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Generalization of Rangaig transform

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Abstract

In this paper, a generalized form of the known Rangaig integral transform has been proposed; the general integral form is presented as a new integral transform called the "general Rangaig integral transform". The General Rangaig integral transform has been studied and proven for some fundamental functions, its applicability and ability to find the exact solution have been proven via utilizing the transform in solving first- and second-order ordinary differential equations.

Keywords: Integral transforms, Rangaig transform, Inverse of Rangaig transform, Ordinary differential equations, Partial differential equations

1. Introduction

Since the invention of integral transforms in 1763 by [2]. Many scientific and engineering fields have exploited the ability of integral transforms to map problems from one domain, in which the problem would be complicated and very difficult to solve, into another domain in which they have become more straightforward to grasp and analyze. It is usually possible to map back the transformed function into its original domain using the inverse format of the used integral transform [5, 3, 12]. Throughout the years, the unremitting efforts of mathematicians have materialized in the proposal of many integral transforms that have been implemented in solving all kinds of differential and integral problems with suitable boundaries and initial conditions [9, 6, 7, 8, 1, 10, 4].

In 2017, [11], introduced an integral transform called Rangaig integral transform. Rangaig integral transform can be written as [11]:

$$\eta\left[h\left(t\right)\right]=\Lambda\left(\mu\right)=\frac{1}{\mu}\int_{-\infty}^{0}e^{(\mu t)}\ h\left(t\right)\ \mathrm{dt}\ ,\qquad \frac{1}{\lambda_1}\leq\mu\leq\frac{1}{\lambda_2}\ .$$

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This work presents a generalization of the Rangaig integral transform. The general form of the Rangaig integral transform is applied and proved for some general functions, and its applicability and efficiency are going to be tested through the implementation of the general Rangaig integral transform on some first-order and second-order ordinary differential equations.

2. General Rangaig Integral Transform

Like any integral transform, general Rangaig integral transform could transform an equation from one domain into another and obtain the exact solution to that equation.
For the set of functions, an exponential order could be defined as:

$$H_g = \Big\{ h(t) : \textit{there exist } N,\ \lambda_1 \text{ and } \lambda_2 > 0\ ,\ |h(t)| > N e^{\lambda_j |t|},\quad t \in (-1)^{j-1} \times (-\infty, 0)\,, \\ \textit{where } j = 1, 2 \} \tag{2.1}$$

For equation (2.1):
$N \equiv$ finite constant.
λ_1 and $\lambda_2 \equiv$ finite or infinite constants.
General Rangaig integral transform that defined for the set H_g in equation (2.1), can be written as:

$$\eta_g \{h(t)\} = \Lambda_g(\mu) = \frac{1}{\mu^n} \int_{t=-\infty}^{0} e^{p(\mu)t}\ h(t)\ \mathrm{dt}\ , \tag{2.2}$$

Where: $\Lambda_g(\mu)$ denote the general Rangaig integral transform of $h(t) \in H_g$.
$\frac{1}{\lambda_1} < \mu < \frac{1}{\lambda_2}$, $n \in \mathbb{Z}$.
n is an integer number.
$p(\mu)$ is a function of parameter μ.
For the function $h(t)$, t is factorized by μ, or the function $h(t)$ is mapped into the function $\Lambda_g(\mu)$ of μ-space.

3. General Rangaig Integral Transform for Some Fundamental Functions

For an existing function $h(t)$, that is an element of H, general Rangaig integral transform is piecewise continuous and has decreasing exponential order if $t \geq 0$ is obtained for the function $h(t)$.

Theorem 3.1. *If $h(t) = 1$, then its general Rangaig integral transform is:*

$$\eta_g \{1\} = \frac{1}{\mu^n} \int_{t=-\infty}^{0} e^{p(\mu)t}\ dt = \frac{1}{\mu^n p(\mu)}\quad ,\ n \in \mathbb{Z}.$$

And, if $h(t) = t$, then its general Rangaig integral transform is:

$$\eta_g \{t\} = \frac{1}{\mu^n} \int_{t=-\infty}^{0} e^{p(\mu)t}\ t\ dt = \frac{-1}{\mu^n [p(\mu)]^2}.$$

Now, general Rangaig integral transform for $h(t) = t^m$, is:

$$\eta_g \{t^m\} = \frac{1}{\mu^n} \int_{t=-\infty}^{0} e^{p(\mu)t}\ t^m\ dt = \frac{(-1)^m m!}{\mu^n [p(\mu)]^{m+1}}.$$

