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Novel solitons through optical fibers for perturbed cubic-quintic-septic nonlinear Schrödinger-type equation

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Abstract

The current analysis employs the Riccati and modified simple equation methods to retrieve new optical solitons for highly dispersive nonlinear Schrödinger-type equation (NLSE). With cubic-quintic-septic law (also known as a polynomial) of refractive index and perturbation terms having cubic nonlinearity, 1-optical solitons in the form of hyperbolic, periodic, and rational are derived. the two schemes offer an influential mathematical tool for solving NLSEs in various areas of applied sciences.

Keywords: Conformable derivative, Riccati simple equation method, Modified simple equation method, Optical soliton solutions
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1. Introduction

The appearance of group velocity dispersion (GVD), inter-modal dispersion (IMD), third-order dispersion (3OD), fourth-order dispersion (4OD), fifth order-dispersion (5OD), and the sixth-order dispersion (6OD) terms has led to deploy the concept of highly dispersive solitons during the few last years [26, 27, 28]. The dynamics of dispersive optical solitons has been studied with four nonlinear forms; Kerr law, quadratic–cubic law, nonlocal law and cubic–quintic–septic (CQS) law. A variety of numeric-analytic mathematical algorithms can process such kind of problems [36, 57, 13, 14, 45, 2, 25, 47, 48, 4, 5, 10, 11, 12, 29, 33, 58, 43, 51, 52, 53, 15, 16, 17, 18, 55, 54, 44, 46, 42, 34, 6, 50, 8, 9, 37, 39, 19, 20, 21, 7], our project's objective is to address the Riccati simple equation method (RSEM) [22, 23] and the modified simple equation method (MSEM) [24, 49] for retrieving dispersive optical solitons when the refractive index is of CQS-type with cubic nonlinear perturbation terms of Hamiltonian type given by [40, 41]

$$i\,\partial_t^\rho q+\sum_{k=1}^{6} i^{k^2} a_k\partial_x^k q+q\sum_{k=1}^{3} b_k|q|^{2k}=i\left(\lambda\partial_x\left(|q|^2q\right)+\theta\,\partial_x\left(|q|^2\right)q+\mu\,\partial_x\left(q\right)|q|^2\right). \tag{1.1}$$

This equation describes the pulses propagation of a variety models in optical fiber. The complex-valued operator $q=q(x,t)$ performs the soliton pulse profile. x and t are the local and temporal coordinates, while a_k's are the inter-modal, group velocity , third, fourth, fifth, and sixth-order dispersion terms respectively. b_k's represent the cubic–quintic–septic law of refractive index. λ is the coefficient of self-steepening term for short pulses, while θ and μ account for nonlinear dispersions. $\partial_t^\rho(\cdot)$, $\rho\in(0,1)$, is the time ρ-fractional differential operator, and $\partial_x^k(\cdot)$ denotes the kth local differential operator.

Kohl et al. [40, 41] applied the semi-inverse variational principle to tackle Eq.(1.1) for $\rho=1$. Bright 1-soliton solutions were derived. Many authors have numeric-analytically studied the non-perturbed version of considered nonlinear model [35, 30, 56, 3, 31, 32, 59].

While the fractional analysis has gained rising popularity, and to extend and generalize the previous results in [40], the time-fractional of perturbed model in Eq.(1.1), in conformable sense, is considered. The basic facts and concepts of conformable calculus are listed as follows [38, 1]:

For a function $q:[0,\infty)\to\mathbb{R}$, the conformable time derivative operator of order ρ for q is defined as

$$\partial_t^\rho q(t)=\lim_{\zeta\to 0}\frac{q(t+\zeta t^{1-\rho})-q(t)}{\zeta},\ \forall\, t>0,\ \rho\in(0,1). \tag{1.2}$$

In addition, If q is ρ-differentiable within an interval $(0,\tau)$ where $\tau>0$ and

$$\lim_{t\to 0} q^{(\rho)}(t), \tag{1.3}$$

exists, we specify

$$q^{(\rho)}(0)=\lim_{t\to 0^+} q^{(\rho)}(t). \tag{1.4}$$

Also:

(i.) $\partial_t^\rho(\theta_1q_1+\theta_2q_2)=\theta_1\partial_t^\rho q_1+\theta_2\partial_t^\rho q_2,\ \forall\ \theta_1,\theta_2\in\mathbb{R}.$

(ii.) $\partial_t^\rho(t^\varrho)=\varrho t^{\varrho-\rho},\ \forall\ \varrho\in\mathbb{R}.$

(iii.) $\partial_t^\rho(q_1 q_2) = q_1 \partial_t^\rho q_2 + q_2 \partial_t^\rho q_1$.
(iv.) $\partial_t^\rho\big(\frac{q_1}{q_2}\big) = \frac{q_2 \partial_t^\rho q_1 - q_1 \partial_t^\rho q_2}{q_2^2}$, provided $q_2 \neq 0$.
(v.) $\partial_t^\rho(K) = 0$, where K is a constant.
(vi.) $\partial_t^\rho q(t) = t^{1-\rho}\frac{\partial q(t)}{\partial t}$, for the differentiable function q.

Where $\rho \in (0,1]$ and q_1, q_2 are ρ-differentiable at a point $t > 0$.

In what follow, this work is prepared as follows: brief description of the used schemes is considered in Section 2. Analytic treatment of Eq. (1.1) by applying the mentioned approaches is discussed in Section 3. Finally, the concluding remarks are dedicated to Section 4.

