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Shannon entropy in generalized order statistics from Pareto-type distributions

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Abstract

In this paper, we derive the exact analytical expressions for the Shannon entropy of generalized order statistics from Pareto-type and related distributions.

Keywords: Shannon entropy; Generalized order statistics; Pareto distribution; Burr distribution.
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1. Introduction and preliminaries

The concept of entropy was first discussed by Shannon [13] in the twentieth century. The Shannon entropy of a random variable X is a mathematical measure of information which measures the average reduction of uncertainty of X, and for a continuous random variable X with probability density function $f(x)$ is defined as

$$H(X) = -\int_{-\infty}^{+\infty} f_X(x) \ln f_X(x) dx \tag{1.1}$$

Renyi [12] , Cover and Tommas [3] , Lazo and Rathie [7] , Kapur [5] , and Kullback [6] are among the researchers who have generalized this definition. Wong and Chen [14] did some research on the entropy of order statistics. Madadi and Tata [8]- [9] have recently computed the Shannon and Rényi information in the record data. Yari and Mohtashami [2] have obtained the entropy for Pareto-type distributions and their order statistics.

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Definition 1.1. *Let F be an absolutely continuous distribution function with density f. Let $n \in N$, $n \geq 2$, $k > 0$, $\widetilde{m} = (m_1, m_2, ..., m_{n-1}) \epsilon R^{n-1}$ and $M_r = \sum_{j=r}^{n-1} m_j$ for $\gamma_r = k + n - r + M_r > 0$ for all $r = 1, 2, ..., n-1$. The random vector $\mathbf{X}(n, \tilde{m}, k) = (X(1, n, \tilde{m}, k), ..., X(n, n, \tilde{m}, k))$ is called the vector of generalized order statistics from F if its density function is of the form*

$$f^{\mathbf{X}(n,\widetilde{m},k)}(\mathbf{x}) = k(\prod_{i=1}^{n-1} \gamma_i)(\prod_{i=1}^{n-1} (1 - F(x_i))^{m_i} f(x_i))(1 - F(x_n))^{k-1} \qquad f(x_n), \tag{1.2}$$

for $F^{-1}(0) < x_1 \leq x_2 \leq ... \leq x_n < F^{-1}(1)$.

The density function of the r^{th} generalized order statistic is given by

$$f^{X(r,n,\widetilde{m},k)}(x) = c_{r-1} \sum_{i=1}^{r} a_i (1 - F(x))^{\gamma_i - 1} f(x), \tag{1.3}$$

where $c_{r-1} = \prod_{i=1}^{r} \gamma_i$ and $a_i(r) = \prod_{j=1,\ j \neq i}^{r} \frac{1}{\gamma_j - \gamma_i}$.

The concept of generalized order statistics was introduced by Kamps [4] to unify several important concepts that have been used in statistics. For example, [2]

(i) the order statistics $X_{1:n}, ..., X_{n:n}$ of a sample $(X_1, ..., X_n)$ of size n from cdf F are generalized order statistics with parameters $m_1 = ... = m_{n-1} = 0$ and $k = 1$;

(ii) the record values in a sequence $X_n, n \geq 1$ of i.i.d. random variables are generalized order statistics with parameters $m_1 = ... = m_{n-1} = -1$ and $k = 1$;

(iii) the $k - th$ record values $Y_1^{(k)}, ..., Y_n^{(k)}$ in an i.i.d. sequence $X_n, n \geq 1$ are generalized order statistics with parameters $m_1 = ... = m_{n-1} = -1$ and $k \geq 1$, a positive integer;

(iv) Pfeifer's record values $X_{\triangle_1}^{(1)}, ..., X_{\triangle_n}^{(n)}$ in an array $X_i^{(j)}, i \geq 1, j \geq 1$ of independent random variables such that $X_i^{(j)}$, $i \geq 1$, are identically distributed with distribution function $F_j(x) = 1 - (1 - F(x))^{\beta_j}$, $j \geq 1$, where $\beta_j > 0$, are generalized order statistics with parameters $m_i = \beta_i - \beta_{i+1} - 1$ and $k = \beta_n$ (cf.Pfeifer (1982));

