Int. J. Nonlinear Anal. Appl. 13 (2022) No. 1, 2455–2465
ISSN: 2008-6822 (electronic)
http://dx.doi.org/10.22075/ijnaa.2022.5945

Liu-Type estimator in gamma regression model based on (r-(k-d)) class estimator

Ahmed Mutlag Algboory[a,*], Zakariya Yahya Algamal[b]

[a] College of Physical Education and Sport Sciences, University of Samarra, Salah Aldeen, Iraq
[b] Department of Statistics and Informatics, University of Mosul, Mosul, Iraq

(Communicated by Madjid Eshaghi Gordji)

Abstract

It is known that when the multicollinearity exists in the gamma regression model, the variance of maximum likelihood estimator is unstable and high. In this article, a new Liu-type estimator based on (r-(k-d)) class estimator in gamma regression model is proposed. The performance of the proposed estimator is studied and comparisons are done with others. Depending on the simulation and real data results in the sense of mean squared error, the proposed estimator is superior to the other estimators.

Keywords: Liu-type estimator; gamma regression model; (r-(k-d)) class estimator, (r – d) class estimator; (r – k) class estimator; (k – d) class estimator.

1. Introduction

The gamma regression is widely used for analyzing real data, particularly in medical felids, automobile insurance claims, and the economics of health-care.([9], [21], [10], [3], [6]). Where the response variable is not distributed normally or is positively skewed, the gamma regression model is used. As a result, gamma regression assumes a gamma distribution for the response variable. ([2]; [3], [27], [4]).

The gamma regression model, like the linear regression model, assumes that the regressors have no correlation. There is a natural correlation between the explanatory variables in many regression model applications. When correlations are strong, estimation of the regression parameters becomes unstable, making it difficult to interpret regression coefficient estimates ([3], [20]).

*Corresponding author
Email addresses: ahmed.mo@uosamarra.edu.iq- ORCID: 0000-0002-9294-2884 (Ahmed Mutlag Algboory), zakariya.algamal@uomosul.edu.iq- ORCID: 0000-0002-0229-7958 (Zakariya Yahya Algamal)

Received: May 2021 *Accepted:* October 2021

When using the maximum likelihood (ML) method to estimate the regression coefficients for a gamma regression model, the estimated coefficients are unstable with high variance, and this lead to reduce statistical significance as a result of previous studies by ([21], [10]). It's difficult to estimate the individual effects of and explanatory variable in a regression model when there's an issue with multicollinearity. Furthermore, the regression coeficients' sampling variance can influence both inference and prediction ([4]).

Several methods have been suggested as a remedial methods to solve the problem of multicollinearity. Hoerl and Kennard (1970)[11] suggested the ridge estimator, which has been shown to be a viable replacement to the "ML estimator". Segerstedt used the ridge estimator in his generalized linear models (GLM) (1992)[24]. Furthermore, the ridge estimator was considered in logistic regression by Segerstedt (1992)[24] and Schaefer, Roi, and Wolfe (1984)[25]. The ridge estimator was also used by Månsson and Shukur (2011)[20] and Månsson (2012)[18] in (Poisson regression and negative binomial regression) samples, respectively. Kurto glu and Ozkale(2016) [16] recently demonstrated the application of the well-known Liu estimator Liu (1993) to GLM and the use of gamma distributed response variables (2016). We also relate to Wu and Asar (2017)[28], Mnsson, Kibria, and Shukur (2012)[19], Urgan and Tez (2008)[26], and Asar et al. for Liu regression (2017). Liu (2003) suggested a new approach to the collinearity problem known as the Liu-type estimator. This estimator has also been well studied in the literature and generalized to some GLM models; we refer to the following reviews for more details: "Inan and Erdogan (2013) [12] , Asar and Genc. (2016)[7], Asar (2018)[6], and Akdeniz and Duran (2010)[1]". Liu (1993) [14] suggested the Liu estimator, which combines the Stein r is estimator and the ridge estimator. "Compared to the ridge estimator, the Liu estimate a linear model of the shrinkage parameter, making shrinkage parameter selection easier than ridge parameter selection." (Ozkale and Kaciranlar (2007)[23]; Yang and Chang (2007))[29] suggested the two-parameter estimator, which is a combination of the ridge and Liu estimators (2010). Alheety and Kibria (2013)[5] proposed a new estimator that combines the Liu estimator with the (r – k) class estimator.

