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Decentralized PID control for nonlinear multivariable systems using decoupler

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Abstract

Interactions between input and output variables are a prevalent challenge in the design of multi-loop controllers for multivariable processes, and they can be a major stumbling block to obtaining good overall performance of a multi loop control system. The deconstructed dynamic interaction analysis is proposed to solve this limitation by decomposing the multi loop control system into a series of n independent SISO systems, each with its own PID controller. The multivariable decoupler and multi loop PID controller is applied to Two Tank Conical Interacting System (TTCIS). This TTCIS is chosen as benchmark problem used by many researchers. Firstly, the Mathematical modelling of TTCIS is derived using First principal model. The non-linear system is linearized using Jacobian matrix and decomposed into multiple SISO systems. The controller design for the process is then obtained, and an RGA matrix is constructed to minimise the interaction effects. To demonstrate the efficiency of the suggested strategy, simulation results using TTCIS are provided.

Keywords: Multi-loop PID controller, Decoupling, Linearisation, Two Tank Conical Interacting System.

1. Introduction

Conical tanks are extensively used in process industries, petrochemical industries, food process industries and wastewater treatment industries. Conical bottom tanks are widely used in process applications where the total drainage of the tank is required as it has varying cross section.
Because of its non-linearities and continually changing area of cross section, controlling conical tanks

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is a difficult task in many applications. The primary goal is to create an appropriate controller design for the conical tank system in order to maintain the target level while removing all interactions. The liquid level control in tank and flow in the tank is a basic problem in all the chemical industries [3]. Process liquids must be pumped, kept in tanks, and then pumped to another tank in the chemical industry. The liquid will then be treated chemically in the tanks, however the amount of the fluid in the tanks must always be monitored. Liquid level control is a critical and common operation in the process industries. Here, the tank is conical shape in which the level of liquid is desired to maintain at a steady value. This is achieved by controlling the input flow to the tanks.

The vast majority of chemical processes are multi-input/multi-output (MIMO) systems. Despite advances in advanced multivariable controllers, multi-loop PID control employing multiple single-input/single-output (SISO) PID controllers remains the industry standard for controlling MIMO systems for all interacting multivariable processes. When SISO tuning methods are applied to multi-loop systems, they frequently result in poor performance and instability. A proportional-integral-derivative controller (PID controller) is a common control loop feedback mechanism used in a variety of control systems [8]. The difference between the measured process variable and a desired set point is used by a PID controller to calculate an error value. By modifying all of the process control inputs, the controller seeks to reduce the mistake. Because the PID controller is simple and reliable, it is frequently employed in the process industries [8].

In the process industries, decentralised control is the most often employed control for all nonlinear MIMO systems [2]. Despite advances in advanced multivariable controllers, decoupling-based decentralised multi-loop PID control using multiple Single-Input/Single-Output (SISO) PID controllers is still the industry standard for controlling nonlinear MIMO systems with low interaction. The rationale for this is because it is a simple, failure-tolerant structure that is straightforward to build and maintain by plant workers [5]. Furthermore, throughout the past two decades, the PID controller and the model predictive controller (MPC) have been the two most extensively used control methods in the process industries. In practise, all MPC systems operate in a supervisory mode, with sampling times that are longer than those of lower-level PID controllers [4].

A multivariable system with n inputs and n output variables is treated as n monovariable systems using decentralised techniques. Multi-variable controllers, on the other hand, are far more difficult to design and tune than single-variable controllers due to process loop interactions. The tuning of one loop cannot be done independently because the controls interact with each other [7].

2. Relative Gain Array

The RGA provides quantitative criteria for the selection of control loops that would lead to minimum interaction among the loops. The following are the steps that must be followed to arrive at the Relative Gain Array.