2. Methodologies description

In this section, the fundamental algorithms for using the RSEM [22, 23] and MSEM [24, 49] are discussed. Given a nonlinear time-fractional PDE in the form:

$$F\big(q,\, \partial_x q,\, \partial_x^2 q,\, \cdots,\, \partial_t^\rho q\big) = 0. \tag{2.1}$$

Accordingly, the wave transformation $q(x,t) = W(\eta)$, $\eta = x - \nu\,\frac{t^\rho}{\rho}$, where ν is an arbitrary constant to be calculated afterwards, is assumed to convert Eq. (2.1) into nonlinear ODE

$$G\big(W(\eta),\, W'(\eta),\, W''(\eta),\, W'''(\eta), \cdots\big) = 0. \tag{2.2}$$

The RSEM assumes the solution of Eq. (2.2) takes the form

$$W(\eta) = \sum_{i=0}^{N} A_i \phi^i(\eta), \quad A_N \neq 0, \tag{2.3}$$

where A_j, $(j = 0, 1, \cdots, N)$ are parameters to be calculated later. The integer N can be found by balancing the highest nonlinear term and the highest-order derivative in Eq. (2.2). The function $\phi(\eta)$ is assumed to satisfy the generalized Riccati equation:

$$\phi'(\eta) = R + P\,\phi(\eta) + Q\,\phi^2(\eta), \tag{2.4}$$

as a simplest equation, where P, Q and R are all variable real constants. Next, substituting Eq. (2.3) into Eq. (2.2), take note of Eq. (2.4). Upon setting all coefficients of $\phi^i(\eta)$ to zero, we obtain some algebraic equations. Solving this algebraic system and putting into Eq. (2.3), also, using the known families of solutions of Eq. (2.4), we finally obtain an explicit solutions of Eq. (2.1).

On the other hand, The MSEM expresses the solution of Eq. (2.2) by making an ansatz for $W(\eta)$ as

$$W(\eta) = \sum_{j=0}^{N} B_i \left(\frac{\varphi'(\eta)}{\varphi(\eta)}\right)^j, \; B_N \neq 0, \tag{2.5}$$

where B_i's are parameters to be calculated, N is a positive integer that can be determined as in the RSEM case. $\varphi(\xi)$ is an unspecified function to be determined subsequently.

Substituting Eq. (2.5) into Eq. (2.2), with the already determined value of N, results a polynomial of φ^{-i} and $\varphi^{(i)}$, $i > 0$. Gathering the items with the same power of φ^{-i}, and equating to zero yields a system of algebraic-differential equations in the B_i's, ϕ and its derivatives. Solving the obtained system to get the values of B_i's and φ and putting the results into Eq. (2.3) will completely determine the exact solutions of Eq. (2.1).

3. Analytic optical solitons; An application

In the current part, the schemes presented in Section 2 and the given conformable rules are employed to process Eq. (1.1) analytically. For this purpose, assume that Eq. (1.1) has a solution of the form:

$$q(x,t) = W(\eta)e^{i\left(-\kappa x+\omega \frac{t^\rho}{\rho}+\theta_0\right)}, \tag{3.1}$$

where $W(\eta)$ is a real-valued function, $\eta = x - \nu\frac{t^\rho}{\rho}$ is the wave transform, ν is speed of wave propagation, κ, ω and θ_0 represent the soliton frequency, wave number and phase constant respectively. Consequently, Eq. (1.1) would be reduced to an ordinary differential equation (ODE). Decomposing the resulted ODE into real and imaginary implies

$$-\delta_1 W + \delta_2 W^3 + b_2 W^5 + b_3 W^7 + \delta_3 W'' + \delta_4 W^{(4)} + a_6 W^{(6)} = 0, \tag{3.2}$$

and

$$\begin{aligned}(a_5 - 6\kappa a_6)W^{(5)} + (a_3 - \kappa(4a_4 - 10\kappa + 20\kappa a_6))W^{(3)}\\ -(6a_6\kappa^5 - 5a_5\kappa^4 - 4a_4\kappa^3 + 3a_3\kappa^2 + 2a_2\kappa - a_1 + \nu + W^2(2\theta + 3\lambda + \mu))W' = 0.\end{aligned} \tag{3.3}$$

Applying the linearly independent rule on Eq. (3.3) by vanishing the derivatives coefficients results

$$\begin{aligned}\mu &= -3\lambda - 2\theta,\\ a_3 &= 4\kappa(10a_6\kappa^2 + a_4),\\ a_5 &= 6\kappa a_6,\\ \nu &= a_1 - 2\kappa(48a_6\kappa^4 + 4a_4\kappa^2 + a_2).\end{aligned} \tag{3.4}$$

The δ_i's in Eq. (3.2) are defined consequently by employing the constraints Eq. (3.4) to be as following:

$$\begin{aligned}\delta_1 &= 35a_6\kappa^6 + 3a_4\kappa^4 + a_2\kappa^2 - a_1\kappa + \omega,\\ \delta_2 &= 2\kappa(\theta + \lambda) + b_1,\\ \delta_3 &= a_2 + \kappa^2(6a_4 + 75\kappa^2 a_6),\\ \delta_4 &= a_4 + 15\kappa^2 a_6,\end{aligned} \tag{3.5}$$

The principle balance Algorithm between the higher derivative and highest nonlinear term in the real part Eq. (3.2) gives

$$N = 1. \tag{3.6}$$

In what follow, highly despersive optical solitons of the governed model using the RSEM and MSEM will be derived.

3.1. Using RSEM

As a result of Eq. (3.6), Eq. (3.2) should get the formal solution

$$W(\eta) = A_0 + A_1\,\phi(\eta), \qquad A_1 \neq 0. \tag{3.7}$$

Inserting Eqs. (3.7) along with (2.4) into Eq. (3.2), gathering the coefficients of $\phi^i(\eta)$ and equating them to zero. The following system of algebraic equations, for $j = 0, 1, \cdots, 7$, is achieved

$$\begin{aligned}
&720a_6Q^6 + A_1^6 b_3 = 0,\\
&360a_6PQ^5 + A_0A_1^5b_3 = 0,\\
&24Q^4(70a_6(2P^2+QR)+\delta_4) + A_1^4(21A_0^2b_3+b_2) = 0,\\
&12PQ^3(35a_6(P^2+2QR)+\delta_4) + A_0A_1^3(7A_0^2b_3+b_2) = 0,\\
&2Q^2(7a_6(43P^4+256P^2QR+88Q^2R^2)+\delta_3\\
&\quad +5\delta_4(5P^2+4QR)) + A_1^2(35A_0^4b_3+10A_0^2b_2+\delta_2) = 0,\\
&3PQ(7a_6(3P^4+56P^2QR+88Q^2R^2)+\delta_3\\
&\quad +5\delta_4(P^2+4QR)) + A_0A_1(21A_0^4b_3+10A_0^2b_2+3\delta_2) = 0,\\
&a_6(P^6+114P^4QR+720P^2Q^2R^2+272Q^3R^3) + 7A_0^6b_3 + 5A_0^4b_2\\
&\quad +3A_0^2\delta_2 - \delta_1 + \delta_3(P^2+2QR) + \delta_4(P^4+22P^2QR+16Q^2R^2) = 0,\\
&A_1PR(a_6(P^4+52P^2QR+136Q^2R^2)+\delta_3\\
&\quad +\delta_4(P^2+8QR)) + A_0^7b_3 + A_0^5b_2 + A_0^3\delta_2 - A_0\delta_1 = 0.
\end{aligned} \tag{3.8}$$

