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# The estimation process in the Bayesian quantile structural equation modeling approach

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**Abstract**

Latent variable models define as a wide class of regression models with latent variables that cannot be directly measured, the most important latent variable models are structural equation models. Structural equation modeling (SEM) is a popular multivariate technique for analyzing the interrelationships between latent variables. Structural equation models have been extensively applied to behavioral, medical, and social sciences. In general, structural equation models includes a measurement equation to characterize latent variables through multiple observable variables and a mean regression type structural equation to investigate how the explanatory latent variables affect the outcomes of interest. Despite the importance of the structural equations model, it does not provide an accurate analysis of the relationships between the latent variables. Therefore, the quantile regression method will be presented within the structural equations model to obtain a comprehensive analysis of the latent variables. we apply the quantile regression method into structural equation models to assess the conditional quantile of the outcome latent variable given the explanatory latent variables and covariates. The posterior inference is performed using asymmetric Laplace distribution. The estimation is done using the Markov Chain Monte Carlo technique in Bayesian inference. The simulation was implemented assuming different distributions of the error term for the structural equations model and values for the parameters for a small sample size. The method used showed satisfactorily performs results.

*Keywords:* structural equations model, Bayesian inference, latent variable models, structural equations model, quantile regression

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## 1. Introduction

### Latent variables

Everett (2013) defines latent variable models as a wide class of regression models with latent variables that cannot be directly measured. For the following reasons, latent variables are frequently used in models. A latent variable, for starters, can be thought of as the underl$y_i$ng attribute that is measured by a number of observable variables. This is a common method for achieving dimensionality reduction in factor analysis models, in which the latent variables are also referred to as factors. Second, in mixed effects models, which are widely employed in the analysis of longitudinal data with repeated measurements on the same subjects, latent variables might indicate unobserved subject-specific variability. The latent variables are also known as random effects in this context. Third, latent variables can be utilized to represent the true values of observed variables in some cases where the observed variables are subject to measurement error.

### Structural equation model

Structural equation model(SEM) are a versatile class of models that allow for complicated modeling of correlated multivariate data in order to examine interrelationships between observable and latent variables. Many extensively used statistical models, such as regression, factor analysis, canonical correlations, and analysis of variance and covariance, are included in this class of models, which is well recognized in the fields of social and psychological sciences. Most applications of SEMs are related to the study of interrelationships among latent variables. In particular, they are useful for examining the effects of explanatory latent variables on outcome latent variables of interest. For such situations, researchers usually have in mind what observed variables should be selected from the whole data set for the analysis, and how these observed variables are grouped to form latent variable [12].

Where it is mentioned that the term of SEM are actually referring to the simultaneous equation model in econometrics, which is not what we commonly refer to as “SEM” because no latent variables are involved in their models. Traditional methods for analyzing SEMs were mainly developed in psychometrics, and have been extensively applied in behavioral, educational, social research During the past years. Recently, SEMs have begun to attract a great deal of attention in medical sciences public health [1],Although it is widely used in many fields, it has rarely been applied in latent variable models, including structural equation models, In Bayesian latent models, [7] presented the QR technique as a comprehensive statistical tool to analyze the relationship between the response variable and the explanatory variables in the entire conditional distribution of the response variable by estimating conditional quantiles Qp(Y|X)} instead of only estimating the conditional mean (E(Y| X)) as in the normal regression. In this technique, the Least Absolute Deviation method (LAD) was used to estimate the model parameters instead of the quadratic loss function used in the normal regression model. Explain the importance of using (QR) technique to estimate regression models that do not assume the state of the non-Gaussian errors. [15] suggested the Bayesian method in analyzing the Linear QR model by assuming an Asymmetric Laplace distribution for random errors in the model in order to link the Bayesian method to the classical method in estimating the model and noting that the minimization of the loss function applied in the method The classic is equivalent to maximizing the possibility function using the skewed Laplace distribution of error,

The median regression method for Bayesian latent response models was proposed by Dunson, Watson, and Taylor (2003), Dunson and Taylor(2005) proposed an approximate method based on a substitution likelihood expressed as a vector of quantiles, Burgette and Reiter (2012) proposed the quantile regression in a factor analysis model to analyze the effects of latent variables on the lower quantiles of the response distribution. The Bayesian inference for the proposed quantile Structural equation model is based on ALD in this article, not only because it is a natural and effective method to model the Bayesian quantile regression, but also because it is a natural and effective way to model the Bayesian quantile regression.

## 2. Quantile Regression

The subject of regression is one of the important statistical topics used in many scientific research and has wide applications in many fields in recent years. Ordinary regression, or what is sometimes called mean regression, is one of the important statistical methods that are used in analyzing the relationship between explanatory variables X and response variable Y, through the mean point in the distribution of the response variable. In other words, in the normal mean regression analysis, the attention is focused on estimating the conditional mean of the distribution of the response variable $\{E(Y|X)\}$ and the loss function used in estimating the normal regression models is the quadratic loss function. It is known that the normal regression analysis is based on the assumptions of the analysis, the most prominent of which is that the random error in the model is distributed independently, normally, with a mean equal to zero, and variance is $\sigma_\epsilon^2$.

The main purpose of quantile regression is to obtain a highly comprehensive analysis of the relationship between variables by using different measures of central tendency and statistical dispersion. A well-known special case is the least absolute deviation, or median regression, where the 50% conditional quantile is estimated [12].

The rest of the paper is organized as follows. In section 3, we present Quantile Structural equation model. In section 4 we present Bayesian inference of QSEM model with finding the conditional distributions of parameters and latent variable within the Bayesian analysis in chapter 5.and in section, In section 6we perform simulation studies to examine the performance of the method used with different error term distributions. We conclude with brief conclusions in section 7.

## 3. Quantile Structural equation model

The structural equation model consists of two components, as follows:

1. Let $y_i = (y_{i1}, y_{i2}, \ldots, y_{ip})^T$ be a $(p)$ vector representing the ith observation in a random sample of size $n$, and $(\omega_{i1}, \omega_{i2}, \ldots, \omega_{ip})^T$ be a $(q \times 1)$ vector of latent variables with $(q < p)$.
   The link between $y_i$ and $\omega_i$ is defined by the following measurement equation:

$$y_i = Ac_i + \Lambda\omega_i i + \varepsilon_i, i = 1, \ldots.., n \tag{3.1}$$

   where A $(p \times r1)$ and $\Lambda(p \times q)$ are matrices of unknown coefficients, $ci(r1 \times 1)$ is a vector of fixed covariates, and $\epsilon i(p \times 1)$ is a random vector of error terms.
2. $\eta_i$ can be assessed in the following structural equation

$$\eta_i = \beta\tau di + \Gamma\tau\zeta_i + \delta i, \qquad i = 1, \ldots.., n \tag{3.2}$$

   Then the quantile SEM is defined by Equations (3.1) and (3.2).

