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On the exact solutions and conservation laws of a generalized (1+2)-dimensional Jaulent-Miodek equation with a power law nonlinearity

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Abstract

In this paper, a generalized (1+2)-dimensional Jaulent-Miodek equation with a power law nonlinearity is examined, which arises in numerous problems in nonlinear science. The computed conservation laws reside in enormously crucial areas both at the foundations of nonlinear science such as biology, physics and other related areas. Exact solutions are acquired using the Lie symmetry method. In addition to exact solutions, we also present conservation laws. The arbitrary functions in the multipliers lead to infinitely many conservation laws.

Keywords: A generalized (1+2)-dimensional Jaulent-Miodek equation with a power law nonlinearity, Lie symmetry method, Conservation laws
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1. Introduction

The scholarship of nonlinear evolution equations has made an ample headway in the last few decades [18, 19, 8, 14, 23, 24, 22, 21, 3, 10, 26, 27, 9, 4, 5, 12, 13, 29, 28, 25, 2, 16, 17, 1, 6, 7, 11, 15]. A plethora of procedures have been used to carry out the integration of such equations. The tanh

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method, Hirota's bilinear method and the Lie symmetry method are one of the most popular methods that are dedicated to the construction of explicit solutions.

Lie symmetry method plays a key role in the production of exact solutions to nonlinear evolution equations. Centered on the innovative work of Lie on continuous groups, Lie symmetry method bids an integrated elucidation for the seemingly wide-ranging and ad hoc integration methods used to solve differential equations.

In (1+2)-dimensions, one finds the invariance of a partial differential equation

$$\Omega(t, x, y, u_t, u_x, u_y, \cdots) = 0 \tag{1.1}$$

under the group of infinitesimal transformations

$$\begin{aligned}
\bar{t} &= t + \xi^1(t, x, y, u)\epsilon + O(\epsilon^2),\\
\bar{x} &= x + \xi^2(t, x, y, u)\epsilon + O(\epsilon^2),\\
\bar{y} &= y + \xi^3(t, x, y, u)\epsilon + O(\epsilon^2),\\
\bar{u} &= u + \eta(t, x, y, u)\epsilon + O(\epsilon^2).
\end{aligned}$$

A set of over-determined system of linear partial differential equations is constructed as a consequence of the above transformations for the infinitesimals ξ^1, ξ^2, ξ^3 and η which when computed, gives rise to the symmetries of (1.1). Basically once a symmetry is computed for a differential equation, invariance of the solution leads to the invariant surface condition

$$\xi^1 u_t + \xi^2 u_x + \xi^3 u_y = \eta. \tag{1.2}$$

Solutions of (1.2) provide a solution ansatz, which, when inserted into (1.1) leads to a reduction of the original equation (1.1). Moreover, the reduction of a partial differential equation with respect to p-dimensional (solvable) subalgebra of its Lie symmetry algebra leads to plummeting the number of independent variables by p.

A (1+2)-dimensional nonlinear model generated by the Jaulent-Miodek hierarchy is given by [20]

$$w_t + \frac{1}{4}\left(w_{xx} - 2w_x^3\right)_x + \frac{3}{4}\left(\frac{1}{4}\partial_x^{-1} w_{yy} + w_x \partial_x^{-1} w_y\right) = 0. \tag{1.3}$$

Multiple kink solutions and multiple singular kink solutions that are characterized by distinct physical structures were reported in [20]. Here ∂_x^{-1} is the inverse of ∂_x with $\partial_x^{-1}\partial_x = \partial_x\partial_x^{-1} = 1$ and $(\partial_x^{-1} f)(x) = \int_{-\infty}^{x} f(t)dt$, under the decaying condition at infinity. Motivated by [20], after cosmetic changes using the potential $w = u_x$, we consider a generalized (1+2)-dimensional Jaulent-Miodek equation with a power law nonlinearity

$$a u_{xt} - u_{xxxx} + b u_x^n u_{xx} - c u_{xx} u_y - d u_x u_{xy} + e u_{yy} = 0. \tag{1.4}$$

Here a, b, c, d and e are nonzero arbitrary constants. We aim to perform Lie symmetry analysis of (1.4) and derive conservation laws of (1.4) using the multiplier approach.

2. Symmetry reductions and exact solutions of (1.4)

The vector field

$$\mathbf{X} = \xi^1(t, x, y, u)\frac{\partial}{\partial t} + \xi^2(t, x, y, u)\frac{\partial}{\partial x} + \xi^3(t, x, y, u)\frac{\partial}{\partial y} + \eta(t, x, y, u)\frac{\partial}{\partial u} \tag{2.1}$$

is a Lie point symmetry of (1.4) if

$$\mathbf{X}^{[4]}\left(au_{xt}-u_{xxxx}+bu_x^nu_{xx}-cu_{xx}u_y-du_xu_{xy}+eu_{yy}\right)\Big|_{(1.4)}=0. \tag{2.2}$$

$\mathbf{X}^{[4]}$ is the fourth prolongation of (2.1). Expanding (2.2) and splitting on the derivatives of u results in the following overdetermined system of linear partial differential equations:

$$\begin{aligned}
&\xi^2_x=0,\\
&\tau_y=0,\\
&\xi^1_u=0,\\
&\tau_x=0,\\
&\xi^2_u=0,\\
&\tau_u=0,\\
&\eta_x=0,\\
&\xi^1_y=0,\\
&\eta_{uu}=0,\\
&\xi^1_{xx}=0,\\
&\eta_{yy}=0,\\
&\eta_{yu}-\xi^1_{xy}=0,\\
&\xi^2_{yy}-2\eta_{yu}=0,\\
&a\xi^1_t+c\eta_y=0,\\
&a\xi^2_t+2e\xi^1_y=0,\\
&e\xi^1_{yy}-a\eta_{tu}+a\xi^1_{tx}=0,\\
&2\xi^1_x-\eta_u-\xi^2_y=0,\\
&2\xi^1_x+\eta_u-\xi^2_y=0,\\
&\tau_t-\xi^1_x+\eta_u-\xi^2_y=0,\\
&n\xi^1_x-n\eta_u+\eta_u-\xi^2_y=0.
\end{aligned}$$

Solving the above equations we obtain the values of ξ^1,ξ^2,ξ^3 and η with the aid of Maple. We now state the result in the following theorem.

