frac{0}{1}$

### *3.1. Correlation analysis of experimental parameters*

Based on Figures 4 to 10, there exists a strong relationship between the ultrasonic wave velocity and the concrete age. Determination of the relationship between the ultrasonic wave velocity at different ages and the water to cement ratio and the gradation of the aggregate is possible. Modulus of elasticity variations with the age of the concrete is examined considering the relationship between the modulus of elasticity and the longitudinal wave velocity. In addition, the ratio of ultrasonic wave velocity to ultrasonic wave velocity of concrete at 28 days and the ratio of modulus of elasticity to

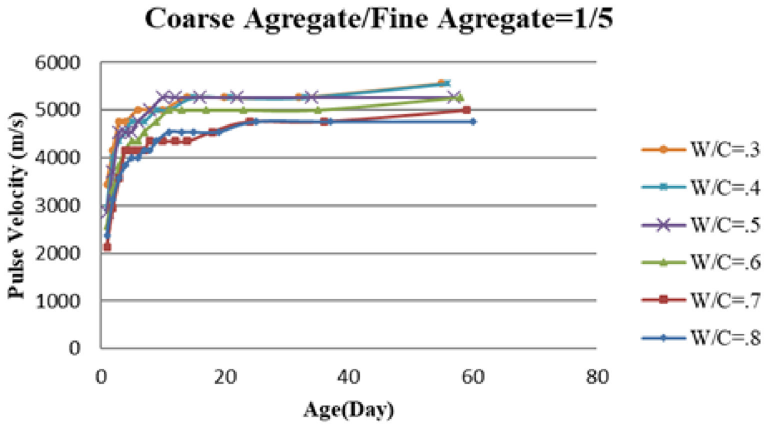


Figure 5: Ultrasound wave velocity versus concrete age in specimens with a coarse aggregate to fine aggregates ratio equal to $\frac{1}{5}$

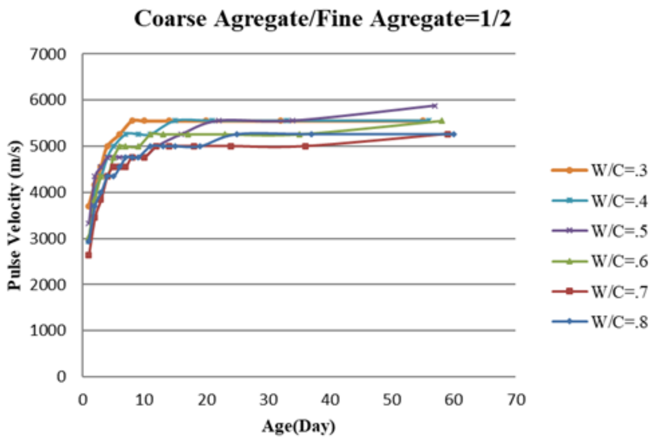


Figure 6: Ultrasound wave velocity versus concrete age in specimens with a coarse aggregate to fine aggregates ratio equal to $\frac{1}{2}$

modulus of elasticity at 28 days have been defined respectively as the relative velocity and relative elasticity modulus. In order to study their relationship with different parameters and significance of this relationship, bivariate correlation analysis based on Pearson correlation coefficient by SPSS software is performed (Pallant 2010). Correlation of longitudinal wave velocity, elasticity modulus, relative velocity and relative elasticity modulus with different parameters including concrete ages, water to cement ratio, and aggregate grading are shown in Tables 1 to 4.

In Tables 1 to 4, the parameters with the significant relationship are marked with **; i.e., the ultrasonic wave velocity is significantly correlated with all the three parameters. This applies to the modulus of elasticity, but the relative wave velocity and the relative elasticity modulus have a significant relationship only with the concrete age.

### *3.2. A constitutive model for ultrasonic wave velocity*

Based on the results of the test, the ultrasonic wave velocity is significantly correlated with all three parameters including concrete age, water to cement ratio, and sand to gravel ratio. First,

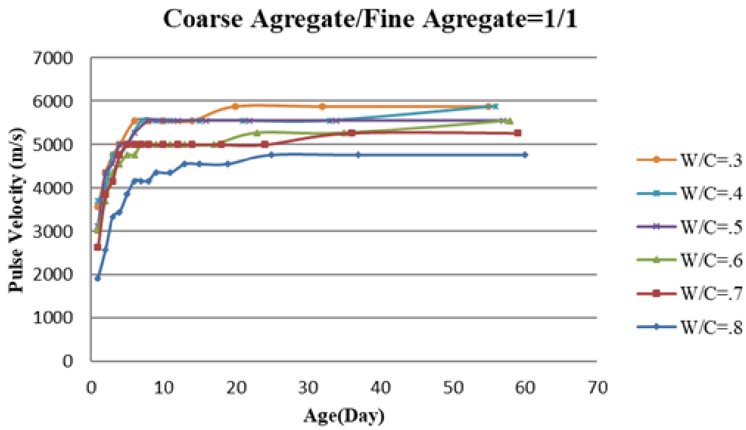


Figure 7: Ultrasound wave velocity versus concrete age in specimens with a coarse aggregate to fine aggregates ratio equal to $\frac{1}{1}$