By solving this system to obtain the complementary solution's existence constraints of Eq. (1.1), and with the aid of Mathematica programming, in what follow, the most simple nontrivial cases are listed.

Case 1. *For $\Delta = P^2 - 4QR \neq 0$, $\omega = \omega$, and $\kappa = \kappa$, we get*

$$\begin{aligned}
A_0 &= \pm\frac{\sqrt{154\delta_1+60\Delta\delta_3-69\Delta^2\delta_4}}{\sqrt{\frac{34\Delta\delta_2}{P^2}}}, \quad A_1 = \frac{2QA_0}{P},\\
a_6 &= -\frac{2(2A_0^2\delta_2+P^2(\delta_3-5\Delta\delta_4))}{77\,P\,\Delta^2},\\
b_2 &= \frac{3P^4(35\Delta\,a_6-2\delta_4)}{4A_0^4}, \quad b_3 = -\frac{720Q^6a_6}{A_1^6}.
\end{aligned}$$

Case 2. *For $\Delta = P^2 - 4QR \neq 0$ and $d_0 = d_0 \neq 0$, we get,*

$$\begin{aligned}
A_1 &= \frac{2QA_0}{P}, \quad b_3 = -\frac{45P^6a_6}{4A_0^6}, \quad \delta_4 = \frac{35}{2}\Delta a_6 - \frac{2b_2A_0^4}{3P^4},\\
\delta_2 &= \frac{147P^4\Delta^2a_6 - 10\Delta b_2A_0^4 - 3P^4\delta_3}{6P^2A_0^2},\\
\delta_1 &= \frac{\Delta(159P^4\Delta^2a_6 - 8\Delta b_2A_0^4 - 6P^4\delta_3)}{12P^4}.
\end{aligned}$$

Equivalently, and out of Eq. (3.5), this case implies that

$$\begin{aligned}
\kappa &= \pm\frac{\sqrt{3p^4(-2a_4+35(p^2-4qr)a_6)-4A^4b_2}}{3p^2\sqrt{10a_6}},\\
\lambda &= -\frac{-147P^4\Delta^2a_6+6P^2(2\theta\kappa+b_1)A_0^2+10\Delta b_2A_0^4+3P^4\delta_3}{12P^2\kappa A_0^2},\\
\omega &= \kappa(a_1-\kappa(a_2+3\kappa^2a_4+35\kappa^4a_6)) + \frac{\Delta(159P^4\Delta^2a_6-8\Delta b_2A_0^4-6P^4\delta_3)}{12P^4}.
\end{aligned}$$

Case 3. *For $\Delta = P^2 - 4QR \neq 0$, $A_1 = A_1 \neq 0$, and $\kappa = \kappa$, we get*

$$A_0 = -\frac{360PQ^5a_6}{b_3A_1^5},\ \ b_3 = -\frac{720Q^6a_6}{A_1^6},\ \ b_2 = \frac{12Q^4(35\Delta a_6 - 2\delta_4}{)}A_1^4,$$
$$\delta_2 = -\frac{Q^2(77\Delta^2a_6 + 2(\delta_3 - 5\Delta\delta_4))}{A_1^2},\ \ \delta_1 = -\frac{1}{4}\Delta(17\Delta^2a_6 + 2\delta_3 - 4\Delta\delta_4).$$

Equivalently, and out of Eq. (3.5), this case implies that

$$b_1 = -\frac{2\kappa(\theta+\lambda)A_1^2 + Q^2(77\Delta^2a_6 + 2(\delta_3 - 5\Delta\delta_4))}{A_1^2},$$
$$\omega = -\frac{17}{4}\Delta^3a_6 + \kappa(a_1 - \kappa(a_2 + 3\kappa^2a_4 + 35\kappa^4a_6)) - \frac{1}{2}\Delta\delta_3 + \Delta^2\delta_4.$$

For the parameters set in Case 3, we list the obtained optical solitons of NLSE Eq. (1.1) among Eq. (3.4)-Eq. (3.5), with the use of Eq. (3.1) and Eq. (3.7) as follows:

Family 1. For $\Delta = P^2 - 4QR > 0$, $PQ \neq 0$ (or $QR \neq 0$) with two non-zero real constants A and B, we get:

$$q_1(x,t) = -\frac{A_1\sqrt{\Delta}\tanh\left(\frac{1}{2}\sqrt{\Delta}\left(\frac{(96a_6\kappa^5+8a_4\kappa^3+2a_2\kappa-a_1)t^\rho}{\rho}+x\right)\right)}{2Q} \times e^{i\left(-x\kappa+\frac{t^\rho\left(-\frac{17}{4}\Delta^3a_6+\kappa\left(a_1-\kappa\left(a_2+3\kappa^2a_4+35\kappa^4a_6\right)\right)-\frac{1}{2}\Delta\delta_3+\Delta^2\delta_4\right)}{\rho}+\theta_0\right)}, \tag{3.9}$$

$$q_2(x,t) = -\frac{A_1\sqrt{\Delta}\coth\left(\frac{1}{2}\sqrt{\Delta}\left(\frac{(96a_6\kappa^5+8a_4\kappa^3+2a_2\kappa-a_1)t^\rho}{\rho}+x\right)\right)}{2Q} \times e^{i\left(-x\kappa+\frac{t^\rho\left(-\frac{17}{4}\Delta^3a_6+\kappa\left(a_1-\kappa\left(a_2+3\kappa^2a_4+35\kappa^4a_6\right)\right)-\frac{1}{2}\Delta\delta_3+\Delta^2\delta_4\right)}{\rho}+\theta_0\right)}, \tag{3.10}$$

