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Fixed point of four maps in generalized b-metric spaces

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Abstract

In this paper, some common fixed point results for four mappings satisfying generalized contractive condition in a generalized b-metric spaces are proved. Advantage of our work in comparison with studies done in the context of b-metric is that, the b-metric functions used in the theorems are not necessarily continuous. So, our results extend and improve several comparable results obtained previously. To show the validity of our work, we also prove that the same results hold even if the space is endowed with two metrics.

Keywords: common fixed point, matrix convergent to zero, weakly compatible
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1. Introduction

Due to the wide applications of fixed point theorems in different fields, the study of existence and uniqueness of fixed points and common fixed points has become a subject of great interest. The main idea is to extend or generalize the famous Banach contraction principle in different directions. Many authors generalized the Banach contraction principle by generalizing the concept of a metric space see [8]. The classical Banach contraction principle was extended for contraction mappings on spaces endowed with vector-valued metrics by Perov [11] in 1964. The concept of b-metric space was introduced by Bakhtin in [4] and extensively used by Czerwik in [7], since then several papers have dealt with fixed point theory in b-metric spaces (see [1], [2], [3], [5], [13]). The aim of this paper is to present some common fixed point results for four mappings satisfying generalized contractive condition in a generalized b-metric space, where the b-metric is not necessarily continuous. Many authors in their work have used the b-metric spaces in which b-metric function is continuous, but the

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techniques used here can be employed in the setup of discontinuous b-metric spaces. From this point of view the results obtained in this paper generalize and extend several comparable existing results in the framework of b-metric spaces. In this paper we focused on Hardy-Rogers type contractions [9] and present some common fixed point results in generalized b-metric spaces for four mappings.

2. Preliminaries

In this section, we present some useful properties and auxiliary results to prove our fixed point theorems in the following section.

Definition 2.1. *[7] Let X be a nonempty set and let $s \geq 1$ be a given real number. A function $d : X \times X \to \mathbb{R}_+^n$ is said to be a vector-valued b-metric on X if and only if for all $x, y, z \in X$ the following conditions are satisfied:*

1. *$d(x, y) = 0$ if and only if $x = y$,*
2. *$d(x, y) = d(y, x)$,*
3. *$d(x, z) \leq s[d(x, y) + d(y, z)]$.*

A pair (X, d) is called a generalized b-metric space.

Remark 2.2. *If $\alpha, \beta \in \mathbb{R}^n$ with $\alpha = (\alpha_1, \alpha_2, \ldots, \alpha_n)$, $\beta = (\beta_1, \beta_2, \ldots, \beta_n)$ and $c \in \mathbb{R}$, then by $\alpha \leq \beta$ (respectively $\alpha < \beta$), we mean that $\alpha_i \leq \beta_i$ (respectively $\alpha_i < \beta_i$), for all $i = \overline{1, n}$ and by $\alpha \leq c$ we mean that $\alpha_i \leq c$, for all $i = \overline{1, n}$.*

It should be noted that the class of generalized b-metric spaces is larger than the class of metric spaces, since a generalized b-metric space is a generalized metric space when $s = 1$ in the third assumption of the above definition. If $n = 1$ in the previous definition, then we get the concept of b-metric introduced by Bakhtin. Following is an example which shows that a b-metric need not be a metric:

Example 2.3. *Let (X, d) be a metric space and $\rho(x, y) = (d(x, y))^p$, where $p > 1$ is a real number. We show that ρ is a b-metric with $s = 2^{p-1}$. Obviously, conditions (1) and (2) of definition 2.1 are satisfied. If $1 < p < \infty$, then convexity of the function $f(x) = x^p (x > 0)$ implies that $\left(\frac{a+b}{2}\right)^p \leq \frac{1}{2}(a^p + b^p)$, that is, $(a + b)^p \leq 2^{p-1}(a^p + b^p)$ holds. Thus for each $x, y, z \in X$, we have:*

$$\begin{aligned} \rho(x, y) &\leq (d(x, z) + d(z, y))^p \\ &\leq 2^{p-1}((d(x, z))^p + (d(z, y))^p) \\ &= 2^{p-1}(\rho(x, z) + \rho(z, y)). \end{aligned}$$

So condition (3) of definition 2.1 holds and ρ is a b-metric. Note that (X, ρ) is not necessarily a metric space. For example, if $X = \mathbb{R}$ be the set of real numbers and $d(x, y) = |x - y|$ a usual metric, then $\rho(x, y) = (x - y)^2$ is a b-metric on $\mathbb{R}$ with $s = 2$, but not a metric on $\mathbb{R}$, as the triangle inequality for a metric does not hold.