1. Loops are opened and the controllers are detached from the process. Keeping u_2 constant, introduce a step input in u_1. That would yield a static gain K that would indicate the direct effect of input on output. The static gain K is given in Equation (2.1).

$$K = \frac{\Delta \bar{y}_1}{\Delta \bar{u}_1} \mid \bar{u}_2 = Constant \tag{2.1}$$

2. Loop 2 is closed and the corresponding controller is attached with the process. A step input u_1 is introduced while maintaining y_2 at its desired set point through the loop 2 controller. That would yield another open loop gain K' that would indicate the direct as well as indirect effect

of u_1 input on output y_1. The static gain K' is given in Equation (2.2).

$$K' = \frac{\Delta \bar{y}_1}{\Delta \bar{u}_1} \mid \bar{y}_2 = Constant \tag{2.2}$$

3. The ratio of the above open loop gains is defined as the relative gain in Equation (2.3).

$$\lambda_{11} = \frac{K = \frac{\Delta \bar{y}_1}{\Delta \bar{u}_1} \mid \bar{u}_2}{K' = \frac{\Delta \bar{y}_1}{\Delta \bar{u}_1} \mid \bar{y}_2} = Constant \tag{2.3}$$

In the similar manner, relative gains between other input-output combinations can be calculated as expressed in the matrix form as in Equation (2.4).

$$\lambda = \begin{bmatrix} \lambda_{11} & \lambda_{12} \\ \lambda_{21} & \lambda_{22} \end{bmatrix} \tag{2.4}$$

The two tank conical interacting system yielded a relative gain array with λ_{11} and λ_{22} having positive values and λ_{12} and λ_{21} having negative values, showing that input u_1 and y_1 are greatly paired and has direct effect and u_2 and y_2 are greatly paired and has direct effect and the interactions are minimum between u_1 and y_2 and the pair u_2 and y_1. Thus the system prevails with minimum interactions.

3. Decoupler Design

When the control system designer is confronted with two strongly interacting loops, new elements called decouplers are introduced in the control systems. The aim of decoupler is to cancel out the interaction effects between the two loops and thus convert into two non-interacting control loops [6]. In order to compensate for process interactions and completely eliminate control loop interactions, a decoupler mechanism in a system necessitates the usage of extra controllers. Decoupling control, in theory, permits set point changes to effect just the controlled variables that are intended. Decoupling controllers are often constructed using a simple process model (e.g., a steady-state model or transfer function model). A 2×2 process can be implemented with one or two decouplers.
Consider the decoupling control system with two decouplers shown in Fig. 1.

The control scheme consists of 4 controllers:

1. Two feedback controllers Gc_1 and Gc_2
2. Two decouplers D_{12} and D_{21}

Decoupler D_{21} is designed to remove the interaction between u_1 and Y_2. Decoupler D_{12} is designed to remove the interaction between u_2 and Y_1. The ideal decouplers are given as,