(v) the progressive type II censored order statistics $X_{1:n:N}^{\widetilde{R}}, ..., X_{n:n:N}^{\widetilde{R}}$, where $\widetilde{R} = (R_1, ..., R_n)$ and $R_i \epsilon_0$, $1 \leq i \leq n$, are generalized order statistics with parameters $m_i = R_i$, $k = R_n + 1$(cf. Balakrishnan et al.(2001));

(vi) the sequential order statistics $X_*^{(1)}, ..., X_*^{(n)}$ of an array of independent random variables $Y_j^{(i)}, 1 \leq i \leq n, 1$ such that $Y_j^{(i)}$, $1 \leq j \leq n - i + 1$, are identically distributed with distribution function $F_i(x) = 1 - (1 - F(x))^{\alpha_i}$ for $1 \leq i \leq n$, are generalized order statistics with parameters $m_i = (n - i + 1)\alpha_i - (n - i)\alpha_{i+1} - 1$ and $k = \alpha_n$.

The Pareto-type distributions are flexible parametric models with applications in reliability, actuarial science, economics, finance and telecommunications. The hierarchy of Pareto distributions has been established starting from the classical Pareto(I) distribution, and subsequent addition of parameters related to location, scale, shape and inequality. The most general model in this family is the Pareto (IV) distribution, with the distribution function

$$F_X(x) = 1 - \Big(1 + (\frac{x - \mu}{\theta})^{\frac{1}{\gamma}}\Big)^{-\alpha} \quad , x > \mu, \tag{1.4}$$

where $-\infty < \mu < +\infty$, $\theta > 0$, $\gamma > 0$ and $\alpha > 0$ are location, scale, inequality and shape parameters, respectively. We denote this distribution by Pareto $(IV)(\mu,\theta,\gamma,\alpha)$. The density function is

$$f_X(x) = \frac{\alpha(\frac{x-\mu}{\theta})^{\frac{1}{\gamma}-1}}{\theta\gamma\Big(1+(\frac{x-\mu}{\theta})^{\frac{1}{\gamma}}\Big)^{\alpha+1}}, \quad x > \mu. \tag{1.5}$$

(i) If $\alpha = 1$ we obtain the Pareto(III) distribution.

(ii) If $\gamma = 1$ we obtain the Pareto(II).

(iii) If $\gamma = 1$ and $\mu = \theta$ we obtain Pareto(I).

(iv) If $\mu = 0$ then we get the Burr(XII) distribution with parameters θ, α and $\frac{1}{\gamma}$.

In section (2) of this paper we obtain the Shannon entropy of generalized order statistics from a Pareto(IV) distribution. The Shannon entropy of generalized order statistics for other types of Pareto distribution are obtained as a consequence. Section (3) contains some numerical results and section (4) conclude the paper.

2. Shannon entropy in generalized order statistics from a Pareto (IV) distribution

Let $\mathbf{X}(n,\tilde{m},k)$ be a vector of generalized order statistics from a Pareto(IV) distribution, and set $Y(r,n,\tilde{m},k) = (\frac{X(r,n,\tilde{m},k)-\mu}{\theta})^{\frac{1}{\gamma}}$, $r = 1,2,...n$. Then, from Eq.(1.2) ,(1.4) and (1.5), we have

$$f^{\mathbf{Y}(n,\tilde{m},k)}(\mathbf{y}) = k\alpha^n \prod_{j=1}^{n-1}\gamma_j \Big(\prod_{i=1}^{n-1}\frac{1}{(1+y_i)^{\alpha m_i+\alpha+1}}\Big)\frac{1}{(1+y_n)^{\alpha k+1}}\ , \tag{2.1}$$

for $0 < y_1 \le y_2 \le ... \le y_n$.