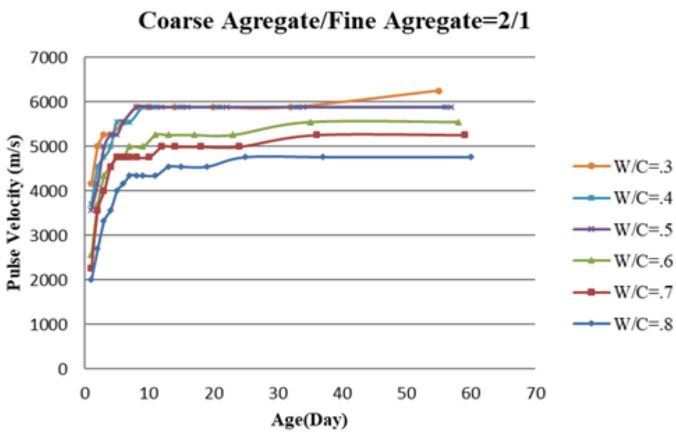


Figure 8: Ultrasound wave velocity versus concrete age in specimens with a coarse aggregate to fine aggregates ratio equal to $\frac{2}{5}$

regardless of the water to cement ratio and aggregates gradation, a correlation between ultrasonic wave velocity and concrete age is obtained by fitting a curve with the minimum residual sum of squares. Thus, by examining different diagrams, it is concluded that a logarithmic relationship can describe this behavior well. This relation is defined as equation (3.1), where $V$ is the ultrasonic wave velocity in $m/s$ and $t$ is the concrete age in day, and $a_1$ and $a_2$ are unknown coefficients which are obtained by minimizing the residual sum of squares. Table 5 is a part of the SPSS output that shows the values obtained for $a_1$ and $a_2$. Variance analysis results are shown in Table 6.

$$V = a_1 ln(t) + a_2 \tag{3.1}$$

As you can see in the table 5, the values of constants are $a_1 = 566.650$ and $a_2 = 3450.836$. Therefore, the relation of the longitudinal wave velocity with the concrete age, regardless of materials mixing ratios is as equation (3.2):

$$V = 566.650 ln(t) + 3450.836, R^2 = 0.513 \tag{3.2}$$

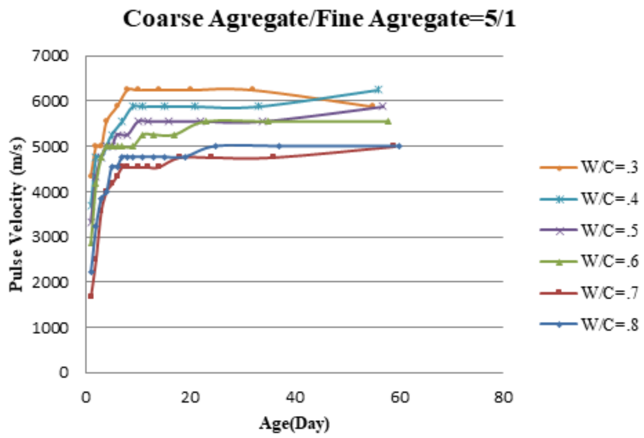


Figure 9: Ultrasound wave velocity versus concrete age in specimens with a coarse aggregate to fine aggregates ratio equal to $\frac{5}{1}$

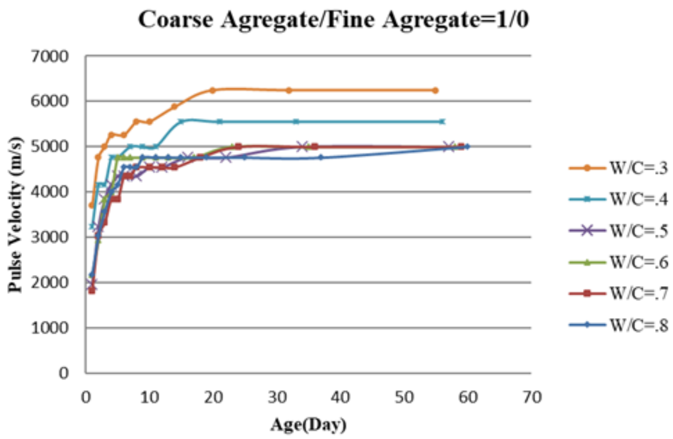


Figure 10: Ultrasound wave velocity versus concrete age in specimens with a coarse aggregate to fine aggregates ratio equal to $\frac{1}{0}$

Where V is the ultrasonic wave velocity in m/s and t is the concrete age in day. Matching of the estimated velocity obtained from equation (3.2) with the measured value in the experiments is shown in Fig. 11.

### 3.3. *A constitutive model for ultrasonic wave velocity considering the concrete mixing ratios*

In another model for the behavior of concrete, we tried to involve other factors, such as water to cement ratio and grading. In this model, the effect of concrete age was also considered as logarithmic. By examining other parameters including water to cement ratio and sand to gravel ratio, we concluded that considering their relationship with the longitudinal wave velocity has acceptable linear accuracy. Therefore, this model was defined as equation (3.3), where V is the velocity of the ultrasonic wave in m/s and t is the concrete age in days, and $(\frac{w}{c})$ is the ratio of water to cement and $(\frac{S}{G})$ is the weight ratio of sand to gravel, and $a_1$, $a_2$, $a_3$ and $a_4$ are fixed coefficients obtained by examining the behavior of the specimens and minimizing the residual sum of squares. Table 7 is a part of the SPSS output that shows the values for $a_1$, $a_2$, $a_3$ and $a_4$. Table 8 also shows the variance analysis results

