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Developing bulk arrival queuing models with the constant batch policy under uncertainty data using (0 - 1) variables

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Abstract

This paper delves into some significant performance measures (PMs) of a bulk arrival queueing system with constant batch size b, according to arrival rates and service rates being fuzzy parameters. The bulk arrival queuing system deals with observation arrival into the queuing system as a constant group size before allowing individual customers entering to the service. This leads to obtaining a new tool with the aid of generating function methods. The corresponding traditional bulk queueing system model is more convenient under an uncertain environment. The α-cut approach is applied with the conventional Zadeh's extension principle (ZEP) to transform the triangular membership functions (Mem. Fs) fuzzy queues into a family of conventional bulk queues. This new model focus on mixed-integer non-linear programming (MINLP) tenders a mathematical computational approach is known as (0 -1) variables. To measures the efficiency of the method, the efficient solution strategy plays a crucial role in the adequate application of these techniques. Furthermore, different stages of the α-cut intervals were analyzed and the final part of the article gives a numerical solution of the proposed model to achieve practical issues.

Keywords: Constant batch size, Uncertainty data, Mixed-integer, Non-linear programming (0 - 1) variables

1. Introduction

A lot of previous literature has gone into presenting controllable queuing models in varying areas of real-life scenarios, such as wireless networks, inventory/ production, and manufacturing systems,

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as seen in the works of [11, 2, 1, 4, 13]. In all these queuing systems, arrivals can be singly or in bulk. This paper concentrates on the latter, that is bulk arrivals, with a constant batch policy considering a queuing system like the manufacturing systems, the customers arrive randomly according to the Poisson process, form a queue, and then, in some queuing systems, the customers were transformed into batches, called batch queues. This implies that after a cumulated number of customers are allowed as a constant batch to enter the server, they depart in batch denoted by b. This type of queue, called a b-constant-batch queue, is implemented to prevent system peak congestion. Numerous researchers have dealt with bulk arrival queuing models like the works of [9, 10] who attempt batch queuing system with a vacation period comprising an idle period and as random setup period, [16] adopted bi-level control policy under batch arrival of hobs to the production line, [24] classified a single-server, two-stages queuing system with a constant-size batch policy and [18] who considered extending batch arrival controlling queuing system with Bernoulli schedule server vacations and random system breakdowns.

In practical terms, it is generally preferable to describe the arrival rates and service rates using linguistic terms. That is, using statements like the mean arrival rate is approximately 6 customers per hour or the mean service rate is around 15 customers per hour. Expressing these parameters i.e., customers and services as linguistic expressions for fuzzy queues is more realistic than the commonly crisp queues [21]. The basic concept of fuzzy sets was first introduced by [28], with its possibility concept investigated by [3], who dealt with the relationship between uncertainty data and queuing systems. Other researchers have also discussed fuzzy queuing systems, including [14] proposing a general procedure to construct (Mem. Fs) of (PMs) in Four types of single fuzzy queues F and the fuzzy exponential time denoted by FM, respectively). Besides, Wang [25] adopted a fuzzy multiple channel queuing system and likewise [5] investigated the fuzzy model with single bulk service rates (FM^K). Other notable works include [20] investigating optimal operating policy for a controllable queuing model in the production line, [17] choose the Markov properties to control arrivals rates with optional service, [27] discussed the F-policy queues using uncertain parameters. Mueen et al., [23] adopting the arrival rates with single or multiple channels queuing systems represented as Hexagonal (Mem. Fs).

Other studies have an emphasis on controllable systems by using varying mathematical computation approaches to transform arrival rates and service rates. One such mathematical approach is the (MINLP) technique adopted in the works of Chen [6, 7] using the fuzzy queues with batch arrival as variety as Geometric distribution with multiple channel queues, [19] investigating the Erlang distribution model, and [22] adopting single queues batch fuzzy arrival (FM^X) with multiple working vacations. The technique is also suitable for application in bulk arrival queuing models, bearing in mind that the constant batch is an important factor for balanced control in the manufacturing systems. This constant batch policy follows the assumptions of customers being cumulated as a constant group size before entering to start the service and then consecutively obtaining the PMs. There exists a dearth of research for adopting this concept under uncertain data, hence the motivation to investigate the fuzzy queues with a constant batch in real-life applications.