The purpose of the measurement equation in an SEM is to relate the latent variables in $\omega$ to the observed variables in y . It represents the link between observed and latent variables, through the specified factor loading matrix $\Lambda$, the vector of measurement error $\epsilon$ is used to take the residual

error into account. The important issue in formulating the measurement equation is to specify the structure of the factor loading matrix $\Lambda$, based on the knowledge of the observed variables in the study. Any element of $\Lambda$ can be a free parameter or fixed parameter with a preassigned value [11]. The positions and the preassigned values of fixed parameter are decided on the basis of the prior knowledge of the observed and latent variables, and they are also related to the interpretations of latent variables [11].
To analyze the interrelationship among latent variables, let partition $\omega_i = (\eta_i^T, \zeta_i^T)^T$, where $\eta_i(q_1 \times 1)$ denote outcome latent variables and $\zeta_i(q_2 \times 1)$ is explanatory latent variables.

to simplify we assume that$q_1 = 1$ . The primary aim of SEM is analyze the behavior of latent variable $\eta_i$ given the information contained in a set of explanatory latent variables $\zeta_i$ . This is done in traditional SEM by calculating the conditional mean of ( $\eta_i \backslash \zeta_i$ ) and fixed covariates $d_i(r2 \times 1)$ as follows: [4]

$$E(\eta_i \backslash \zeta_i, di) = Bdi + \Gamma \zeta_i, \qquad i = 1, \ldots.., n \tag{3.3}$$

Where $B(q1 \times r2)$ and $\Gamma(q1 \times q2)$ are the matrices of unknown coefficients to be estimated.
The conditional mean does not provide a complete description of the interrelationship among latent variables. A more comprehensive analysis can be achieved from a combination of $Q(\eta_i \backslash \zeta_i)$ with fixed covariates $di(r2 \times 1)$ the conditional quantile of $\eta_i$, under a number of different quantiles $\tau \in (0, 1)$, as follows:

$$Q\tau(\eta_i \backslash \zeta_i, di) = B_\tau di + \Gamma_\tau \zeta_i, \qquad i = 1, \ldots.., n \tag{3.4}$$

The coefficients matrices $B_\tau$ and $\Gamma_\tau$ have a subscript $\tau$ because they might not be equal for different quantiles.
Unlike in conventional SEMs, here the distribution of $\delta_i$ is undefined The only assumption is that the $\tau-$ quantile of $\delta_i$ is 0 to guarantee that equation (3.4) holds. Unlike the structural equation, the measurement equation is restricted to a median regression model rather than quantile regression with an arbitrary $\tau$ , because its main role is to relate highly correlated observed variables to latent factors. Therefore, a quantile regression is meaningless here. Instead, a median regression model is employed to achieve robust estimates of parameters in the measurement equation while persisting its parsimonious [12].

As it was previously shown that the quantile SEM can overcome many of the problems faced by the traditional SEM, which is first by focusing on the conditional mean and since the traditional SEM ignore all the information about the tails of the distribution that may be important in the research, and on the contrary, the quantile SEM is Of more comprehensive importance is the relationships between the latent variables in all response levels, including the upper and lower tails and the central tendency, as well as in many cases the error term distribution is not normally consistent with equal variances, and this fails the traditional SEM assumptions, leading to inaccurate results.

Conventional(SEM) are sensitive to error terms with heavy-tailed distributions and extreme outliers, which can distort the results significantly. On the contrary, quantile regression is widely regarded as a robust method, which is less vulnerable to outliers and extreme distributions of error terms. It should be emphasized that quantile SEM is not a replacement for conventional SEM but rather a supplement to it. In substantive study, if the effects of explanatory variables on the entire distribution of the response is of interest, quantile SEM is a good choice. Alternatively, one could conduct both conventional SEM and quantile SEM and use the results for a highly comprehensive analysis [12].

## 4. Bayesian inference of QSEM model

The Bayesian approach to SEM has increase in popularity in recent years for establishing efficient and rigorous statistical approaches, and solving practical problems. As a result, we use the Bayesian method to analyze quantile SEM due to its ease of use when dealing with complex models and the convenience of inference with Markov chain Monte Carlo (MCMC) methods.

However, because the distribution of the error term $\delta_i$ in structural equation (3.2) is not specified, the likelihood function, which is essential in Bayesian analysis, is also unspecified. Therefore, the ALD is introduced to address this problem, where [15] assumed (regardless of the real distribution of the data) the Asymmetric Laplace Distribution

A random variable y is distributed as Asymmetric Laplace Distribution ($AL(\mu, \sigma, \tau)$), the probability density function It can be expressed in the following form [12]

$$f\left(y \backslash \mu, \sigma, \tau\right) = \frac{\tau(1-\tau)}{\sigma} \exp\left\{-\rho_\tau\left(\frac{y-\mu}{\sigma}\right)\right\}$$

Where $\mu$ is the location parameter, $\sigma$ is the scale parameter and $\tau$ is the skewness parameter,

the $\rho_\tau(x) = x(\tau - I(x < 0))$ is called check function.

As mentioned earlier [15] discovered the link between quantile regression and Asymmetric Laplace Distribution (ALD) in the following quantile regression model:

$$Q_\tau(y_i \backslash X_i) = X_i^T \beta_\tau, \qquad i = 1, \ldots, n$$

Where $y_i$ is the response variable and $X_i$ is the explanatory variable , $Q_\tau(y_i \backslash X_i)$ is the conditional $\tau$- quantile of $(y_i \backslash X_i)$, and $\beta_\tau$ contains quantile- specific regression coefficients.
According to Koenker and Bassett(1978) [7],the frequentist approach to coefficient estimation is to solve the following optimization problem [4].

$$\min_\beta \sum_{i=1}^{n} \rho_\tau\left(y_i - x_i^T \beta\right) \tag{4.1}$$

The likelihood function for the model is showed by [15] by assuming that the error terms are independent and identically distributed is$AL(0, 1, \tau)$ as following:

$$L\left(\beta; y, \tau\right) = \tau^n (1-\tau)^n \exp\left\{-\sum_{i=1}^{n} \rho_\tau(y_i - x_i^T \beta)\right\}$$

Apparently the maximum likelihood estimation of $\beta$ is equivalent to the solution of equation (4.1).