Table 1: Correlation analysis of longitudinal wave velocity with concrete age, water to cement ratio and sand to gravel ratio

| | | **V** | **Age** | $\frac{w}{c}$ | $\frac{S}{G}$ |
|---|---|---|---|---|---|
| **V** | Pearson Correlation | 1 | 0.488** | -0.428** | -0.264** |
| | Sig. (2-tailed) | | 0.000 | 0.000 | 0.000 |
| **Age** | Pearson Correlation | 0.488** | 1 | 0.000 | -0.015 |
| | Sig. (2-tailed) | 0.000 | | 0.992 | 0.719 |
| $\frac{w}{c}$ | Pearson Correlation | -0.428** | 0.000 | 1 | -0.004 |
| | Sig. (2-tailed) | 0.000 | 0.992 | | 0.927 |
| $\frac{w}{c}$ | Pearson Correlation | -0.264** | -0.015 | -0.004 | 1 |
| | Sig. (2-tailed) | 0.000 | 0.719 | 0.927 | |

**. Correlation is significant at the 0.01 level (2-tailed).

Table 2: Correlation analysis of elasticity modulus with concrete age, water to cement ratio and sand to gravel ratio

| | | **E** | **Age** | $\frac{w}{c}$ | $\frac{S}{G}$ |
|---|---|---|---|---|---|
| **E** | Pearson Correlation | 1 | 0.478** | -0.485** | -0.292** |
| | Sig. (2-tailed) | | 0.000 | 0.000 | 0.000 |
| **Age** | Pearson Correlation | 0.478** | 1 | 0.000 | -0.015 |
| | Sig. (2-tailed) | 0.000 | | 0.992 | 0.719 |
| $\frac{w}{c}$ | Pearson Correlation | -0.485** | 0.000 | 1 | -0.004 |
| | Sig. (2-tailed) | 0.000 | 0.992 | | 0.927 |
| $\frac{w}{c}$ | Pearson Correlation | -0.292** | -0.015 | -0.004 | 1 |
| | Sig. (2-tailed) | 0.000 | 0.719 | 0.927 | |

**. Correlation is significant at the 0.01 level (2-tailed).

Table 3: Correlation analysis of relative wave velocity with concrete age, water to cement ratio and sand to gravel ratio

| | | **Vr** | **Age** | $\frac{w}{c}$ | $\frac{S}{G}$ |
|---|---|---|---|---|---|
| **Vr** | Pearson Correlation | 1 | 0.580** | -0.080 | -0.015 |
| | Sig. (2-tailed) | | 0.000 | 0.057 | 0.719 |
| **Age** | Pearson Correlation | 0.580** | 1 | 0.000 | -0.015 |
| | Sig. (2-tailed) | 0.000 | | 0.992 | 0.719 |
| $\frac{w}{c}$ | Pearson Correlation | -0.080 | 0.000 | 1 | -0.004 |
| | Sig. (2-tailed) | 0.057 | 0.992 | | 0.927 |
| $\frac{w}{c}$ | Pearson Correlation | -0.015 | -0.015 | -0.004 | 1 |
| | Sig. (2-tailed) | 0.719 | 0.719 | 0.927 | |

**. Correlation is significant at the 0.01 level (2-tailed).

for this analysis.

Table 4: Correlation analysis of relative elasticity modulus with concrete age, water to cement ratio and sand to gravel ratio

| | | **Er** | **Age** | $\frac{w}{c}$ | $\frac{S}{G}$ |
|---|---|---|---|---|---|
| **Er** | Pearson Correlation | 1 | 0.629** | -0.073 | -0.018 |
| | Sig. (2-tailed) | | 0.000 | 0.080 | 0.677 |
| **Age** | Pearson Correlation | 0.629** | 1 | 0.000 | -0.015 |
| | Sig. (2-tailed) | 0.000 | | 0.992 | 0.719 |
| $\frac{w}{c}$ | Pearson Correlation | -0.073 | 0.000 | 1 | -0.004 |
| | Sig. (2-tailed) | 0.080 | 0.992 | | 0.927 |
| $\frac{w}{c}$ | Pearson Correlation | -0.018 | -0.015 | -0.004 | 1 |
| | Sig. (2-tailed) | 0.677 | 0.719 | 0.927 | |

**. Correlation is significant at the 0.01 level (2-tailed).

Table 5: The coefficients $a_1$ and $a_2$ estimated by SPSS

| | | | **95 % Confidence Interval** | |
|---|---|---|---|---|
| **Parameter** | **Estimate** | **Std. Error** | **Lower Bound** | **Upper Bound** |
| $a_1$ | 566.650 | 23.236 | 521.012 | 612.289 |
| $a_2$ | 3450.836 | 54.727 | 3343.342 | 3558.330 |

Table 6: Variance analysis of ultrasonic wave velocity vs. concrete age

| Source | Sum of Squares | df | Mean Squares |
|---|---|---|---|
| **Regression** | 1.240E10 | 2 | 6.199E9 |
| **Residual** | 2.038E8 | 565 | 360679.105 |
| **Uncorrected Total** | 1.260E10 | 567 | |
| **Corrected Total** | 4.183E8 | 566 | |

Dependent variable: V

R squared =1 - (Residual Sum of Squares) / (Corrected Sum of Squares) = 0.513.

$$V = a_1 ln(t) + a_2\frac{w}{c} + a_3\frac{S}{G} + a_4 \tag{3.3}$$

As you can see in the table 7, the constants are $a_1$= 578.823, $a_2$= -2297.793, $a_3$= -65.793 and $a_4$= 4914.584. Therefore, the relation between the longitudinal wave velocity and the age of concrete with respect to its mixing design is as equation (3.4).

$$V = 578.823(t) + 2297.793\frac{w}{c} + 65.793\frac{S}{G} + 4914.584, R^2 = 0.785 \tag{3.4}$$

