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The effects of outlier on some Bayesian survival estimators for Burr-X distribution with a Covid-19 as a case study

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Abstract

Survival functions estimators can be affected by outlier, and thus these estimations move away from their real values, especially with the increasing in the outlier ratios within the sample of the random variable. The research included a comparison of a number of Bayesian methods for the estimations of survival functions of burr- X distribution with the percentages of different outliers within the sample. Simulation results showed the effect of the estimation methods by sample size and the percentage of outliers, and the real values of the parameters distribution.

Mean square error was adopted as a measure to compare the estimation methods with a number of simulation experiments. The research also included a case study of Covid-19 for practical application. Other estimation methods can be taken (maximum likelihood estimation method, moment method, and shrinkage method) to note the possibility of being affected by outlier values

Keywords: Survival function, Bayesian estimators, integrated mean square error, Burr-X Distribution, Outlier values, covid-19, mean square error

1. General Introduction

Statistical distributions were and still one of the applied topics, which are represented in many life experiments, so there was a need to estimate the survival function of the Burr-X distribution when there were polluting values within the studied sample in case of difference (distribution parameter , sample size, polluted percentages) in many Bayesian estimation methods .Simulation and experimental results showed the effected of the estimation method by (parameter values, sample size,

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polluted percentage). research also included an estimate of survival for a sample of Coved-19 patients. The survival functions can be estimated for a number of other distributions (gamma, beta) within different percentages of pollution. The survival functions can also be estimated by a number of Non-Bayesian Survival Estimators methods and the results can be compared.

2. Problem of Research

The different percentages of pollution can lead to the deviation of the estimated values of survival functions from their real values, and thus the appearance of a large mean square error due to the effect of these polluting percentages.

3. Objective of research

The aim of the research is to estimate the survival function and its mean squares of error for Burr-X Distribution within (parameter values, sample size, polluted percentage) for a number of Bayesian methods.

4. Importance of Research

Survival function estimators are very important as they are included in many life experiments, and Bayesian methods are considered one of the most important methods for estimating survival functions in many statistical distributions.

5. Burr-X Distribution

It is one of the continuous statistical distributions of non-negative (positive) random variables, and it belongs to the family of generalized log-logistic distributions. It was presented by (Burr) in (1942) and the distribution has the following characteristics:
the probability density function can be[4, 7, 8]

$$f\left(x,\alpha,\beta\right)=\frac{2\alpha x}{\beta^2}\; e^{-\left(\frac{x}{\beta}\right)^2}\left(1-e^{-\left(\frac{x}{\beta}\right)^2}\right)^{\alpha-1},\quad x,\alpha,\beta>0 \tag{5.1}$$

the cumulative density function are

$$F\left(x,\alpha,\beta\right)=\left(1-e^{-\left(\frac{x}{\beta}\right)^2}\right)^{\alpha} \tag{5.2}$$

the survival function are[2]

$$S\left(x,\alpha,\beta\right)=1-\left(1-e^{-\left(\frac{x}{\beta}\right)^2}\right)^{\alpha} \tag{5.3}$$

the hazard function are[1, 3]

$$h\left(x,\alpha,\beta\right)=\frac{\frac{2\alpha x}{\beta^2}\; e^{-\left(\frac{x}{\beta}\right)^2}\left(1-e^{-\left(\frac{x}{\beta}\right)^2}\right)^{\alpha-1}}{1-\left(1-e^{-\left(\frac{x}{\beta}\right)^2}\right)^{\alpha}} \tag{5.4}$$

the generation of (Burr-X) distribution with (n) random sample and $(\alpha,\beta=1)$ parameters can be[5, 6]

$$x=\left[-Ln(1-U^{\frac{1}{\alpha}}\;)\right]^{\frac{1}{2}} \tag{5.5}$$

With (U) represent uniform distribution random variable

6. Bayesian Estimation Method

There are a number of Bayesian methods that can be adopted to estimate the parameters of the Pure X distribution and thus estimate the survival function of this distribution[10].