$$q_3(x,t) = -\frac{iA_1\sqrt{\Delta}\left(\operatorname{sech}\left(\sqrt{\Delta}\left(\frac{(96a_6\kappa^5+8a_4\kappa^3+2a_2\kappa-a_1)t^\rho}{\rho}+x\right)\right) - i\tanh\left(\sqrt{\Delta}\left(\frac{(96a_6\kappa^5+8a_4\kappa^3+2a_2\kappa-a_1)t^\rho}{\rho}+x\right)\right)\right)}{2Q} \times e^{i\left(-x\kappa+\frac{t^\rho\left(-\frac{17}{4}\Delta^3a_6+\kappa\left(a_1-\kappa\left(a_2+3\kappa^2a_4+35\kappa^4a_6\right)\right)-\frac{1}{2}\Delta\delta_3+\Delta^2\delta_4\right)}{\rho}+\theta_0\right)}, \tag{3.11}$$

$$q_4(x,t) = \frac{iA_1\sqrt{\Delta}\left(\operatorname{sech}\left(\sqrt{\Delta}\left(\frac{(96a_6\kappa^5+8a_4\kappa^3+2a_2\kappa-a_1)t^\rho}{\rho}+x\right)\right) + i\tanh\left(\sqrt{\Delta}\left(\frac{(96a_6\kappa^5+8a_4\kappa^3+2a_2\kappa-a_1)t^\rho}{\rho}+x\right)\right)\right)}{2Q} \times e^{i\left(-x\kappa+\frac{t^\rho\left(-\frac{17}{4}\Delta^3a_6+\kappa\left(a_1-\kappa\left(a_2+3\kappa^2a_4+35\kappa^4a_6\right)\right)-\frac{1}{2}\Delta\delta_3+\Delta^2\delta_4\right)}{\rho}+\theta_0\right)}, \tag{3.12}$$

$$q_5(x,t) = -\frac{A_1\sqrt{\Delta}\tanh\left(\frac{1}{4}\sqrt{\Delta}\left(\frac{(96a_6\kappa^5+8a_4\kappa^3+2a_2\kappa-a_1)t^\rho}{\rho}+x\right)\right)\left(\coth^2\left(\frac{1}{4}\sqrt{\Delta}\left(\frac{(96a_6\kappa^5+8a_4\kappa^3+2a_2\kappa-a_1)t^\rho}{\rho}+x\right)\right)+1\right)}{4Q} \times e^{i\left(-x\kappa+\frac{t^\rho\left(-\frac{17}{4}\Delta^3a_6+\kappa\left(a_1-\kappa\left(a_2+3\kappa^2a_4+35\kappa^4a_6\right)\right)-\frac{1}{2}\Delta\delta_3+\Delta^2\delta_4\right)}{\rho}+\theta_0\right)}, \tag{3.13}$$

$$q_6(x,t) = \frac{A_1\left(\sqrt{\Delta\left(A^2+B^2\right)} - A\sqrt{\Delta}\cosh\left(\sqrt{\Delta}\left(\frac{\left(96a_6\kappa^5+8a_4\kappa^3+2a_2\kappa-a_1\right)t^\rho}{\rho}+x\right)\right)\right)}{2Q\left(A\sinh\left(\sqrt{\Delta}\left(\frac{\left(96a_6\kappa^5+8a_4\kappa^3+2a_2\kappa-a_1\right)t^\rho}{\rho}+x\right)\right)+B\right)} \times e^{i\left(-x\kappa+\frac{t^\rho\left(-\frac{17}{4}\Delta^3a_6+\kappa\left(a_1-\kappa\left(a_2+3\kappa^2a_4+35\kappa^4a_6\right)\right)-\frac{1}{2}\Delta\delta_3+\Delta^2\delta_4\right)}{\rho}+\theta_0\right)}, \tag{3.14}$$

$$q_7(x,t) = -\frac{A_1\left(A\sqrt{\Delta}\cosh\left(\sqrt{\Delta}\left(\frac{\left(96a_6\kappa^5+8a_4\kappa^3+2a_2\kappa-a_1\right)t^\rho}{\rho}+x\right)\right)+\sqrt{\Delta\left(A^2+B^2\right)}\right)}{2Q\left(A\sinh\left(\sqrt{\Delta}\left(\frac{\left(96a_6\kappa^5+8a_4\kappa^3+2a_2\kappa-a_1\right)t^\rho}{\rho}+x\right)\right)+B\right)} \times e^{i\left(-x\kappa+\frac{t^\rho\left(-\frac{17}{4}\Delta^3a_6+\kappa\left(a_1-\kappa\left(a_2+3\kappa^2a_4+35\kappa^4a_6\right)\right)-\frac{1}{2}\Delta\delta_3+\Delta^2\delta_4\right)}{\rho}+\theta_0\right)}, \tag{3.15}$$

$$q_8(x,t) = \frac{1}{2}A_1\left(\frac{P}{Q} - \frac{4R}{P-\sqrt{\Delta}\tanh\left(\frac{1}{2}\sqrt{\Delta}\left(\frac{\left(96a_6\kappa^5+8a_4\kappa^3+2a_2\kappa-a_1\right)t^\rho}{\rho}+x\right)\right)}\right) \times e^{i\left(-x\kappa+\frac{t^\rho\left(-\frac{17}{4}\Delta^3a_6+\kappa\left(a_1-\kappa\left(a_2+3\kappa^2a_4+35\kappa^4a_6\right)\right)-\frac{1}{2}\Delta\delta_3+\Delta^2\delta_4\right)}{\rho}+\theta_0\right)}, \tag{3.16}$$