2. Statistical methodology

2.1. Gamma regression model

Let y_i be the response parameter, which is based on the gamma distribution with non-negative shape variable k and non-negative scale parameter τ, ie, $y_i \sim$Gamma $(k,\ \tau)$; the After that, the probability density problem is described as $y_i \sim$Gamma $(k,\ \tau)$.

$$f\left(y_i;k,\tau\right)=\frac{{y_i}^{k-1}\ e^{-y_i/\tau}}{\Gamma(k)(\tau)^k},\ y_i\geq 0 \text{ and } k,\tau>0 \tag{2.1}$$

where k is the non-negative shape parameter and τ is the scale parameter such that $E\left(Y_i\right)=\mu_i= k\tau=\theta_i$ which is also known as the canonical parameter and $\operatorname{var}\left(Y_i\right)= k\tau^2= 1/(k{\theta_i}^2)$, $\theta_i= \exp({x_i}^T\beta)$ where $x_i=(x_{i1}, x_{i2}, \ldots, x_{ip})^T$, $i = 1, 2, \ldots, n$ and $j = 1,2, \ldots, p$ where n is the sample size n and p is the number of explanatory variables ($n>p$). The parameters are usually obtained using maximum likelihood estimation.

Maximize the following log-likelihood function with respect to β in order to do so. β

$$l\left(\beta\right)=\sum_{i=1}^{n}\left[(k-1)\log y\ -y/\tau-k\log\tau-\log(\Gamma\left(k\right))\right] \tag{2.2}$$

We can use some iterative methods to obtain the solutions because the obtained equations are non-linear in b. As a result, the following iterations can be described using the Fisher Scoring process.

$$\hat{\beta}^{t+1}=\hat{\beta}^{t}-\left\{E[H_1(\beta)]_{\beta=\beta^t}\right\}\left[\frac{\partial l(\beta)}{\beta}\right]_{\beta=\beta^t} \tag{2.3}$$

where $H_1(\beta)=-\frac{1}{\phi}X^TWX$ is the Hessian matrix such that $\phi= 1/k$ is the dispersion parameter and

$$\frac{\partial l(\beta)}{\beta}=\phi\sum_{i=1}^{n}\left[y_i-\frac{1}{{x_i}^T\beta}\right]x_i. \tag{2.4}$$

Therefore, Eq. (2.3) can be as following:

$$\hat{\beta}^{t+1}=\hat{\beta}^{t}-\left\{(X^T\widehat{W}X)^{-1}\ X^T\widehat{W}\widehat{z}\ \right\}_{\beta=\beta^t} \tag{2.5}$$

where $\widehat{W}=\text{diag}({\theta_i}^2)$and $\widehat{z}=\widehat{\theta}_i+\frac{y_i-\widehat{\theta}_i}{\widehat{\theta}_i^2}$ is the ith part of the "vector z". This duplicated method continues till the sequential appreciation assembles to, say, $\widehat{\beta}_{MLE}$, at which point we get $\widehat{\beta}_{MLE}=(X^T\widehat{W}X)^{-1}\ X^T\widehat{V}\widehat{z}$, where$\widehat{W}$ and $\widehat{z}$ are computed at the final iteration.
The covariance matrix of $\widehat{\beta}_{MLE}$; cov $\left(\widehat{\beta}_{MLE}\right)=\phi\ (X^T\widehat{W}X)^{-1}$is well-known for being ill-conditioned, causing the variance of the regression coefficients to be inflated ([17]; [24]). The maximum likelihood estimator's (MLE) mean squared error (MSE) is given by:

$$\begin{aligned} MSE\left(\hat{\beta}_{MLE}\right) &= E\left(\widehat{\beta}_{MLE}-\beta\right)^T\left(\widehat{\beta}_{MLE}-\beta\right) \\ &= tr\left[\phi\ (X^T\widehat{W}X)^{-1}\right]=\phi\sum_{j=1}^{p}\frac{1}{\lambda_j} \end{aligned} \tag{2.6}$$

tr (.) is the trace of a matrix, and λ_j is the jth eigenvalue of the matrix $D=X^T\widehat{W}X$.
The eigenvalue decomposition of the matrix M is also taken into account as follows: $D=Q^T\Lambda Q$, where Q is the orthogonal matrix consisting of the eigenvectors identical, to the eigenvalues of (D), and $\Lambda=\text{diag}(\lambda_1,\lambda_2,\ldots,\lambda_p)$is the rectangular matrix consisting of the eigenvectors identical to the eigenvalues of (D).
It's easy that if one value or more of the eigenvalues are relative to zero, the MSE of the MLE inflates, so the "regression coefficients" suffer as a result.
If there is an orthogonal matrix T = $(t_1,\ldots,t_p)$ such that $T'DT=\Lambda$, the gamma ridge estimator (GRE) is used to resolve the problem of multicollinearity:

$$\widehat{\beta}_{GRE}=(D+kI)^{-1}\ X'\widehat{W}\widehat{z} \tag{2.7}$$

The covariance matrix and bias (**b**) vector of GRE respectively, may be results from by

$$\text{cov}\left(\widehat{\beta}_{GRE}\right)=\phi D_k^{-1}D\ D_k^{-1} \tag{2.8}$$

$$\boldsymbol{b}_{GRE}=bias\left(\widehat{\beta}_k\right)=-kD_k^{-1}\ \beta \tag{2.9}$$