The parameters of equation (3.4) are: $t$: concrete age in days $fracwc$: water to cement ratio $fracSG$: ratio of fine-grained to coarse-grained aggregates $V$: ultrasonic wave velocity in $fracms$

Matching of the estimated velocity obtained from equation (3.4) with the measured value in the experiments is shown in Fig. 12. Comparison of Fig. 11 and Fig. 12 shows that the estimated wave

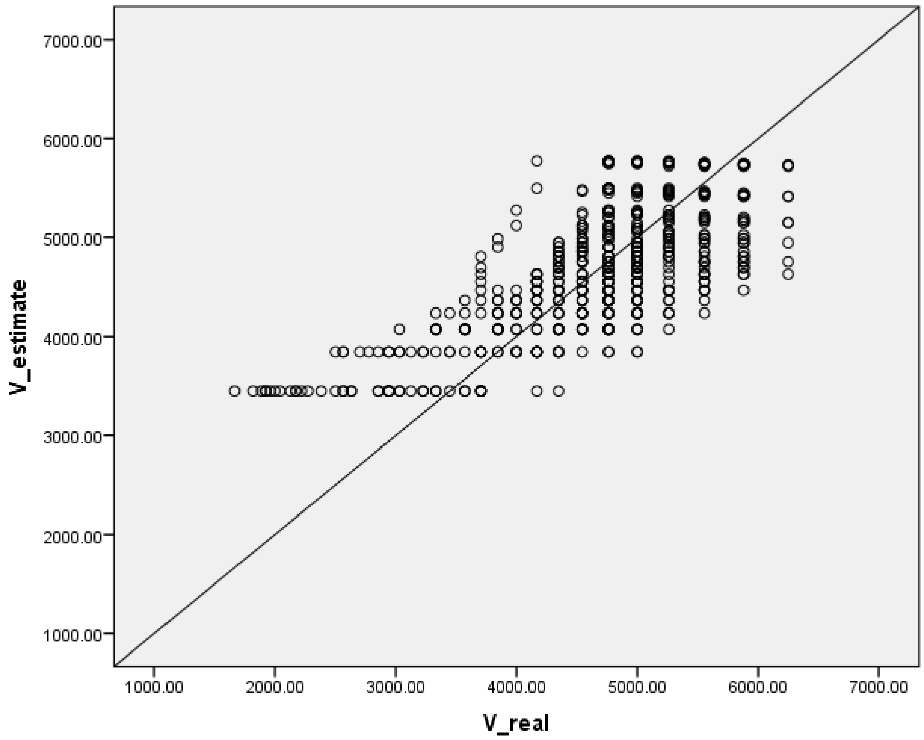


Figure 11: Comparison between the results of suggested equation with and measured velocities in experiments

Table 7: The coefficients $a_1$ and $a_2$ estimated by SPSS

| | | | 95 % Confidence Interval | |
|---|---|---|---|---|
| **Parameter** | **Estimate** | **Std. Error** | **Lower Bound** | **Upper Bound** |
| $a_1$ | 578.823 | 15.460 | 548.456 | 609.190 |
| $a_2$ | -2297.793 | 99.059 | -2492.364 | -2103.222 |
| $a_3$ | -65.793 | 4.951 | -75.518 | -56.069 |
| $a_4$ | 4914.584 | 67.694 | 4781.620 | 5047.548 |

Table 8: Analysis of the variance of ultrasonic wave velocity vs. concrete age

| Source | Sum of Squares | df | Mean Squares |
|---|---|---|---|
| **Regression** | 1.251E10 | 4 | 3.128E9 |
| **Residual** | 89796943.793 | 563 | 159497.236 |
| **Uncorrected Total** | 1.260E10 | 567 | |
| **Corrected Total** | 4.183E8 | 566 | |

Dependent variable: V

R squared =1 - (Residual Sum of Squares) / (Corrected Sum of Squares) = 0.785.

velocity from equation (3.4) is more consistent with the measured value in the experiments. The equation (3.4) is more accurate than the equation (3.2) because the effect of the materials mixing ratios on the wave velocity is considered.

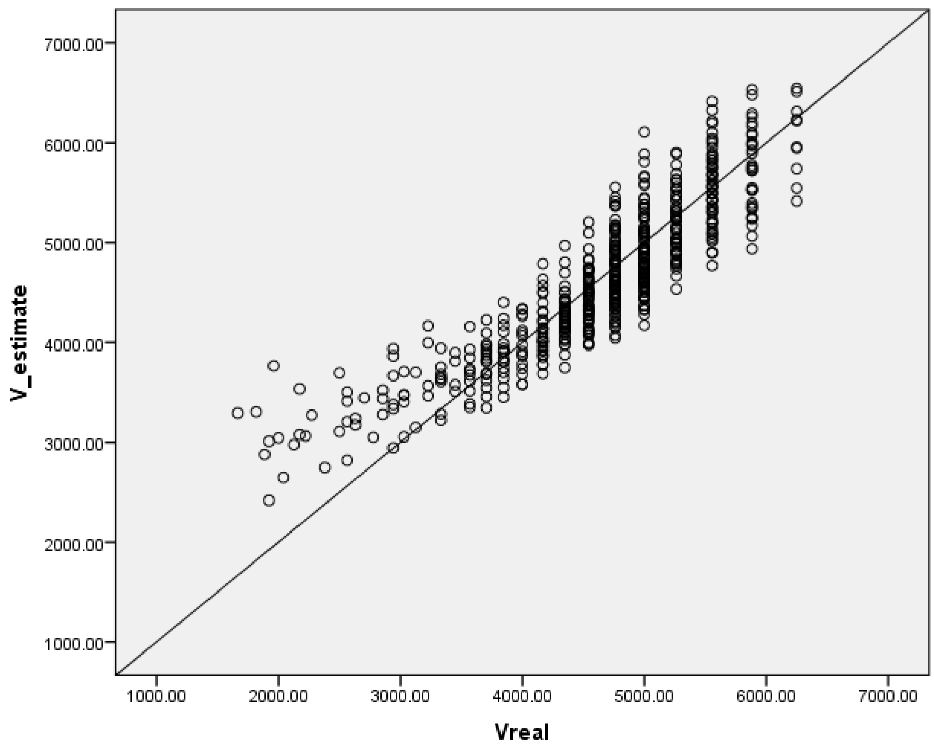


Figure 12: Comparison between the results of suggested equation and measured velocities in experiments