$$q_9(x,t) = \frac{1}{2}A_1\left(\frac{P}{Q} - \frac{4R}{P-\sqrt{\Delta}\coth\left(\frac{1}{2}\sqrt{\Delta}\left(\frac{\left(96a_6\kappa^5+8a_4\kappa^3+2a_2\kappa-a_1\right)t^\rho}{\rho}+x\right)\right)}\right) \times e^{i\left(-x\kappa+\frac{t^\rho\left(-\frac{17}{4}\Delta^3a_6+\kappa\left(a_1-\kappa\left(a_2+3\kappa^2a_4+35\kappa^4a_6\right)\right)-\frac{1}{2}\Delta\delta_3+\Delta^2\delta_4\right)}{\rho}+\theta_0\right)}, \tag{3.17}$$

$$q_{10}(x,t) = \frac{1}{2}A_1\left(\frac{P}{Q} - \frac{4R\cosh\left(\sqrt{\Delta}\left(\frac{\left(96a_6\kappa^5+8a_4\kappa^3+2a_2\kappa-a_1\right)t^\rho}{\rho}+x\right)\right)}{P\cosh\left(\sqrt{\Delta}\left(\frac{\left(96a_6\kappa^5+8a_4\kappa^3+2a_2\kappa-a_1\right)t^\rho}{\rho}+x\right)\right)-\sqrt{\Delta}\left(\sinh\left(\sqrt{\Delta}\left(\frac{\left(96a_6\kappa^5+8a_4\kappa^3+2a_2\kappa-a_1\right)t^\rho}{\rho}+x\right)\right)+i\right)}\right) \times e^{i\left(-x\kappa+\frac{t^\rho\left(-\frac{17}{4}\Delta^3a_6+\kappa\left(a_1-\kappa\left(a_2+3\kappa^2a_4+35\kappa^4a_6\right)\right)-\frac{1}{2}\Delta\delta_3+\Delta^2\delta_4\right)}{\rho}+\theta_0\right)}, \tag{3.18}$$

$$q_{11}(x,t) = \frac{A_1\left(\sqrt{\Delta}P\cosh\left(\frac{1}{2}\sqrt{\Delta}\left(\frac{\left(96a_6\kappa^5+8a_4\kappa^3+2a_2\kappa-a_1\right)t^\rho}{\rho}+x\right)\right)-\Delta\sinh\left(\frac{1}{2}\sqrt{\Delta}\left(\frac{\left(96a_6\kappa^5+8a_4\kappa^3+2a_2\kappa-a_1\right)t^\rho}{\rho}+x\right)\right)\right)}{2Q\left(\sqrt{\Delta}\cosh\left(\frac{1}{2}\sqrt{\Delta}\left(\frac{\left(96a_6\kappa^5+8a_4\kappa^3+2a_2\kappa-a_1\right)t^\rho}{\rho}+x\right)\right)-P\sinh\left(\frac{1}{2}\sqrt{\Delta}\left(\frac{\left(96a_6\kappa^5+8a_4\kappa^3+2a_2\kappa-a_1\right)t^\rho}{\rho}+x\right)\right)\right)} \times e^{i\left(-x\kappa+\frac{t^\rho\left(-\frac{17}{4}\Delta^3a_6+\kappa\left(a_1-\kappa\left(a_2+3\kappa^2a_4+35\kappa^4a_6\right)\right)-\frac{1}{2}\Delta\delta_3+\Delta^2\delta_4\right)}{\rho}+\theta_0\right)}. \tag{3.19}$$

Family 2. For $\Delta = P^2 - 4\,QR < 0$, $PQ \neq 0$, $(or QR \neq 0)$ with two non-zero real constants A and B satisfy $A^2 - B^2 > 0$, we get:

$$q_{12}(x,t) = \frac{A_1\sqrt{-\Delta}\tan\left(\frac{1}{2}\sqrt{-\Delta}\left(\frac{\left(96a_6\kappa^5+8a_4\kappa^3+2a_2\kappa-a_1\right)t^\rho}{\rho}+x\right)\right)}{2Q} \times e^{i\left(-x\kappa+\frac{t^\rho\left(-\frac{17}{4}\Delta^3a_6+\kappa\left(a_1-\kappa\left(a_2+3\kappa^2a_4+35\kappa^4a_6\right)\right)-\frac{1}{2}\Delta\delta_3+\Delta^2\delta_4\right)}{\rho}+\theta_0\right)}, \tag{3.20}$$

$$q_{13}(x,t) = -\frac{A_1\sqrt{-\Delta}\cot\left(\frac{1}{2}\sqrt{-\Delta}\left(\frac{(96a_6\kappa^5+8a_4\kappa^3+2a_2\kappa-a_1)t^\rho}{\rho}+x\right)\right)}{2Q} \times e^{i\left(-x\kappa+\frac{t^\rho\left(-\frac{17}{4}\Delta^3a_6+\kappa\left(a_1-\kappa\left(a_2+3\kappa^2a_4+35\kappa^4a_6\right)\right)-\frac{1}{2}\Delta\delta_3+\Delta^2\delta_4\right)}{\rho}+\theta_0\right)}, \tag{3.21}$$

$$q_{14}(x,t) = \frac{A_1\sqrt{-\Delta}\left(\tan\left(\sqrt{-\Delta}\left(\frac{(96a_6\kappa^5+8a_4\kappa^3+2a_2\kappa-a_1)t^\rho}{\rho}+x\right)\right)+\sec\left(\sqrt{-\Delta}\left(\frac{(96a_6\kappa^5+8a_4\kappa^3+2a_2\kappa-a_1)t^\rho}{\rho}+x\right)\right)\right)}{2Q} \times e^{i\left(-x\kappa+\frac{t^\rho\left(-\frac{17}{4}\Delta^3a_6+\kappa\left(a_1-\kappa\left(a_2+3\kappa^2a_4+35\kappa^4a_6\right)\right)-\frac{1}{2}\Delta\delta_3+\Delta^2\delta_4\right)}{\rho}+\theta_0\right)}, \tag{3.22}$$