They also proposed a Bayesian approach by imposing an improper uniform prior for all of the components of $\beta$, resulting in a proper joint posterior distribution. The posterior mode is also demonstrated to be equal to equation 5's solution. As a result, using Asymmetric Laplace Distribution (ALD) for the error terms is a natural technique to approach the Bayesian quantile regression problem.

The ALD is not a standard distribution and dose not have a conjugate prior, unlike the gamma or Gaussian distributions. As a result of the complexity of the likelihood function, the resultant posterior density for $\beta$ is not analytically tractable. For posterior inference, different MCMC methods have been proposed. There are many attempts by researchers to use different types of MCMC methods, the first of which is [15] attempts, where they used a random-walk Metropolis algorithm to simulate

from posterior distributions, By employing a mixture representation of ALD, [9], considered Gibbs sampling approaches for quantile regression.
Then, if y is a random variable distributed $AL(\mu, \sigma, \tau)$ then it can be represented as following:

$$y = \ \mu + k_1 e + \sqrt{k_2 \sigma e} \varsigma$$

Where $k_1 = \frac{(1-2\tau)}{(\tau(1-\tau))}, \quad k_2 = \frac{2}{\tau(1-\tau)}$
$c \sim N[0,1], \qquad e \sim \exp(1/\sigma)$
The resulting conditional distribution of y is normal, with a mean ( $\mu + k_1 e$) and variance ($k_2 \sigma e$).

And the latent variable e is augmented to y. therefore, the augmentation makes it possible to use the usual normal prior of the coefficient $\beta$. According to [9], the developed Gibbs sampling algorithm is preferable to the Metropolis algorithm because it avoids the inconvenience of choosing the proposal distribution, and thus improves the estimation result as well as the efficiency of the MCMC sampler [12].

## 5. Bayesian inference

In order to speed up and increase the performance of the Bayesian method in the analysis of the QSEM model, and for the reasons mentioned previously, this research was based on the proposal of [12] in using the mixed representation of the skewed Laplace distribution (AL) for random error in the model. According to Wang's [12] assumption that will be adopted in this research for the error terms, specifically $\epsilon_{ij}$ the $k^{th}$ component of the error terms $\epsilon_i$ is distributed $AL(0, \sigma_{yk}, 0.5)$ for measurement equation (3.1) the median regression, and $\sigma_i$ is distributed $AL(0, \sigma_{yk}, \tau))$ for structural equation (3.2) the $\tau$-quantile regression. Noting that the variables $e_{y_{ik}}$ and $e_{\eta_i}$ are the nuisance variables for augmenting $\epsilon_{ij}$ and $\delta_i$.
Let $\theta_1 y$ the unknown parameters in equation (3.1), and $\theta_\omega$ unknown parameters in equation (3.2), and $\theta = (\theta_1 y, \theta_1 \omega)$ ,then the Bayesian for Quantile SEM by the following hierarchical representation:

$$(y_i \backslash \eta_i, \xi_i, \theta 1_y, e_{y_i}) \sim N_p \ \ (Ac_i + \Lambda \omega_i \ , \Psi_i) \tag{5.1}$$

$$(\eta_i \backslash \xi_i \ , \theta 2_\omega, e_{\eta_i}) \sim N \ ( \ B_\tau d_i + \Gamma_\tau \xi_i + k_1 e_{\eta_i}, k_2 \sigma_\eta e_{\eta_i}) \tag{5.2}$$

The unknown parameters in the measurement equation (3.1) let $\theta 1_y = (A, \Lambda) = (\lambda_{ykj})$ and in the structural equation (3.2), the unknown parameters are $\theta 2 \omega \tau = (B_\tau, \Gamma_\tau)$.
Some elements of $\theta 1_y$ must be fixed for identification purposes ,for measurement equation, an index matrix $M = (I_{ykj})$ as its identification matrix is created as follows

$$I_{ykj} = \begin{cases} 1 & \lambda_{ykj} \quad is \ (free) \\ 0 & \lambda_{ykj} \quad is \ (fixed) \end{cases}$$

When $I_{ykj} = 1$ if $\Lambda_{ykj}$ is subject to estimation and $I_{ykj} = 0$ if the value of $\Lambda_{ykj}$ is prefixed for identification purpose

$$e_{yik} \sim \ \ Exponential \ (\sigma_{yk})$$
$$e_{\eta i} \sim \ \ Exponential \ (\sigma_\eta)$$
$$\zeta_i \sim N(0, \Phi)$$

Where $e_{y_i k} = (e_{y_i 1}, \ldots, e_{y_i p})^T, \psi = diag(8\sigma_{y1} e_{y_i 1}, \ldots, 8\sigma_{yp} e_{y_i p}), \ and \ e_\eta = (e_{\eta_1}, \ldots, e_{\eta_n})^T$
The following conjugate prior distribution in Bayesian quantile SEM are:

- For measurement equation as follows :

$$\begin{aligned} \theta 1_{yk} &\sim N_{r1+q}(\Lambda_{0yk}, H_{0yk}) \\ \sigma_{yk}^{-1} &\sim Gamma(a_{0yk}, b_{0yk}) \end{aligned} \tag{5.3}$$

- For structural equation as follows :

$$\begin{aligned} \theta 2_{\omega_\tau} &\sim N_{r2+q2}(\Lambda_{0\omega}, H_{0\omega}) \\ \sigma_\eta^{-1} &\sim Gamma(a_{0\sigma}, b_{0\sigma}) \\ \Phi^{-1} &\sim \ Wishart\ (R_0, \rho_0) \end{aligned} \tag{5.4}$$

where $(\Lambda_{0yk}, a_{0yk}, b_{0yk}, \Lambda_{0\omega}, a_{0\sigma}, b_{0\sigma}$ are hyperparameters and the positive-definite $H_{0yk}, H_{0\omega}$ are also hyperparameters, Noting that the values are given from previous research or professional knowledge. let$Y = (y_1, \ldots, y_n), C = (c_1, \ldots, c_n), D = (d_1, \ldots . d_n)$ *and* $\omega = (\omega_1, \ldots, \omega_n)$ be the matrix of latent variable. Given the complexity of the model, direct inference of the common posterior distribution $p(\omega, \theta \backslash Y, C, D, e_\eta)$ is difficult and complex. However, the full conditional distributions of the latent variables and all parameters are common distributions. Therefore, the Gibbs sampling method is used as an easy and uncomplicated method in obtaining Bayesian estimators so that the Gibbs sampling tool can be implemented easily, and a Bayesian estimate is taken for each parameter to be the average of the sample random observations derived from each iteration.