Theorem 2.1. *The Shannon entropy of the vector* $\mathbf{Y}(n,\tilde{m},k) = (Y(1,n,\tilde{m},k),$ $...,Y(n,n,\tilde{m},k))$ *is*

$$\begin{aligned} H(\mathbf{Y}(n,\tilde{m},k)) &= -\ln k - n\ln\alpha - \sum_{j=1}^{n-1}\ln\gamma_j + \sum_{i=1}^{n-1}\frac{\alpha m_i+\alpha+1}{\alpha}\sum_{j=1}^{i}\frac{1}{\gamma_j} \\ &\quad + \frac{\alpha k+1}{\alpha}\sum_{j=1}^{n}\frac{1}{\gamma_j}. \end{aligned} \tag{2.2}$$

Proof . From Eq.(1.2) and (2.1)

$$\begin{aligned} H(\mathbf{Y}(n,\tilde{m},k)) &= -\ln k - n\ln\alpha - \sum_{i=1}^{n-1}\ln\gamma_i + \sum_{i=1}^{n-1}(\alpha m_i+\alpha+1) \\ &\quad \times E(\ln(1+Y_i)) + (\alpha k+1)E(\ln(1+Y_n)). \end{aligned}$$

But from Eq.(1.3)

$$\begin{aligned}
E(\ln(1+Y_i)) &= \int_0^{+\infty}(\ln(1+y_i))\alpha c_{i-1}(\sum_{j=1}^{i}\frac{a_j(i)}{(1+y_i)^{\alpha\gamma_j+1}})dy_i \\
&= \alpha c_{i-1}\sum_{j=1}^{i}a_j(i)\int_0^{+\infty}\frac{\ln(1+y_i)}{(1+y_i)^{\alpha\gamma_j+1}}dy_i \\
&= \frac{1}{\alpha}c_{i-1}(\sum_{j=1}^{i}\frac{a_j(i)}{\gamma_j^2}) \\
&= \frac{1}{\alpha}(\sum_{j=1}^{i}\frac{1}{\gamma_j})(cf. Balakrishnan et al. (2001),\ p.\ 309).
\end{aligned}$$

This completes the proof. □

Remark 2.2. *By Theorem 2.1 and change of variable* $Y(r,n,\tilde{m},k) = (\frac{X(r,n,\tilde{m},k)-\mu}{\theta})^{\frac{1}{\gamma}}$, $r = 1,2,...n$, *we can obtain the Shannon entropy of* $\mathbf{X}(n,\tilde{m},k)$ *in the form*

$$\begin{aligned}
H(\mathbf{X}(n,\widetilde{m},k)) &= H(\mathbf{Y}(n,\tilde{m}m,k)) + n\ln\theta\gamma \\
&+(\gamma-1)c_{n-1}\sum_{j=1}^{n}a_j(i)\frac{\Psi(1)-\Psi(\alpha\gamma_j)}{\gamma_j} \\
&+(\gamma-1)\sum_{i=1}^{n-1}c_{i-1}\sum_{j=1}^{i}a_j(i)\frac{\Psi(1)-\Psi(\alpha\gamma_j)}{\gamma_j}
\end{aligned} \tag{2.3}$$

where $\Psi(z) = \frac{d}{dz}\ln\Gamma(z)$ and Γ is the gamma function. The Shannon entropy in some models of ordered random variables are given in Remark 2.3

Remark 2.3. *(i) Putting* $k = 1$ *and* $m_1 = m_2 = ... = m_{n-1} = 0$ *in relation (2.3), we obtain Shannon entropy of order statistics for* $Pareto(IV)$ *distribution as*

$$\begin{aligned}
H(X_{1:n},...,X_{n:n}) &= \ln(\frac{\theta\gamma}{\alpha})^n - \sum_{j=1}^{n-1}\ln(n-j+1) + \frac{\alpha+1}{\alpha} \\
&\times \sum_{j=1}^{n}\frac{1}{n-j+1} + (\gamma-1) \\
&\times \sum_{i=1}^{n}\Big[\Big(\prod_{j=1}^{i}(n-j+1)\Big)\sum_{j=1}^{i}\Big(\prod_{l=1,\ l\neq j}^{i}\frac{1}{j-l}\Big) \\
&\times \Big(\frac{\Psi(1)-\Psi(\alpha n-\alpha j+\alpha)}{n-j+1}\Big)\Big]
\end{aligned}$$