6.1. Uniform Bayesian Method(UBM)

This method depends on finding the estimator of the distribution parameter based on the uniform distribution so that it is the known prior distribution and that the probability is closer to one[9, 11]

$$p(\alpha) \ltimes 1$$

The posterior distribution will be under this hypothesis

$$p(\alpha/x) = \frac{[\sum_{i=1}^{n} Ln(1-e^{-x_i^2})^{-1}]^{n+1}}{\Gamma(n-1)} \quad \alpha^n e^{-\alpha \sum_{i=1}^{n} Ln(1-e^{-x_i^2})^{-1}} \tag{6.1}$$

The Bayesian estimator and the risk function by adopting a square error loss function will be

$$\alpha_{S-UBM} = \frac{n+1}{\sum_{i=1}^{n} Ln(1-e^{-x_i^2})^{-1}} \tag{6.2}$$

$$R(\alpha_{S-UBM}) = \frac{n+1}{[\sum_{i=1}^{n} Ln(1-e^{-x_i^2})^{-1}]^2} \tag{6.3}$$

and Bayesian estimator and the risk function by adopting a quadratic error loss function will be

$$\alpha_{Q-UBM} = \frac{n-1}{\sum_{i=1}^{n} Ln(1-e^{-x_i^2})^{-1}} \tag{6.4}$$

$$R(\alpha_{Q-UBM}) = \frac{1}{n} \tag{6.5}$$

and Bayesian estimator and the risk function by adopting a weighted error loss function will be[13, 14]

$$\alpha_{W-UBM} = \frac{n}{\sum_{i=1}^{n} Ln(1-e^{-x_i^2})^{-1}} \tag{6.6}$$

$$R(\alpha_{W-UBM}) = \frac{1}{\sum_{i=1}^{n} Ln(1-e^{-x_i^2})^{-1}} \tag{6.7}$$

6-2 MCMC Estimation Method

It is one of the estimation methods that are adopted to find the Bayesian and modified Bayesian estimators for the distribution parameter (α) and the reliability function related to it ($R(t)$).
The posterior probability density function for the distribution parameter can be [12, 15, 16]

$$\pi^*(\alpha/x) = \begin{cases} \alpha^{s+\alpha-1} e^{-b\alpha} [-\omega^{\alpha}{}_{x_{i:n}}]^{n-s} \prod_{i=1}^{s} \omega^{\alpha-1}{}_{x_{i:n}} & P = 0, \ldots, s-1 \\ \alpha^{P+\alpha-1} e^{-b\alpha} [-\omega^{\alpha}{}_{T}]^{n-P} \prod_{i=1}^{P} \omega^{\alpha-1}{}_{x_{i:n}} & P = s, \ldots, u-1 \\ \alpha^{u+\alpha-1} e^{-b\alpha} [-\omega^{\alpha}{}_{x_{u:n}}]^{n-u} \prod_{i=1}^{u} \omega^{\alpha-1}{}_{x_{i:n}} & P = u \end{cases} \tag{6.8}$$

the estimation method can be applied by the following algorithm

i- *strating by intial parameter value* (α_0)

ii- *generat* (α_j) *by using* (6.8)

iii- *generate a random sample with spesefic distribution*

iv- *applying the following acceptance probability*

$$a(\alpha^{j-1}/\alpha_j) = Min\,(1, \emptyset\,(\pi^*))\ \textit{such that}\ \emptyset\,(\pi^*) = \frac{\pi^*(\alpha^{(j)}/x)}{\pi^*(\alpha^{(j-1)}/x)}$$

v- *if* $A(\alpha^{j-1}/\alpha_j) > \Delta$ *then* $\alpha^{(y)} = \alpha_j$ *else setting* $\alpha_0 = \alpha_j$ *and re-calculate steps algorithm* Δ *=given thresoulting value*

vi- *re-calculate the previouse steps* (N) *times till getting* $\left(\alpha^1, \alpha^2, \ldots\ldots, \alpha^N\right)$ *then (MCMC) estimators of* (α) *under (SEL) function will be*

$$\alpha_{SEL-MCMC} = \frac{\sum_{y=M+1}^{N} \alpha^{(y)}}{N - M} \tag{6.9}$$

(MCMC) estimators of (α) under (LINEX) function will be

$$\alpha_{\text{LINEX}-MCMC} = \frac{Ln[(\sum_{y=M+1}^{N} e^{-h\alpha^{(y)}})/(N - M)]}{h} \tag{6.10}$$