As is well known the Bayesian estimate of parameters are obtained from the joint posterior distribution $p(\Omega, \theta | Y, C, D, e_\eta)$ by drawing samples iteratively for parameters and latent variables, each component of the posterior distribution is generated by the Gibbs sampling method from its full conditional posterior distribution in an iteratively. The Bayesian estimates of $\theta$ and $\Omega$ is taken to be the sample mean of the random observations generated.

As mentioned earlier, the main objective is to use MCMC methods to obtain the Bayesian estimates of $\theta$ and $\Omega$, for this reason a sequence of random observations from the joint posterior distribution $[\theta, \Omega / Y]$ will be generated via the Gibbs sampler which is implemented as follows: At the jth iteration with current value $\theta^{(j)}$ [11] :

(a) Generate a random variate $\Omega_{(j+1)}$ from the condition $\{\omega / y, \theta_j\}$
(b) Generate a random variate $\theta(j+1)$ from the condition $\{\theta / y, \Omega\}$ and return to step a if necessary

Then the full conditional posterior distribution for Bayesian quantile SEM (BQSEM) as follows:

The Gibbs sampling algorithm is implementing with following full conditional posterior distribution of parameters and latent variable [12]. Let $\theta 1_y = (A, \Lambda), \theta 2_\omega = (B_\tau, \Gamma_\tau), ui = (ci^T, \omega_i^T)^T, vi = (di^T, \zeta_i^T)^T, U = (u1, \ldots, ui)$ where $U_k$ be its submatrix with rows corresponding to $I_y k_j = 0$ are deleted, $Y_k^* = (y_{1k}^*, \ldots, y_{nk}^*)$ where

$$y_{ik}^* = \ y_{ik} - \ \sum_{j=1}^{r_1+q} \lambda_{\mathrm{ykj}}\ u_{\mathrm{ij}}\ (1 - \ I_{\mathrm{ykj}})$$

1- The full conditional posterior distribution of the latent variable $\Omega$
As shown in equation (5.1), the y distribution ia as follows:

$$(y_i \backslash \theta 1_y, \eta_i, \xi_i, e_{y_i}) \sim \ N_p \ \left(Ac_i + \Lambda \omega_i \ , \Psi_i\right)$$

$$p(Y \backslash \theta 1_y, \eta_i,\ \xi_i,\ e_{y_i})\eta_i \ = (\Psi_i)^{\frac{-n}{2}} \exp\left\{ -\frac{1}{2} \sum_{i=1}^{n} (\ y_i - \theta 1_y u_i)^T \Psi_e^{-1}\ (y_i - \theta 1_y u_i) \right\}$$

It is known that

$$p(\omega_i/\theta 1_{y_i},\ \theta_y)\ \propto p(\omega_i/\theta_y)\ p(y_i/\omega_i,\theta_y)$$

Then, The full conditional posterior distribution of the latent variable is

$$(\ \omega_i \backslash y_i \sigma_{y_i} e_{y_i} \theta 1_y \sigma_\eta e_{\eta_i} \Lambda_\omega \Phi) \sim\ \ N_q(\ \mu_i, \Sigma_i^{-1*})$$

Where

$$\mu_i =\ \Sigma_i^{*-1}\Lambda^T\psi_i^{-1}(y_i - Ac_i) + \Sigma_i^{*-1}\Sigma_{\omega_i}^{-1}\begin{pmatrix} B_\tau d_i + k_1 e_{\eta_i} \\ 0 \end{pmatrix}$$

$$\Sigma_i^* = \Sigma_{\omega_i}^{-1} + \Lambda^T\psi_i^{-1}\Lambda$$

$$\Sigma_{\omega_i} = \begin{pmatrix} \Gamma_\tau\Phi\Gamma_\tau^T + k_2\sigma_\eta e_{\eta_i} & \Gamma_\tau\Phi \\ \Phi\Gamma^T & \Phi \end{pmatrix} \qquad \psi_i = diag(8\sigma_{y1}e_{y_{i1}}, \ldots, 8\sigma_{yp}e_{y_{io}})$$

2- The full conditional posterior distribution of the $e_{y_ik} : for\ (i = 1, \ldots, n, k = 1, \ldots, p)$

$$p\left(e_{y_ik}^{-1} \backslash y_{ik},\ \omega_i,\ \theta 1_{yk},\ \sigma_{yk}\right) \propto f\left(y_{ik},\ \omega_i,\ \theta 1_{yk}, \sigma_{yk}\right) f(e_{y_ik} \backslash \sigma_{yk})$$

$$p\left(e_{y_ik}^{-1} \backslash y_{ik}, \omega_i, \theta 1_{yk}, \sigma_{yk}\right) \propto \left\{\frac{2\sigma_{yk}^{-1}}{2\pi(e_{y_ik}^{-1})^3}\right\}^{\frac{1}{2}} \exp\left\{\frac{2\sigma_{yk}^{-1}\left(e_{y_ik}^{-1} - \frac{4}{|y_{ik}-\theta 1_{yk}u_i|}\right)^2}{2\left[4\left|y_{ik} - \theta 1_{yk}u_i\right|^{-1}\ \right]^2 e_{y_ik}^{-1}}\right\}$$

Thus, the full conditional distribution of $e_{y_ih}$ is a inverse Gaussian distribution $(4\left|y_ik - \theta 1_{yk}u_i\right|^{-1}, 2\sigma_{yk}^{-1})$

3- The full conditional posterior distribution of the $\theta 1_y : for\ (k = 1, \ldots, p)$

$$p\left(\theta 1_{yk} \backslash Y, e_{y_{ik}}, \sigma_{yk}\right) \propto \left(\Sigma_{\theta 1k}^{-1}\right)^{\frac{-1}{2}} \exp\left(\frac{-1}{2}\left(\theta 1_{yk} - Mu_{\Lambda k}\right)^T\left(\Sigma_{\Lambda k}^{-1}\right)^{-1}\left(\theta 1_{yk} - Mu_{\Lambda k}\right)\right)$$