### 3.4. *A constitutive model for modulus of elasticity*

Based on the results of the tests, the modulus of elasticity of concrete is significantly related to all of the three parameters including concrete age, water to cement ratio and sand to gravel ratio. At first, correlation between the elasticity modulus and concrete age is obtained by fitting a curve with minimum residual sum of squares and materials mixing ratios are neglected. thus different kind of graphs is examined and concluded that the logarithmic relationship describes this behavior successfully. This relation is defined as equation (3.5), where, $E$ is the modulus of elasticity of the concrete in MPa and t is the concrete age in days and $a_1$ and $a_2$ are unknown parameters which are obtained by examining the behavior of the specimens and minimizing the residual sum of squares. Table 9 is a part of the SPSS output that shows the values obtained for $a_1$ and $a_2$. The results of the variance analysis are shown in Table 10.

$$E = a_1 ln(t) + a_2 \tag{3.5}$$

Based on the table 9, the constant values are $a_1$= 9986.649, $a_2$= 25546.595. Therefore, the relationship between the concrete modulus of elasticity with concrete age is equation (3.6). Materials mixing ratios are neglected in equation (3.6)

$$E = 9986.649 ln(t) + 25546.595, R^2 = 0.447 \tag{3.6}$$

where $E$ is the modulus of elasticity of concrete in $MPa$ and $t$ is the concrete age in days. Matching of the estimated modulus of elasticity obtained from equation (3.6) with the measured value in the experiments is shown in Fig. 13.

Table 9: The coefficients $a_1$ and $a_2$ estimated by SPSS

| | | | **95 % Confidence Interval** | |
|---|---|---|---|---|
| **Parameter** | **Estimate** | **Std. Error** | **Lower Bound** | **Upper Bound** |
| $a_1$ | 9986.649 | 467.708 | 9067.990 | 10905.308 |
| $a_2$ | 25546.595 | 1101.604 | 23382.857 | 27710.333 |

Table 10: Variance analysis of modulus of elasticity vs. concrete age

| **Source** | **Sum of Squares** | **df** | **Mean Squares** |
|---|---|---|---|
| **Regression** | 1.288E12 | 2 | 6.442E11 |
| **Residual** | 8.257E10 | 565 | 1.461E8 |
| **Uncorrected Total** | 1.371E12 | 567 | |
| **Corrected Total** | 1.492E11 | 566 | |

Dependent variable: E

R squared =1 - (Residual Sum of Squares) / (Corrected Sum of Squares) = 0.447.

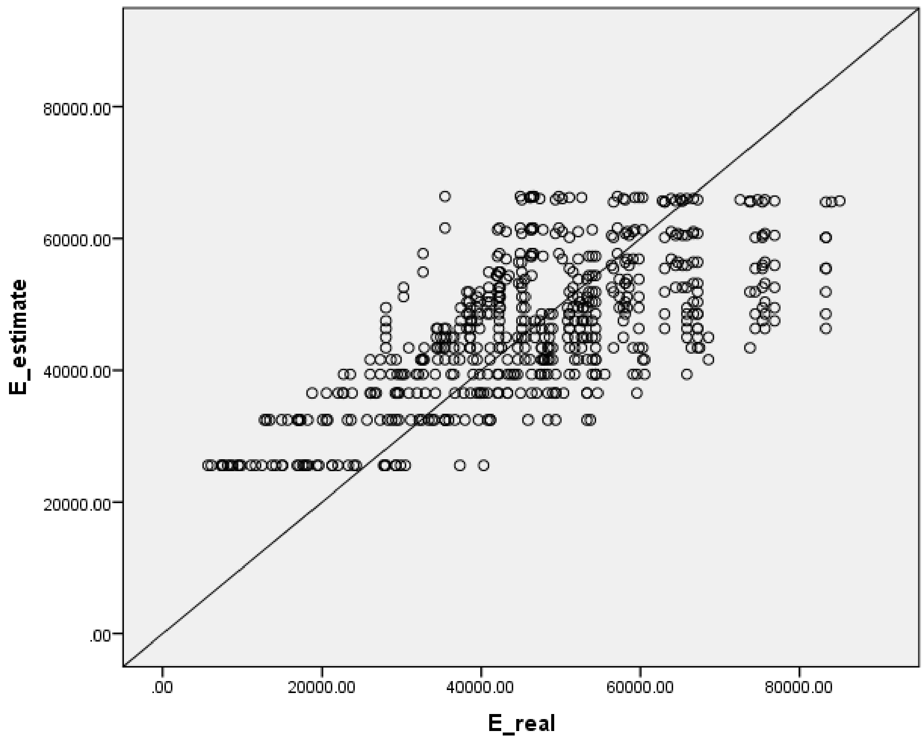


Figure 13: Comparison between the measured modulus of elasticities in experiments and results from suggested equation