The gamma Liu estimator (GLE) is known as

$$\widehat{\beta}_{GLE}=T_d\ \ \widehat{\beta}_{MLE} \tag{2.10}$$

$\boldsymbol{T}_d=(D+I)^{-1}(D+dI)$ and 0<d<1 covariance matrix and bias vector calculated using the following formulas.

$$\operatorname{cov}\left(\widehat{\beta}_d\right)=\phi\ \boldsymbol{T}_d D^{-1}\boldsymbol{T}_d{}^T \tag{2.11}$$

$$\boldsymbol{b}_{GLE}=bias\left(\widehat{\beta}_d\right)=-(1-d)(D+I)^{-1}\ \beta \tag{2.12}$$

2.2. The proposed estimator

Baye and Parker (1984)[8] suggested a ridge estimation and principal components regression (PCR) estimator.

$$\widehat{\beta}_r(k)=T_r(T_r'\grave{X}WXT_r+kI_r)^{-1}\ T_r'\grave{X}y \tag{2.13}$$

The (r – k) estimator is also known as the (r – k) class estimator. Kaciranlar and Sakallioglu (2001)[13] proposed a new estimator dependent on Liu estimation and principal components regression (PCR). This explained as:

$$\widehat{\beta}_r\left(d\right)=T_r\left(T_r'\grave{X}WXT_r+I_r\right)^{-1}\ \left(T_r'\grave{X}y+dT_r'\widehat{\beta}_r\right)\ 0<d<1, \tag{2.14}$$

where $\widehat{\beta}_r=T_r\left(T_r'X\grave{W}XT_r\right)^{-1}T_r'\grave{X}y$ *is PCR.* $\widehat{\beta}_r\left(d\right)$ is define as the (r-d) class estimator. Alheety and Kibria (2013)[5] proposed a new estimator that combines the Liu estimator with the (r – k) class estimator.

This type of estimator was applied to a gamma regression model in this paper. The latest estimate is as follows:

$$\widehat{\beta}_r\left(k,d\right)=T_r\left(T_r'\grave{X}WXT_r+I_r\right)^{-1}\ \left(T_r'\grave{X}y+dT'{}_r\widehat{\beta}_r(k)\right) \tag{2.15}$$

$k>0,\ -\infty<d<\infty$, and is define as the (r - (k-d)) class estimator.

3. Comparison of the Estimators by MSE Criterion

The mean squares error matrix of this estimator's $\widehat{\beta}$ is introduced as follows:

$$MSE\left(\widehat{\beta}\right)=E\left(\widehat{\beta}-\beta\right)^T\left(\widehat{\beta}-\beta\right)=Var\left(\widehat{\beta}\right)+\left(Bias\left(\widehat{\beta}\right)\right)\left(Bias\left(\widehat{\beta}\right)\right)' \tag{3.1}$$

Where
$Bias\left(\widehat{\beta}\right)=E\left(\widehat{\beta}\right)-\beta$ is the bias of $\widehat{\beta}$ and $Var\left(\widehat{\beta}\right)=E[\left(\widehat{\beta}-E\left(\widehat{\beta}\right)\right)\left(\widehat{\beta}-E\left(\widehat{\beta}\right)\right)']$
is a variance of β. is chosen over an alternative for a given value of $\beta,\widehat{\beta}_1$When $MSE\left(\widehat{\beta}_2\right)-MSE(\widehat{\beta}_1)$ is a non-negative definite matrix definite matrix, then $mse\left(\widehat{\beta}_2\right)-\ mse\left(\widehat{\beta}_1\right)\ \geq\ 0$

3.1. Comparison between the (r – (k – d) Class Estimator and (r – d) Class Estimator

The matrix (MSE) and scalar (mse) for the $\widehat{\beta}_r\left(k,d\right)$ and $\widehat{\beta}_r\left(d\right)$ are known as:

$$\begin{aligned}MMSE\left(\widehat{\beta}_r\left(k,d\right)\right)=&\phi\ T_rD_r^{-1}(1)\left(I_r+dD_r^{-1}\left(k\right)\right)T_r'D_r^{-1}T_r\left(I_r+dD_r^{-1}\left(k\right)\right)D_r^{-1}(1)T_r'\\&+\left(T_rD_r^{-1}(1)\left(I_r+dD_r^{-1}\left(k\right)T_r'D_rT_r\right)T_r'+T_{p-r}'T_{p-r}\right)\beta\beta'\\&\times\left(T_rD_r^{-1}\left(1\right)\left(I_r+dT_r'D_rT_rD_r^{-1}\left(k\right)\right)T_r'+T_{p-r}'T_{p-r}\right).\end{aligned} \tag{3.2}$$

$$mse\left(\widehat{\beta}_r(k,d)\right)=\sum_{i=1}^{r}\frac{\phi\lambda_i\ \ (\lambda_i+k+d)^2+\ (\lambda_i+k-d\lambda_i)^2\alpha_i^2}{(\lambda_i+k)^2\ (\lambda_i+1)^2}+\sum_{i=p-r}^{p}\alpha_i^2 \tag{3.3}$$

where $D_r^{-1}(1)=(\Lambda_r-I_r)$.
By minimize mse $\widehat{\beta}_r(k,d)$ with respect to d we get

$$d_{opt}=\frac{\sum_{i=1}^{r}\ \lambda_i(\alpha_i^2-\phi)/(\lambda_i+k)\ (\lambda_i+1)^2}{\sum_{i=1}^{r}\ \lambda_i(\lambda_i\alpha_i^2-\phi)/(\lambda_i+k)^2\ (\lambda_i+1)^2} \tag{3.4}$$