Proof . *From general Rangaig integral transform definition:*

$$\eta_g\left\{t^m\right\}=\frac{1}{\mu^n}\int_{t=-\infty}^{0} e^{p(\mu)t}\ t^m\ dt=\frac{1}{\mu^n}\left(\left[\frac{t^m e^{p(\mu)t}}{p(\mu)}\right]_{-\infty}^{0}-\frac{m}{p(\mu)}\int_{t=-\infty}^{0} e^{p(\mu)t}\ t^{m-1}\ dt\right),$$

$$=\left[\frac{e^{p(\mu)t}}{\mu^n p(\mu)}\left(t^m-\frac{t^{m-1}m}{p(\mu)}+\frac{m(m-1)t^{m-2}}{[p(\mu)]^2}-\ldots+\frac{m!(-1)^m}{[p(\mu)]^m}\right]_{-\infty}^{0}=\frac{m!(-1)^m}{\mu^n[p(\mu)]^{m+1}}.$$

□

Theorem 3.2. *For an exponential function* $h(t)=e^{at}$, $a\equiv$ *constant , then its general Rangaig integral transform is:*

$$\eta_g\left\{e^a\right\}=\frac{1}{\mu^n}\int_{t=-\infty}^{0} e^{p(\mu)t}\ e^a\ dt=\frac{1}{\mu^n(p(\mu)+a)}.$$

Theorem 3.3. *The general Rangaig integral transform for trigonometric functions, which are* $h(t)=\sin(t)$ *and* $h(t)=\cos(t)$, *are:*

$$\eta_g\left\{\sin(t)\right\}=\frac{1}{\mu^n}\left(\frac{1}{[p(\mu)]^2+1}\right),$$

and

$$\eta_g\left\{\cos(t)\right\}=\frac{1}{\mu^n}\left(\frac{p(\mu)}{[p(\mu)]^2+1}\right).$$

Proof . *It is possible to write the trigonometric functions in the exponential format as follows:* $\sin t=\frac{1}{2i}\left(e^{it}-e^{-it}\right)$, $i=\sqrt{-1}$, *then*

$$\eta_g\left\{\sin(t)\right\}=\frac{1}{\mu^n}\int_{t=-\infty}^{0} e^{p(\mu)t}\sin(t)\ dt=\frac{1}{2i\ \mu^n}\int_{t=-\infty}^{0} e^{p(\mu)t}\left(e^{it}-e^{-it}\right)\ dt,$$

$$=\frac{1}{2i\ \mu^n}\int_{t=-\infty}^{0} e^{(p(\mu)+i)t}\ dt-\frac{1}{2i\ \mu^n}\int_{t=-\infty}^{0} e^{-(i-p(\mu))t}\ dt$$

$$=\frac{1}{2i\ \mu^n}\left[\frac{1}{p(\mu)+i}+\frac{1}{i-p(\mu)}\right]=\frac{1}{2i\ \mu^n}\left[\frac{i-p(\mu)+i+p(\mu)}{(p(\mu)+i)(i-p(\mu))}\right]$$

$$=\frac{1}{\mu^n}\left[\frac{i-p(\mu)+i+p(\mu)}{ip(\mu)-[p(\mu)]^2-1-ip(\mu)}\right]=\frac{1}{\mu^n\ \left([p(\mu)]^2+1\right)}.$$

Similarly, for

$$\eta_g\left\{\cos(t)\right\}=\frac{1}{2\ \mu^n}\int_{t=-\infty}^{0} e^{p(\mu)t}\left(e^{it}+e^{-it}\right)\ dt=\frac{1}{2\ \mu^n}\int_{t=-\infty}^{0} e^{(p(\mu)+i)t}\ dt+\frac{1}{2\ \mu^n}\int_{t=-\infty}^{0} e^{-(i-p(\mu))t}\ dt$$

$$=\frac{1}{2\ \mu^n}\left[\frac{1}{p(\mu)+i}-\frac{1}{i-p(\mu)}\right]=\frac{1}{\mu^n}\left[\frac{i-p(\mu)-i-p(\mu)}{ip(\mu)-[p(\mu)]^2-1-ip(\mu)}\right]$$

$$=\frac{p(\mu)}{\mu^n\ \left([p(\mu)]^2+1\right)}.$$

□

Theorem 3.4 (General Rangaig integral transform for derivatives). *If* $h(t), h'(t), \ldots, h^{(m)}(t)\in H_g$, $m\geq 0$, *then*

(i) $\eta_g\left\{h^{'}(t)\right\} = \frac{1}{\mu^n}h(0) - p(\mu)\,\eta_g\{h(t)\}$.

(ii) $\eta_g\left\{h^{''}(t)\right\} = \frac{1}{\mu^n}h^{'}(0) - \frac{p(\mu)}{\mu^n}h(0) + [p(\mu)]^2\,\eta_g\{h(t)\}$.