$$D_{21} = -\frac{G_{21}}{G_{22}} \tag{3.1}$$

$$D_{12} = -\frac{G_{12}}{G_{11}} \tag{3.2}$$

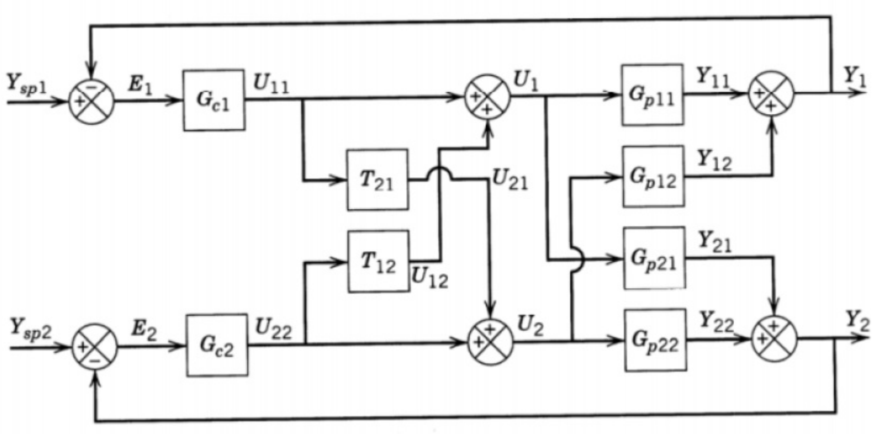

Figure 1: 2×2 Decoupling Control Scheme

4. Two Tank Conical Interacting System

4.1. System Description

Two conical tanks in the shape of an inverted cone, manufactured from sheet metal, are used in the proposed system. Figure 2 depicting the TTCIS. The TTCIS is a platform for studying nonlinear multivariable feedback control schemes both theoretically and empirically. TTCIS is made up of two conical tanks (Tank1 and Tank2), two independent pumps (Pump1 and Pump2), and two control valves (CV1 and CV2) that supply the liquid flows Fin1 and Fin2 to Tank1 and Tank2. These two tanks are interconnected at the bottom through a manually controlled valve, MV12 with a valve co-efficient It features a reservoir for storing water, which is then delivered to the tanks via pumps. At the top and bottom of the tank (8), there are provisions for water inflow and outflow, respectively. The level of water in the tanks is maintained by connecting two gate valves, one at the outflow of tank1 and the other at the outflow of tank2. Variable The water is discharged from the reservoir tank to the process tanks using a speed pump as an actuator. The input voltage has a direct relationship with the pump speed. It is made up of a differential pressure transmitter that measures the bottom pressure caused by the water level and displays the height in milliamps. The two output flows from Tank1 and Tank2 are Fout1 and Fout2 through manual control valves MV1 and MV2 with valve coefficients. The coefficients, represent the resistance of the opening aperture of the corresponding valves. Equation 7 can be used to calculate the valve coefficient.

$$\beta_i = v_i a_i \sqrt{2g} \tag{4.1}$$

4.2. Mathematical Model

Mathematical model is the one which gives complete description about the process under consideration using some mathematical concepts [6]. The process that involves in developing a mathematical model is termed as mathematical modelling [1]. Mathematical modelling of the TTCIS is derived using the Total Mass Balance Equation as given in Equations (4.2) and (4.3).

$$Accumulation = Input–Output \tag{4.2}$$

$$\frac{d(Ah)}{dt} = F_{IN} - F_{OUT} \tag{4.3}$$

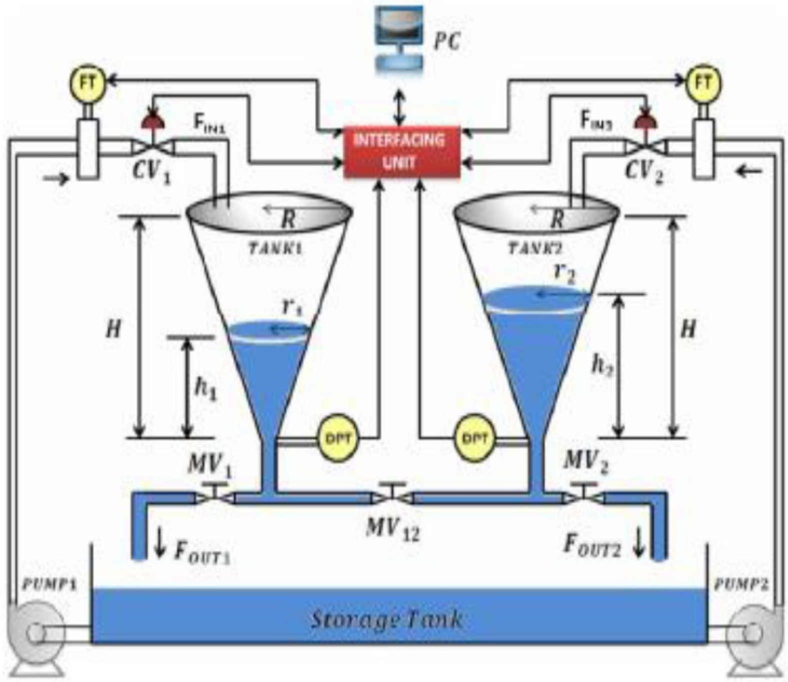

Figure 2: Schematic of TTCIS

The mathematical model of TTCIS is given by the Equations (4.4) and (4.5).