Theorem. The infinitesimal symmetries of (1.4) constitute the principal Lie algebra spanned by the following linearly independent operators:

$$\begin{aligned}
&\mathbf{X_1}=\frac{\partial}{\partial t},\\
&\mathbf{X_2}=\frac{\partial}{\partial y},\\
&\mathbf{X_{f(t)}}=-cf(t)\frac{\partial}{\partial x}+yaf'(t)\frac{\partial}{\partial u},\\
&\mathbf{X_{g(t)}}=g(t)\frac{\partial}{\partial u},
\end{aligned}$$

For specific value of the parameter $n=2$, the principal Lie algebra can be inflated. This results in the following:

$$\mathbf{X_3} = 3t\frac{\partial}{\partial t} + x\frac{\partial}{\partial x} + 2y\frac{\partial}{\partial y},$$
$$\mathbf{X_4} = 2yba\frac{\partial}{\partial x} + t(cd - 4be + d^2)\frac{\partial}{\partial y} - ax(c+d)\frac{\partial}{\partial u}.$$

$\mathbf{X_{f(t)}}$ gives rise to the invariants

$$J_1 = y, J_2 = t, W_1 = u + \frac{yaf'(t)x}{f(t)c}. \tag{2.3}$$

Thus the invariant solution of (1.3) under $\mathbf{X_{f(t)}}$ is given by

$$u(x,y,t) = \frac{h(t)f(t)yc - yaxf'(t) + p(t)f(t)c}{f(t)c}, \tag{2.4}$$

where

$$f(t) = \left(\frac{d\,(C_1\,t + C_2)}{c}\right)^{\frac{c}{d}}.$$

$\mathbf{X_3}$ gives rise to the following invariants:

$$f = \frac{t}{y^{\frac{3}{2}}}, r = \frac{x}{\sqrt{y}}, \theta = u. \tag{2.5}$$

Treating θ as the new dependent variable and f, r as new the independent variables we have

$$\begin{aligned} &9ef^2\theta_{ff} + 6efr\theta_{fr} + 6cf\theta_{rr}\theta_f + 6df\theta_{fr}\theta_r + er^2\theta_{rr} + 2cr\theta_{rr}\theta_r + 2dr\theta_{rr}\theta_r \\ &+4b\theta_{rr}\theta_r^2 + 15ef\theta_f + 3er\theta_r + 2d\theta_{rr}^2 + 4a\theta_{fr} - 4\theta_{rrrr} = 0. \end{aligned} \tag{2.6}$$

The above equation has the following symmetries:

$$\mathbf{\Gamma}_1 = -3cf^{\frac{1}{3}}\frac{\partial}{\partial r} + af^{-\frac{2}{3}}\frac{\partial}{\partial \theta},$$
$$\mathbf{\Gamma}_2 = \frac{\partial}{\partial \theta}.$$

A linear combination $A\mathbf{\Gamma}_1 + B\mathbf{\Gamma}_2$ of the above symmetries, where A and B are arbitrary constants yields the following invariants:

$$k = f, \phi = \frac{1}{3}\frac{3A\theta cf^{\frac{4}{3}} + raAf^{\frac{1}{3}} + rBf}{cAf^{\frac{4}{3}}}.$$

This in turn leads to a second order ordinary differential equation

$$81Ak^{\frac{13}{3}}c^2e\phi''(k) + 135Ac^2ek^{\frac{10}{3}}\phi'(k) - 4Aa^2k^{\frac{1}{3}}(d-3c) - 4Bka(d-c) = 0$$

whose the general solution is given by

$$\phi\,(k) = -\frac{3}{2}k^{-\frac{2}{3}}C_1 + \frac{a^2}{18eck^2}\left(\frac{d-3c}{3c}\right) + \frac{Bak^{-\frac{4}{3}}}{18ecA}\left(\frac{d-c}{c}\right) + C_2.$$

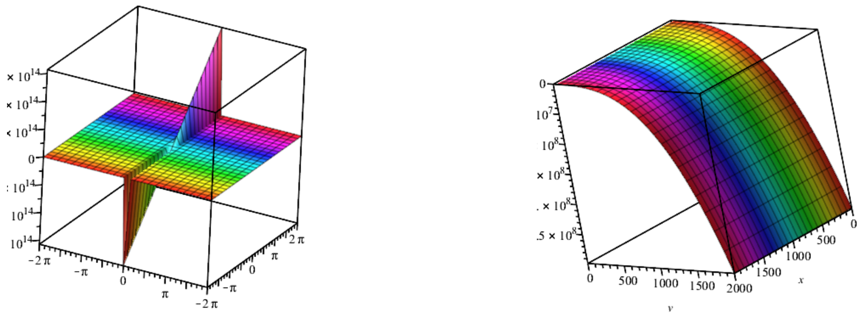

Figure 1: A profile of the solutions (2.4) and (2.7) respectively.