Proof .

$$(i)\ \ \eta_g\left\{h^{'}(t)\right\} = \frac{1}{\mu^n}\int_{t=-\infty}^{0} e^{p(\mu)t}h^{'}(t)\ \ dt,$$

Integrating by parts,

$$Let\ \ u = e^{p(\mu)t}\ ,\ dv = h^{'}(t)\,dt,$$

$$du = p(\mu)\,e^{p(\mu)t}\ ,\ v = h(t)\,.$$

$$\eta_g\left\{h^{'}(t)\right\} = \frac{1}{\mu^n}\ \ [h(t)\ \ e^{p(\mu)t}|\ _{-\infty}^{0} - p(\mu)\int_{t=-\infty}^{0} h(t)\ \ e^{p(\mu)t}dt] = \frac{1}{\mu^n}h(0) - p(\mu)\ \ \eta_g\{h(t)\}\,.$$

(ii) Integrating by parts,

$$Let\ \ u = e^{p(\mu)t}\ ,\ dv = h^{''}(t)\,dt,$$

$$du = p(\mu)\,e^{p(\mu)t}\ ,\ v = h^{'}(t)\,.$$

$$\eta_g\left\{h^{''}(t)\right\} = \frac{1}{\mu^n}\ \ [e^{p(\mu)}h^{'}(t)\ \ e^{p(\mu)t}|\ _{-\infty}^{0} - p(\mu)\int_{t=-\infty}^{0} h^{'}(t)\ \ e^{p(\mu)t}dt] = \frac{1}{\mu^n}h^{'}(0) - p(\mu)\ \ \eta_g\left\{h^{'}(t)\right\}$$

$$= \frac{1}{\mu^n}h^{'}(0) - \frac{p(\mu)}{\mu^n}h(0) + [p(\mu)]^2\,\eta_g\{h(t)\}\,.$$

In general:

$$\eta_g\left\{h^{(m)}(t)\right\} = \frac{1}{\mu^n}\sum_{k=0}^{m-1}(-1)^k\,[p(\mu)]^k h(0)^{(m-1-k)} + (-1)^m\,[p(\mu)]^m\,\eta_g\{h(t)\}\,.$$

□

4. Applications of General Rangaig Integral Transform

To demonstrate the implementation capability of the general Rangaig integral transform on ordinary differential equations, some first- and second-order ordinary differential equations examples are going to be discussed.

Application (1) For the following 1^{st} order linear ordinary differential equation: $\frac{\mathrm{dh}}{\mathrm{dt}} + h(t) = g(t)$, with initial condition $h(0) = 1$, it is possible to use general Rangaig integral transform to solve this equation as:
By taking the general Rangaig integral transform for both sides of the equation:

$$\eta_g\left\{h^{'}(t)\right\} + \eta_g\{h(t)\} = \eta_g\{g(t)\}\,,$$

$$\frac{1}{\mu^n}h(0) - p(\mu)\,\eta_g\{h(t)\} + \eta_g\{h(t)\} = \eta_g\{g(t)\}\,,$$

$$[1 - p(\mu)]\,\eta_g\{h(t)\} = \eta_g\{g(t)\} + \frac{1}{\mu^n},$$

For $g(t) = 0$, Taking inverse general Rangaig integral transform, provides the exact required solution, which is: $h(t) = e^{-t}$.

Application (2) For the following 2^{nd} order linear ordinary differential equation:$y'' + y = 1$, with initial conditions $y'(0) = 0 \; and \; y(0) = 0$, it is possible to use general Rangaig integral transform to solve this equation as:
By taking the general Rangaig integral transform for both sides of the equation:

$$\eta_g \left\{ y'' \right\} + \eta_g \left\{ y \right\} = \eta_g \left\{ 1 \right\},$$
$$\frac{1}{\mu^n} y'(0) - \frac{p(\mu)}{\mu^n} y(0) + \left[p(\mu) \right]^2 \eta_g \left\{ y(t) \right\} + \eta_g \left\{ y(t) \right\} = \frac{-1}{\mu^n \left[p(\mu) \right]^2},$$

then

$$\eta_g \left\{ y(t) \right\} = \frac{-1}{\mu^n \left[p(\mu) \right]^2 \left(p(\mu) + 1 \right)},$$

After simple computations and taking inverse Rangaig integral transform, the exact solution is obtained:

$$y(t) = \cos(t) + \sin(t) + 1.$$

5. Discussion and Conclusions

The general Rangaig integral transform is a derived transform from the Rangaig integral transform that provides a generalization to the Rangaig integral transform.
The generalized factor in the general Rangaig integral transform comes from replacing the variable μ inside the integration by the function $p(\mu)$, and μ outside the integration by μ^n.
The properties of the proposed transform have been studied through applying the transform on some fundamental functions. Using the general Rangaig integral transform in solving some ordinary differential equations of the first- and second-order has proven the capable and efficiency of the transform to be utilized in such important equations that are used in many scientific fields.

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