Now it's time to fix k. So

$$\begin{aligned}mse\left(\widehat{\beta}_r(k,d)\right)-mse\left(\widehat{\beta}_r(d)\right)=&d^2\sum_{i=1}^{r}\frac{(\lambda_i\alpha_i^2+\phi)+\left(\lambda_i^2-\ (\lambda_i+k)^2\right)}{\lambda_i(\lambda_i+k)^2\ (\lambda_i+1)^2}\\&+2dk\sum_{i=1}^{r}\frac{(\alpha_i^2-\phi)}{(\lambda_i+k)\ \ (\lambda_i+1)^2}\end{aligned} \tag{3.5}$$

Since k , λ_i , α_i^2 *and* σ^2 are positive numbers, so:

$$\sum_{i=1}^{r}\frac{(\lambda_i\alpha_i^2+\phi)+\left(\lambda_i^2-\ (\lambda_i+k)^2\right)}{\lambda_i(\lambda_i+k)^2\ (\lambda_i+1)^2}<0\rightarrow\sum_{i=1}^{r}\frac{(\lambda_i\alpha_i^2+\phi)+\left(\ (\lambda_i+k)^2-\lambda_i^2\right)}{\lambda_i(\lambda_i+k)^2\ (\lambda_i+1)^2}>0 \tag{3.6}$$

Let $M_1=\sum_{i=1}^{r}\frac{(\lambda_i\alpha_i^2+\phi)+\left(\lambda_i^2-\ (\lambda_i+k)^2\right)}{\lambda_i(\lambda_i+k)^2\ (\lambda_i+1)^2}$ and $M_2=k\sum_{i=1}^{r}\frac{\left(\alpha_i^2-\phi\right)}{(\lambda_i+k)\ (\lambda_i+1)^2}$ then,

$$mse\left(\widehat{\beta}_r(k,d)\right)-mse\left(\widehat{\beta}_r(d)\right)=d^2M_1+2dM_2 \tag{3.7}$$

Now, when M2 >0, we need to know the conditions that to create $mse\left(\widehat{\beta}_r(k,d)\right)-mse\left(\widehat{\beta}_r(d)\right)>0$.

$$mse\left(\widehat{\beta}_r(k,d)\right)-mse\left(\widehat{\beta}_r(d)\right)=d^2M_1+2dM_2=d(dM_1+2M_2)$$

so, $mse\left(\widehat{\beta}_r(k,d)\right)-mse\left(\widehat{\beta}_r(d)\right)$ will be positive when d <0 and $dM1+\ 2M2\ <\ 0$. In this case,

$$dM_1+\ 2M_2<\ 0\Leftrightarrow dM_1<-2M_2\ \Leftrightarrow\ d\left(-M_1\right)>2M_2\Leftrightarrow\ d>\frac{2M_2}{(-M_1)}=d^*>0\ .$$

Also, $mse\left(\widehat{\beta}_r(k,d)\right)-mse\left(\widehat{\beta}_r(d)\right)$ will be positive when d >0 and $dM_1+\ 2M_2\ >\ 0$. In this case,

$$dM_1+\ 2M_2>\ 0\Leftrightarrow dM_1>-2M_2\ \Leftrightarrow\ d\left(-M_1\right)<2M_2\Leftrightarrow\ d<d^*>0\ .$$

So, $mse\left(\widehat{\beta}_r-(k,d)\right)-mse\left(\widehat{\beta}_r(d)\right)>0$ for $0\ <\ d\ <d^*$.
Now, we are looking for the state that makes$\left(\widehat{\beta}_r(k,d)\right)-mse\left(\widehat{\beta}_r(d)\right)<0$. This inequity, will be held when $d\ <\ 0$ and, $dM_1+\ 2M_2\ >\ 0\ \Leftrightarrow\ d\ <\ d^*>\ 0$. So , $mse\left(\widehat{\beta}_r(k,d)\right)-mse\left(\widehat{\beta}_r(d)\right)<0$ for $<0\ \ or\ \ d\ <d^*$.
at the same method, when $M_2\ <\ 0$, , $mse\left(\widehat{\beta}_r(k,d)\right)-mse\left(\widehat{\beta}_r(d)\right)>0$ for $d^*\ <\ d\ <\ 0$.
Also, $mse\left(\widehat{\beta}_r(k,d)\right)-mse\left(\widehat{\beta}_r(d)\right)<0$ for $d>0\ \ and\ \ d>d^*$. so, we may state the Theory 3.1.