This document has developed a new step that may support fuzzy (PMs) for bulk arrival queues with constant group size. The (Mem. Fs) of (PMs) can be derived completely by applying α-cut and (ZEP). Two couple of constraints of (MINLP) models are formulated, which can be summarized as (0 - 1) constraints to calculate the bottom-bound and top-bound. Furthermore, the (Mem. Fs) of the system performance are derived analytically.

The outline of this paper is organized as follows; the model was described in Section 2, the (MINLP) approach constructed (Mem. Fs) with (0 - 1) constraints in section 3. While section 4 presents a numerical example to demonstrate the validity of the newly proposed method under uncertainty data. Finally, sections 5 and 6 support the finding and discussion of results and conclude the paper.

2. Fuzzy Arrival Queues with Constant Batch Size

Consider a queuing system in which customers arrive as a Poisson process at a single server facility in a constant batch of size b with fuzzy arrival rates $\widetilde{\lambda}$, and fuzzy service rates $\widetilde{\mu}$ following an exponential distribution (note that the conventional symbols are not used for these parameters in queuing systems). The arrival of customers into the system via a first-come-first-service (F-C-F-S) pattern, and the size of the system and population is infinite. Hence, this model will be indicated as (FMb/FM/1-FCFS). In this model, the arrival rates and service rates are quasi known and represented by convex fuzzy sets. Note that a fuzzy set $\widetilde{A}$, in the universal set V. The convex of $\widetilde{\mu}_A(\varphi v_1 + (1-\varphi) v_2) \geq min\{\widetilde{\mu}_A(v_1), \widetilde{\mu}_A(v_2)\}$, where $\widetilde{\mu}_A$ is its (Mem. Fs) $\varphi \in [0,1]$, and $v_1, v_2 \in V$. Let $\mu_{\widetilde{\lambda}}(x)$, and $\mu_{\widetilde{\mu}}(y)$ denote the (Mem. Fs) of the arrival rates and service rates set, respectively, defined as.

$$\widetilde{\lambda} = \{(x, \mu_{\widetilde{\lambda}}(x))/x \in R(\widetilde{\lambda})\} \tag{2.1}$$

$$\widetilde{\mu} = \{(y, \mu_{\widetilde{\mu}}(y))/y \in R(\widetilde{\mu})\} \tag{2.2}$$

Where, $R(\widetilde{\lambda})$, $R(\widetilde{\mu})$ provides of $\widetilde{\lambda}$ and $\widetilde{\mu}$, which denote the universal sets of arrival and service rates. Let $f(x,y)$ indicate the system of (PMs) of interest: when the arrival and service rates are fuzzy parameters, it is clear that $f(\widetilde{\lambda}, \widetilde{\mu})$ will be fuzzy as well. Recalling into (ZEP) [26]. The (Mem. Fs) off $(\widetilde{\lambda}, \widetilde{\mu})$ are formulated as.

$$\mu_{f(\widetilde{\lambda},\widetilde{\mu})}(v) = \sup\{\min \mu_{\widetilde{\lambda}}(x), \mu_{\widetilde{\mu}}(y)/v = f(x,y)\} \tag{2.3}$$

If the α-cut off $(\widetilde{\lambda}, \widetilde{\mu})$ at all alpha values degenerate to the same point, then the values of the system (PMs) are crisp, and if otherwise, they are fuzzy values.