For examples which show that a b-metric need not to be a metric see [16] and for the notions of convergence, closedness and completeness in a b-metric space see [6]. In general a b-metric function $d(x, y)$ for $s > 1$ is not jointly continuous in all two of its variables for examples, see [10].

Throughout this paper we denote by $\mathcal{M}_{n\times n}(\mathbb{R}_+)$ the set of all $n \times n$ matrices with positive elements, by Θ the zero $n \times n$ matrix and by I the identity $n \times n$ matrix.

Definition 2.4. *[15] A matrix $C \in \mathcal{M}_{n\times n}(\mathbb{R}_+)$ is said to be convergent to zero if and only if*

$$C^n \to \Theta, \quad \text{as } n \to \infty.$$

For other examples and considerations on matrices which converge to zero, see Turinici [14].

Notice that, for the proof of the main results, we need the following theorem, part of which being a classical result in matrix analysis.

Theorem 2.5. *[12] Let $C \in \mathcal{M}_{n\times n}(\mathbb{R}_+)$. The following statements are equivalent:*

(i) *C is convergent towards zero.*

(ii) *The eigenvalues of C are in the open unit disc, that is, $|\lambda| < 1$, for every $\lambda \in \mathbb{C}$ with $\det(C - \lambda I) = 0$.*

(iii) *The matrix $(I - C)$ is nonsingular and*

$$(I - C)^{-1} = I + C + C^2 + \ldots + C^n + \ldots.$$

(iv) *The matrix $(I - C)$ is nonsingular and $(I - C)^{-1}$ has nonnegative elements.*

(v) *$C^n q \to 0$ and $qC^n \to 0$ as $n \to \infty$, for each $q \in \mathbb{R}^n$.*

We need also the following simple definitions.

Definition 2.6. *Let f and g be self-mapping of a generalized b-metric space (X, d). An element $x \in X$ is said to be a common fixed point of f and g if and only if $x = f(x) = g(x)$.*

Definition 2.7. *Let f and g be self-mapping of a generalized b-metric space (X, d). f and g are said to be weakly compatible if they commute at their coincidence points, the equality $fu = gu$ for some $u \in X$ implies that $fgu = gfu$.*

3. Main Results

Theorem 3.1. *Let f, g, S and T be self-mappings of a generalized b-metric space (X, d) satisfying the following conditions:*

$$f(X) \subset T(X), \quad g(X) \subset S(X). \tag{3.1}$$

Suppose that one of $S(X), T(X), f(X)$, and $g(X)$ is a complete subspace of X and the pairs (S, f) and (T, g) are weakly compatible. There exists matrices $M, N, P \in \mathcal{M}_{n\times n}(\mathbb{R}_+)$ with:

(i) $(I - N - Ps)$ *is nonsingular and* $(I - N - Ps)^{-1} \in \mathcal{M}_{n\times n}(\mathbb{R}_+)$*;*

(ii) sC *is convergent towards zero, where* $C = (I - N - Ps)^{-1}(M + N + Ps)$*;*

(iii) $(I - M - 2P)$ *is nonsingular and* $(I - M - 2P)^{-1} \in \mathcal{M}_{n\times n}(\mathbb{R}_+)$*;*

(iv) $d(fx, gy) \le Md(Sx, Ty) + N[d(Sx, fx) + d(Ty, gy)] + P[d(Sx, gy) + d(Ty, fx)]$*, for all* $x, y \in X$.

Then f, g, S, T have a unique common fixed point z.

Proof . Let x_0 be an arbitrary point in X. By (3.3), we can define inductively a sequence $\{y_n\}$ in X such that

$$y_{2n} = Tx_{2n+1} = fx_{2n}$$
$$y_{2n+1} = Sx_{2n+2} = gx_{2n+1}$$

We have:

$$\begin{aligned} d(y_{2n}, y_{2n+1}) &= d(fx_{2n}, gx_{2n+1}) \\ &\leq Md(Sx_{2n}, Tx_{2n+1}) + N[d(Sx_{2n}, fx_{2n}) + d(Tx_{2n+1}, gx_{2n+1})] \\ &\quad +P[d(Sx_{2n}, gx_{2n+1}) + d(Tx_{2n+1}, fx_{2n})] \\ &= Md(y_{2n-1}, y_{2n}) + N[d(y_{2n-1}, y_{2n}) + d(y_{2n}, y_{2n+1})] + Pd(y_{2n-1}, y_{2n+1}) \\ &\leq Md(y_{2n-1}, y_{2n}) + N[d(y_{2n-1}, y_{2n}) + d(y_{2n}, y_{2n+1})] \\ &\quad +Ps[d(y_{2n-1}, y_{2n}) + d(y_{2n}, y_{2n+1})] \end{aligned}$$