Theorem 3.1
(a) When $\sum_{i=1}^{r} \frac{\left(\alpha_i^2-\phi\right)}{(\lambda_i+k)\ (\lambda_i+1)^2} > 0$ then:
(1) $mse\left(\widehat{\beta}_r\left(k,d\right)\right) > mse\left(\widehat{\beta}_r\left(d\right)\right)$ for $0 < d < d^*$.
(2) $mse\left(\widehat{\beta}_r\left(k,d\right)\right) < mse\left(\widehat{\beta}_r\left(d\right)\right)$ for $< 0\ \ or\ \ d < d^*$.
(b) When $\sum_{i=1}^{r} \frac{\left(\alpha_i^2-\phi\right)}{(\lambda_i+k)\ (\lambda_i+1)^2} < 0$ then:
(1) $mse\left(\widehat{\beta}_r\left(k,d\right)\right) > mse\left(\widehat{\beta}_r\left(d\right)\right)$ for $d^* < d < 0$.
(2) $mse\left(\widehat{\beta}_r\left(k,d\right)\right) < mse\left(\widehat{\beta}_r\left(d\right)\right)$ for $> 0\ \ and\ \ d > d^*$.
Where

$$d^* = \frac{2k\sum_{i=1}^{r} \frac{\left(\widehat{\alpha}_i^2-\phi\right)}{(\lambda_i+k)\ (\lambda_i+1)^2}}{\sum_{i=1}^{r} \frac{\left(\ (\lambda_i+k)^2-\lambda_i^2\right)\left(\lambda_i\alpha_i^2+\phi\right)}{\lambda_i\ (\lambda_i+1)^2(\lambda_i+k)^2}}$$

3.2. Comparison between (r –(k – d)) and (r – k) Class Estimator
The MSE and mse of (r – k) type estimator respectively, are:-

$$MSE\left(\widehat{\beta}_r\left(k\right)\right) = \phi\ T_r D_r^{-1}\left(k\right)\Lambda_r D_r^{-1}\left(k\right)T_r' + [T_r D_r^{-1}\left(k\right)\Lambda_r T_r' - I_p]\beta\beta'[T_r D_r^{-1}\left(k\right)\Lambda_r T_r' - I_p] \tag{3.8}$$

$$mse\left(\widehat{\beta}_r\left(k\right)\right) = mse\left(\widehat{\beta}_r\left(k,1-k\right)\right) = \sum_{i=1}^{r} \frac{\phi\lambda_i + k^2\alpha_i^2}{\left(\lambda_i+k\right)^2} + \sum_{i=p-r}^{p} \alpha_i^2 \tag{3.9}$$

When d = 1 – k in (3.3), we obtain mse of the (r – k) class estimator. Anyway, $mse\left(\widehat{\beta}_r\left(k,d\right)\right)$ is minimized at d_{opt}, so, we may state the following theorem.

Theorem 3.2. When

$$d = \frac{\sum_{i=1}^{r}\ \lambda_i(\alpha_i^2-\phi)/(\lambda_i+k)\ (\lambda_i+1)^2}{\sum_{i=1}^{r}\ \lambda_i(\lambda_i\alpha_i^2-\phi)/(\lambda_i+k)^2\ (\lambda_i+1)^2}$$

$$mse\left(\widehat{\beta}_r\left(k,d\right)\right) \le mse\left(\widehat{\beta}_r\left(k\right)\right).$$

This theory show that the comparison results are influenced by the unknown parameters α. As a result, we cannot rule out the possibility that our theorem results may not hold, and that the results can change. As a result, we replace it (α) with their unbiased estimators. so (d) is based on (k) and the unknown parameters (α) , we replace k by its estimator "(in this analysis, k is calculated using the estimator proposed by Hoerl and Kennard (1970a)"[11] and we denote d by its estimator. $\hat{k}_{HK} = \frac{\widehat{\phi}}{\sum_{i=1}^{r}\widehat{\alpha}_i^2}$
We'd like to point out that there are a variety of estimators that researchers have suggested for estimating the ridge parameter k; for more details, see Kibria (2003)[15] and Muniz and Kibria (2009)[22]. As a result, the approximate $\hat{d}_{opt}$ would be as follows:

$$\hat{d}_{opt} = \frac{\sum_{i=1}^{r}\ \lambda_i(\widehat{\alpha}_i^2-\widehat{\phi})/(\lambda_i+\hat{k}_{HK})\ (\lambda_i+1)^2}{\sum_{i=1}^{r}\ \lambda_i(\lambda_i\widehat{\alpha}_i^2-\widehat{\phi})/(\lambda_i+\hat{k}_{HK})^2\ (\lambda_i+1)^2} \tag{3.10}$$

4. Simulation study

Monte Carlo simulation experiment was using in this paper, to test the suggested estimator's efficiency under different stats of multicollinearity.