From the literary knowledge of the conventional queuing model under the utilization of the server $\rho = \frac{\mathrm{bx}}{y}$ being less than 1, and from the results of the generation function method [23], the four different mathematical expressions of this model are termed as the (PMs) of the system and can be defined as follows. The first measure is the expected mean waiting time of customers in the queue Wq is defined.

$$W_q = \frac{(b + 2\rho - 1)}{2y(1-\rho)} \tag{2.4}$$

The following Equation (2.3), the (Mem. Fs) for $\widetilde{W_q}$ is

$$\mu_{\widetilde{W_q}}(v) = \sup\left\{\min \mu_{\widetilde{\lambda}}(x), \mu_{\widetilde{\mu}}/v = \frac{(b+2\rho-1)}{2y(1-\rho)}\right\} \tag{2.5}$$

Other (PMs) can be obtained according to little's formula in the same manner. These measures include the expected number of customers in the queue, Lq, expected mean waiting time of customers in system Ws, and the number of customers in the system Ls. The (Mem. Fs) for $\widetilde{L}_q$, $\widetilde{W_S}$, and $\widetilde{L}_s$

are given in the following Equations.

$$\mu_{\widetilde{L}_q}(v) = \sup\left\{\min \mu_{\widetilde{\lambda}}(x), \mu_{\widetilde{\mu}}/z = \frac{\rho(b-1+2\rho)}{2(1-\rho)}\right\} \tag{2.6}$$

$$\mu_{\widetilde{W}_s}(v) = \sup\left\{\min \mu_{\widetilde{\lambda}}(x), \mu_{\widetilde{\mu}}/v = \frac{(1+b)}{2y(1-\rho)}\right\} \tag{2.7}$$

$$\mu_{\widetilde{L}_s}(v) = \sup\left\{\min \mu_{\widetilde{\lambda}}(x), \mu_{\widetilde{\mu}}/v = \frac{\rho(1+b)}{2(1-\rho)}\right\} \tag{2.8}$$

Theoretically, Equations (2.5)-(2.8) are expressed in a complex pattern. The challenges to inferring the side of the (Mem. Fs) ;$\mu_{\widetilde{W}_q}(v)$, $\mu_{\widetilde{L}_q}(v)$, $\mu_{\widetilde{W}_s}(v)$, *and* $\mu_{\widetilde{L}_s}(v)$. Hence, the development of a mathematical programming technique is based on (0-1) constraints, which indicate the maximum value and minimum value of the alpha interval.

2.1. Mixed-Integer Nonlinear Programming Approach

This subsection begins by relaxing the requirements for integrality of variables (0 - 1), leading to a continuous and non-linear problem of programming optimization [12]. To construct the (Mem. Fs) of $\mu_{p(\widetilde{\lambda},\widetilde{\mu})}$ is based on deriving the alpha levels of $\mu_{p(\widetilde{\lambda},\widetilde{\mu})}$, where $\widetilde{\lambda}$ and $\widetilde{\mu}$are accomplished with.

$$\lambda_\alpha = \left[x_\alpha^{BB}, x_\alpha^{TB}\right] = \left[\min_{x\in X}\left\{x\middle|\mu_{\widetilde{\lambda}}(x) \geq \alpha\right\}, \max_{x\in X}\left\{x\middle|\mu_{\widetilde{\lambda}}(x) \geq \alpha\right\}\right] \tag{2.9}$$

$$\mu_\alpha = \left[y_\alpha^{BB}, y_\alpha^{TB}\right] = \left[\min_{y\in Y}\left\{y|\mu_{\widetilde{\mu}}(y) \geq \alpha\right\}, \max_{y\in Y}\left\{y|\mu_{\widetilde{\mu}}(y) \geq \alpha\right\}\right] \tag{2.10}$$