this implies that:

$$d(y_{2n}, y_{2n+1}) \leq (I - N - Ps)^{-1}(M + N + Ps)d(y_{2n-1}, y_{2n}) = Cd(y_{2n-1}, y_{2n}).$$

Similarly, we have:

$$\begin{aligned} d(y_{2n+1}, y_{2n+2}) &= d(fx_{2n+2}, gx_{2n+1}) \\ &\leq Md(Sx_{2n+2}, Tx_{2n+1}) + N[d(Sx_{2n+2}, fx_{2n+2}) + d(Tx_{2n+1}, gx_{2n+1})] \\ &\quad +P[d(Sx_{2n+2}, gx_{2n+1}) + d(Tx_{2n+1}, fx_{2n+2})] \\ &= Md(y_{2n+1}, y_{2n}) + N[d(y_{2n+1}, y_{2n+2}) + d(y_{2n}, y_{2n+1})] + Pd(y_{2n}, y_{2n+2}) \\ &\leq Md(y_{2n}, y_{2n+1}) + N[d(y_{2n+1}, y_{2n+2}) + d(y_{2n}, y_{2n+1})] \\ &\quad +Ps[d(y_{2n}, y_{2n+1}) + d(y_{2n+1}, y_{2n+2})]. \end{aligned}$$

Thus

$$d(y_{2n+1}, y_{2n+2}) \leq (I - N - Ps)^{-1}(M + N + Ps)d(y_{2n}, y_{2n+1}) = Cd(y_{2n}, y_{2n+1}).$$

We obtain that:

$$d(y_n, y_{n+1}) \leq C^n d(y_0, y_1), \quad \text{for each } n \in \mathbb{N}.$$

To prove that $(y_n)_{n\in\mathbb{N}}$ is a Cauchy sequence, we estimate $d(y_n, y_{n+p})$ using the triangle inequality:

$$\begin{aligned} d(y_n, y_{n+p}) &\leq sd(y_n, y_{n+1}) + s^2 d(y_{n+1}, y_{n+2}) + \cdots + s^{p-2} d(y_{n+p-3}, y_{n+p-2}) \\ &\quad +s^{p-1} d(y_{n+p-2}, y_{n+p-1}) + s^{p-1} d(y_{n+p-1}, y_{n+p}) \\ &\leq sC^n d(y_0, y_1) + s^2 C^{n+1} d(y_0, y_1) + \cdots + s^{p-2} C^{n+p-3} d(y_0, y_1) \\ &\quad +s^{p-1} C^{n+p-2} d(y_0, y_1) + s^{p-1} C^{n+p-1} d(y_0, y_1) \\ &= sC^n [I + sC + \cdots + s^{p-2} C^{p-2} + s^{p-2} C^{p-1}] d(y_0, y_1) \\ &\leq sC^n [I + sC + \cdots + s^{p-2} C^{p-2} + s^{p-1} C^{p-1}] d(y_0, y_1) \\ &\leq sC^n (I - sC)^{-1} d(y_0, y_1) \\ &\leq (sC)^n (I - sC)^{-1} d(y_0, y_1). \end{aligned}$$

Note that $(I - sC)$ is nonsingular since sC is convergent to zero. This implies that the sequence $\{y_n\}$ is a Cauchy sequence in X, therefore, the subsequence $\{y_{2n}\} = \{fx_{2n}\} \subset f(X)$ is a Cauchy sequence in $f(X)$. Since $T(X)$ is complete, it converges to a point $z = Tv$ for some $v \in X$.

Therefore, the sequence $\{y_n\}$ converges also to z and the subsequences $\{Sx_{2n+2}\}, \{gx_{2n+1}\}$ and $\{fx_{2n}\}$ converge to z.