$$\frac{dh_1}{dt} = \frac{Fin_1 - \beta_1 a_1 \sqrt{2gh_1} - sign(h_1 - h_2)\beta_{12} a_{12} \sqrt{2g \mid h_1 - h_2 \mid}}{\Pi R_1^2 \frac{h_1^2}{H}} \tag{4.4}$$

$$\frac{dh_2}{dt} = \frac{Fin_2 - \beta_2 a_2 \sqrt{2gh_2} - sign(h_1 - h_2)\beta_{12} a_{12} \sqrt{2g \mid h_1 - h_2 \mid}}{\Pi R_2^2 \frac{h_2^2}{H}} \tag{4.5}$$

The operating system parameters and its values are given in Table 1.

Table 1: Parameters of TTCIS

Parameter	Description	Value
R	Top radius of the conical tank	20 cm
H	Maximum height of tank 1 and tank 2	50 cm
β_1	Valve co-efficient of MV1	50 $\frac{cm^2}{s}$
β_2	Valve co-efficient of MV2	50 $\frac{cm^2}{s}$
β_2	Valve co-efficient of MV12	35 $\frac{cm^2}{s}$
a_1, a_{12}, a_2	Cross section area of pipe	1.227 cm^2
h_{1s}	Level of water in tank 1	2.5 cm
h_{2s}	Level of water in tank 2	2.1 cm
Fin_1	Inlet flow of tank 1	165 $\frac{cm^3}{sec}$
Fin_2	Inlet flow of tank 2	82.5 $\frac{cm^3}{sec}$

4.3. Linearisation

Linearisation is the process by which we approximate the linear systems with non-linear ones and analyse the behaviour of a nonlinear function near a given point [6]. Linearisation can be done by two methods:

- Calculus method
- Non calculus method

The Taylor series expansion is used in the calculus approach of linearisation. The first order term of a function's Taylor expansion around the point of interest is its linearisation. Equation (4.6) is used to define a system.

$$\frac{dx}{dt} = F(x,t) \tag{4.6}$$

The linearised system can be written as in Equation (4.7),

$$\frac{dx}{dt} = F(x_0,t) + DF(x_0,t)(x - x_0) \tag{4.7}$$

Where x_0 is the point of interest and $DF(x_0,t)$ is the Jacobian of $F(x)$ evaluated at x_0. The Jacobian in matrix form for a second order system in generalized term is given as follows,

$$\begin{pmatrix} \dot{x}_1 \\ \dot{x}_2 \end{pmatrix} = \begin{pmatrix} \frac{\partial f}{\partial S} & \frac{\partial f}{\partial I} \\ \frac{\partial f}{\partial S} & \frac{\partial f_2}{\partial S} \end{pmatrix} = \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} \tag{4.8}$$

The state space general equation of any system is given as,

$$\dot{X} = AX + BU \tag{4.9}$$

$$Y = CX + DU \tag{4.10}$$

Where A is system matrix, B is input matrix, C is output matrix and D is transition matrix. These matrices are defined in Jacobian linearization as,

$$A = \begin{pmatrix} \frac{\partial f_1}{\partial h_1} & \frac{\partial f_1}{\partial h_2} \\ \frac{\partial f_2}{\partial h_1} & \frac{\partial f_2}{\partial h_2} \end{pmatrix} \tag{4.11}$$

$$B = \begin{pmatrix} \frac{\partial f_1}{\partial f_{in1}} & \frac{\partial f_1}{\partial f_{in2}} \\ \frac{\partial f_2}{\partial f_{in1}} & \frac{\partial f_2}{\partial f_{in2}} \end{pmatrix} \tag{4.12}$$