Where $Mu_{\Lambda_k} =\ \Sigma_{\Lambda_k}^{-1}(\ \H_{0y}^{-1}\Lambda_{oy} + \sum_{i=1}^n \frac{y_{ik}u_i}{8\sigma_{yk}e_{y_{ik}}})$ and $\Sigma_{\Lambda k} =\ \H_{0y}^{-1} +\ \sum_{i=1}^n \frac{u_i u_i^T}{8\sigma_{yk}e_{y_{ik}}}$

Thus, The full conditional posterior distribution of the $\theta 1_y$ is a normal distribution $(Mu_{\Lambda_k}, \sigma_{\Lambda_k}^{-1})$

4- The full conditional posterior distribution of the $\sigma_{yk} : for\ (k = 1, \ldots, p)$

$$p\left(\sigma_{yk}^{-1} \backslash Y, U, \Lambda_{yk}\right) \propto \left(\sigma_{yk}^{-1}\right)^{n+a_{oyk-1}} \exp\left\{\left(b_{oyk} + \frac{1}{2}\sum_{i=1}^n \left|y_{ik} - \theta 1_{yk}u_i\right|\right)\sigma_{yk}^{-1}\right\}$$

Thus, The full conditional posterior distribution of the $\sigma_{yk}$ is Gamma distribution

$$(n + a_{oyk},\ b_{oyk} + \frac{1}{2}\sum_{i=1}^n |y_{ik} -\ \theta 1_{yk}u_i|)$$

5- The full conditional posterior distribution of the $\Phi$ :

$$p(\Phi \backslash \Omega_2) \propto p(\Phi)\prod_{i=1}^n p(\xi_i \backslash \Phi)$$
$$p(\Phi \backslash \Omega_2) \propto |\Phi|^{\frac{-(n+\rho_0+q_2+1)}{2}} \exp\left\{-\frac{1}{2}\ \mathrm{tr}\left[\Phi^{-1}(\Omega_2\Omega_2^T + R_0^{-1})\right]\right\} \tag{5.5}$$

Since the right-hand side of (5.4) is proportional to the density function of an inverted Wishart distribution (Zellner,1971), it follows that the conditional posterior distribution of $(\Phi\backslash\Omega_2)$ is given by

$$[\Phi\backslash\Omega_2] \sim \mathrm{IW}_{q2}\left[\left(\Omega_2\Omega_2^T + R_0^{-1}\right),\ n+\rho_0\right]$$

6- The full conditional posterior distribution of the $e_{\eta_i} : for\ \ (i=1,\ldots,n)$

$$p\left(e_{\eta_i}^{-1}\backslash\ \omega_i,\ \theta2_\omega,\ \sigma_\eta\right) \propto f\left(\ \omega_i,\ \theta2_\omega, e_{\eta_i}^{-1}, \sigma_\eta\right) f(e_{\eta_i}^{-1}\backslash\sigma_{yk})$$

$$\left(e_{\eta_i}^{-1}\backslash\omega_i, \theta2_\omega, \sigma_\eta\right) \propto \left\{\frac{\frac{k_2}{4\sigma_\eta}}{2\pi(e_{\eta_i}^{-1})^3}\right\}^{\frac{1}{2}} \exp\left\{\frac{\frac{k_2}{4\sigma_\eta}\left(e_{\eta_i}^{-1} - \frac{k_2}{2|\eta_i - B_\tau d_i - \Gamma_\tau\xi_i|}\right)^2}{2\left[\frac{k_2}{2|\eta_i - B_\tau d_i - \Gamma_\tau\xi_i|}\right]^2 e_{\eta_i}^{-1}}\right\}$$

Thus, The full conditional posterior distribution of the $e_{\eta_i}$ is a inverse Gaussian distribution

$$\left(\frac{k_2}{2\left|\eta_i - B_\tau d_i - \Gamma_\tau\xi_i\right|}, \frac{k_2}{4\sigma_\eta}\right)$$

7- The full conditional posterior distribution of the $\theta2_{\omega\tau}$

$$p\left(\theta2_{\omega\tau}\backslash\Omega, e_\eta, \sigma_\eta\right) \propto \left(\Sigma_{\theta2\omega}^{-1}\right)^{\frac{-1}{2}} \exp\left(\frac{-1}{2}\left(\theta2_{\omega_\tau} - Mu_{\theta2\omega}\right)^T \left(\Sigma_{\theta2\omega}^{-1}\right)^{-1}\left(\theta2_{\omega_\tau} - Mu_{\theta2\omega}\right)\right)$$

where $Mu_{\theta2\omega} = \Sigma_{\theta2\omega}^{-1}\left(\mathrm{H}_{o\omega}^{-1}\theta2_{\omega\tau} + \sum_{i=1}^n \frac{(\eta_i - k1e_{\eta_i})v_i}{k2\sigma_\eta e_{\eta_i}}\right)$ and $\Sigma_{\theta2\omega} = \mathrm{H}_{0\omega}^{-1} + \sum_{i=1}^n \frac{v_i v_i^T}{k2\sigma_\eta e_{\eta_i}}$

Thus, The full conditional posterior distribution of the $\theta2_\omega$ is a normal distribution $(\mu_{\theta2\omega\tau}, \sigma_{\theta2\omega\tau}^{-1})$

8- - The full conditional posterior distribution of the $\sigma_\eta$ :

$$p\left(\sigma_\eta^{-1}\backslash\Omega,\theta2_{\omega_\tau}\right) \propto (\sigma_\eta^{-1})^{n+a_{0\delta}-1} \exp\left\{\left(b_{0\delta} + \sum_{i=1}^n \rho_\tau\left|\eta_i - \theta2_{\omega_\tau}v_i\right|\right)\sigma_\eta^{-1}\right\}$$

Thus, The full conditional posterior distribution of the $\sigma_\eta$ is Gamma distribution $(n + a_{o\delta}, b_{o\delta} + \sum_{i=1}^n \rho_\tau\left|\eta_i - \theta2_{\omega\tau}v_i\right|)$

## 6. Simulation study

In this section, We employ simulation to evaluate the Bayesian quantile SEM's empirical performance. We generated the data set from SEM:

$$y_i = Aci + \Omega\omega_i + \epsilon_i$$
$$\eta_i = b1di + \gamma_1 + \xi_{i1} + \gamma_2\zeta_{i2} + \delta_i \quad Where\ p = 9, q = 3\ \ (q_1 = 1, q_2 = 2)\ \ and\ \ r_1 = r_2 = 1.$$

Where $y_i = (y_{1i}, \ldots, , y_{9i})^T, A = (a_1, \ldots, a_9)^T, ci = c1i, \epsilon_i = (\epsilon_{1i}, \ldots, \epsilon_{9i})\ and\ \omega_i = (\eta_i, \zeta_i)$

The simulation study's main purpose is to estimate the quantile regression coefficients b1,$\gamma_1$ and$\gamma_2$ under different quantiles with small sample size and compare them to their theoretical values.