If $z \neq gv$, using the contraction condition, we obtain

$$\begin{aligned} d(fx_{2n}, gv) &\leq Md(Sx_{2n}, Tv) + N[d(Sx_{2n}, fx_{2n}) + d(Tv, gv)] \\ &\quad + P[d(Sx_{2n}, gv) + d(Tv, fx_{2n})] \\ &= Md(y_{2n-1}, z) + N[d(y_{2n-1}, y_{2n}) + d(z, gv)] \\ &\quad + Ps[d(y_{2n-1}, z) + d(z, gv)] + Pd(z, y_{2n}). \end{aligned}$$

Letting $n \to \infty$, we obtain

$$(I - N - Ps)d(z, gv) \leq 0$$

taking into account that $(I - N - Ps)$ is nonsingular and $(I - N - Ps)^{-1} \in \mathcal{M}_{n\times n}(\mathbb{R}_+)$, we get that $z = Tv = gv$. Since $g(X) \subset S(X)$, there exists an $u \in X$ such that $z = Su = gv$.

If $z \neq fu$, using the contraction condition, we have:

$$\begin{aligned} d(fu, gv) &\leq Md(Su, Tv) + N[d(Su, fu) + d(Tv, gv)] \\ &\quad + P[d(Su, gv) + d(Tv, fu)] \\ &= Md(z, z) + N[d(z, fu) + d(z, z)] \\ &\quad + P[d(z, z) + d(z, fu)]. \end{aligned}$$

We get

$$(I - N - P)d(fu, z) \leq 0.$$

Since $(I - N - P)$ is nonsingular and $(I - N - P)^{-1} \in \mathcal{M}_{n\times n}(\mathbb{R}_+)$, we get that $z = Su = fu$. Since the pairs (S, f) and (T, g) are weakly compatible, we get $fz = Sz$ and $gz = Tz$.

Now we prove that $z = fz = Sz$,

$$\begin{aligned} d(fz, z) &= d(fz, gv) \\ &\leq Md(Sz, Tv) + N[d(Sz, fz) + d(Tv, gv)] \\ &\quad + P[d(Sz, gv) + d(Tv, fz)] \\ &= Md(fz, z) + P[d(fz, z) + d(z, fz)]. \end{aligned}$$

Since $(I - M - 2P)$ is nonsingular and $(I - M - 2P)^{-1} \in \mathcal{M}_{n\times n}(\mathbb{R}_+)$ this implies that $z = fz = Sz$. Similarly, we can prove that $z = gz = Tz$. Hence z is a common fixed point of f, g, S and T.

If there exists another common fixed point w in X for f, g, S and T, then

$$\begin{aligned} d(w, z) &= d(fw, gz) \\ &\leq Md(Sw, Tz) + N[d(Sw, fw) + d(Tz, gz)] \\ &\quad + P[d(Sw, gz) + d(Tz, fw)] \\ &= Md(w, z) + P[d(w, z) + d(z, w)]. \end{aligned}$$

Since $(I - M - 2P)$ is nonsingular and $(I - M - 2P)^{-1} \in \mathcal{M}_{n\times n}(\mathbb{R}_+)$ this implies that z is a unique common fixed point of f, g, S and T. □

We get the concept of generalized metric space in the previous theorem if $s = 1$, in this case we have:

Corollary 3.2. *Let f, g, S and T be self-mappings of a generalized metric space (X, d) satisfying the following conditions:*

$$f(X) \subset T(X), \quad g(X) \subset S(X). \tag{3.2}$$

Suppose that one of $S(X), T(X), f(X)$, and $g(X)$ is a complete subspace of X and the pairs (S, f) and (T, g) are weakly compatible. There exists matrices $M, N, P \in \mathcal{M}_{n\times n}(\mathbb{R}_+)$ with:

(i) $(I - N - P)$ *is nonsingular and* $(I - N - P)^{-1} \in \mathcal{M}_{n\times n}(\mathbb{R}_+)$;

(ii) C *is convergent towards zero, where* $C = (I - N - P)^{-1}(M + N + P)$;

(iii) $(I - M - 2P)$ *is nonsingular and* $(I - M - 2P)^{-1} \in \mathcal{M}_{n\times n}(\mathbb{R}_+)$;

(iv) $d(fx, gy) \le Md(Sx, Ty) + N[d(Sx, fx) + d(Ty, gy)] + P[d(Sx, gy) + d(Ty, fx)]$, *for all* $x, y \in X$.

Then f, g, S, T have a unique common fixed point z.

For the proof of this corollary, we follow the same steps as in theorem (3.1) but the difference here is that the generalized metric function d is continuous. As we see in theorem (3.1), we have not supposed supplementary conditions to prove the existence and the uniqueness of the fixed point despite that we have not the continuity of the function d which prove that the techniques used in theorem (3.1) can be employed in the setup of discontinuous b-metric spaces.