By solving the required differentials for the TTCIS modelled Equations (4.4) and (4.5) and substituting system parameters and standard conditions as values given in Table 1, we get the following matrices as shown in Equations (4.13) to (4.16).

$$A = \begin{bmatrix} \frac{\partial f_1}{\partial h_1} & \frac{\partial f_1}{\partial h_2} \\ \frac{\partial f_2}{\partial h_1} & \frac{\partial f_2}{\partial h_2} \end{bmatrix} = \begin{bmatrix} -6.9 & 4.3 \\ 6.4 & -10.8 \end{bmatrix} \tag{4.13}$$

$$B = \begin{bmatrix} \frac{\partial f_1}{\partial f_{in1}} & \frac{\partial f_1}{\partial f_{in2}} \\ \frac{\partial f_2}{\partial f_{in1}} & \frac{\partial f_2}{\partial f_{in2}} \end{bmatrix} = \begin{bmatrix} 7.7 & 0 \\ 0 & 11.5 \end{bmatrix} \tag{4.14}$$

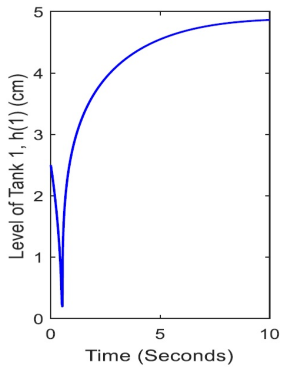

Figure 3: Open loop response of tank 1

$$C = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \tag{4.15}$$

$$D = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} \tag{4.16}$$

5. Results And Discussion

The two tank non-linear conical system modelled by the first principle model is linearized by the Jacobian matrix and substituting the parameters of the system and assuming standard conditions as given in Table 1 results in SISO systems. They are then subjected to corresponding inputs to obtain the open loop and closed loop responses with and without decoupler. The matrices obtained after linearization are converted into equivalent transfer functions using inbuilt command ss2tf in Matlab are shown in the Equations (5.1) to (5.4).

$$H_{11} = \frac{0.0064s + 0.0401}{s^2 + 8.82s + 14.5774} \tag{5.1}$$

$$H_{12} = \frac{0.0086}{s^2 + 8.82s + 14.5774} \tag{5.2}$$

$$H_{21} = \frac{0.0086}{s^2 + 8.82s + 14.5774} \tag{5.3}$$

$$H_{22} = \frac{0.0090s + 0.0227}{s^2 + 8.82s + 14.5774} \tag{5.4}$$

5.1. Open Loop Response

The two tank conical interacting system is simulated under open loop conditions. Open loop conditions of the system are analyzed as it will give the actual characteristics and nature of the system without any controller's intervention. The Fig. 3. shows the open loop response of tank 1. The level of the tank 1 increases slowly and reaches 4.9 cm at 10 sec. When the time is increased, it will settle at some value. The Fig. 4. shows the open loop response of tank 2. The level of the tank increases suddenly to 12 cm and then decreases slowly. It can be seen that the level of tank 2 is greater than that of tank 1.