4.1. Simulation design

The response variable of n obtained from gamma regression type known as, $y_i\sim$ Gamma(k,τ);, where $\tau=$ (0.50, 1.5) and $\theta_i= exp({x_i}^T\beta)$, $\boldsymbol{\beta}= (\beta_1,\ldots,\beta_p)$ with $\sum_{j=1}^{p}\beta_j^2=1$ and $(\beta_1{=}\beta_2=\cdots=\beta_p)$. The explanatory variables $x_i^T= (x_{i1},x_{i2},\ldots,\ x_{in})$ created from the formula below:

$$\boldsymbol{x_{ij}{=}(1{-}\rho^2)^{1/2}w_{ij}+\rho w_{ip}\quad i=1,2,\ldots,n,\quad j=1,2,\ldots,p}$$

where ρ is the correlation between the explanatory variables and w_{ij}'s are independent standard normal pseudo-random numbers. Because the sample size has direct impact on the prediction accuracy, three representative values of the sample size are considered: 50, 100, and 150 are considered. Furthermore, p=3 and p=7 are used to determine the number of explanatory variables. Since a larger number of explanatory variables will leading to a bigger MSE. Furthermore, since we are attentive in the impact of multicollinearity, where the degrees of correlation are more significant, three values of the pairwise correlation are considered with $\rho=$ (0.90 , 0.95 , 0.99).
The produced data is repeated 1000 times for incorporation of these values of n, p, , and the average MSE calculated as:

$$MSE\left(\widehat{\boldsymbol{\beta}}\right)=\frac{1}{1000}\sum_{i=1}^{1000}\left(\widehat{\boldsymbol{\beta}}-\boldsymbol{\beta}\right)^T\ \left(\widehat{\boldsymbol{\beta}}-\boldsymbol{\beta}\right) \tag{4.1}$$

4.2. Results of the simulation

Tables 1 and 2 summarize the averaged MSE for all combinations of n,τ,p, and ρ,. The best averaged MSE's value is highlighted in bold. We can deduce the following conclusions from the tables:

1. The new estimator, r-(k-d), has the best performance in all of the situations considered. Moreover, performance of r-(k-d) is better for larger values of the correlation coefficient.

2. It is noted from Tables 1 and 2 that r-(k-d) ranks first with respect to MSE. In the second rank, r-d estimator performs better than both GR and r-k estimators. Additionally, GR estimator has the worst performance among r-k, r-d, and r-(k-d) which is significantly impacted by the multicollinearity.

3. Regarding the number of explanatory variables, it is easily seen that there is a negative impact on MSE, where there are increasing in their values when the p increasing from three variables to seven variables. In Addition, in terms of the sample size n, the MSE values decrease when n increases, regardless the value of ρ,τ and p.

Clearly, in terms of the dispersion parameter τ, both bias and MSE values are decreasing when τ increasing **.**

Table 1: MAveraged MSE values for the four estimators when $\phi = 0.5$

n	p	ρ	GR	r-k	r-d	r-(k-d)
50	3	0.90	4.733	4.492	4.153	**4.039**
		0.95	4.777	4.542	4.203	**4.089**
		0.99	5.043	4.808	4.469	**4.355**
	7	0.90	4.847	4.612	4.273	**4.159**
		0.95	4.897	4.662	4.323	**4.209**
		0.99	5.163	4.928	4.589	**4.475**
100	3	0.90	4.485	4.25	3.911	**3.797**
		0.95	4.535	4.3	3.961	**3.847**
		0.99	4.801	4.566	4.227	**4.113**
	7	0.90	4.611	4.37	4.031	**3.917**
		0.95	4.655	4.42	4.081	**3.967**
		0.99	4.921	4.686	4.347	**4.233**
150	3	0.90	4.434	4.199	3.86	**3.746**
		0.95	4.484	4.249	3.91	**3.797**
		0.99	4.75	4.515	4.176	**4.062**
	7	0.90	4.554	4.319	3.98	**3.867**
		0.95	4.604	4.369	4.03	**3.916**
		0.99	4.87	4.635	4.296	**4.182**

Table 2: Averaged MSE values for the four estimators when $\phi = 1.5$

n	p	ρ	GR	r-k	r-d	r-(k-d)
50	3	0.90	4.624	4.389	4.05	**3.936**
		0.95	4.673	4.438	4.099	**3.985**
		0.99	4.94	4.705	4.366	**4.252**
	7	0.90	4.744	4.509	4.17	**4.056**
		0.95	4.793	4.558	4.219	**4.104**
		0.99	5.06	4.825	4.486	**4.372**
100	3	0.90	4.382	4.147	3.808	**3.694**
		0.95	4.432	4.196	3.857	**3.743**
		0.99	4.698	4.463	4.124	**4.01**
	7	0.90	4.502	4.267	3.928	**3.814**
		0.95	4.552	4.317	3.978	**3.864**
		0.99	4.818	4.583	4.244	**4.13**
150	3	0.90	4.331	4.096	3.757	**3.643**
		0.95	4.38	4.145	3.807	**3.692**
		0.99	4.647	4.412	4.073	**3.959**
	7	0.90	4.451	4.216	3.877	**3.763**
		0.95	4.5	4.265	3.927	**3.812**
		0.99	4.767	4.532	4.193	**4.079**