Consequentially the invariant solution takes this particular form

$$
\begin{aligned}
u(x,y,t) = \quad & \frac{a^2y^3}{18cet^2}\left(\frac{d-3c}{3c}\right) - \frac{3}{2}\left(\frac{t}{y^{\frac{3}{2}}}\right)^{-\frac{2}{3}} C_1 - \frac{1}{3}\left(\frac{t}{y^{\frac{3}{2}}}\right)^{-1} ac^{-1}xy^{-\frac{1}{2}} \\
& + \frac{1}{18}\left(\frac{t}{y^{\frac{3}{2}}}\right)^{-\frac{4}{3}} BaA^{-1}e^{-1}c^{-1}\left(\frac{d-c}{c}\right) - \frac{1}{3}\left(\frac{t}{y^{\frac{3}{2}}}\right)^{-\frac{4}{3}} BA^{-1}c^{-1}txy^{-2} \\
& + C_2. \qquad (2.7)
\end{aligned}
$$

The figures above depicts a group invariant solution that contains some arbitrary elements. In many applications, group invariant solutions capture the limiting behaviour of problems that far away from their initial or boundary conditions.

3. Conservation laws of (1.4)

A local conservation law for equation (1.4) is a space-time divergence

$$D_tT^t + D_xT^x + D_yT^y = 0$$

which holds for all formal solutions $u(x,y,t)$ of equation (1.4) where the conserved density T^t and the spatial fluxes T^x,T^y are functions of t,x,y,u and derivatives of u. Furthermore, if there exists a nontrivial differential function Λ, called a 'multiplier' such that $E_u\,(\Lambda G)$=0, then ΛG is a total divergence, i.e $\Lambda G = D_tT^t + D_xT^x + D_yT^y$, for some (conserved) vector $[T^t, T^x, T^y]$ and E_u is the Euler-Lagrange operator. Thus, knowledge of each multiplier Λ leads to a conserved vector computed by a homotopy operator. If u and its derivatives tend to zero as x and y approaches infinity, the conserved quantities are obtained by $\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} T^t dxdy$.

The above analysis prompts the following Lemmas and associated conservation laws:

Lemma 1. *Equation (1.4) establishes the conservation law multiplier of the form*

$$\Lambda = f'(t)y - \frac{f(t)(c-d)}{a}u_x + C_1 u_x + g(t),$$

whenever $n = 2$.

$$D_t T_1^t + D_x T_1^x + D_y T_1^y = 0,$$

$$\begin{aligned}
T_1^t &= \frac{1}{4}\left(a u_x^2 - a u u_{xx}\right),\\
T_1^x &= \frac{1}{12(n+2)}\Big(3anuu_{tx} + 6auu_{tx} + 4cnuu_xu_{xy} + 8cuu_xu_{xy} - 4dnuu_xu_{xy} - 8duu_xu_{xy}\\
&\quad +6enuu_{yy} + 12euu_{yy} + 3anu_tu_x + 6au_tu_x + 12bu_x^{(n+2)} - 4cnu_x^2u_y - 8cu_x^2u_y - 2dnu_x^2u_y\\
&\quad -4du_x^2u_y - 12nu_xu_{xxx} + 6nu_{xx}^2 - 24u_{xxx}u_x + 12u_{xx}^2\Big),\\
T_1^y &= \frac{1}{6}\left(-2cu_{xx}u_xu + 2du_{xx}u_xu - 3eu_{xy}u - du_x^3 + 3eu_xu_y\right);
\end{aligned}$$

$$D_t T_2^t + D_x T_2^x + D_y T_2^y = 0,$$

$$\begin{aligned}
T_2^t &= \frac{1}{4}\left(cf(t)uu_{xx} - df(t)uu_{xx} + 2ayf'(t)u_x - cf(t)u_x^2 + df(t)u_x^2\right),\\
T_2^x &= \frac{1}{12a(n+1)(n+2)}\Big(24abyf'{u_x}^{n+1} + 12abnyf'{u_x}^{n+1} - 12bcf(t){u_x}^{n+2} + 12bdf(t){u_x}^{n+2}\\
&\quad -12bcnf(t){u_x}^{n+2} + 12bdnf(t){u_x}^{n+2} + 8c^2f(t)u_y{u_x}^2 - 4d^2f(t)u_y{u_x}^2 + 4c^2n^2f(t)u_y{u_x}^2\\
&\quad -2d^2n^2f(t)u_y{u_x}^2 - 2cdn^2f(t)u_y{u_x}^2 - 4cdf(t)u_y{u_x}^2 + 12c^2nf(t)u_y{u_x}^2 - 6d^2nf(t)u_y{u_x}^2\\
&\quad -6cdnf(t)u_y{u_x}^2 + 3acn^2uf'u_x + 6acuf'u_x + 9acnuf'u_x - 6acn^2yf'u_yu_x - 3adn^2yf'u_yu_x\\
&\quad -12acyf'u_yu_x - 6adyf'u_yu_x - 18acnyf'u_yu_x - 9adnyf'u_yu_x - 8c^2f(t)uu_{xy}u_x\\
&\quad -8d^2f(t)uu_{xy}u_x - 4c^2n^2f(t)uu_{xy}u_x - 4d^2n^2f(t)uu_{xy}u_x + 8cdn^2f(t)uu_{xy}u_x + 16cdf(t)uu_{xy}u_x\\
&\quad -12c^2nf(t)uu_{xy}u_x - 12d^2nf(t)uu_{xy}u_x + 24cdnf(t)uu_{xy}u_x + 12cn^2f(t)u_{xxx}u_x\\
&\quad -12dn^2f(t)u_{xxx}u_x + 24cf(t)u_{xxx}u_x - 24df(t)u_{xxx}u_x + 36cnf(t)u_{xxx}u_x - 36dnf(t)u_{xxx}u_x\\
&\quad -3acn^2f(t)u_tu_x + 3adn^2f(t)u_tu_x - 6acf(t)u_tu_x + 6adf(t)u_tu_x - 9acnf(t)u_tu_x\\
&\quad +9adnf(t)u_tu_x - 6cn^2f(t){u_{xx}}^2 + 6dn^2f(t){u_{xx}}^2 - 12cf(t){u_{xx}}^2 + 12df(t){u_{xx}}^2 - 18cnf(t){u_{xx}}^2\\
&\quad +18dnf(t){u_{xx}}^2 - 12a^2yuf'' - 6a^2n^2yuf'' - 18a^2nyuf'' - 6cen^2f(t)uu_{yy} + 6den^2f(t)uu_{yy}\\
&\quad -12cef(t)uu_{yy} + 12def(t)uu_{yy} - 18cenf(t)uu_{yy} + 18denf(t)uu_{yy} + 6acn^2yuf'u_{xy}\\
&\quad -3adn^2yuf'u_{xy} + 12acyuf'u_{xy} - 6adyuf'u_{xy} + 18acnyuf'u_{xy} - 9adnyuf'u_{xy} - 12an^2yf'u_{xxx}\\
&\quad -24ayf'u_{xxx} - 36anyf'u_{xxx} + 12a^2yf'u_t + 6a^2n^2yf'u_t + 18a^2nyf'u_t - 3acn^2f(t)uu_{tx}\\
&\quad +3adn^2f(t)uu_{tx} - 6acf(t)uu_{tx} + 6adf(t)uu_{tx} - 9acnf(t)uu_{tx} + 9adnf(t)uu_{tx}\Big),\\
T_2^y &= \frac{1}{12a}\Big(-6acyf'uu_{xx} + 3adyf'uu_{xx} + 4c^2f(t)uu_xu_{xx} - 8cdf(t)uu_xu_{xx} + 6cef(t)uu_{xy}\\
&\quad +4d^2f(t)uu_xu_{xx} - 6def(t)uu_{xy} - 12aef'u - 3adyf'{u_x}^2 + 12aeyf'u_y + 2cdf(t){u_x}^3\\
&\quad -6cef(t)u_xu_y - 2d^2f(t){u_x}^3 + 6def(t)u_xu_y\Big);
\end{aligned}$$