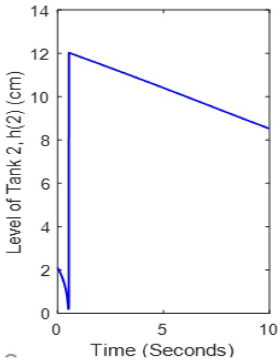

Figure 4: Open loop response of tank 2

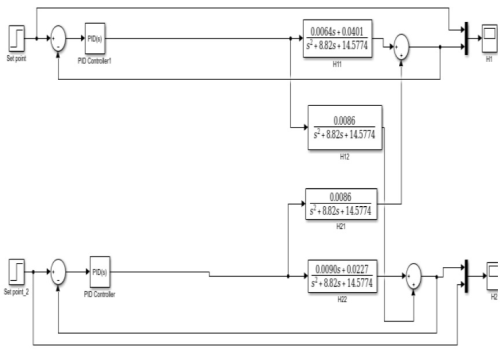

Figure 5: Simulink Model of Closed Loop Response of TTCIS

5.2. Closed Loop Response

The two tank conical interacting system is simulated under closed loop conditions with negative feedback and a tuned PID Controller. The four transfer functions yielded by linearizing MIMO interacting system by the Jacobian Matrix is used with two tuned controllers for loop 1 and loop 2 as shown in Fig. 5. The PID Controller helps in tracking the set point with desired time domain and steady state performances, output is also result of the interactions between tank 1 and tank 2 of the TTCIS.

The response of the tank 1 for the simulink model in Fig. 5. is as shown in the Fig. 6. It can be observed that the response is much better in spite of the interactions between the two tanks and non-linearity. This is because of the tuned PID Controller separate for the two tanks. The PID Controller brings about the desired closed loop response by tracking the set point after a considerable overshoot. The time domain specifications for the obtained response were calculated.

The tank 2 response for the Simulink Model in Fig 3 is depicted in Fig 5. Despite the interactions between the two tanks and non-linearity, the response is significantly better. This is because of the tuned PID Controller separate for the two tanks. The PID Controller brings about the desired set point tracking after a considerable overshoot. The time domain specifications for the obtained response were calculated.

5.3. Closed Loop Response With Decoupler

The two tank conical interacting system is simulated under closed loop conditions with decoupler. The transfer functions of the decoupler are obtained from the transfer functions of the plant. The transfer functions D_{12} and D_{21} are as given in Equations (5.5) and (5.6).

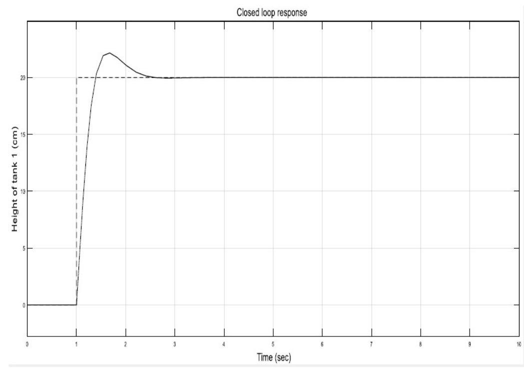

Figure 6: TTCIS tank 1's closed loop response

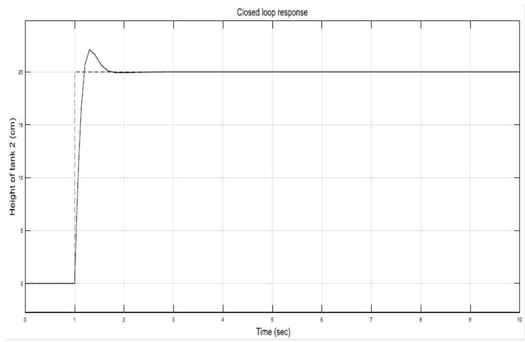

Figure 7: Closed loop response of tank 2 of TTCIS

$$D_{12} = \frac{-H_{12}}{H_{11}} = \frac{-0.0086}{0.0064s + 0.0401} \tag{5.5}$$

$$D_{21} = \frac{-H_{21}}{H_{22}} = \frac{-0.0086}{0.0090s + 0.0227} \tag{5.6}$$

The block diagram of the closed loop response with decoupler simulated in Simulink is as shown in the Fig. 8.

The response of the tank 1 for the above block diagram is as shown in the Fig.9. It can be observed that the response is much superior compare to that obtained without decoupler. The decoupler is employed to eliminate the interactions and to obtain an enhanced overall performance of the system. The time domain specifications for the obtained response were calculated.