5. The Application Real Data

To explain the capacity of the r-(k-d) estimator in true application, we found here a chemical data with $(n,\ p)\ =\ (212, 10)$, where n is the number of antifungal agents. The antimicrobial effectiveness was metric as pMIC (the logarithm of reciprocal of MIC, the MIC is lower inhibitory condensation against *C. albicans* in mM/L). While p it indicates the number of molecular descriptors, which are concerned as explanatory variables ([3]).

In chemometrics, the analysis of quantitative structure-activity relationships (QSAR) has gained a lot of traction. The aim of QSAR is to model many biological activities using structural properties of a series of chemical materials. One of mainly methods for building a QSAR model is the use of regression models. Table 3 gives a rundown of the explanatory variables that were used. The variables are all numerical in nature.

First, The Chi-square test is used to determine if the response variable belongs to the gamma distribution. The test yielded a result of 10.0286 with a p-value of 0.9117. The gamma distribution suits this response variable very well, as shown by this result. 0.0153 is the approximate dispersion parameter. Second, after adequate the gamma regression model with the function of log link and an approximate dispersion variable of 0.0153, the eigenvalues of the matrix $X^T \hat{W} X$ which were used to evaluate for multicollinearity, and obtained as 1.97×10^9,3.74×10^6, 1.21×10^4, 1.34×10^3, 1.22×10^3,1.07×10^3, 4.63×10^2, 2.08×10^1, 10.68, and 1.57.

The evaluated condition number $\text{CN} = \sqrt{\lambda_{\max}/\lambda_{\min}}$ when is the data (35422.83) indicating that the acute multi-collinearity matter is exist.

Table 4 displays the approximate gamma regression coefficients and MSE values for the GR, r-k, k-d, and r-(k-d) estimators. The r-(k-d) effectively shrinks the number of the estimated coefficients, as shown in Table 4.

Also, there is a significant reduce in the MSE in favour of the r- (k-d). The MSE of the r-(k-d) estimator was approximately 39.486%, 24.080%, and 21.337% lower than that of the GR, r-k, and k-d estimators, respectively.

Table 3: Description of the used explanatory variables

Variable name's	description
IC3	Information Content index (neighborhood symmetry of 3-order)
ATS8v	Broto-Moreau autocorrelation of lag 8 (log function) weighted by van der Waals volume
MATS7v	Moran autocorrelation of lag 7 weighted by van der Waals volume
MATS2s	Moran autocorrelation of lag 2 weighted by I-state
GATS4p	Geary autocorrelation of lag 4 weighted by polarizability
SpMax8_Bh(p)	largest eigenvalue n. 8 of Burden matrix weighted by polarizability
SpMax3_Bh(s)	largest eigenvalue n. 3 of Burden matrix weighted by I-state
P_VSA_e_3	P_VSA-like on Sanderson electronegativity, bin 3
TDB08m	3D Topological distance based descriptors - lag 8 weighted by mass
Mor21e	signal 21 / weighted by Sanderson electronegativity

Table 4: The estimated coefficients and MSE values for the four used estimators.

	Estimators			
	GR	r-k	k-d	r-(k-d)
$\hat{\beta}_{\mathrm{IC3}}$	1.1278	1.1064	1.0875	1.0818
$\hat{\beta}_{\mathrm{ATS8v}}$	2.1326	2.1112	2.0923	2.0866
$\hat{\beta}_{\mathrm{MATS7v}}$	0.9087	0.8873	0.8684	0.8627
$\hat{\beta}_{\mathrm{MATS2s}}$	-1.1533	-1.1747	-1.1936	-1.1993
$\hat{\beta}_{\mathrm{GATS4p}}$	-1.7531	-1.7745	-1.7934	-1.7991
$\hat{\beta}$ SpMax8_Bh (p)	0.1387	0.1173	0.0984	0.0927
$\hat{\beta}$ SpMax3_Bh (s)	-1.1851	-1.2065	-1.2254	-1.2311
$\hat{\beta}$ P_VSA_e_3	-.2108	-0.2322	-0.2511	-0.2568
$\hat{\beta}$ TDB08m	-1.1532	-1.1746	-1.1935	-1.1992
$\hat{\beta}$ Mor21e	3.1056	3.0842	3.0653	3.0596
MSE	4.1087	3.2749	3.1607	**2.4863**

6. Conclusion

In this paper we studied the theoretical properties of our propose estimator, (r-(k-d)) class estimator, in gamma regression model. Comparison of our proposed estimator to other estimator has been studied using the MSE. Depending on the simulation and real data results in the concept of mean squared error, the proposed estimator reveals a superior performance to the other estimators.