These intervals denote where the group arrival rates and service rates lie at possibility level α with λ_α and $\mu_\alpha$being conventional sets rather than fuzzy sets. According to the concept of α-cuts, the embedded fuzzy Markov chain in the (FMb/FM/1) can be decomposed into a family of ordinary Markov chains with different transition probability matrices, which are also parameterized by α. The arrival rate and service rate can also be viewed by different stages of the confidence interval. Consequently, the (FMb/FM/1) model can be reduced to a family of traditional (Mb/M/1) models with different alpha level sets $\{\lambda_\alpha|0 < \alpha \leq 1\}$ and $\{\mu_\alpha|0 < \alpha \leq 1\}$. These two sets cause nested structures for expressing the rapport between ordinary sets and fuzzy sets and they represent sets of movable boundaries [15, 14, 20, 11, 27, 9, 10, 29]. Without loss of generality, the fuzzy arrival rates $\widetilde{\lambda}$, and fuzzy service rates $\widetilde{\mu}$ as crisp values can be represented by degenerated (Mem. Fs) that only have one value in their domain. By the convexity of a fuzzy number, the bounds of these intervals are functions of alpha. Thus, it can be obtained from $x_\alpha^{BB} = \min \mu_{\widetilde{\lambda}}^{-1}(\alpha)$, $x_\alpha^{TB} = \max \mu_{\widetilde{\lambda}}^{-1}(\alpha), y_\alpha^{BB} = \min \mu_{\widetilde{\mu}}^{-1}(\alpha)$, $y_\alpha^{TB} = \max \mu_{\widetilde{\mu}}^{-1}(\alpha)$. Therefore, as defined in Equation (2.3) above, the (Mem. Fs) of $p\left(\widetilde{\lambda}, \widetilde{\mu}\right)$ is also parameterized by alpha. Thus, the parameter alpha cut can be used to constructing the (Mem. Fs).

Consider the (Mem. Fs) of the expected waiting time of customers in the queue $\widetilde{W_q}$, through Equation (2.4) $\mu_{\widetilde{W}_q}(f)$ is the minimum of $\mu_{\widetilde{\lambda}}(x)$ and $\mu_{\widetilde{\mu}}(y)$. Hence, it's mandatory $\mu_{\widetilde{\lambda}}(x) = \alpha$, and $\mu_{\widetilde{\mu}}(y) \geq \alpha$ such that $v = \frac{(b+2\rho-1)}{2y(1-\rho)}$ to satisfy $\mu_{\widetilde{w}_q}(f) = \alpha$.

To calculate the membership function of $\mu_{\widetilde{W}_q}(v)$, it suffices to find the left side and the right side function of$\mu_{\widetilde{W}_q}(v)$, which is equivalent to finding the bottom-bound V_α^{BB} and the top-bound $V_\alpha^{TB}$of the α-cut of $\mu_{\widetilde{W}_q}(v)$. Since requiring of $\mu_{\widetilde{\lambda}}(x) = \alpha$, represented by $x = x_\alpha^{BB}$, thus can be formulated as the constraint of $x = \beta_1 x_\alpha^{\mathrm{BB}} + (1-\beta_1)\, x_\alpha^{\mathrm{BB}}$, where $\beta_1 = 0$ or 1. Similarly, $\mu_{\widetilde{\mu}}(y) = \alpha$ Hence, to formulate the constraint of $y = \beta_2 y_\alpha^{\mathrm{TB}} + (1-\beta_2)\, y_\alpha^{\mathrm{TB}}$, where $\beta_2 = 0$. Moreover, from the definition of λ_α, and μ_α in Equations (9-10), $x \in \lambda_\alpha$and $y \in \mu_\alpha$ can be replaced into $x \in \left[x_\alpha^{\mathrm{BB}},\ \ y_\alpha^{\mathrm{TB}}\right]$, and $y \in \left[y_\alpha^{\mathrm{BB}},\ \ y_\alpha^{\mathrm{TB}}\right]$. Moreover, under these two stages, the (Mem. Fs) of $\mu_{\widetilde{W}_q}$ can be constructed by bottom-bound and top-bound as $(W_q)_\alpha^{\mathrm{BB}} = \min\left\{(W_q)_\alpha^{BB1},\ (W_q)_\alpha^{BB2}\right\}$, and $(W_q)_\alpha^{\mathrm{TB}} = \max\left\{(W_q)_\alpha^{TB1},\ (W_q)_\alpha^{TB2}\right\}$ respectively, where.