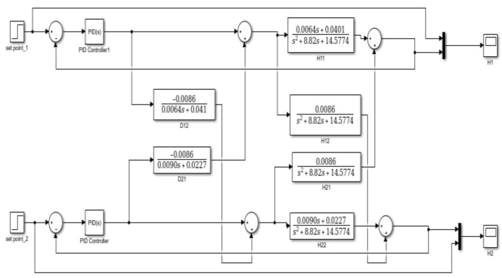

Figure 8: Simulink Model of closed loop response with decoupler

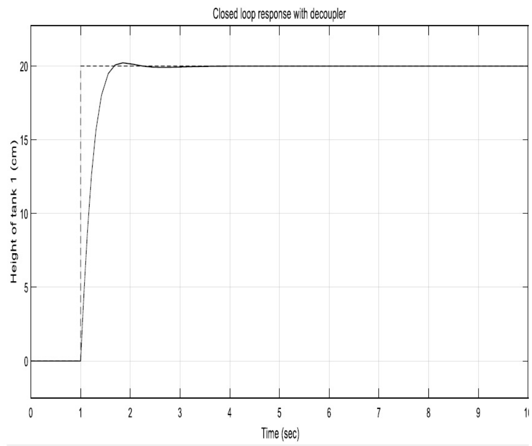

Figure 9: Tank 1's closed loop reaction with decoupler

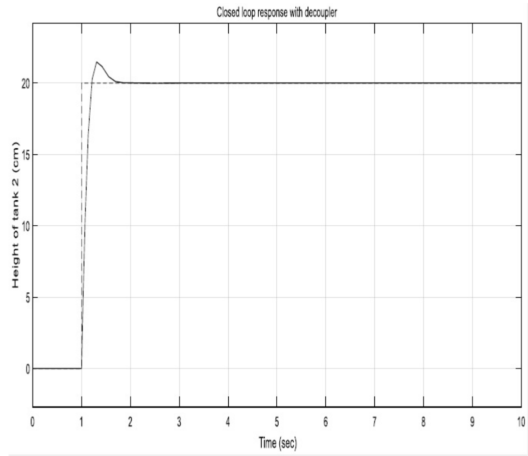

Figure 10: Closed loop response of tank 2 with decoupler

The tank 2's response to the aforesaid block diagram is depicted in Fig. 10. It can be observed that the response is much superior compare to that obtained without decoupler. The response is much faster compared to that of the response obtained for tank 2 without decoupler.

5.4. Comparison Of Responses

Time domain specifications help in assessing the performance of the system. The assessing parameters be rise time t_r, Peak time t_p, settling time t_s, delay time t_d, and maximum peak overshoot M_p. These time domain specifications of the closed loop response with and without decoupler of the two conical interacting tanks are calculated and compared in Table 2.

6. Conclusion

Here, the two tank conical interacting system is identified as a non-linear system. The mathematical model of TTCIS is obtained by using first principle law. The interactions between the two tanks were quantified using RGA. The non-linear model was linearised using Jacobian matrix, which yielded four matrices A, B, C and D. The transfer functions of the plant were found from the state space equations using ss2tf function in Matlab. The decoupler was designed from the plant transfer function. The open loop response of the TTCIS was simulated for understanding the plant. The

Table 2: Comparison of Time Domain Specifications

TIME DOMAIN SPECIFICATION	WITHOUT DECOUPLER		WITH DECOUPLER	
	TANK 1 (sec)	TANK 2 (sec)	TANK 1 (sec)	TANK 2 (sec)
Rise Time	1.7	1.2	1.4	1.2
Peak Time	1.9	1.4	1.7	1.3
Settling Time	2.2	2	2	1.9
Delay Time	1.1	1	1.15	1.05
Maximum Peak Overshoot	40 %	40 %	-	25 %

closed loop response without decoupler was obtained by ZN tuning the PID controller in it. Then, the better response was obtained by incorporating decoupler into it. The responses were compared by calculating the time domain parameters.

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