$$D_tT_3^t + D_xT_3^x + D_yT_3^y = 0,$$

$$\begin{aligned}
T_3^t &= \frac{1}{2}ag(t)u_x,\\
T_3^x &= -\frac{1}{4(n+1)}\Bigg(-2cng(t)uu_{xy} - 2cg(t)uu_{xy} + dng(t)uu_{xy} + dg(t)uu_{xy} + 2ang'u\\
&\quad +2ag'u - 2ang(t)u_t - 2ag(t)u_t - 4bg(t){u_x}^{n+1} + 2cng(t)u_xu_y + 2cg(t)u_xu_y + dng(t)u_xu_y\\
&\quad +dg(t)u_xu_y + 4ng(t)u_{xxx} + 4g(t)u_{xxx}\Bigg),\\
T_3^y &= \frac{1}{4}\left(-2cg(t)uu_{xx} + dg(t)uu_{xx} - dg(t)u_x^2 + 4eg(t)u_y\right).
\end{aligned}$$

Lemma 2 *Equation (1.4) establishes the conservation law multiplier of the form*

$$\Lambda = f(t)y + C_1u_x + g(t),$$

whenever $n = 2$ and $d = c$.

$$D_tT_1^t + D_xT_1^x + D_yT_1^y = 0,$$

$$\begin{aligned}
T_1^t &= \frac{1}{2}au_x^2,\\
T_1^x &= \frac{1}{4}bu_x^4 - \frac{1}{3}cuu_xu_{xy} - \frac{2}{3}cu_yu_x^2 - u_xu_{xxx} + \frac{1}{2}u_{xx}^2 + \frac{1}{2}euu_{yy},\\
T_1^y &= \frac{1}{3}cuu_xu_{xx} - \frac{1}{2}euu_{xy} + \frac{1}{2}eu_yu_x;
\end{aligned}$$

$$D_tT_2^t + D_xT_2^x + D_yT_2^y = 0,$$

$$\begin{aligned}
T_2^t &= af(t)yu_x,\\
T_2^x &= \frac{1}{3}y\left(bf(t)u_x^3 - 3cf(t)u_xu_y - 3af'(t)u - 3f(t)u_{xxx}\right),\\
T_2^y &= ef(t)yu_y - ef(t)u;
\end{aligned}$$

$$D_tT_3^t + D_xT_3^x + D_yT_3^y = 0,$$

$$\begin{aligned}
T_3^t &= af(t)u_x,\\
T_3^x &= \frac{1}{3}bg(t)u_x^3 - cg(t)u_xu_y - ag'(t)u - g(t)u_{xxx},\\
T_3^y &= eg(t)u_y.
\end{aligned}$$

Lemma 3 *Equation (1.4) establishes the conservation law multiplier of the form*

$$\Lambda = \frac{y(cyu_x + 2ex)}{2e}C_1 + \frac{(cyu_x + ex)}{e}C_2 + C_3u_x + f(t)y + g(t),$$

whenever $b = c^2/2e$.