$$\begin{aligned} (W_q)_\alpha^{BB1} &= \min_{\substack{x\in X\\ y\in Y}} \left\{\frac{(b+2\rho-1)}{2y(1-\rho)}\right\}, \qquad \text{subject to} \\ & x = d_1 x_\alpha^{BB1} + (1-d_1)\, x_\alpha^{TB}, \\ & y_\alpha^{BB} \le y \le y_\alpha^{TB}, \\ & d_1 = 0 \quad Or \quad 1, \end{aligned} \tag{2.11}$$

$$\begin{aligned} (W_q)_\alpha^{BB2} &= \min_{\substack{x\in X\\ y\in Y}} \left\{\frac{(b+2\rho-1)}{2y(1-\rho)}\right\}, \qquad \text{subject to} \\ Subj.To: \quad & y = d_2 y_\alpha^{\mathrm{BB}} + (1-d_2)\, y_\alpha^{\mathrm{TB}}, \\ & x_\alpha^{\mathrm{BB}} \le x \le x_\alpha^{\mathrm{TB}}, \\ & d_2 = 0 \quad Or \quad 1, \end{aligned} \tag{2.12}$$

$$\begin{aligned} (W_q)_\alpha^{TB1} &= \min_{\substack{x\in X\\ y\in Y}} \left\{\frac{(b+2\rho-1)}{2y(1-\rho)}\right\}, \qquad \text{subject to} \\ & x = d_3 x_\alpha^{\mathrm{BB}} + (1-d_3)\, x_\alpha^{\mathrm{TB}}, \\ & y_\alpha^{\mathrm{BB}} \le y \le y_\alpha^{\mathrm{TB}}, \\ & d_3 = 0 \quad Or \quad 1, \end{aligned} \tag{2.13}$$

$$\begin{aligned} (W_q)_\alpha^{TB2} &= \max_{\substack{x\in X\\ y\in Y}} \left\{\frac{(b+2\rho-1)}{2y(1-\rho)}\right\}, \qquad \text{subject to} \\ & y = d_4 y_\alpha^{\mathrm{BB}} + (1-d_4)\, y_\alpha^{\mathrm{TB}} \\ & x_\alpha^{\mathrm{BB}} \le x \le x_\alpha^{\mathrm{TB}}, \\ & d_4 = 0 \quad Or \quad 1, \end{aligned} \tag{2.14}$$

The crisp interval $\left[(W_q)_\alpha^{\mathrm{BB}},\ \ (W_q)_\alpha^{\mathrm{TB}}\right]$ obtained by solving Equations (2.11) and (2.14) represents the α-cut of $\widetilde{W}_q$. An attractive feature of the alpha cut approach is that all alpha sections form a structure embedded in α. From (ZEP), $\widetilde{W}_q$ defined in Equation (2.5) is a fuzzy number that possesses convexity. Therefore, the values α_1 and α_2 such as that $0 < \alpha_2 < \alpha_1 \le 1$; $(W_q)_{\alpha_1}^{\mathrm{BB}} \ge\ (W_q)_{\alpha_2}^{\mathrm{BB}}$ and $(W_q)_{\alpha_1}^{\mathrm{TB}} \le\ (W_q)_{\alpha_2}^{\mathrm{TB}}$.

The other notations, $(W_q)_\alpha^{\mathrm{BB}}$ is non-decreasing to α and $(W_q)_\alpha^{\mathrm{TB}}$is non-increasing to α. This property is based on the convexity of $\widetilde{W}_q$ and the (Mem. Fs) $\mu_{\widetilde{\mathrm{Wq}}}(v)$ can be calculated from the solutions of Equations (2.11)-(2.14). If both $(W_q)_\alpha^{\mathrm{BB}}$ and $(W_q)_\alpha^{\mathrm{TB}}$are invertible to α, then the left side shape function $\mathrm{LS}(v) = \left[(W_q)_\alpha^{\mathrm{BB}}\right]^{-1}$ and the right side shape function $\mathrm{RS}(v) = \left[(W_q)_\alpha^{\mathrm{TB}}\right]^{-1}$ with the