$$q_{15}(x,t) = \frac{A_1\sqrt{-\Delta}\left(\tan\left(\sqrt{-\Delta}\left(\frac{(96a_6\kappa^5+8a_4\kappa^3+2a_2\kappa-a_1)t^\rho}{\rho}+x\right)\right)-\sec\left(\sqrt{-\Delta}\left(\frac{(96a_6\kappa^5+8a_4\kappa^3+2a_2\kappa-a_1)t^\rho}{\rho}+x\right)\right)\right)}{2Q} \times e^{i\left(-x\kappa+\frac{t^\rho\left(-\frac{17}{4}\Delta^3a_6+\kappa\left(a_1-\kappa\left(a_2+3\kappa^2a_4+35\kappa^4a_6\right)\right)-\frac{1}{2}\Delta\delta_3+\Delta^2\delta_4\right)}{\rho}+\theta_0\right)}, \tag{3.23}$$

$$q_{16}(x,t) = -\frac{A_1\sqrt{-\Delta}\tan\left(\frac{1}{4}\sqrt{-\Delta}\left(\frac{(96a_6\kappa^5+8a_4\kappa^3+2a_2\kappa-a_1)t^\rho}{\rho}+x\right)\right)\left(\cot^2\left(\frac{1}{4}\sqrt{-\Delta}\left(\frac{(96a_6\kappa^5+8a_4\kappa^3+2a_2\kappa-a_1)t^\rho}{\rho}+x\right)\right)-1\right)}{4Q} \times e^{i\left(-x\kappa+\frac{t^\rho\left(-\frac{17}{4}\Delta^3a_6+\kappa\left(a_1-\kappa\left(a_2+3\kappa^2a_4+35\kappa^4a_6\right)\right)-\frac{1}{2}\Delta\delta_3+\Delta^2\delta_4\right)}{\rho}+\theta_0\right)}, \tag{3.24}$$

$$q_{17}(x,t) = \frac{A_1\left(\sqrt{-\Delta\left(A^2-B^2\right)}-A\sqrt{-\Delta}\cos\left(\sqrt{-\Delta}\left(\frac{(96a_6\kappa^5+8a_4\kappa^3+2a_2\kappa-a_1)t^\rho}{\rho}+x\right)\right)\right)}{2Q\left(A\sin\left(\sqrt{-\Delta}\left(\frac{(96a_6\kappa^5+8a_4\kappa^3+2a_2\kappa-a_1)t^\rho}{\rho}+x\right)\right)+B\right)} \times e^{i\left(-x\kappa+\frac{t^\rho\left(-\frac{17}{4}\Delta^3a_6+\kappa\left(a_1-\kappa\left(a_2+3\kappa^2a_4+35\kappa^4a_6\right)\right)-\frac{1}{2}\Delta\delta_3+\Delta^2\delta_4\right)}{\rho}+\theta_0\right)}, \tag{3.25}$$

$$q_{18}(x,t) = -\frac{A_1\left(A\sqrt{-\Delta}\cos\left(\sqrt{-\Delta}\left(\frac{(96a_6\kappa^5+8a_4\kappa^3+2a_2\kappa-a_1)t^\rho}{\rho}+x\right)\right)+\sqrt{-\Delta(A^2-B^2)}\right)}{2Q\left(A\sin\left(\sqrt{-\Delta}\left(\frac{(96a_6\kappa^5+8a_4\kappa^3+2a_2\kappa-a_1)t^\rho}{\rho}+x\right)\right)+B\right)} \times e^{i\left(-x\kappa+\frac{t^\rho\left(-\frac{17}{4}\Delta^3a_6+\kappa\left(a_1-\kappa\left(a_2+3\kappa^2a_4+35\kappa^4a_6\right)\right)-\frac{1}{2}\Delta\delta_3+\Delta^2\delta_4\right)}{\rho}+\theta_0\right)}, \tag{3.26}$$

$$q_{19}(x,t) = \frac{1}{2}A_1\left(\frac{P}{Q}-\frac{4R}{\sqrt{-\Delta}\tan\left(\frac{1}{2}\sqrt{-\Delta}\left(\frac{(96a_6\kappa^5+8a_4\kappa^3+2a_2\kappa-a_1)t^\rho}{\rho}+x\right)\right)+P}\right) \times e^{i\left(-x\kappa+\frac{t^\rho\left(-\frac{17}{4}\Delta^3a_6+\kappa\left(a_1-\kappa\left(a_2+3\kappa^2a_4+35\kappa^4a_6\right)\right)-\frac{1}{2}\Delta\delta_3+\Delta^2\delta_4\right)}{\rho}+\theta_0\right)}, \tag{3.27}$$

$$q_{20}(x,t) = \frac{1}{2}A_1\left(\frac{P}{Q}-\frac{4R}{P-\sqrt{-\Delta}\cot\left(\frac{1}{2}\sqrt{-\Delta}\left(\frac{(96a_6\kappa^5+8a_4\kappa^3+2a_2\kappa-a_1)t^\rho}{\rho}+x\right)\right)}\right) \times e^{i\left(-x\kappa+\frac{t^\rho\left(-\frac{17}{4}\Delta^3a_6+\kappa\left(a_1-\kappa\left(a_2+3\kappa^2a_4+35\kappa^4a_6\right)\right)-\frac{1}{2}\Delta\delta_3+\Delta^2\delta_4\right)}{\rho}+\theta_0\right)}, \tag{3.28}$$

$$q_{21}(x,t)=\frac{1}{2}A_1\left(\frac{P}{Q}-\frac{4R\cos\left(\sqrt{-\Delta}\left(\frac{(96a_6\kappa^5+8a_4\kappa^3+2a_2\kappa-a_1)t^\rho}{\rho}+x\right)\right)}{P\cos\left(\sqrt{-\Delta}\left(\frac{(96a_6\kappa^5+8a_4\kappa^3+2a_2\kappa-a_1)t^\rho}{\rho}+x\right)\right)+\sqrt{-\Delta}\left(\sin\left(\sqrt{-\Delta}\left(\frac{(96a_6\kappa^5+8a_4\kappa^3+2a_2\kappa-a_1)t^\rho}{\rho}+x\right)\right)+1\right)}\right)$$
$$\times e^{i\left(-x\kappa+\frac{t^\rho\left(-\frac{17}{4}\Delta^3a_6+\kappa\left(a_1-\kappa\left(a_2+3\kappa^2a_4+35\kappa^4a_6\right)\right)-\frac{1}{2}\Delta\delta_3+\Delta^2\delta_4\right)}{\rho}+\theta_0\right)}, \tag{3.29}$$