$$D_tT_1^t + D_xT_1^x + D_yT_1^y = 0,$$

$$
\begin{aligned}
T_1^t &= -\frac{1}{8}\frac{ay\left(cyuu_{xx}-cyu_x^2-4exu_x+4eu\right)}{e},\\
T_1^x &= \frac{1}{48e^2}\Bigg(3c^3y^2u_x^4+8c^2exyu_x^3-12c^2ey^2u_x^2u_y+6acey^2uu_{tx}+12ce^2xyuu_{xy}+12c^2e^2y^2uu_{yy}\\
&\quad +6acey^2u_tu_x-36ce^2xyu_yu_x+12ce^2xuu_x+24ce^2yuu_y+24ae^2xyu_t-24cey^2u_xu_{xxx}\\
&\quad +12cey^2u_{xx}^2-48e^2xyu_{xxx}+48e^2yu_{xxx}\Bigg),\\
T_1^y &= -\frac{1}{12e}\Bigg(c^2y^2u_x^3+3cexyuu_{xx}+3cey^2uu_{xy}+3cexyu_x^2-3cey^2u_xu_y+3ceyuu_x\\
&\quad -12e^2xyu_y+12e^2xu\Bigg);
\end{aligned}
$$

$$D_tT_2^t+D_xT_2^x+D_yT_2^y=0,$$

$$
\begin{aligned}
T_2^t &= -\frac{1}{4}\frac{a\left(cyuu_{xx}-cyu_x^2-2exu_x+2eu\right)}{e},\\
T_2^x &= \frac{1}{24e^2}\Bigg(3c^3yu_x^4+4c^2exu_x^3-12c^2eyu_x^2u_y+6aceyuu_{tx}+6ce^2xuu_{xy}+12ce^2yuu_{yy}\\
&\quad +6aceyu_tu_x-18ce^2xu_yu_x+12ce^2uu_y+12ae^2xu_t-24ceyu_xu_{xxx}\\
&\quad +12ceyu_{xx}^2-24e^2xu_{xxx}+24e^2u_{xxx}\Bigg),\\
T_2^y &= -\frac{1}{12e}\Bigg(2c^2yu_x^3+3cexuu_{xx}+6ceyuu_{xy}+3cexu_x^2-6ceyu_xu_y+3ceuu_x\\
&\quad -12e^2xu_y\Bigg);
\end{aligned}
$$

$$D_tT_3^t+D_xT_3^x+D_yT_3^y=0,$$

$$
\begin{aligned}
T_3^t &= -\frac{1}{4}a\left(uu_{xx}-u_x^2\right),\\
T_3^x &= \frac{1}{8e}\Bigg(c^2u_x^4-4ceu_yu_x^2+2aeuu_{tx}+4e^2uu_{yy}+2aeu_tu_x-8eu_xu_{xxx}+4eu_{xx}^2\Bigg),\\
T_3^y &= -\frac{1}{6}cu_x^3-\frac{1}{2}euu_{xy}+\frac{1}{2}eu_xu_y;
\end{aligned}
$$

$$D_tT_4^t+D_xT_4^x+D_yT_4^y=0,$$

$$
\begin{aligned}
T_4^t &= \frac{1}{2}af(t)u_{xy},\\
T_4^x &= \frac{1}{12e}\Bigg(2c^2yf(t)u_x^3+3ceyf(t)uu_{xy}-9cef(t)yu_xu_y+3cef(t)uu_x+6aeyf(t)u_t-6aeyf'(t)u\\
&\quad -12f(t)eyu_{xxx}\Bigg),\\
T_4^y &= -\frac{1}{4}f(t)\left(cyuu_{xx}+cyu_x^2-4eyu_y+4eu\right);
\end{aligned}
$$

$$D_tT_5^t + D_xT_5^x + D_yT_5^y = 0,$$

$$\begin{aligned}
T_5^t &= \frac{1}{2}ag(t)u_x,\\
T_5^x &= \frac{1}{12e}\Big(2c^2g(t)u_x^3 + 3ceg(t)uu_{xy} - 9ceg(t)u_xu_y + 6aeg(t)u_t - 6aeg'(t)u\\
&\quad -12g(t)eu_{xxx}\Big),\\
T_5^y &= -\frac{1}{4}g(t)\left(cuu_{xx} + cu_x^2 - 4eu_y\right).
\end{aligned}$$

Lemma 4 *Equation (1.4) establishes the conservation law multiplier of the form*

$$\Lambda = \Big(x + \frac{cy}{e}u_x\Big)C_1 + f'(t)y + \frac{(2be - c^2)}{2ac}f(t)u_x + C_2u_x + g(t),$$

whenever $n = 2$ and $d = (2be + c^2)/2c$.

$$D_tT_1^t + D_xT_1^x + D_yT_1^y = 0,$$

$$\begin{aligned}
T_1^t &= -\frac{a}{4e}\Big(cyuu_{xx} - cyu_x^2 - 2exu_x + 2eu\Big),\\
T_1^x &= \frac{1}{24ce}\Big(6bc^2yu_x^4 - 8bceyuu_xu_{xy} + 4c^3yuu_xu_{xy} + 8cexu_x^3 - 4bceyu_yu_x^2 - 10c^3yu_yu_x^2\\
&\quad +6c^2ayuu_{tx} - 4bceuu_x^2 + 2c^3uu_x^3 - 6be^2xuu_{xy} + 9c^2exuu_{xy} + 12c^2eyuu_{yy} + 6c^2ayu_tu_x\\
&\quad -6be^2xu_xu_y - 15c^2exu_xu_y + 12c^2euu_y + 12acexu_t - 24c^2yu_xu_{xxx} + 12c^2yu_{xx}^2\\
&\quad -24cexu_{xxx} + 24ceu_{xx}\Big),\\
T_1^y &= \frac{1}{24ce}\Big(8bceyuu_xu_{xx} - 4c^3yuu_xu_{xx} - 4bceyu_x^3 - 2c^3yu_x^3 + 6be^2xuu_{xx} - 9c^2exuu_{xx}\\
&\quad -12c^2eyuu_{xy} - 6be^2xu_x^2 - 3c^2exu_x^2 + 12c^2eyu_xu_y + 6be^2uu_x - 9c^2euu_x + 24ce^2xu_y\Big);
\end{aligned}$$