We are choose three small sample size $n = (25, 50, 100)$ and the quantile we choose $\tau = 0.25, 0.5, 0.75$.

The factor loading matrix $\Lambda$ has the common non-overlapping structure

$$\Lambda^T = \begin{bmatrix} 1^* & \lambda_{21} & \lambda_{31} & 0^* & 0^* & 0^* & 0^* & 0^* & 0^* \\ 0^* & 0^* & 0^* & 1^* & \lambda_{52} & \lambda_{62} & 0^* & 0^* & 0^* \\ 0^* & 0^* & 0^* & 0^* & 0^* & 0^* & 1^* & \lambda_{83} & \lambda_{93} \end{bmatrix}$$

Where the zero and ones marked with an asterisk(*) are fixed in advance to allow for a clear interpretation of latent variables and model identification, while the other Λjk are unknown parameters. The true vales of parameters $\Lambda_{jk}$ and $aj$ in the measurement equation are taken to be $\Lambda_{21} = \Lambda_{31} = \Lambda_{52} = \Lambda_{62} = \Lambda_{83} = \Lambda_{93} = 0.7$, then the factor loading matrix Λ will be in the following

$$\Lambda^T = \begin{bmatrix} 1^* & 0.7 & 0.7 & 0^* & 0^* & 0^* & 0^* & 0^* & 0^* \\ 0^* & 0^* & 0^* & 1^* & 0.7 & 0.7 & 0^* & 0^* & 0^* \\ 0^* & 0^* & 0^* & 0^* & 0^* & 0^* & 1^* & 0.7 & 0.7 \end{bmatrix}$$

And $A = a_1 = \ldots = a_9 = 0.5$. The true values of parameters$B_\tau = (b1) = (0.1)$ and $\Gamma_\tau = [\gamma_1, \gamma_2] = [0.1, 0.3]$, and the explanatory latent variable $\zeta_i = (\zeta_{i_1}, \zeta_{i_2})^T$ is assumed to follow a normal distribution $N(0, \Phi)$ where $\Phi = \begin{bmatrix} 2 & 0.3 \\ 0.3 & 2 \end{bmatrix}$, and the fixed covariates $c_{1i}$ and $d_i$ are independently generated from standard normal distribution $N(0, 1)$.

Also, the prior distributions that specified within equation (5.2) and (5.3), and the hyperparameters are as follows:

for the conjugate prior of $\Lambda_{yk} \sim N_{r1+q}(\Lambda_{0yk}, Ḫ_{0yk})$, the free elements in the prior mean $\Lambda_{0yk}$ and $Ḫ_{0yk}$ is taken as a diagonal matrix with diagonal elements (10), As well for structural equation the $\Lambda_\omega$ the prior mean $\Lambda_{0\omega} = (1, 0.7, 0.7)$ and the covariance matrix $Ḫ_{0\omega} = 10$ an identity matrix.

And for the conjugate inverse gamma prior of $\sigma_{yk} = (a_{0yk}, b_{0yk}) = (1, 1)$ *and* $\sigma_\eta = (a_{0\sigma}, b_{0\sigma}) = (1, 1)$.

And for the inverse Wishart prior of Φ , we set $\rho 0 = 4, R0 = 5I_2$.

For the error terms for SEM $\epsilon_i$ and $\delta_i$, we consider the following different distribution

(i) $\epsilon_i$ and $\delta_i$ 's follow the normal distribution$N(0, 0.4)$.

(ii) $\epsilon_i$ and $\delta_i$ 's are distributed as the heavy- tailed central t-distribution t(5)

(iii) $\epsilon_i$ and $\delta_i$ 's are distributed as the skewed $\ln N(0, 0.35)$.

In the case (i), the normal distribution was chosen for the error terms as it aligns with that of traditional SEM., In the case (ii), the heavy-tailed t-distribution is used to assess the quantile SEM's performance in the presence of outliers in both the observed and latent variables. In Case (iii), the quantile SEM with skewed outcome latent variables is evaluated using a log-normal distribution we run10, 000 iterations with the initial 2,000 observations dropped in the burn-in phase on the basis of 100 replications where the program was written in R language. The performance of the Bayesian quantile structured equation model (QSEM) is assessed using the bias and root mean square error (RMS) ,where:

Bias $\left(\widehat{\theta}\right) = E\left(\widehat{\theta}\right) - \theta$

and root mean square error (RMS) is:

RMS $\left(\widehat{\theta}\right) = \left\{\frac{1}{n}\sum_{i=1}^{n}\left(\widehat{\theta} - \theta\right)\right\}^{\frac{1}{2}}$

the estimation result is compared with that of the conventional linear structured equation model. For it the estimates of regression coefficients in the structural equation, Which is the main goal are presented in Table 1 and 3 in comparison to the result of the conventional structural equation model. The estimates of other parameters are shown in Table 2 and 4

***Table 1: Bayesian estimates of the regression coefficients for structural equation for sample size n=25 $\epsilon i \sim N\ (0,\ 0.4)$***