References

[1] F. Akdeniz and E. A. Duran, *Liu-type estimator in semiparametric regression models* , J. Stat. Comput. Simul., 80(8)(2010) 853–871.

[2] A. M. Al-Abood and D. H. Young, *Improved deviance goodness of fit statistics for a gamma regression model* , Commun. Stat. Methods, 15(6)(1986) 1865–1874.

[3] Z. Y. Algamal, *Developing a ridge estimator for the gamma regression model* , J. Chemom., 32(10)(2018) e3054.

[4] Z. Algamal, *Shrinkage estimators for gamma regression model* , Electron. J. Appl. Stat. Anal., 11(1)(2018) 253–268, `https://doi.org/10.1285/i20705948v11n1p253`.

[5] M. I. Alheety and B. M. Golam Kibria, *Modified Liu-type estimator based on (r− k) class estimator* , Commun. Stat. Methods, 42(2)(2013) 304–319.

[6] Y. Asar, *Liu-type negative binomial regression: A comparison of recent estimators and applications* , In: Trends Perspect. Linear Stat. Inference , Springer, 2018. pp. 23–39.

[7] Y. Asar and A. Genç, *New shrinkage parameters for the Liu-type logistic estimators* , Commun. Stat. Comput., 45(3)(2016) 1094–1103.

[8] M. R. Baye and D. F. Parker, *Combining ridge and principal component regression: a money demand illustration* , Commun. Stat. Methods, 13(2)(1984) 197–205.

[9] P. De Jong and G. Z. Heller, *Generalized linear models for insurance data* , Cambridge Books, 2008.

[10] E. Dunder, S. Gumustekin, and M. A. Cengiz, *Variable selection in gamma regression models via artificial bee colony algorithm* , J. Appl. Stat., 45(1)(2018) 8–16.

[11] A. E. Hoerl and R. W. Kennard, *Ridge regression: applications to nonorthogonal problems* , Technometrics, 12(1)(1970) 69–82.

[12] D. Inan and B. E. Erdogan, *Liu-type logistic estimator* , Commun. Stat. Comput., 42(7)(2013) 1578–1586.

[13] S. Kaçıranlar and S. Sakallıoğlu, *Combining the Liu estimator and the principal component regression estimator* , 2001.

[14] L. Kejian, *A new class of blased estimate in linear regression* , Commun. Stat. Methods, 22(2)(1993) 393–402.

[15] B. M. G. Kibria, *Performance of some new ridge regression estimators* , Commun. Stat. Comput., 32(2)(2003) 419–435.

[16] F. Kurtoğlu and M. R. Özkale, *Liu estimation in generalized linear models: application on gamma distributed response variable* , Stat. Pap., 57(4)(2016) 911–928.

[17] M. J. Mackinnon and M. L. Puterman, *Collinearity in generalized linear models* , Commun. Stat. methods, 18(9)(1989) 3463–3472.

[18] K. Månsson, *On ridge estimators for the negative binomial regression model* , Econ. Model., 29(2)(2012) 178–184.

[19] K. Mansson, B. M. G. Kibria, P. Sjolander, and G. Shukur, *Improved Liu estimators for the Poisson regression model* , Int. J. Stat. Probab., 1(1)(2012) 2.

[20] K. Månsson and G. Shukur, *A Poisson ridge regression estimator* , Econ. Model., 28(4)(2011) 1475–1481.

[21] A. S. Malehi, F. Pourmotahari and K. A. Angali, *Statistical models for the analysis of skewed healthcare cost data: a simulation study* , Health Econ. Rev., 5(1)(2015) 1–16.

[22] G. Muniz and B. M. G. Kibria, *On some ridge regression estimators: An empirical comparisons* , Commun. Stat. Comput., 38(3)(2009) 621-630 .

[23] M. R. Özkale and S. Kaciranlar, *The restricted and unrestricted two-parameter estimators* , Commun. Stat. Methods, 36(15)(2007) 2707–2725.

[24] B. Segerstedt, *On ordinary ridge regression in generalized linear models* , Commun. Stat. Methods, 21(8)(1992) 2227–2246.

[25] R. L. Schaefer, L. D. Roi, and R. A. Wolfe, *A ridge logistic estimator* , Commun. Stat. Methods, 13(1)(1984) 99–113.

[26] N. N. Urgan and M. Tez, *Liu estimator in logistic regression when the data are collinear* , In: 20th Euro Mini conf., 2008, pp. 323–327.

[27] M. Wasef Hattab, *A derivation of prediction intervals for gamma regression* , J. Stat. Comput. Simul., 86(17)(2016) 3512–3526.

[28] J. Wu and Y. Asar, *More on the restricted Liu estimator in the logistic regression model* , Commun. Stat. Comput., 46(5)(2017) 3680–3689.

[29] H. Yang and X. Chang, *A new two-parameter estimator in linear regression* , Commun. Stat. Methods, 39(6)(2010) 923–934.