7. Simulation Results

number of simulation experiments were carried out according to various (n) and (a) values by adopting formula (5.5) and applying the formulas for the estimators of the distribution parameter according to formulas (6.2), (6.4), (6.6), (6.9), (6.10) and (5.3), the following tables and graphs were obtained. Comparing estimation methods by using (MSE) b using the following formula

$$MSE = \frac{\sum_{i=1}^{K} \left[\widehat{\alpha}_i - \alpha\right]^2}{k} \tag{7.1}$$

With $\widehat{\alpha}_i$ represent the estimator for iteration i
k represent No. of iteration

Table 1: the (α) estimator for each estimation method according to simulation parameters with (0.05) outlier pollution and ($M_1 = \alpha_{S-UBM}, M_2 = \alpha_{Q-UBM}, M_3 = \alpha_{W-UBM}, M_4 = \alpha_{SEL-MCMC}, M_5 = \alpha_{\text{LINEX}-MCMC}$)

α	n	M_1	M_2	M_3	M_4	M_5
0.25	30	0.309168	0.248772	0.250689	0.264723	0.264159
	50	0.275749	0.249744	0.249723	0.271553	0.271475
	80	0.260172	0.250002	0.25002	0.256995	0.257
0.50	30	0.575913	0.500739	0.499769	0.544735	0.54469
	50	0.603209	0.499844	0.499912	0.597305	0.597305
	80	0.577026	0.499991	0.500009	0.571658	0.571665
0.75	30	0.966621	0.74776	0.751181	0.901607	0.903259
	50	0.804255	0.749867	0.750145	0.798786	0.798873
	80	0.80808	0.749999	0.750005	0.803508	0.803477

From table 1 estimators shows that the estimation methods give different outputs in spite of the input parameters are same.

Table 2: the (α) estimator for each estimation method according to simulation parameters with (0.10) outlier pollution and ($M_1 = \alpha_{S-UBM}, M_2 = \alpha_{Q-UBM}, M_3 = \alpha_{W-UBM}, M_4 = \alpha_{SEL-MCMC}, M_5 = \alpha_{\text{LINEX}-MCMC}$)

α	n	M_1	M_2	M_3	M_4	M_5
0.25	30	0.365457	0.256814	0.304291	0.353173	0.271931
	50	0.317627	0.255562	0.295616	0.352805	0.340899
	80	0.342839	0.329558	0.274175	0.295376	0.283495
0.50	30	0.587326	0.58033	0.508798	0.563116	0.63696
	50	0.634215	0.578223	0.571501	0.638015	0.638747
	80	0.61228	0.570354	0.538225	0.584507	0.601689
0.75	30	1.057945	0.75353	0.807563	0.9892	0.962309
	50	0.807224	0.757224	0.817496	0.894253	0.816752
	80	0.812184	0.797658	0.822313	0.861319	0.854256

From table 2 estimators shows that the estimation methods give different outputs in spite of the input parameters are same with another pollution ratio.

Table 3: the Mean Square Error for (α) estimator for each estimation method according to simulation parameters with (0.05) outlier pollution ($MSE_1 = MSE_{S-UBM}, MSE_2 = MSE_{Q-UBM}$, $MSE_3 = MSE_{W-UBM}, MSE_4 = MSE_{SEL-MCMC}, MSE_5 = MSE_{\text{LINEX}-MCMC}, Best = Min(MSE1, \ldots, MSE5)$)

α	n	MSE_1	MSE_2	MSE_3	MSE_4	MSE_5	$Best$	$Index$
0.25	30	4.93E-03	2.42E-05	6.76E-06	2.92E-03	2.66E-03	6.76E-06	3
	50	2.91E-03	2.38E-07	2.54E-07	2.91E-03	1.88E-03	2.38E-07	2
	80	2.79E-04	8.78E-10	2.58E-09	2.14E-04	2.14E-04	8.78E-10	2
0.50	30	3.11E-02	2.07E-05	1.71E-05	1.65E-02	2.99E-02	1.71E-05	3
	50	1.63E-02	2.40E-07	1.31E-07	1.63E-02	1.14E-02	1.31E-07	3
	80	1.22E-02	1.79E-09	2.37E-09	1.14E-02	0.009958	1.79E-09	2
0.75	30	8.29E-02	2.32E-05	1.87E-05	1.23E-02	5.87E-02	1.87E-05	3
	50	1.13E-02	9.40E-08	1.46E-07	1.13E-02	0.01045	9.40E-08	2
	80	0.010717	1.58E-09	1.24E-09	1.00E-02	1.00E-02	1.24E-09	3