| n= 25 | | | | | | | |
|---|---|---|---|---|---|---|---|
| *Error terms* | $\tau$ | $b_{1\tau}$ | | $\Gamma_{1\tau}$ | | $\Gamma_{2\tau}$ | |
| | | *RMS* | *Bias* | *RMS* | *Bias* | *RMS* | *Bias* |
| $\delta i \sim N(0,\ 0.4)$ | | | | | | | |
| *BQSEM* | 0.25 | 0.07907737 | -0.02253345 | 0.08403401 | 0.07595055 | 0.56830961 | 0.56685200 |
| | 0.5 | 0.10115009 | 0.09701302 | 0.07514715 | 0.07511159 | 0.61023296 | 0.60613099 |
| | 0.75 | 0.27155771 | 0.22295662 | 0.08985901 | 0.08632475 | 0.72082010 | 0.71788708 |
| *Classical SEM* | | 0.060251 | 0.081995 | 0.006198 | 0.0700285 | 0.5117301 | 0.5501241 |
| $\delta i \sim t(5)$ | | | | | | | |
| *BQSEM* | 0.25 | 0.07625216 | -0.02185594 | 0.08619419 | 0.07389584 | 0.55859595 | 0.55707620 |
| | 0.5 | 0.09111530 | 0.089224047 | 0.07659086 | 0.07625003 | 0.61088063 | 0.604798187 |
| | 0.75 | 0.2825732 | 0.2308178 | 0.1181471 | 0.1111511 | 0.7243099 | 0.7209314 |
| *Classical SEM* | | 0.294428 | -0.0190437 | 0.120462 | 0.1199032 | 0.766801 | 0.7938144 |
| $\delta i \sim \ln N(0,\ 0.5)$ | | | | | | | |
| *BQSEM* | 0.25 | 0.07815844 | -0.02175021 | 0.08833374 | 0.07366192 | 0.55181876 | 0.55076762 |
| | 0.5 | 0.10682500 | 0.103767993 | 0.09279538 | 0.092011884 | 0.61308815 | 0.608375589 |
| | 0.75 | 0.2879212 | 0.2423950 | 0.1181316 | 0.1128416 | 0.7388622 | 0.7341831 |
| *Classical SEM* | | 0.3100367 | -0.0118537 | 0.211769 | 0.1308377 | 0.744418 | o. 481077 |

***Table 2: Bayesian estimates of the parameters for measurement equation for sample size n= 25 with $\epsilon i \sim N\ (0,\ 0.4)$***

| | *Bayesian Quantile SEM (BQSEM)* | | | | | | *Classical SEM* | |
|---|---|---|---|---|---|---|---|---|
| | $\tau = 0.25$ | | $\tau = 0.5$ | | $\tau = 0.75$ | | | |
| *parameter* | *RMS* | *Bias* | *RMS* | *Bias* | *RMS* | *Bias* | *RMS* | *Bias* |
| $\lambda 21$ | 0.2697962 | 0.2235111 | 0.2790897 | 0.2318790 | 0.262098 | 0.2217202 | 0.259562 | 0.219983 |
| $\lambda 31$ | 0.1846035 | 0.1523480 | 0.1978101 | 0.1625291 | 0.1903749 | 0.1588405 | 0.184100 | 0.159322 |
| $\lambda 52$ | 0.1936832 | 0.1936815 | 0.1975364 | 0.1974831 | 0.192408 | 0.1923511 | 0.1923901 | 0.1921141 |
| $\lambda 62$ | 0.1143613 | 0.1071646 | 0.1221041 | 0.1142222 | 0.1128328 | 0.1082963 | 0.112995 | 0.1064492 |
| $\lambda 83$ | 0.2313131 | 0.2311828 | 0.2243311 | 0.2243307 | 0.2163074 | 0.2163022 | 0.215840 | 0.2159948 |
| $\lambda 93$ | 0.4306284 | 0.4305967 | 0.4201381 | 0.4200725 | 0.4100501 | 0.4095631 | 0.409223 | 0.4004823 |
| *a1* | 0.6346445 | -0.6139620 | 0.8111362 | -0.8054472 | 0.7677844 | -0.7647434 | 0.600572 | -0.9207726 |
| *a2* | 0.6478146 | -0.6010030 | 0.7913873 | -0.7633368 | 0.7500643 | -0.7281257 | 0.630446 | -0.7707318 |
| *a3* | 0.7026507 | - 0.6493849 | 0.8284873 | -0.7970920 | 0.7943271 | -0.7675296 | 0.700572 | -0.8005831 |
| *a4* | 0.6188674 | -0.5748930 | 0.6918025 | -0.6350983 | 0.6306503 | -0.5940988 | 0.601764 | -0.6504419 |
| *a5* | 0.5517148 | -0.5252306 | 0.6152347 | -0.5774228 | 0.5609859 | -0.5409852 | 0.550584 | -0.7848160 |
| *a6* | 0.7383686 | -0.7084734 | 0.7947926 | -0.7573358 | 0.7519067 | -0.7259338 | 0.730992 | -0.7603381 |
| *a7* | 0.6785300 | -0.6615303 | 0.7662372 | -0.7459725 | 0.6999516 | -0.6867111 | 0.670336 | -0.8104472 |
| *a8* | 0.5930336 | -0.5529379 | 0.6650815 | -0.6233569 | 0.6108447 | -0.5804727 | 0.588301 | -0.6503291 |
| *a9* | 0.6625122 | -0.6489197 | 0.7658687 | -0.7476199 | 0.7038860 | -0.6944282 | 0.652876 | -0.7801562 |
| $\Phi 11$ | 0.9826032 | 0.6626182 | 0.9513067 | 0.6437265 | 0.9625991 | 0.6609021 | 0.950773 | -0.6399106 |
| $\Phi 11$ | 0.7359126 | -0.2509661 | 0.7365012 | -0.2743960 | 0.7420674 | -0.2494504 | 0.729180 | -0.2807718 |
| $\Phi 21$ | 0.7359126 | -0.2509661 | 0.7365012 | -0.2743960 | 0.7420674 | -0.2494504 | 0.730712 | -0.2807718 |
| $\Phi 22$ | 0.8450573 | 0.7958273 | 0.8277118 | 0.7889141 | 0.8653089 | 0.8188990 | 0.750711 | 0.7206687 |