$$D_tT_2^t + D_xT_2^x + D_yT_2^y = 0,$$

$$\begin{aligned}
T_2^t &= -\frac{1}{4}a\left(uu_{xx} - u_x^2\right),\\
T_2^x &= \frac{1}{12c}\Big(3bcu_x^4 - 4beuu_xu_{xy} + 2c^2uu_xu_{xy} - 2beu_yu_x^2 - 5c^2u_yu^2 + 3acuu_{tx} + 6ceuu_{yy} + 3acu_tu_x\\
&\quad -12cu_xu_{xxx} + 6cu_{xx}^2\Big),\\
T_2^y &= -\frac{1}{12c}\Big(4beuu_xu_{xx} - 2c^2uu_xu_{xx} - 2beu_x^3 - c^2u_x^3 - 6ceuu_{xy} + 6ceu_xu_y\Big);
\end{aligned}$$

$$D_tT_3^t + D_xT_3^x + D_yT_3^y = 0,$$

$$
\begin{aligned}
T_3^t &= -\frac{1}{8}\Big(2f(t)beuu_{xx} - f(t)c^2uu_{xx} - 2f(t)beu_x^2 + f(t)c^2u_x^2 - 4f'(t)acyu_x\Big),\\
T_3^x &= -\frac{1}{24ac^2}\Big(8f(t)b^2e^2uu_xu_{xy} + 8f(t)bec^2u_yu_x^2 - 12f(t)bce^2uu_{yy} + 24f(t)bceu_xu_{xxx}\\
&\quad -5f(t)c^4u_yu_x^2 - 12f(t)c^3u_xu_{xxx} + 12f''(t)a^2c^2yu - 6f'(t)ac^3uu_x - 12f'(t)a^2c^2yu_t\\
&\quad +24ac^2yf'(t)u_{xxx} - 8bec^2f(t)uu_xu_{xy} - 6abcef(t)uu_{tx} - 6abcef(t)u_tu_x\\
&\quad +3ac^3f(t)uu_{tx} + 6c^3ef(t)u_{yy} + 3c^3af(t)u_tu_x - 12bcef(t)u_{xx}^2 - 6b^2cef(t)u_x^4\\
&\quad +2c^4f(t)uu_xu_{xy} + 4b^2e^2u_yu_x^2f(t) + 3f(t)bc^3u_x^4 - 8abc^2f'(t)yu_x^3 - 9ac^3f'(t)yuu_{xy}\\
&\quad +15ac^3f'(t)yu_xu_y + 6abcef'(t)yuu_{xy} + 6abcef'(t)yu_xu_y + 6c^3f(t)u_{xx}^2\Big),\\
T_3^y &= \frac{1}{24ac^2}\Big(6abcef'(t)yuu_{xx} - 9ac^3f'(t)yuu_{xx} - 6abcef'(t)yu_x^2 - 3ac^3f'(t)yu_x^2 + 8b^2c^2f(t)uu_xu_{xx}\\
&\quad -8bec^2f(t)uu_xu_{xx} + 2c^4f(t)uu_xu_{xx} - 4b^2e^2f(t)u_x^3 + c^4f(t)u_x^3 + 24aec^3f'(t)yu_y\\
&\quad -12bce^2f(t)uu_{xy} + 6ec^3f(t)uu_{xy} + 12bce^2f(t)u_xu_y - 6ec^3f(t)u_xu_y - 24aec^2f'(t)u\Big);
\end{aligned}
$$

$$D_tT_4^t + D_xT_4^x + D_yT_4^y = 0,$$

$$
\begin{aligned}
T_4^t &= \frac{1}{2}ag(t)u_x,\\
T_4^x &= -\frac{1}{24c}\Big(-8bcg(t)u_x^3 + 6beg(t)uu_{xy} - 9c^2g(t)uu_{xy} + 6beg(t)u_xu_y + 15c^2g(t)u_xu_y\\
&\quad +12cag'(t)u - 12acg(t)u_t + 24cg(t)u_{xxx}\Big),\\
T_4^y &= \frac{1}{8c}g(t)\Big(2beuu_{xx} - 3c^2uu_{xx} - 2beu_x^2 - c^2u_x^2 + 8ceu_y\Big);
\end{aligned}
$$

Lemma 5 *Equation (1.4) establishes the conservation law multiplier of the form*

$$\Lambda = f(t)x + \frac{cf(t)y}{e}u_x - \frac{af'(t)y^2}{2e} + g(t)u_x - \frac{ag'(t)y}{c} + +h(t),$$

whenever $b = c^2/2e$.