Table 4: the Mean Square Error for (α) estimator for each estimation method according to simulation parameters with (0.10) outlier pollution ($MSE_1 = MSE_{S-UBM}, MSE_2 = MSE_{Q-UBM}$, $MSE_3 = MSE_{W-UBM}, MSE_4 = MSE_{SEL-MCMC}, MSE_5 = MSE_{\text{LINEX}-MCMC}, Best = Min(MSE1, \ldots, MSE5)$)

α	n	MSE_1	MSE_2	MSE_3	MSE_4	MSE_5	$Best$	$Index$
0.25	30	1.35E-02	2.76E-03	8.26E-03	5.10E-03	1.01E-02	2.76E-03	2
	50	7.05E-03	5.32E-03	7.95E-03	9.27E-03	8.30E-03	5.32E-03	2
	80	6.78E-03	3.95E-03	8.41E-03	3.81E-03	9.79E-03	3.81E-03	4
0.50	30	3.52E-02	9.79E-04	5.39E-03	2.27E-02	3.41E-02	9.79E-04	2
	50	2.07E-02	7.78E-03	4.22E-03	1.76E-02	2.03E-02	4.22E-03	3
	80	1.70E-02	8.40E-03	8.83E-03	1.56E-02	1.57E-02	8.40E-03	2
0.75	30	8.89E-02	4.06E-03	6.67E-03	2.07E-02	6.01E-02	4.06E-03	2
	50	1.59E-02	7.86E-03	1.15E-03	1.48E-02	1.15E-02	1.15E-03	3
	80	1.62E-02	2.12E-04	9.22E-03	1.49E-02	1.89E-02	2.12E-04	2

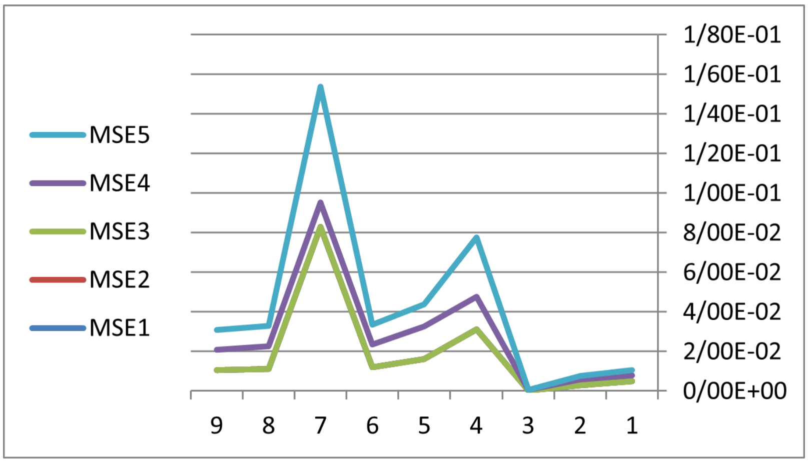

Figure 1: the Mean Square Error for (α) estimator for each estimation method according to simulation parameters with (0.05) outlier pollution $MSE_1 = MSE_{S-UBM}, MSE_2 = MSE_{Q-UBM}$, $MSE_3 = MSE_{W-UBM}, MSE_4 = MSE_{SEL-MCMC}, MSE_5 = MSE_{\text{LINEX}-MCMC}$