### 3.5. *A constitutive model for modulus of elasticity considering concrete mixing ratios*

Here, another model for concrete behavior is presented to determine modulus of elasticity. In this model, other affecting parameters like water to cement ratio and aggregates grading are considered which are neglected in equation (3.7). Here, the effect of concrete age is considered as a logarithmic curve. By examining other parameters, it is observed that a linear relationship would have acceptable accuracy. Therefore, this model was defined as equation (3.8), where E is the concrete modulus of elasticity in MPa, t is the concrete age in day, $(\frac{w}{c})$ is water to cement ratio, $(\frac{S}{G})$ is the sand to gravel weight ratio and $a_1$, $a_2$, $a_3$ and $a_4$ are unknown coefficients obtained by examining the specimen's behavior and minimizing the residual sum of squares. Table 11 is a part of the SPSS output that shows the estimated values of $a_1$, $a_2$, $a_3$ and $a_4$. Analysis variance results are shown in Table 12.

$$E = a_1 ln(t) + a_2\frac{w}{c} + a_3\frac{S}{G} + a_4 \tag{3.7}$$

Table 11: The coefficients $a_1$ and $a_2$ estimated by SPSS

| | | | **95 % Confidence Interval** | |
|---|---|---|---|---|
| **Parameter** | **Estimate** | **Std. Error** | **Lower Bound** | **Upper Bound** |
| $a_1$ | 10244.259 | 291.685 | 9671.335 | 10817.183 |
| $a_2$ | -48629.342 | 1868.915 | -52300.239 | -44958.445 |
| $a_3$ | -1367.409 | 93.407 | -1550.877 | -1183.941 |
| $a_4$ | 56457.822 | 1277.162 | 53949.237 | 58966.408 |

Table 12: Variance analysis of modulus of elasticity vs. concrete age

| **Source** | **Sum of Squares** | **df** | **Mean Squares** |
|---|---|---|---|
| **Regression** | 1.339E12 | 4 | 3.348E11 |
| **Residual** | 3.196E10 | 563 | 56772976.334 |
| **Uncorrected Total** | 1.371E12 | 567 | |
| **Corrected Total** | 1.492E11 | 566 | |

Dependent variable: V
R squared =1 - (Residual Sum of Squares) / (Corrected Sum of Squares) = 0.786.

Based on Table 11, the constant values are $a_1$= 10244.259, $a_2$= -48629.342, $a_3$= -1367.409 and $a_4$=56457.822. Thus, the relationship between the concrete modulus of elasticity and the concrete age considering materials mixing ratios is as equation (3.8).

$$E = 10244.2(t) - 48629.3\frac{w}{c} - 1367.4\frac{S}{G} + 56457.8, R^2 = 0.786 \tag{3.8}$$

Where: $t$: concrete age in days $fracwc$: water to cement ratio $fracSG$: ratio of fine-grained to coarse-grained aggregates $E$: Modulus of elasticity of concrete in $MPa$

Matching of the estimated modulus of elasticity obtained from equation (3.8) with the measured value in the experiments is shown in Fig. 14. Comparison of Fig. 13 and Fig. 14 shows that the estimated wave modulus of elasticity from equation (3.8) is more consistent with the measured value in the experiments. The equation (3.8) is more accurate than the equation (3.6) because the effect of the materials mixing ratios on the modulus of elasticity is considered.

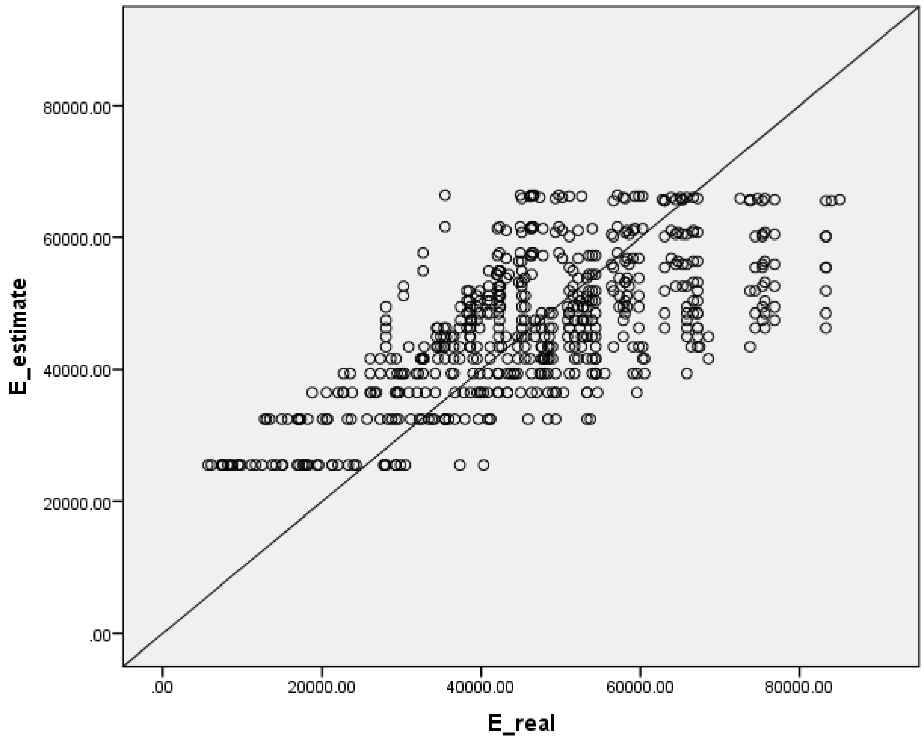


Figure 14: Comparison between the results of the suggested equation and measured modulus of elasticities in experiments

The elasticity modulus obtained from the suggested equations is matched with the dynamical elasticity modulus of concrete specimens experimented by the researchers in (Han and Kim 2004; Kocab et al. 2017). Rate of increase in elasticity over time in these studies are similar to the results obtained here, but in the aforementioned references, the number of specimens is limited and no empirical relationship is presented to estimate the elasticity modulus in practical works.