If $n = 1$ in theorem (3.1) then we get the concept of b-metric introduced by Bakhtin.

To show the validity of our work, we prove next that the same results hold even if the space is endowed with two metrics.

Theorem 3.3. *Let (X, δ) be a complete generalized b-metric space and d another vector-valued b-metric on X. Assume that the operators f, g, S and T satisfy the following conditions:*

$$f(X) \subset T(X), \quad g(X) \subset S(X), \tag{3.3}$$

Suppose that one of $S(X), T(X), f(X)$, and $g(X)$ is a complete subspace of X and the pairs (S, f) and (T, g) are weakly compatible.

(a) There exists a matrix $U \in \mathcal{M}_{n\times n}(\mathbb{R}_+)$ *such that* $\delta(x, y) \le U \cdot d(x, y)$, *for all* $x, y \in X$;

(b) f *is* (δ, δ)*-continuous;*

(c) There exists matrices $M, N, P \in \mathcal{M}_{n\times n}(\mathbb{R}_+)$ *with:*

- *(i)* $(I - N - Ps)$ *is nonsingular and* $(I - N - Ps)^{-1} \in \mathcal{M}_{n\times n}(\mathbb{R}_+)$;
- *(ii)* sC *is convergent towards zero, where* $C = (I - N - Ps)^{-1}(M + N + Ps)$;
- *(iii)* $(I - M - 2P)$ *is nonsingular and* $(I - M - 2P)^{-1} \in \mathcal{M}_{n\times n}(\mathbb{R}_+)$;
- *(iv)* $d(fx, gy) \le Md(Sx, Ty) + N[d(Sx, fx) + d(Ty, gy)] + P[d(Sx, gy) + d(Ty, fx)]$, *for all* $x, y \in X$.

Then f, g, S, T have a unique common fixed point z.

Proof . As in the proof of theorem 3.1, we obtain that $(x_n)_{n\in\mathbb{N}}$ is d-Cauchy. It follows from (a) that $(x_n)_{n\in\mathbb{N}}$ is δ-Cauchy sequence. Therefore, the subsequence $\{y_{2n}\} = \{fx_{2n}\} \subset f(X)$ is a Cauchy sequence in $f(X)$. Since $T(X)$ is complete, it converges to a point $z = Tv$ for some $v \in X$.

Therefore, the sequence $\{y_n\}$ converges also to z and the subsequences $\{Sx_{2n+2}\}$, $\{gx_{2n+1}\}$ and $\{fx_{2n}\}$ converge to z.

If $z \neq gv$, using the contraction condition, we obtain

$$\begin{aligned} d(fx_{2n}, gv) &\leq Md(Sx_{2n}, Tv) + N[d(Sx_{2n}, fx_{2n}) + d(Tv, gv)] \\ &\quad +P[d(Sx_{2n}, gv) + d(Tv, fx_{2n})] \\ &= Md(y_{2n-1}, z) + N[d(y_{2n-1}, y_{2n}) + d(z, gv)] \\ &\quad +Ps[d(y_{2n-1}, z) + d(z, gv)] + Pd(z, y_{2n}). \end{aligned}$$

Letting $n \to \infty$, we obtain

$$(I \ - N - Ps)d(z,\ gv) \leq 0$$

taking into account that $(I - N - Ps)$ is nonsingular and $(I - N - Ps)^{-1} \in \mathrm{M}_{n\times n}(\mathbb{R}_+)$, we get that $z = Tv = gv$. Since $g(X) \subset S(X)$, there exists an $u \in X$ such that $z = Su = gv$.

If $z \neq fu$, using the contraction condition, we have:

$$\begin{aligned} d(fu, gv) &\leq Md(Su, Tv) + N[d(Su, fu) + d(Tv, gv)] \\ &\quad +P[d(Su, gv) + d(Tv, fu)] \\ &= Md(z, z) + N[d(z, fu) + d(z, z)] \\ &\quad +P[d(z, z) + d(z, fu)]. \end{aligned}$$

We get

$$(I \ - N - P)d(fu,\ z) \leq 0.$$

Since $(I - N - P)$ is nonsingular and $(I - N - P)^{-1} \in \mathrm{M}_{n\times n}(\mathbb{R}_+)$, we get that $z = Su = fu$. Since the pairs (S, f) and (T, g) are weakly compatible, we get $fz = Sz$ and $gz = Tz$.