$$q_{22}(x,t)=\frac{A_1\left(\sqrt{-\Delta}P\cos\left(\frac{1}{2}\sqrt{-\Delta}\left(\frac{(96a_6\kappa^5+8a_4\kappa^3+2a_2\kappa-a_1)t^\rho}{\rho}+x\right)\right)-\Delta\mathrm{Sin}\left(\frac{1}{2}\sqrt{-\Delta}\left(\frac{(96a_6\kappa^5+8a_4\kappa^3+2a_2\kappa-a_1)t^\rho}{\rho}+x\right)\right)\right)}{2Q\left(\sqrt{-\Delta}\cos\left(\frac{1}{2}\sqrt{-\Delta}\left(\frac{(96a_6\kappa^5+8a_4\kappa^3+2a_2\kappa-a_1)t^\rho}{\rho}+x\right)\right)-P\sin\left(\frac{1}{2}\sqrt{-\Delta}\left(\frac{(96a_6\kappa^5+8a_4\kappa^3+2a_2\kappa-a_1)t^\rho}{\rho}+x\right)\right)\right)}$$
$$\times e^{i\left(-x\kappa+\frac{t^\rho\left(-\frac{17}{4}\Delta^3a_6+\kappa\left(a_1-\kappa\left(a_2+3\kappa^2a_4+35\kappa^4a_6\right)\right)-\frac{1}{2}\Delta\delta_3+\Delta^2\delta_4\right)}{\rho}+\theta_0\right)}. \tag{3.30}$$

Family 3. For $PQ\neq 0$ and $R=0$:

$$q_{23}(x,t)=\frac{P\left(-1+\frac{2}{1+e^{P\left(x+\frac{t^\rho(-a_1+2\kappa a_2+8\kappa^3a_4+96\kappa^5a_6)}{\rho}\right)}\alpha}\right)A_1}{2Q}\times e^{i\left(-x\kappa+\frac{t^\rho\left(-\frac{17}{4}\Delta^3a_6+\kappa\left(a_1-\kappa\left(a_2+3\kappa^2a_4+35\kappa^4a_6\right)\right)-\frac{1}{2}\Delta\delta_3+\Delta^2\delta_4\right)}{\rho}+\theta_0\right)}, \tag{3.31}$$

$$q_{24}(x,t)=\frac{P\left(-1+\frac{2\alpha}{e^{P\left(x+\frac{t^\rho(-a_1+2\kappa a_2+8\kappa^3a_4+96\kappa^5a_6)}{\rho}\right)}+\alpha}\right)A_1}{2Q}\times e^{i\left(-x\kappa+\frac{t^\rho\left(-\frac{17}{4}\Delta^3a_6+\kappa\left(a_1-\kappa\left(a_2+3\kappa^2a_4+35\kappa^4a_6\right)\right)-\frac{1}{2}\Delta\delta_3+\Delta^2\delta_4\right)}{\rho}+\theta_0\right)}. \tag{3.32}$$

Family 4. For $P=R=0$ and $Q\neq 0$:

$$q_{25}(x,t)=\frac{P\left(-1+\frac{2\alpha}{e^{P\left(x+\frac{t^\rho(-a_1+2\kappa a_2+8\kappa^3a_4+96\kappa^5a_6)}{\rho}\right)}+\alpha}\right)A_1}{2Q}\times e^{i\left(-x\kappa+\frac{t^\rho\left(-\frac{17}{4}\Delta^3a_6+\kappa\left(a_1-\kappa\left(a_2+3\kappa^2a_4+35\kappa^4a_6\right)\right)-\frac{1}{2}\Delta\delta_3+\Delta^2\delta_4\right)}{\rho}+\theta_0\right)}, \tag{3.33}$$

where α is arbitrary constant, and $\rho\in(0,1]$.

3.2. Using MSEM

To handle the underlying model by the MSEM, and making use of Eq. (3.6), the Eq. (2.5) would be

$$W(\eta)=B_0+B_1\frac{\varphi'(\eta)}{\varphi(\eta)},\qquad B_1\neq 0. \tag{3.34}$$

Inserting Eq.(3.34) into (3.2), collecting different powers of $\varphi^{-i}(\eta)$ where $i=0,1,\cdots,7$, with $\varphi'\neq 0$, and equating them to zero, a set of algebraic-differential equations is obtained as

$$720a_6+b_3B_1^6=0, \tag{3.35}$$

$$b_3B_0^7+b_2B_0^5+B_0^3\delta_2-B_0\delta_1=0. \tag{3.36}$$

$$b_3B_0B_1^5\varphi'(\eta)-360a_6\varphi''(\eta)=0, \tag{3.37}$$

$$2520a_6\varphi''(\eta)^2+840a_6\varphi^{(3)}(\eta)\varphi'(\eta)+(21b_3B_0^2B_1^4+b_2B_1^4+24\delta_4)\varphi'(\eta)^2=0, \tag{3.38}$$

$$\begin{aligned}&\varphi'(\eta)^2(-42a_6\varphi^{(4)}(\eta)-12\delta_4\varphi''(\eta))-126a_6\varphi''(\eta)^3\\&-252a_6\varphi^{(3)}(\eta)\varphi'(\eta)\varphi''(\eta)+(7b_3B_0^3B_1^3+b_2B_0B_1^3)\varphi'(\eta)^3=0,\end{aligned} \tag{3.39}$$

$$\begin{aligned}&\varphi'(\eta)^2(42a_6\varphi^{(5)}(\eta)+20\delta_4\varphi^{(3)}(\eta))+\varphi'(\eta)(140a_6\varphi^{(3)}(\eta)^2\\&+210a_6\varphi^{(4)}(\eta)\varphi''(\eta)+30\delta_4\varphi''(\eta)^2)+210a_6\varphi^{(3)}(\eta)\varphi''(\eta)^2\\&+(35b_3B_1^2B_0^4+10b_2B_1^2B_0^2+B_1^2\delta_2+2\delta_3)\varphi'(\eta)^3=0,\end{aligned} \tag{3.40}$$