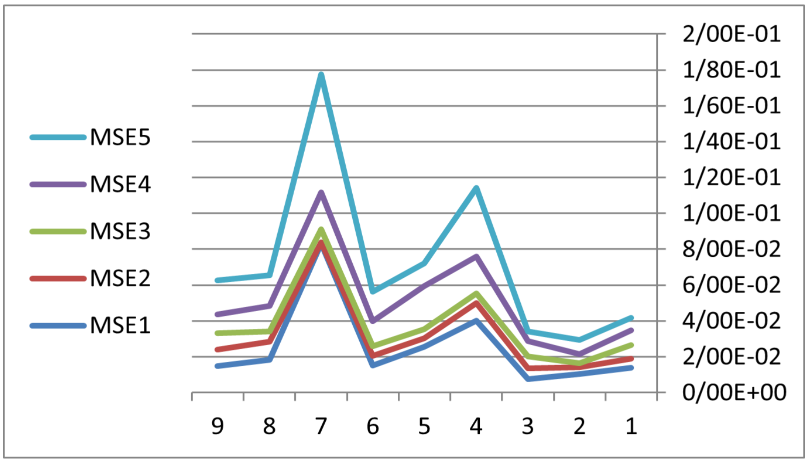

Figure 2: the Mean Square Error for (α) estimator for each estimation method according to simulation parameters with (0.10) outlier pollution $MSE_1 = MSE_{S-UBM}, MSE_2 = MSE_{Q-UBM}$, $MSE_3 = MSE_{W-UBM}, MSE_4 = MSE_{SEL-MCMC}, MSE_5 = MSE_{\text{LINEX}-MCMC}$

From table 3 and fig 1 results shows that the best estimation method for each simulation experiment are deferent and depend on (input parameter, sample size) and pollution ration (0.05) the best estimation methods was (MSE_2, MSE_3)
From table 4 and fig 2 results shows that the best estimation method for each simulation experiment are different and depend on (input parameter, sample size) and pollution ration (0.10) the best estimation methods was (MSE_2, MSE_3, MSE_4)

Table 5: the 1^{st} five Survival estimator for each estimation method according to simulation parameters with outlier pollution ration (0.05) ($t = the\ reall\ time$, $S_1 = S_{S-UBM}, S_2 = S_{Q-UBM}, S_3 = S_{W-UBM}, S_4 = S_{SEL-MCMC}, S_5 = S_{\text{LINEX}-MCMC}$)

α	n	i	t_i	S_{true}	S_1	S_2	S_3	S_4	S_5
0.25	30	1	3.48E-05	0.994104	0.998251	0.993954	0.994187	0.995642	0.995592
		2	1.40E-03	0.96264	0.982839	0.962032	0.962977	0.969216	0.968986
		3	5.22E-03	0.927737	0.961198	0.926798	0.928258	0.938096	0.937728
		4	7.52E-03	0.913308	0.951401	0.91226	0.91389	0.924936	0.92452
		5	5.34E-02	0.769049	0.836739	0.76738	0.769979	0.788146	0.787444
	50	1	9.52E-04	0.969151	0.97844	0.969041	0.969032	0.977144	0.977119
		2	1.11E-02	0.894792	0.91657	0.89455	0.894529	0.913357	0.913296
		3	1.19E-02	0.89084	0.913107	0.890593	0.890572	0.909815	0.909753
		4	1.25E-02	0.888234	0.910815	0.887983	0.887962	0.907474	0.907411
		5	1.52E-02	0.876751	0.900657	0.876487	0.876464	0.897104	0.897037
	80	1	6.44E-04	0.974626	0.978149	0.974627	0.974633	0.977105	0.977107
		2	1.51E-03	0.961177	0.965984	0.961178	0.961187	0.96455	0.964552
		3	1.63E-03	0.959601	0.964546	0.959602	0.959611	0.96307	0.963072
		4	6.57E-03	0.918915	0.926794	0.918917	0.918931	0.924419	0.924423
		5	6.72E-03	0.918052	0.925983	0.918054	0.918068	0.923592	0.923596
0.5	30	1	0.019102	0.9809	0.989527	0.981011	0.980865	0.986596	0.986591
		2	4.24E-02	0.957581	0.973746	0.957779	0.957519	0.968028	0.968019
		3	0.060867	0.939189	0.960248	0.939441	0.939111	0.952665	0.952653
		4	9.10E-02	0.909186	0.936908	0.909508	0.909086	0.926728	0.926712
		5	0.104087	0.896194	0.926403	0.896541	0.896085	0.915238	0.91522
	50	1	1.07E-02	0.98927	0.995792	0.989255	0.989262	0.995561	0.995561
		2	2.01E-02	0.979905	0.991029	0.97988	0.979891	0.990606	0.990606
		3	2.38E-02	0.976224	0.989011	0.976196	0.976209	0.988515	0.988515
		4	0.034473	0.965538	0.982804	0.965501	0.965517	0.982106	0.982106
		5	5.07E-02	0.949294	0.9726	0.949247	0.949268	0.971618	0.971618
	80	1	1.76E-03	0.998239	0.999337	0.998239	0.99824	0.999291	0.999291
		2	4.69E-03	0.995311	0.997947	0.99531	0.995311	0.997826	0.997826
		3	2.63E-02	0.973673	0.984966	0.973671	0.973675	0.984368	0.984369
		4	3.39E-02	0.966122	0.979888	0.96612	0.966124	0.979144	0.979145
		5	4.58E-02	0.954249	0.971553	0.954247	0.954252	0.970596	0.970597
0.75	30	1	3.49E-02	0.993473	0.998474	0.993374	0.993524	0.99764	0.997666
		2	4.88E-02	0.989241	0.997094	0.989095	0.989318	0.995696	0.995738
		3	7.51E-02	0.979471	0.993318	0.979231	0.979596	0.990641	0.990721
		4	0.1191	0.959115	0.983762	0.958723	0.95932	0.978576	0.978726
		5	0.119285	0.959021	0.983713	0.958628	0.959226	0.978517	0.978667
	50	1	5.07E-02	0.988593	0.991746	0.988584	0.988602	0.991473	0.991477
		2	9.75E-02	0.969643	0.976424	0.969624	0.969663	0.975816	0.975825
		3	0.152995	0.940679	0.951642	0.940649	0.940711	0.950636	0.950652
		4	0.172313	0.929262	0.941596	0.929228	0.929298	0.940457	0.940476
		5	0.19999	0.911891	0.92609	0.911853	0.911932	0.924769	0.92479
	80	1	0.02479	0.996098	0.99746	0.996098	0.996098	0.997373	0.997372
		2	5.58E-02	0.986832	0.990583	0.986832	0.986832	0.990332	0.99033
		3	0.104786	0.966219	0.974015	0.966219	0.96622	0.973472	0.973469
		4	0.108724	0.964309	0.972428	0.964308	0.964309	0.971862	0.971858
		5	0.113447	0.961973	0.970479	0.961973	0.961974	0.969884	0.96988