(ii) *Putting* $\gamma = 1$ *in relation (2.3) obtain Shannon entropy of the Pareto(II) distribution is*

$$H(\mathbf{X}(n,\tilde{m},k)) = \ln(\frac{\theta}{\alpha})^n - \ln k - \sum_{j=1}^{n-1} \ln\gamma_j + \sum_{j=1}^{n-1} \frac{\alpha m_i + \alpha + 1}{\alpha} \times \Big(\sum_{j=1}^{i} \frac{1}{\gamma_j}\Big) + \frac{\alpha k + 1}{\alpha}\Big(\sum_{j=1}^{n}\frac{1}{\gamma_j}\Big), \tag{2.4}$$

now if $m_1 = m_2 = ... = m_{n-1} = -1$ *and* $k \geq 1$ *we have that the Shannon entropy of* $k-$*record values for Pareto(II) distribution is*

$$H(X_1^{(k)},...,X_n^{(k)}) = \frac{n(n+1+2\alpha k)}{2\alpha k} + \ln(\frac{\theta}{\alpha k})^n, \tag{2.5}$$

which is the result by Madadi and Tata [9].

(iii) *one can obtain the Shannon entropy of Pfeifer's record values for Pareto (IV) distribution by putting* $m_i = \beta_i - \beta_{i+1} - 1$ *and* $k = \beta_n$ *in relation (2.3).*

$$\begin{aligned} H(X_{\triangle_1}^{(1)},...,X_{\triangle_n}^{(n)}) &= n\ln\frac{\theta\gamma}{\alpha} - \ln\beta_n - \sum_{j=1}^{n-1}\ln\beta_j \\ &+ (\gamma-1)\sum_{i=1}^{n}\Big[\prod_{l=1}^{i}\beta_l\sum_{j=1}^{i}\Big(\prod_{l=1,\ l\neq j}^{i}\frac{1}{\beta_l-\beta_j}\Big) \\ &\times \Big(\frac{\Psi(1)-\Psi(\alpha\beta_j)}{\beta_j}\Big)\Big] \\ &+ \sum_{i=1}^{n-1}\frac{\alpha(\beta_i-\beta_{i+1}-1)+\alpha+1}{\alpha}\Big(\sum_{j=1}^{i}\frac{1}{\beta_j}\Big) \\ &+ \frac{\alpha\beta_n+1}{\alpha}\Big(\sum_{j=1}^{n}\frac{1}{\beta_j}\Big) \end{aligned}$$

(iv) *The Shannon entropy of progressive type* II *censored order statistics for a Pareto* (IV) *distribution is obtained by setting* $m_i = R_i$ *and* $k = R_n + 1$ *in relation (2.3).*

(v) *Similarly putting* $m_i = (n-i+1)\alpha_i - (n-i)\alpha_{i+1} - 1$ *and* $k = \alpha_n$ *in relation (2.3) we obtian the Shannon entropy of sequential order statistics for a Pareto*(IV) *distribution.*

Remark 2.4. *The Shannon entropy in generalized order statistics from a Pareto*(III) *distribution is*

$$\begin{aligned} H(\mathbf{X}(n,\tilde{m},k)) &= n\ln\theta\gamma - \ln k + (\gamma-1)\sum_{i=1}^{n-1}c_{i-1}\sum_{j=1}^{i}a_j(i)\frac{\Psi(1)-\Psi(\gamma_j)}{\gamma_j} \\ &+ (\gamma-1)c_{n-1}\sum_{j=1}^{n}a_j(i)\frac{\Psi(1)-\Psi(\gamma_j)}{\gamma_j} - \sum_{j=1}^{n-1}\ln\gamma_j \\ &+ (k+1)\sum_{j=1}^{n}\frac{1}{\gamma_j} + \sum_{i=1}^{n-1}(m_i+2)\sum_{j=1}^{i}\frac{1}{\gamma_j}. \end{aligned}$$