(Mem. Fs) $\mu_{\widetilde{\mathrm{Wq}}}$ as:

$$\mu_{\widetilde{W}_q}(v) = \begin{cases} LS\,(v)\,, & v_1 \le v \le v_2 \\ 1 & v = \ v_2 \\ RS\,(v) & v_2 \le v \le v_3 \end{cases} \tag{2.15}$$

such that;

$$LS(v_1) = RS(v_3) = 0$$
$$LS(v_2) = RS(v_2) = 1.$$

In the same path, the other measures of (PMs) derived into new (Mem. Fs). A numerical partition is given in the following section for better clarification.

2.2. Numerical Illustration

Pay attention to a load manufacturing system in which trucks using single queues with single-channel loading trucks arrived in the facility by a discrete process and service rates follow as continuous distribution. No appointments are allowed in this system and customers follow as F-C-F-S. The arrival and service rates approximately are known and it's more convenient described as Triangular fuzzy numbers, represented by $\widetilde{\lambda} \ = [4,\ 5,\ 6]$ and $\widetilde{\mu} \ = [11, \ \ 12,\ 13]$ per hour, respectively, with constant batch size b equals two. The manager of the facility endeavors to determine the influence and concerns for the expected waiting time in the queue for any truck. The system can be described as a single queue system integrated with one server. The mathematical process stated in the previous sections will be followed to compute the required (PMs). To start building the model in the system can be characterized by the FM/FM/1-FCFS model.

$\left[x_\alpha^{\mathrm{BB}},\ \mathrm{x}_\alpha^{\mathrm{TB}}\right]$ & $\left[y_\alpha^{\mathrm{BB}},\ \mathrm{y}\ _\alpha^{\mathrm{TB}}\right]$ are obtained easily to be $\left[x_\alpha^{\mathrm{BB}},\ \mathrm{x}_\alpha^{\mathrm{TB}}\right] = \left[\min \mu_{\widetilde{\lambda}}^{-1}(\alpha)\,,\ \ \max \mu_{\widetilde{\lambda}}^{-1}(\alpha)\right] = [4+\alpha,\ \ 3-\alpha]\,;\ \left[y_\alpha^{\mathrm{BB}},\ \ y_\alpha^{\mathrm{TB}}\right] = \left[\min \mu_{\widetilde{\mu}}^{-1}(\alpha)\,,\ \ \max \mu_{\widetilde{\mu}}^{-1}(\alpha)\right] = [11+\alpha,\ \ 13-\alpha]$.

Thus, the following Equations (2.11) and (2.14). The parametric integer nonlinear programming approach is compound with the model to deriving the membership function of $\widetilde{W_q}$ as.

$$\begin{aligned} (W_q)_\alpha^{BB} &= \min_{\substack{x\in X\\ y\in Y}} \left\{ \frac{(2+2\rho-1)}{2y(1-\rho)} \right\}, \qquad \text{subject to} \\ & x = d_1(4+\alpha) + (1-d_1)\,(6-\alpha), \\ & (1+\alpha) \le y \le (13-\alpha), \\ & d_1 = 0 \quad Or \quad 1, \end{aligned} \tag{2.16}$$

$$\begin{aligned} (W_q)_\alpha^{TB} &= \min_{\substack{x\in X\\ y\in Y}} \left\{ \frac{(2+2\rho-1)}{2y(1-\rho)} \right\}, \qquad \text{subject to} \\ & y = d_3(11+\alpha) + (1-d_3)\,(13-\alpha), \\ & (4+\alpha) \le x \le (6-\alpha), \\ & d_3 = 0 \quad Or \quad 1, \end{aligned} \tag{2.17}$$

$$\begin{aligned} (W_q)_\alpha^{BB} &= \max_{\substack{x\in X\\ y\in Y}} \left\{ \frac{(2+2\rho-1)}{2y(1-\rho)} \right\}, \qquad \text{subject to} \\ & x = d_2(4+\alpha) + (1-d_2)\,(6-\alpha), \\ & (11+\alpha) \le y \le (13-\alpha), \\ & d_2 = 0 \quad Or \quad 1, \end{aligned} \tag{2.18}$$