$$D_tT_1^t + D_xT_1^x + D_yT_1^y = 0,$$

$$\begin{aligned}
T_1^t &= -\frac{a}{4e}\bigg(cf(t)yuu_{xx} - cf(t)yu_x^2 + af'(t)y^2u_x - 2ef(t)xu_x + 2ef(t)u\bigg),\\
T_1^x &= \frac{1}{24e^2}\bigg(- 3c^3f(t)yu_x^4 + 2ac^2f'(t)y^2u_x^3 + 8f(t)ec^2yuu_xu_{xy} - 4ec^2f(t)xu_x^3 - 8ec^2f(t)yu_yu_x^2\\
&\quad -6acef'(t)y^2uu_{xy} + 6acef'(t)y^2u_xu_y + 6a^2ef''(t)y^2u + 6acef(t)yuu_{tx} + 4ec^2f(t)uu_x^2\\
&\quad +12ce^2f(t)xuu_{xy} + 12ce^2f(t)yuu_{yy} + 6acef(t)yu_tu_x - 12ce^2f(t)xu_xu_y - 6acef'(t)yuu_x\\
&\quad -6a^2ef'(t)y^2u_t + 12ce^2f(t)uu_y + 12ae^2f(t)xu_t - 24cef(t)yu_xu_{xxx} + 12cef(t)yu_{xx}^2\\
&\quad -12ae^2f'(t)xu + 12aef'(t)y^2u_{xxx} - 24e^2f(t)xu_{xxx} + 24e^2f(t)u_{xx}\bigg),\\
T_1^y &= -\frac{1}{12e}\bigg(4c^2f(t)yuu_xu_{xx} - 3acf'(t)y^2uu_{xx} + 6cef(t)xuu_{xx} + 6cef(t)yuu_{xy} - 6cef(t)yu_xu_y\\
&\quad +6aef'(t)y^2u_y + 6cef(t)uu_x - 12e^2f(t)xu_y - 12aef'(t)yu\bigg);
\end{aligned}$$

$$D_tT_2^t + D_xT_2^x + D_yT_2^y = 0,$$

$$\begin{aligned}
T_2^t &= -\frac{a}{4c}\bigg(cg(t)uu_{xx} - cg(t)u_x^2 - 2ag'(t)yu_x\bigg),\\
T_2^x &= \frac{1}{24ce}\bigg(- 3c^3g(t)u_x^4 + 4ac^2g'(t)yu_x^3 + 8g(t)ec^2uu_xu_{xy} - 8ec^2g(t)u_yu_x^2 - 12ecag'(t)yu_{xy}\\
&\quad +12aceg'(t)yu_xu_y + 6aceg(t)uu_{tx} + 12ce^2g(t)uu_{yy} + 6aceg(t)u_tu_x - 6aceg'(t)uu_x\\
&\quad -12ea^2g'(t)yu_t + 12ea^2g(t)yu - 24ceg(t)u_xu_{xxx} + 12ceg(t)u_{xx}^2 + 24aeg'(t)yu_{xxx}\bigg),\\
T_2^y &= -\frac{1}{6e}\bigg(2c^2g(t)uu_xu_{xx} - 3acg'(t)yuu_{xx} + 3ceg(t)uu_{xy} - 3ceg(t)u_xu_y + 6aeg'(t)yu_y\\
&\quad -6aeg'(t)u\bigg);
\end{aligned}$$

$$D_tT_3^t + D_xT_3^x + D_yT_3^y = 0,$$

$$\begin{aligned}
T_3^t &= \frac{1}{2}ah(t)u_x,\\
T_3^x &= \frac{1}{6e}\bigg(- c^2h(t)u_x^3 + 3ceh(t)uu_{xy} - 3ceh(t)u_xu_y + 3aeh(t)u_t - 3aeh'(t)u - 6eh(t)u_{xxx}\bigg),\\
T_3^y &= -\frac{1}{2}h(t)\bigg(cuu_{xx} - 2eu_y\bigg);
\end{aligned}$$

Lemma 6 *Equation (1.4) establishes the conservation law multiplier of the form*

$$\Lambda = \left(x + \frac{cy}{e}u_x\right)C_1 + f'(t)y - \frac{2c}{a}f(t)u_x + C_2u_x + g(t),$$

whenever $b = -3c^2/2e$.

$$D_tT_1^t + D_xT_1^x + D_yT_1^y = 0,$$

$$
\begin{aligned}
T_1^t &= -\frac{a}{4e}\Big(cyuu_{xx}-cyu_x^2-2exu_x+2eu\Big),\\
T_1^x &= \frac{1}{24e^2}\Big(-9c^3yu_x^4+16c^2eyuu_xu_{xy}-12c^2exu_x^3-4c^2eyu_yu_x^2+6aceyuu_{tx}+8ec^2uu_x^2\\
&\quad+18ce^2xuu_{xy}+12ce^2yuu_{yy}+6aceyu_tu_x-6ce^2xu_xu_y+12ce^2uu_y+12ae^2xu_t\\
&\quad-24ceyu_xu_{xxx}+12ceyu_{xx}^2-24e^2xu_{xxx}+24e^2u_{xx}\Big),\\
T_1^y &= -\frac{1}{12e}\Big(8c^2yuu_xu_{xx}-2c^2yu_x^3+9cexuu_{xx}+6ceyuu_{xy}-3cexu_x^2-6ceyu_xu_y+9ceuu_x\\
&\quad-12e^2xu_y\Big);
\end{aligned}
$$

$$D_tT_2^t+D_xT_2^x+D_yT_2^y=0,$$

$$
\begin{aligned}
T_2^t &= -\frac{1}{4}a\left(uu_{xx}-u_x^2\right),\\
T_2^x &= \frac{1}{24e}\Big(-9c^2u_x^4+16ceuu_xu_{xy}-4ceu_yu_x^2+6aeuu_{tx}+12e^2uu_{yy}+6aeu_xu_t-24eu_xu_{xxx}\\
&\quad+12eu_{xx}^2\Big),\\
T_2^y &= -\frac{2}{3}cuu_xu_{xx}+\frac{1}{6}cu_x^3-\frac{1}{2}euu_{xy}+\frac{1}{2}eu_xu_y;
\end{aligned}
$$