For a Pareto(II) distribution we have

$$
\begin{aligned}
H(\mathbf{X}(n,\widetilde{m},k)) &= \ln(\frac{\theta}{\alpha})^n - \ln k - \sum_{j=1}^{n-1}\ln\gamma_j + \sum_{j=1}^{n-1}\frac{\alpha m_i+\alpha+1}{\alpha}(\sum_{j=1}^{i}\frac{1}{\gamma_j}) \\
&+\frac{\alpha k+1}{\alpha}(\sum_{j=1}^{n}\frac{1}{\gamma_j}). \qquad (2.6)
\end{aligned}
$$

Since the entropy for the Pareto family does not depends on the location parameter μ, the Shannon entropy for Pareto distribution of type(II) and (I) are equal.

Corollary 2.5. *The Shannon entropy in the generalized order statistics from the Burr(XII) distribution is*

$$
\begin{aligned}
H(\mathbf{X}(n,\tilde{m},k)) &= -\ln k + ln(\frac{\theta}{\alpha\gamma})^n + \frac{\alpha k+1}{\alpha}(\sum_{j=1}^{n}\frac{1}{\gamma_j}) \\
&+(\frac{1}{\gamma}-1)(\sum_{i=1}^{n-1}c_{i-1}(\sum_{j=1}^{i}a_j(i)\frac{\Psi(1)-\Psi(\alpha\gamma_j)}{\gamma_j})) \\
&-\sum_{j=1}^{n-1}\ln\gamma_j + (\frac{1}{\gamma}-1)c_{n-1}\sum_{j=1}^{n}a_j(i)\frac{\Psi(1)-\Psi(\alpha\gamma_j)}{\gamma_j} \\
&+\sum_{i=1}^{n-1}\frac{\alpha m_i+\alpha+1}{\alpha}(\sum_{j=1}^{i}\frac{1}{\gamma_j}).
\end{aligned}
$$

3. Numerical results

In this section we compute the Shannon entropy of generalized order statistics from a Pareto (VI) distribution for various values of the parameters and sample of various sizes.
We see from Table 1, with increasing θ, the Shannon entropy of order statistics for Pareto (IV) distribution increases and with increasing α this entropy decreases. Table 2 shows that the Shannon entropy of record values for Pareto (I) and(II) distributions has increases with θ and n, but decreases as α increases. Tables 3 and 4 show that the Shannon entropy of k-record values has increases with k and α. The Shannon entropy also is always greater for $\theta = 2$. Table 5 shows that the Shannon entropy of Pfeifer's record values for Pareto (IV) distribution increases with θ and n, but decreases as increases α.

4. Conclusion.

In this paper, we have obtained the Shannon entropy of generalized order statistics from Pareto-type and Burr (XII) distributions. Also some numerical tables are presented to display the variation of the Shannon entropy in generalized order statistics with respect to the parameters.

Table 1: The Shannon entropy of order statistics for Pareto (IV)distribution for $\gamma = 1$.