$$(W_q)_\alpha^{TB} = \max_{\substack{x\in X\\ y\in Y}} \left\{ \frac{(2+2\rho-1)}{2y(1-\rho)} \right\}, \qquad \text{subject to}$$
$$y = d_4(11+\alpha) + (1-d_4)(13-\alpha), \tag{2.19}$$
$$(4+\alpha) \le x \le (6-\alpha),$$
$$d_4 = 0 \quad Or \quad 1,$$

Where, $\rho = \frac{2x}{y}$.

Each objective function from Equations (16-19) has a positive derivative to ρ, that is a positive partial derivative to x, for $4 \le x \le 6$, and negative partial derivatives to y, for $11 \le y \le 13$. Recalling to the knowledge of calculus operations, the objective functions of these Equations attain their extreme at the bounds and consequently, the smallest number of customers in the system is.

$$(W_q)_\alpha^{\mathrm{BB}} = \frac{\alpha+21}{4\alpha^2-70\alpha+234} \tag{2.20}$$
$$(W_q)_\alpha^{\mathrm{TB}} = \frac{23-\alpha}{4\alpha^2+54\alpha+110} \tag{2.21}$$

The next calculation is corresponding with the (Mem. Fs) is defined as.

$$\mu_{\widetilde{W}_q}(v) = \begin{cases} \frac{(70v+1)-(156v^2+476v+1)^{\frac{1}{2}}}{8v}, & 0.089 \le v \le 0.130 \\ 1, & v = 0.130 \\ \frac{-(54v+1)+(156v^2+476v+1)^{\frac{1}{2}}}{8v}, & 0.130 \le v \le 0.209 \end{cases} \tag{2.22}$$

Similarly, the (Mem. Fs) of the other tools can be constructed to estimate the whole system, as shown in $\mu_{\widetilde{L}_q}(v), \mu_{\widetilde{W}_S}(v)$, and $\mu_{\widetilde{L}_S}(v)$. Figure 1 depicts the rough side of all values.

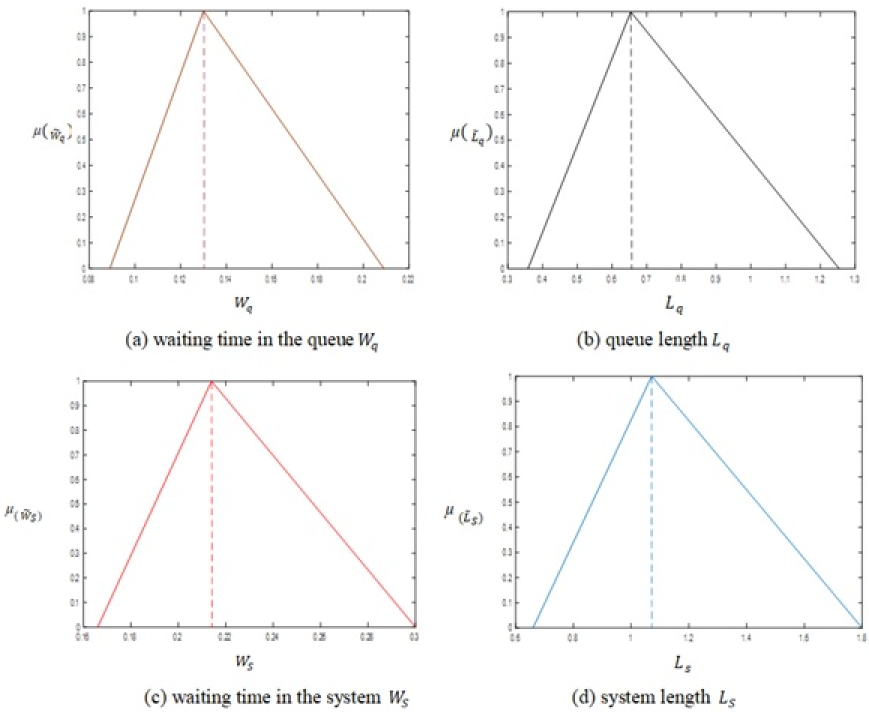

Figure 1: The Membership Functions of (a) $\mu_{(\widetilde{W}_q)}$, (b) $\mu_{(\widetilde{L}_q)}$, (c) $\mu_{(\widetilde{W}_S)}$, and (d) $\mu_{(\widetilde{L}_S)}$