$$D_tT_3^t+D_xT_3^x+D_yT_3^y=0,$$

$$
\begin{aligned}
T_3^t &= \frac{1}{2}\Big(cf(t)uu_{xx}-cf(t)u_x^2+af'(t)yu_x\Big),\\
T_3^x &= -\frac{1}{12ea}\Big(-9c^3f(t)u_x^4+6ac^2f'(t)yu_x^3+16c^2ef(t)uu_xu_{xy}-4ec^2f(t)u_yu_x^2-9acef'(t)yuu_{xy}\\
&\quad+3acef(t)yu_xu_y+6acef(t)uu_{tx}+12ce^2uu_{yy}+6acef(t)u_xu_t-3acef'(t)uu_x-6ea^2f'(t)yu_t\\
&\quad+6ea^2f''(t)yu-24ceu_xu_{xxx}+12cef(t)u_{xx}^2+12aef'(t)yu_{xxx}\Big),\\
T_3^y &= \frac{1}{12a}\Big(16c^2f(t)uu_xu_{xx}-4c^2f(t)u_x^3-9acf'(t)yuu_{xx}+3acf'(t)yu_x^2+12cef(t)uu_{xy}\\
&\quad-12cef(t)u_xu_y+12aef'(t)yu_y-12aef'(t)u\Big);
\end{aligned}
$$

$$D_tT_4^t+D_xT_4^x+D_yT_4^y=0,$$

$$
\begin{aligned}
T_4^t &= \frac{1}{2}ag(t)u_x,\\
T_4^x &= \frac{1}{4e}\Big(-2c^2g(t)u_x^3+3ceg(t)uu_{xy}-ceg(t)u_xu_y+2aeg(t)u_t-2aeg'(t)u-4eg(t)u_{xxx}\Big),\\
T_4^y &= -\frac{1}{4}g(t)\left(3cuu_{xx}-cu_x^2-4eu_y\right);
\end{aligned}
$$

Lemma 7 *Equation (1.4) establishes the conservation law multiplier of the form*

$$\Lambda = C_1 u_t + C_2 u_y + f(t),$$

whenever $n = 2$ and $d = 2c$.

$$D_t T_1^t + D_x T_1^x + D_y T_1^y = 0,$$

$$\begin{aligned}
T_1^t &= \frac{1}{4} buu_x^2 u_{xx} - \frac{2}{3} cuu_x u_{xy} - \frac{1}{3} cuu_y u_{xx} + \frac{1}{2} euu_{yy} - \frac{1}{2} uu_{xxxx} + \frac{1}{4} au_x u_t + \frac{1}{4} auu_{tx}, \\
T_1^x &= \frac{1}{4} bu_t u_x^3 - \frac{1}{4} buu_{tx} u_x^2 + \frac{1}{4} au_t^2 - \frac{1}{2} u_t u_{xxx} + \frac{1}{2} u_{tx} u_{xx} - \frac{1}{2} u_x u_{txx} + \frac{1}{2} uu_{txxx} - \frac{1}{4} auu_{tt} \\
&\quad + \frac{1}{3} cuu_x u_{ty} + \frac{1}{3} cuu_y u_{tx} - \frac{2}{3} cu_t u_x u_y, \\
T_1^y &= \frac{1}{3} cuu_x u_{tx} - \frac{1}{3} cu_t u_x^2 - \frac{1}{2} euu_{ty} + \frac{1}{2} eu_t u_y;
\end{aligned}$$

$$D_t T_2^t + D_x T_2^x + D_y T_2^y = 0,$$

$$\begin{aligned}
T_2^t &= -\frac{1}{4} auu_{xy} + \frac{1}{4} au_x u_y, \\
T_2^x &= \frac{1}{4} au_y u_t - \frac{2}{3} cu_x u_y^2 + \frac{1}{4} bu_y u_x^3 - \frac{1}{2} u_y u_{xxx} + \frac{1}{2} u_{xx} u_{xy} - \frac{1}{2} u_x u_{xxy} - \frac{1}{2} uu_{xxxy} \\
&\quad - \frac{1}{4} auu_{ty} + \frac{1}{3} cuu_x u_{yy} - \frac{1}{4} buu_{xy} u_x^2 + \frac{1}{3} cuu_y u_{xy}, \\
T_2^y &= \frac{1}{4} buu_x^2 u_{xx} - \frac{1}{3} cuu_x u_{xy} - \frac{1}{3} cuu_y u_{xx} - \frac{1}{3} cu_y u_x^2 + \frac{1}{2} auu_{tx} + \frac{1}{2} eu_y^2 - \frac{1}{2} uu_{xxxx};
\end{aligned}$$

$$D_t T_3^t + D_x T_3^x + D_y T_3^y = 0,$$

$$\begin{aligned}
T_3^t &= \frac{1}{2} af(t) u_x, \\
T_3^x &= \frac{1}{2} af(t) u_t - f(t) u_{xxx} + \frac{1}{3} bf(t) u_x^3 - cf(t) u_x u_y - \frac{1}{2} af'(t) u, \\
T_3^y &= -\frac{1}{2} cf(t) u_x^2 + ef(t) u_y.
\end{aligned}$$

We succinctly discuss the significance and physical illumination that ascend from the computed conservation laws. Conservation laws reside in enormously crucial areas both at the foundations of nonlinear science, such as biology, physics and other related areas and in its applications. Mathematical expressions of physical laws, such as conservation of energy, momentum and mass are fundamentally conservation laws. Imperative physical information about the complex behaviour in non-linear systems is confined in conservation laws. The arbitrary functions in the multipliers leads to an infinitely many conservation laws.

4. Concluding remarks

In today's work, a generalized (1+2)-dimensional Jaulent-Miodek equation with a power law nonlinearity was examined, which arises in numerous problems in nonlinear science. Exact solutions were acquired using the Lie symmetry method. In addition to exact solutions, we also presented conservation laws and their physical ramifications were also discussed. It is anticipated that the exact solutions and conservation laws computed in this paper can be used as yardsticks for numerical simulations in biology, physics and other related areas. In future, the computed conserved quantities will be utilized for the construction of closed form solutions and would be reported elsewhere.

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