		$\theta = \frac{2}{5}$	$\theta = \frac{4}{5}$	1	2	3	4
$\alpha = \frac{2}{5}$	$n = 5$	3.20	6.70	7.78	11.25	13.28	14.72
	10	-4.85	2.10	4.31	11.24	15.30	18.17
	20	-29.74	-15.88	-11.42	2.45	10.55	16.31
	40	-95.35	-67.62	-58.69	-30.97	-14.75	-3.24
$\alpha = \frac{4}{5}$	5	-3.12	0.35	1.47	4.93	6.96	8.40
	10	-15.45	-8.51	-6.28	0.65	4.70	7.58
	20	-48.10	-34.24	-29.78	-15.91	-7.81	-2.05
	40	-128.42	-100.69	-91.77	-64.04	-47.82	-36.32
$\alpha = 1$	5	-4.80	-1.34	-0.22	3.24	5.27	6.71
	10	-18.40	-11.48	-9.25	-2.33	1.74	4.62
	20	-53.50	-39.60	-35.14	-21.28	-13.17	-7.41
	40	-138.4	-110.69	-101.76	-74.04	-57.82	-46.31
$\alpha = 2$	5	-9.40	-5.94	-4.83	-1.36	0.66	2.10
	10	-26.81	-19.87	-17.64	-10.71	-6.66	-3.78
	20	-69.13	-55.26	-50.80	-36.94	-28.83	-23.08
	40	-168.28	-140.55	-131.63	-103.90	-87.68	-76.18
$\alpha = 3$	5	-11.82	-8.35	-7.20	-3.80	-1.70	-0.30
	10	-31.35	-24.42	-22.20	-15.30	-11.20	-8.30
	20	-77.84	-63.97	-59.50	-45.60	-37.50	-31.80
	40	-185.21	-157.49	-148.60	-120.80	-104.60	-93.10
$\alpha = 4$	5	-13.45	-9.98	-8.86	-5.40	-3.37	-1.93
	10	-34.47	-27.54	-25.31	-18.37	-14.32	-11.44
	20	-83.89	-70.03	-65.56	-51.70	-43.59	-37.84
	40	-197.08	-169.35	-160.42	-132.70	-116.48	-104.97

Table 2: The Shannon entropy of record values for Pareto (I) and (II)distribution.

		$\theta = \frac{2}{5}$	$\theta = \frac{4}{5}$	1	2	3	4
$\alpha = \frac{2}{5}$	$n = 5$	42.50	45.97	47.08	50.55	52.57	54.01
	10	147.50	154.43	156.66	163.59	167.45	170.53
	20	545.00	558.86	563.33	577.19	585.30	591.05
	40	2090.00	2117.73	2126.65	2154.38	2170.60	2182.10
$\alpha = \frac{4}{5}$	5	20.28	23.75	24.87	28.33	30.36	31.80
	10	71.82	78.75	80.98	87.91	91.97	94.84
	20	268.64	282.50	286.96	300.83	308.94	314.69
$\alpha = 1$	5	15.42	18.88	20.00	23.47	25.49	26.93
	10	55.84	62.77	65.00	71.93	75.99	78.86
	20	211.67	225.54	230.00	243.86	251.97	257.73
	40	823.35	851.07	860.00	887.73	903.94	915.45
$\alpha = 2$	5	4.45	7.92	9.03	12.50	14.53	15.97
	10	21.41	28.34	30.57	37.50	41.55	44.43
	20	92.81	106.67	111.14	125.00	133.11	138.86
	40	385.62	413.35	422.27	450.00	466.22	477.73
$\alpha = 3$	5	-0.07	3.39	4.51	7.97	10.00	11.44
	10	8.17	15.12	17.35	24.28	28.33	31.21
	20	49.70	63.56	68.03	81.89	90.00	95.75
	40	232.74	260.46	269.39	297.11	313.33	324.84
$\alpha = 4$	5	-2.76	0.70	1.82	5.28	7.31	8.75
	10	0.72	7.66	9.89	16.82	20.87	23.75
	20	26.45	40.31	44.77	58.64	66.75	72.50
	40	152.90	180.62	189.55	217.27	233.49	245.00

Table 3: The Shannon entropy of k-record values for Pareto (I) and (II)distribution with assumption $\theta = 1$.

		$k = 2$	3	4
	$n = 5$	24.87	16.59	12.02
$\alpha = \frac{2}{5}$	10	80.98	54.01	39.67
	20	286.96	191.35	141.85
	40	1037.93	716.04	533.70
	5	12.02	6.87	3.87
$\alpha = \frac{4}{5}$	10	39.67	24.16	15.56
	20	141.85	90.00	62.36
	40	533.70	346.65	249.72
	5	9.03	4.51	1.82
$\alpha = 1$	10	30.57	17.35	9.89
	20	111.14	68.03	44.77
	40	422.27	269.39	189.55
	5	1.82	-1.46	-3.52
$\alpha = 2$	10	9.89	1.25	-3.92
	20	44.77	19.16	4.66
	40	189.55	105.00	59.32
	5	-1.46	-4.32	-6.17
$\alpha = 3$	10	1.25	-5.86	-10.27
	20	19.16	-0.61	-12.20
	40	105.00	43.22	8.94
	5	-3.52	-6.17	-7.93
$\alpha = 4$	10	-3.92	-10.27	-14.29
	20	4.66	-12.20	-22.33
	40	59.32	8.94	-19.65