Table 1 displays the value of α-cut possibilities at eleven location distinct values [0: 0.1: 1] for the possibilities of conventional values at each value of the (PMs).

Table 1: The α-cut intervals of (PMs) for the system.

α-cut	$(W_q)_\alpha^{BB}$	$(W_q)_\alpha^{TB}$	$(L_q)_\alpha^{BB}$	$(L_q)_\alpha^{TB}$	$(W_s)_\alpha^{BB}$	$(W_s)_\alpha^{TB}$	$(L_s)_\alpha^{BB}$	$(L_s)_\alpha^{TB}$
0.0	0.089	0.209	0.358	1.254	0.166	0.300	0.666	1.800
0.1	0.092	0.198	0.381	1.170	0.170	0.288	0.698	1.730
0.2	0.096	0.188	0.404	1.093	0.174	0.277	0.732	1.666
0.3	0.099	0.179	0.429	1.022	0.178	0.267	0.767	1.607
0.4	0.103	0.170	0.455	0.957	0.182	0.258	0.804	1.551
0.5	0.107	0.163	0.483	0.896	0.187	0.250	0.843	1.500
0.6	0.111	0.155	0.513	0.840	0.192	0.241	0.884	1.451
0.7	0.116	0.148	0.545	0.789	0.197	0.234	0.927	1.406
0.8	0.120	0.142	0.579	0.741	0.202	0.227	0.972	1.363
0.9	0.125	0.136	0.615	0.696	0.208	0.220	1.020	1.323
1.0	0.130	0.130	0.654	0.654	0.214	0.214	1.071	1.071

3. Findings and Discussions

There are some benefits of using the new MINLP mathematical approach, like being able to obtain the interval of (PMs) at various possibilities levels from the α-cuts of the (Mem. Fs) for $\widetilde{W}_q$, $\widetilde{L}_q$, $\widetilde{W}_S$, and $\widetilde{L}_S$. This range indicates that the expected waiting time will never exceed 0.209 approximately is equal to 13 Minutes or fall below 0.089 like 5 Minutes (mentioned in Table 1), while the number of customers in the system will never exceed two customers or fall below one customer. Furthermore, the measures of waiting for customers in the system will not exceed 0.300 is equal to 18 Minutes or fall less than 0.166 approximately 10 Minutes. This information obtained from analyzing the system will be very compulsory for designing a queuing system, which includes one or a combination of several decisions, such as the efficiency of the servers, the size of the constant bulk of customers, and the system capacity. The major objective shown in this paper is that the recent approach advances in (MINLP) are a suitable approach for adoption to rigorous queuing models, and also it possesses the novel advantage for obtaining the optimal crisp points inside closed intervals.

4. Conclusions

Bulk arrival queue systems have a wide variety of applications in real-world scenarios, such as production lines and service mechanism systems. In this paper, a non-linear integer-integer parametric programming approach was introduced to new triangular (Mem. Fs) of (PMs) when arrival rates and service rates are fuzzy parameters with constant batch sizes. The mathematical concept is grounded into basic constraints, including two variables (arrival rates and service rates) under minimum possibilities and maximum possibilities accessing into the best optimum point inside this interval. The numerical results gained to show the adequacy and ingenuity of this method for estimating the system as the equivalent the maximum value of the customer in the system is approximately equal with constant batch is chosen. In the future, it would be interesting to find other areas of the queuing system with an extension of multi-channel scanning across a constant batch.

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