$$\begin{aligned}&\varphi''(\eta)(-21a_6\varphi^{(5)}(\eta)-10\delta_4\varphi^{(3)}(\eta))-35a_6\varphi^{(3)}(\eta)\varphi^{(4)}(\eta)+(21b_3B_1B_0^5\\&+3B_1B_0\delta_2)\varphi'(\eta)^2+\varphi'(\eta)(-7a_6\varphi^{(6)}(\eta)-5\delta_4\varphi^{(4)}(\eta)-3\delta_3\varphi''(\eta))=0,\end{aligned} \tag{3.41}$$

$$a_6\varphi^{(7)}(\eta)+(7b_3B_0^6+5b_2B_0^4+3B_0^2\delta_2-\delta_1)\varphi'(\eta)+\delta_4\varphi^{(5)}(\eta)+\delta_3\varphi^{(3)}(\eta)=0. \tag{3.42}$$

Treating this mixed-system implies the following two cases:

Case 4. *With $B_0=0$, Eq.(3.37) results*

$$\varphi(\eta)=C_1\eta+C_2, \tag{3.43}$$

where C_1 and C_2 are arbitrary constants. Substitute into remaining equations to get

$$B_1=\mp\sqrt{-\frac{2\delta_3}{\delta_2}},\ \ a_6=\frac{b_3\delta_3^3}{90\delta_2^3},\ \ b_2=-\frac{36(\delta_2^2-1)}{\delta_3^2},\ \ \delta_1=0,\ \ \delta_4=6-\frac{6}{\delta_2^2},$$

provided that $\delta_3\delta_2<0$. Equivalently, and out of Eq. (3.5), this case implies that

$$\kappa=\mp\frac{\sqrt{-6(a_4-6)\delta_2^3-36\delta_2}}{\delta_3^{\frac{3}{2}}\sqrt{b_3}},\ \ \omega=3a_4\kappa^4+a_2\kappa^2-a_1\kappa+\frac{7b_3\delta_3^3\kappa^6}{18\delta_2^3}.$$

Accordingly, the exact solution of Eq. (3.2) is achieved as

$$W(\eta)=\frac{C_1\sqrt{-\frac{2\delta_3}{\delta_2}}}{C_1\eta+C_2},$$

and therefore, the formal optical soliton of Eq. (1.1) is

$$q_{26}(x,t)=\frac{C_1\sqrt{-\frac{2\delta_3}{\delta_2}}e^{i\left(\theta_0+\frac{\omega\,t^\rho}{\rho}-\kappa\,x\right)}}{C_1\left(x-\frac{t^\rho\left(a_1-2\kappa\left(4a_4\kappa^2+a_2+\frac{8b_3\delta_3^3\kappa^4}{15\delta_2^3}\right)\right)}{\rho}\right)+C_2}. \tag{3.44}$$

Case 5. *With nontrivial arbitrary B_0, B_1, and κ Eq. (3.37) results*

$$\varphi(\eta)=C_2-\frac{B_1C_1e^{-\frac{2B_0\eta}{B_1}}}{2B_0}, \tag{3.45}$$

where C_1 and C_2 are arbitrary constants. Substitute into remaining equations to get

$$b_2 = \frac{3\left(112\delta_4 - \frac{5\left(B_1^4\delta_2 + 2B_1^2\delta_3\right)}{B_0^2}\right)}{11B_1^4}, \quad b_3 = \frac{45\left(B_1^4\delta_2 + 2B_1^2\delta_3 - 40B_0^2\delta_4\right)}{77B_0^4 B_1^4},$$
$$a_6 = -\frac{B_1^6\delta_2 + 2B_1^4\delta_3 - 40B_0^2 B_1^2\delta_4}{1232B_0^4}, \quad \delta_1 = \frac{B_0^2\left(17B_1^4\delta_2 - 120B_1^2\delta_3 + 552B_0^2\delta_4\right)}{77B_1^4}.$$

Equivalently, and out of Eq. (3.5), this case implies that

$$\omega = \frac{3696a_4\kappa^4 + \frac{8B_0^2\delta_4\left(175B_1^6\kappa^6 - 1104B_0^6\right) - 5B_1^2\left(7B_1^6\kappa^6\left(B_1^2\delta_2 + 2\delta_3\right) - 384B_0^6\delta_3\right)}{B_0^4 B_1^4} - 272B_0^2\delta_2}{1232} + a_2\kappa^2 - a_1\kappa.$$

The solution of Eq. (3.2) is carried out as

$$W(\eta) = B_0\left(\frac{1}{\frac{B_0 C_2 e^{\frac{2B_0\eta}{B_1}}}{B_1 C_1} - \frac{1}{2}} + 1\right),$$

and therefore, the formal optical soliton of Eq. (1.1) is

$$q_{27}(x,t) = B_0 e^{i\left(\theta_0 + \frac{\omega t^\rho}{\rho} - \kappa x\right)}\left(\frac{1}{\frac{B_0 C_2 \exp\left(\frac{2B_0\left(x - \frac{t^\rho\left(\frac{2}{77}\kappa\left(\frac{3B_1^2\kappa^4\left(B_1^4\delta_2 + 2B_1^2\delta_3 - 40B_0^2\delta_4\right) - 308a_4 B_0^4\kappa^2}{B_0^4} - 77a_2\right) + a_1\right)}{\rho}\right)}{B_1}\right)}{B_1 C_1} - \frac{1}{2}} + 1\right). \tag{3.46}$$

4. Conclusion

This paper studied the time-conformable perturbed highly dispersive optical solitons of NLSE with six dispersion terms and CQS law of refractive index. Two integration schemes, known by RSEM and MSEM, with different algorithms have been successfully employed for this purpose. The existence constraints of obtained solitons are included. Various bright, dark, and singular formal solitons are derived. To the best of our knowledge, the results obtained here didn't appear in any other works. All the solutions obtained are checked with the aid of Mathematica symbolic computation program. Undoubtedly, in describing and understanding certain physical characteristics of the considered models in various scientific fields, these existing solutions may play a prominent role.

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