### 3.6. *A constitutive model for relative wave velocity*

Based on the results of the tests, the relative velocity of the ultrasonic wave has a significant relationship only with concrete age. A curve is fitted between ultrasonic wave velocity and concrete age. Examining different diagrams are shown that a logarithmic relationship describes this behavior well. This relation is defined as equation (3.9), where $V_r$ is the relative velocity and $t$ is the concrete age in day, and $a_1$ and $a_2$ are unknown coefficients. $a_1$ and $a_2$ is obtained by examining the specimen's behavior and minimizing the residual sum of squares. Table 13 is a part of the SPSS output that shows the values obtained for $a_1$ and $a_2$. Variance analysis results of specimens are shown in Table 14.

$$V_r = a_1 ln(t) + a_2 \tag{3.9}$$

Based on table 13 the constants are $a_1$= 0.112 and $a_2 = 0.658$. Thus, relationship between relative velocity and concrete age is as equation (3.10).

$$V_r = 0.112ln(t) + 0.658, R^2 = 0.749 \tag{3.10}$$

where $V_r$ is the relative velocity of the ultrasonic wave and $t$ is the concrete age in days.

Table 13: The coefficients $a_1$ and $a_2$ estimated by SPSS

| | | | 95 % Confidence Interval | |
|---|---|---|---|---|
| **Parameter** | **Estimate** | **Std. Error** | **Lower Bound** | **Upper Bound** |
| $a_1$ | 0.112 | 0.003 | 0.107 | 0.117 |
| $a_2$ | 0.658 | 0.006 | 0.645 | 0.671 |

Table 14: Variance analysis of relative wave velocity vs. concrete age

| Source | Sum of Squares | df | Mean Squares |
|---|---|---|---|
| **Regression** | 459.484 | 2 | 229.742 |
| **Residual** | 2.809 | 565 | 0.005 |
| **Uncorrected Total** | 462.293 | 567 | |
| **Corrected Total** | 11.173 | 566 | |

Dependent variable: E

R squared =1 - (Residual Sum of Squares) / (Corrected Sum of Squares) = 0.749.

### *3.7. A constitutive model for relative elasticity modulus of concrete*

Based on the results of these tests, the concrete relative modulus of elasticity a significant relation has only with the concrete age. Therefore, regardless of the water to cement ratio and aggregate gradation, we found a relationship between the relative elasticity modulus and the concrete age by fitting a curve with the minimum residual sum of squares. Thus, by examining different diagrams, we concluded that a logarithmic relationship could describe this behavior well. This relation is defined as equation (3.11), where $E_r$ is the relative modulus elasticity and $t$ is the concrete age in days and $a_1$ and $a_2$ are unknown coefficients which are obtained by minimizing residual sum of squares. The values obtained for $a_1$ and $a_2$ are shown in Table 15. Results of variance analysis are shown in Table 16.

$$E_r = a_1 ln(t) + a_2 \tag{3.11}$$

Table 15: The coefficients $a_1$ and $a_2$ estimated by SPSS

| | | | 95 % Confidence Interval | |
|---|---|---|---|---|
| **Parameter** | **Estimate** | **Std. Error** | **Lower Bound** | **Upper Bound** |
| $a_1$ | 0.182 | 0.004 | 0.175 | 0.190 |
| $a_2$ | 0.435 | 0.009 | 0.417 | 0.452 |

Based on table 15, the constant values are $a_1$= 0.182 and $a_2$= 0.435. Therefore, the relationship between the relative elasticity modulus of concrete and the concrete age is as equation (3.12).

$$E_r = 0.182 ln(t) + 0.435, R^2 = 0.800 \tag{3.12}$$

Where, $E_r$ is the relative elasticity modulus of concrete and $t$ is the concrete age in days.

Table 16: Variance analysis of relative modulus of elasticity vs. concrete age

| Source | Sum of Squares | df | Mean Squares |
|---|---|---|---|
| **Regression** | 399.087 | 2 | 199.544 |
| **Residual** | 5.546 | 565 | .010 |
| **Uncorrected Total** | 404.634 | 567 | |
| **Corrected Total** | 27.711 | 566 | |

Dependent variable: E

R squared =1 - (Residual Sum of Squares) / (Corrected Sum of Squares) = 0.800.

## 4. Conclusion

There exists a significant relationship between the longitudinal wave velocity and the concrete age. Determination of the relationship between the ultrasonic wave velocity at different ages considering water to cement ratio and the aggregate gradation is possible. This applies to the modulus of elasticity, but the relative wave velocity and the relative elasticity modulus have a significant relationship only with the concrete age. Ultrasonic wave velocity has been increased faster at early ages of specimens where concrete setting process is fast and in last days, rates of increasing the longitudinal wave velocity decreases. An increasing in the water to cement ratio leads to increases the longitudinal wave velocity over time. A series of empirical relations have been proposed for estimating longitudinal wave velocities and modulus of elasticity of concrete at different ages. These relationships have been formulated as logarithmic curves. In some of these equations, the effects of the materials mixing ratios are also considered to be more precise. These empirical equations can be used in the analytical and numerical analysis of structures. These models can be used to determine the loading time of concrete structures and to predicting their other physical and mechanical properties, such as strength and stiffness.