***Table 3: Bayesian estimates of the regression coefficients for structural equation for sample size n=50***

| n=50 | | | | | | | |
|---|---|---|---|---|---|---|---|
| *Error terms* | $\tau$ | $b_{1\tau}$ | | $\Gamma_{1\tau}$ | | $\Gamma_{2\tau}$ | |
| | | *RMS* | *Bias* | *RMS* | *Bias* | *RMS* | *Bias* |
| $\delta i \sim N(0,\ 0.4)$ | | | | | | | |
| *BQSEM* | 0.25 | 0.3659611 | 0.2540730 | 0.4114689 | 0.3356063 | 0.6496669 | 0.6367243 |
| | 0.5 | 0.35147395 | 0.276279100 | 0.43911876 | 0.361449468 | 0.58924238 | 0.588156606 |
| | 0.75 | 0.3955633 | 0.3556238 | 0.4166909 | 0.3605078 | 0.6065419 | 0.6050769 |
| *Classical SEM* | | 0.340291 | 0.241855 | 0.3016691 | 0.285319 | 0.510996 | 0.4928783 |
| $\delta i \sim t(5)$ | | | | | | | |
| *BQSEM* | 0.25 | 0.3612821 | 0.2476817 | 0.4150795 | 0.3354939 | 0.6470759 | 0.6330410 |
| | 0.5 | 0.34096969 | 0.26675727 | 0.44275604 | 0.36310566 | 0.59626770 | 0.59530398 |
| | 0.75 | 0.3871850 | 0.3423514 | 0.4114402 | 0.3505616 | 0.6278267 | 0.6260782 |
| *Classical SEM* | | 0.410663 | 0.590772 | 0.570992 | 0.460339 | 0.7118923 | 0.722691 |
| $\delta i \sim \ln N(0,\ 0.3)$ | | | | | | | |
| *BQSEM* | 0.25 | 0.3739504 | 0.2574705 | 0.4260287 | 0.3448913 | 0.6412085 | 0.6293570 |
| | 0.5 | 0.35954568 | 0.28115723 | 0.43140434 | 0.35165318 | 0.59729556 | 0.5965017 |
| | 0.75 | 0.3947921 | 0.3551088 | 0.4090277 | 0.3456804 | 0.6252271 | 0.6233558 |
| *Classical SEM* | | 0.4210591 | 0.3905588 | 0.4804912 | 0.3901178 | 0.7744193 | 0.6948271 |

The results will be compared with the results of the traditional structural equations model in the Bayesian method, It is shown in Tables 1, 2, 3 and 4. The normally distributed error terms ($\epsilon_i$ and $\delta_i \sim N\ (0,\ 0.4)$ in Case 1 that show in table 1 and 2 is matches with the conventional structured equation model's assumption, whereas the error terms in the quantile structured equation model are misspecified, so the conventional structured equation model performs slightly better than expected, but the results are very close For all assumed quantiles and for all sample sizes.

The estimates of the Classical structural equation model in Cases 2 and 3 are clearly skewed due to the heavy-tailed or skewed error terms. The quantile structured equation model, on the other hand, produces estimates with substantially lower bias and RMS, demonstrating the robustness of

***Table 4: Bayesian estimates of the parameters for measurement equation for sample size n= 50 with*** $\epsilon i \sim N\ (0,\ 0.4)$

| | Bayesian Quantile SEM (BQSEM) | | | | | | Classical BSEM | |
|---|---|---|---|---|---|---|---|---|
| | $\tau = 0.25$ | | $\tau = 0.5$ | | $\tau = 0.75$ | | | |
| parameter | RMS | Bias | RMS | Bias | RMS | Bias | RMS | Bias |
| $\lambda 21$ | 0.2411926 | .2383786 | 0.2507788 | 0.2483686 | 0.2494235 | 0.2480190 | 0.2406693 | 0.2201729 |
| $\lambda 31$ | 0.2170544 | 0.2165038 | 0.2222833 | 0.2219468 | 0.2205673 | 0.2199341 | 0.2205582 | 0.2318964 |
| $\lambda 52$ | 0.1939486 | 0.1924313 | 0.2027062 | 0.2011214 | 0.2043317 | 0.2015585 | 0.1900587 | 0.1901958 |
| $\lambda 62$ | 0.1906013 | 0.1689870 | 0.2046297 | 0.1801278 | 0.1985721 | 0.1751165 | 0.1984131 | 0.1938617 |
| $\lambda 83$ | 0.1946569 | 0.1925195 | 0.1921702 | 0.1903019 | 0.1935480 | 0.1926372 | 0.1948472 | 0.9200580 |
| $\lambda 93$ | 0.3183845 | 0.3066871 | 0.3167914 | 0.3050555 | 0.3133657 | 0.3041963 | 0.3011863 | 0.3050811 |
| $a_1$ | 0.6041172 | -0.6026840 | 0.7495838 | -0.7486536 | 0.8133972 | -0.8089450 | 0.6006817 | -0.8206944 |
| $a_2$ | 0.6197687 | -0.6195902 | 0.7589573 | -0.7587590 | 0.8116690 | -0.8090528 | 0.61186397 | -0.792677 |
| $a_3$ | 0.5644576 | -0.5558977 | 0.6924816 | -0.6850060 | 0.7436408 | -0.7285041 | 0.7903729 | -0.730118 |
| $a_4$ | 0.6419527 | -0.6398631 | 0.6901847 | -0.6860199 | 0.6851028 | -0.7285041 | 0.693193 | -0.731883 |
| $a_5$ | 0.5128443 | -0.5120956 | 0.5479911 | -0.5479478 | 0.5460172 | -0.5457094 | 0.545902 | -0.593910 |
| $a_6$ | 0.6319135 | -0.6195669 | 0.6652681 | -0.6572180 | 0.6658061 | -0.6563945 | 0.633119 | -0.705583 |
| $a_7$ | 0.6563760 | -0.6549768 | 0.7305370 | -0.7299345 | 0.7035828 | -0.7029385 | 0.7308811 | -0.732976 |
| $a_8$ | 0.5932387 | -0.5921227 | 0.6610297 | -0.6590103 | 0.6412507 | -0.6394662 | 0.589532 | -0.6700319 |
| $a_9$ | 0.6297601 | -0.6263057 | 0.7060993 | -0.7041201 | 0.6895409 | -0.6873600 | 0.620048 | -0.710489 |
| $\Phi 11$ | 0.5411528 | 0.5117659 | 0.5197178 | 0.4872096 | 0.5203293 | 0.4864295 | 0.5077318 | 0.472954 |
| $\Phi 11$ | 0.2682921 | -0.2600433 | 0.2690989 | -0.2609211 | 0.2783134 | -0.2672446 | 0.259736 | -0.270281 |
| $\Phi 21$ | 0.2682921 | -0.2600433 | 0.2690989 | -0.2609211 | 0.2783134 | -0.2672446 | 0.259187 | -0.270281 |
| $\Phi 22$ | 0.6912210 | 0.6844864 | 0.6913565 | 0.6844438 | 0.6914934 | 0.6792883 | 0.683711 | 0.661184 |

the suggested approach. When the sample size is increased from 25 to 50, the bias and RMS almost always decrease.

## 7. Conclusion

In this article, a quantitative structured equation model (SEM) technique was used to provide a comprehensive analysis of the interrelationships between latent variables. A Bayesian approach based on ALD theory was used for statistical inference. The simulation study shows that the quantitative values are SEM with small sample sizes with computational efficiency and lead satisfactorily in estimating parameters when the error term distribution is different distributions and non-normal distribution and this matches with the quantile regression hypothesis so it is more efficient

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