Table 4: The Shannon entropy of k-record values for Pareto (I) and (II)distribution with assumption $\theta = 2$.

		$k = 2$	3	4
$\alpha = \frac{2}{5}$	$n = 5$	28.33	20.05	15.49
	10	87.91	60.94	46.61
	20	300.83	205.22	155.71
	40	1101.65	743.77	561.43
$\alpha = \frac{4}{5}$	5	15.49	10.34	7.34
	10	46.61	31.09	22.49
	20	155.71	103.85	76.22
	40	561.43	374.37	277.45
$\alpha = 1$	5	12.50	7.97	5.28
	10	37.50	24.28	16.82
	20	125.00	81.89	458.64
	40	450.00	297.11	217.27
$\alpha = 2$	5	5.28	2.01	-0.06
	10	16.82	8.18	3.01
	20	58.64	33.03	18.52
	40	217.27	132.72	87.05
$\alpha = 3$	5	2.01	-0.85	-2.71
	10	8.18	1.07	-3.33
	20	33.03	13.25	1.66
	40	132.72	70.95	36.66
$\alpha = 4$	5	-0.06	-2.71	-4.46
	10	3.01	-3.33	-7.36
	20	18.52	1.66	-8.46
	40	87.05	36.66	8.07

Table 5: The Shannon entropy of Pfeifer's record values for Pareto (IV)distribution with assumption $\beta_i = i$ and $\gamma = 1$.

		$\theta = \frac{2}{5}$	$\theta = \frac{4}{5}$	1	2	3	4
$\alpha = \frac{2}{5}$	$n = 5$	21.95	25.43	26.54	30.01	32.04	33.48
	10	50.44	57.37	59.61	66.54	70.59	73.47
	20	116.55	130.41	134.87	148.73	156.84	162.60
	40	268.23	295.96	304.88	332.61	348.83	360.33
$\alpha = \frac{4}{5}$	5	7.62	11.09	12.20	15.67	17.70	19.13
	10	15.74	22.67	24.90	31.83	35.89	38.76
	20	33.24	47.11	51.57	65.43	73.54	79.28
	40	71.22	98.95	107.88	−135.61	151.82	163.33
$\alpha = 1$	5	4.33	7.80	8.91	12.38	14.41	15.84
	10	7.95	14.88	17.11	24.05	28.10	30.98
	20	14.89	28.75	33.32	47.08	55.19	60.94
	40	28.45	56.17	65.10	92.83	109.04	120.55
$\alpha = 2$	5	−3.48	−0.02	1.10	4.56	6.59	8.03
	10	−10.09	−3.16	−0.93	6.00	10.06	12.94
	20	−26.75	−12.89	−8.42	5.44	13.55	19.30
	40	−66.99	−39.26	−30.34	−2.61	13.61	25.12
$\alpha = 3$	5	−6.96	−3.50	−2.38	1.09	3.11	4.55
	10	−17.85	−10.92	−8.68	−1.75	2.30	5.18
	20	−44.12	−30.25	−25.79	−11.93	−3.82	1.94
	40	−105.78	−78.05	−69.13	−41.40	−25.18	−13.67
$\alpha = 4$	5	−9.13	−5.66	−4.54	−1.08	0.95	2.39
	10	−22.58	−15.64	−13.41	−6.48	−2.43	0.45
	20	−54.50	−40.64	−36.17	−22.31	−14.20	−8.45
	40	−128.57	−100.84	−91.92	−64.19	−47.97	−36.47

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