## References

[1] C. ASTM, 597, *Standard Test method for Pulse Velocity Through Concrete*, Annual Book of ASTM Standards, 2000.

[2] A. Boumiz, C. Vernet and F.C. Tenoudji, *Mechanical properties of cement pastes and mortars at early ages: evolution with time and degree of hydration*, Adv. Cement Based Mate. 3(3) (1996) 94–106.

[3] BS 1881: Part 207, *Recommendations for the assessment of concrete strength by near-to-surface tests*, J. Model. Engin. (1992).

[4] T. Gudra and B. Stawiski, *Non-destructive strength characterization of concrete using surface waves*, NDT and E Int. 33(1) (2000) 1–6.

[5] S.H. Han and J.K. Kim, *Effect of temperature and age on the relationship between dynamic and static elastic modulus of concrete*, Cement Conc. Res. 34(7) (2004) 1219–1227.

[6] IAEA, *Ultrasonic Testing of Materials at Level 2*, Training course series.

[7] J. Keating, D. Hannant and A. Hibbert, *Correlation between cube strength, ultrasonic pulse velocity and volume change for oil well cement slurries*, Cement Conc. Res. 19(5) (1989) 715–726.

[8] J. Krautkrämer and H. Krautkrämer, *Ultrasonic Testing of Materials*, Springer Science and Business Media, 2013.

[9] D. Kocab, B. Kucharczykova, P. Misak, P. Zitt and M. Kralikova, *Development of the elastic modulus of concrete under different curing conditions*, Procedia Engin. 195 (2017) 96–101.

[10] D.S. Lane, *Evaluation of Concrete Characteristics for Rigid Pavements*, Virginia Transportation Research Council, (No. VTRC-98-R24), 1998.

[11] V. Malhotra and P.K. Mehta, *Concrete Technology: Past, Present, and Future*, Proc. V. Mohan Malhtra Symp. Amer. Conc. Inst. 1994.

[12] V.M. Malhotra, *In Situ/nondestructive Testing of Concrete*, American Concrete Institute, 1984.

[13] V.M. Malhotra and N.J. Carino, *CRC Handbook on Nondestructive Testing of Concrete*, CRC press, 1991.

[14] J. Pallant, *SPSS Survaival Manual: A Step by Step Guide to Data Analysis Using SPSS*, McGraw-Hill Education, 2010.

[15] R.E. Philleo, *Comparison of results of three methods for determining young's modulus of elasticity of concrete*, J. Proc. (1955) 461–470.

[16] S. Popovics, *Analysis of the concrete strength versus ultrasonic pulse velocity relationship*, Mate. Evaluat. 59(2) (2001) 123–130.

[17] S. Popovics, *Analysis of concrete strength versus water-cement ratio relationship*, Mate. J. 87(5) (1990) 517–529.

[18] J. Popovics, J. Achenbach and W.J. Song, , *Application of new ultrasound and sound generation methods for testing concrete structures*, Mag. Conc. Res. 51(1) (1999) 35–44.

[19] S. Popovics, N.M. Bilgutay, M. Caraoguz and T. Akgul, *High-frequency ultrasound technique for testing concrete*, Mate. J. 97(1) (2000) 58–65.

[20] H.Y. Qasrawi and I.A. Marie, *The use of USPV to anticipate failure in concrete under compression (In Persian)*, Cement Conc. Res. 33(12) (2003) 2017–2021.

[21] M. Sabagh and A. Ghalandarzadeh, *Centrifugal modeling of continuous shallow tunnels at active normal faults intersection*, Transp. Geotech. 22 (2020) 100325.

[22] M. Sabagh and A. Ghalandarzadeh, *Centrifuge experiments for shallow tunnels at active reverse fault intersection*, Front. Struct. Civil Engin. 14(3) (2020) 731–745.

[23] M. Sabagh and A. Ghalandarzadeh, *Numerical modelings of continuous shallow tunnels subject to reverse faulting and its verification through a centrifuge*, Comput. Geotech. 128 (2020) 103813.

[24] M. Sabagh, M. Kazemi and M. Asgari, *Constitutive model for estimating concrete strength using ultrasonic test considering mixing ratios (In Persian)*, J. Model. Engin. 17 (2019) 367–374.

[25] S. Sahu, S. Badger, N. Thaulow and R. Lee, *Determination of water–cement ratio of hardened concrete by scanning electron microscopy*, Cement Conc. Comp. 26(8) (2004) 987–992.

[26] M. Sahmaran, O. Kasap, K. Duru and I. Yaman, *Effects of mix composition and water–cement ratio on the sulfate resistance of blended cements*, Cement Conc. Comp. 29(3) (2007) 1795–1802.

[27] C. Sayers and A. Dahlin, *Propagation of ultrasound through hydrating cement pastes at early times*, Adv. Cement Based Mate. 1(1) (1993) 12–21.

[28] C. Sayers and R. Grenfell, *Ultrasonic propagation through hydrating cements*, Ultrason. 31(3) (1993) 147–153.

[29] G. Trtnik and M. Gams, *Ultrasonic assessment of initial compressive strength gain of cement based materials*, Cement and Concrete Research, 67 (2015) 148–155.

[30] J. Zhu, S.H. Kee, D. Han and Y.T. Tsai, *Effects of air voids on ultrasonic wave propagation in early age cement pastes*, Cement Conc. Res. 41(8) (2011) 872–881.