Now we prove that $z = fz = Sz$,

$$\begin{aligned} d(fz, z) &= d(fz, gv) \\ &\leq Md(Sz, Tv) + N[d(Sz, fz) + d(Tv, gv)] \\ &\quad +P[d(Sz, gv) + d(Tv, fz)] \\ &= Md(fz, z) + P[d(fz, z) + d(z, fz)]. \end{aligned}$$

Since $(I - M - 2P)$ is nonsingular and $(I - M - 2P)^{-1} \in \mathrm{M}_{n\times n}(\mathbb{R}_+)$ this implies that $z = fz = Sz$. Similarly, we can prove that $z = gz = Tz$. Hence z is a common fixed point of f, g, S and T.

If there exists another common fixed point w in X for f, g, S and T, then

$$\begin{aligned} d(w, z) &= d(fw, gz) \\ &\leq Md(Sw, Tz) + N[d(Sw, fw) + d(Tz, gz)] \\ &\quad +P[d(Sw, gz) + d(Tz, fw)] \\ &= Md(w, z) + P[d(w, z) + d(z, w)]. \end{aligned}$$

Since $(I - M - 2P)$ is nonsingular and $(I - M - 2P)^{-1} \in \mathrm{M}_{n\times n}(\mathbb{R}_+)$ this implies that z is a unique common fixed point of f, g, S and T.

□

It is of great interest to give fixed point results on a set endowed with vector-valued metrics or norms. Therefore, we may conclude that for different types of estimations, the use of the vector-valued norm and, correspondingly, of the matrices convergent to zero, is more appropriate when treating systems of equations. Application of the previous results is possible for a system of operatorial equations.

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References

[1] A. Aliouche, *Common fixed point theorems via implicit relations*, Miskolc Math. Notes 11(1) (2010) 3–12.
[2] A. Aliouche and T. Hamaizia, *Common fixed point theorems for multivalued mappings in b-metric spaces with an application to integral inclusions*, J. Anal. 2021 (2021) 1–20.
[3] H. Aydi, M. Bota, E. Karapinar and S. A Moradi, *Common fixed point for weak ϕ-contractions on b-metric spaces*, Fixed Point Theory 13(2) (2012) 337–346.
[4] I.A. Bakhtin, *The contraction mapping principle in almost metric space,* Funct. Anal, Unianowsk Gos. Ped. Inst. 30 (1989) 26–37.
[5] V. Berinde, *Generalized contractions in quasimetric spaces*, Seminar on Fixed Point Theory 93 (1993) 3–9.
[6] M. Boriceanu, *Fixed point theory on spaces with vector-valued b-metrics*, Demonstratio Math. 42(4) (2009) 825–835.
[7] S. Czerwik, *Nonlinear set-valued contraction mappings in b-metric spaces*, Atti Sem. Mat. Fis. Univ. Modena. 46(2) (1998) 263–276.
[8] L. Guran, M-F. Bota and A. Naseem, *Fixed point problems on generalized metric spaces in Perov's sense*, Symmetry 12 (2020) 856.
[9] G.E. Hardy and T.D. Rogers, *A generalization of a fixed point theorem of Reich*, Canad. Math. Bull. 16 (1973) 201–206.
[10] N. Hussain and M. H. Shah, *KKM mappings in cone b-metric spaces*, Comput. Math. Appl. 62 (2011) 1677–1684.
[11] A. I. Perov, *On the Cauchy problem for a system of ordinary differential equations*, Pviblizhen. Met. Reshen. Differ. Uvavn. 2 (1964) 115–134.
[12] R. Precup, *The role of matrices that are convergent to zero in the study of semilinear operator systems*, Math. Comput. Model. 49 (2009) 703–708.
[13] J.R. Roshan, N. Shobkolaei, S. Sedghi and M. Abbas, *Common fixed point of four maps in b-metric spaces*, Hacettepe J. Math. Stat. 43(4) (2014) 613–624.
[14] M. Turinici, *Finite-dimensional vector contractions and their fixed points*, Studia Universitatis Babes-Bolyai Math. 35 (1990) 30–42.
[15] R.S. Varga, *Matrix Iterative Analysis*, Springer Series in Computational Mathematics, Springer, Berlin, Germany, 2000.
[16] Z. Yanga, H. Sadatib, S. Sedghib and N. Shobec, *Common fixed point theorems for non-compatible self-maps in b-metric spaces*, J. Nonlinear Sci. App. 8 (2015) 1022–1031.