Table 6: the 1^{st} five Survival estimator for each estimation method according to simulation parameters with outlier pollution ration (0.10) ($t = the\ reall\ time$, $S = true, S_1 = S_{S-UBM}$, $S_2 = S_{Q-UBM}$, $S_3 = S_{W-UBM}, S_4 = S_{SEL-MCMC}, S_5 = S_{\text{LINEX}-MCMC}$)

α	n	i	t_i	S_{true}	S_1	S_2	S_3	S_4	S_5
0.25	30	1	4.14E-03	0.992237	0.995253	0.991428	0.986936	0.993561	0.994877
		2	8.28E-03	0.954921	0.978173	0.959192	0.960049	0.964399	0.960492
		3	9.09E-03	0.921218	0.9512	0.917155	0.924182	0.932382	0.928362
		4	1.69E-02	0.903587	0.94577	0.907096	0.91217	0.92224	0.916659
		5	6.18E-02	0.764204	0.831813	0.760823	0.764429	0.783198	0.780356
	50	1	5.07E-03	0.963445	0.971519	0.965223	0.968054	0.968126	0.970022
		2	1.17E-02	0.884958	0.915106	0.889196	0.890489	0.905121	0.910209
		3	1.76E-02	0.887796	0.904736	0.886711	0.886401	0.90419	0.908606
		4	2.09E-02	0.880335	0.910179	0.882569	0.880484	0.900132	0.905853
		5	1.82E-02	0.869843	0.89389	0.875073	0.86757	0.88964	0.890726
	80	1	9.90E-03	0.967168	0.972727	0.970361	0.9672	0.976668	0.972239
		2	1.11E-02	0.958523	0.961246	0.955906	0.955516	0.961232	0.96354
		3	6.92E-03	0.958808	0.962739	0.953574	0.958053	0.959801	0.960841
		4	1.11E-02	0.910701	0.923216	0.91559	0.916459	0.91611	0.916985
		5	9.89E-03	0.915556	0.919644	0.912822	0.914513	0.920186	0.920882
0.5	30	1	2.31E-02	0.979325	0.980215	0.971998	0.979842	0.980659	0.983777
		2	4.87E-02	0.947911	0.964397	0.953088	0.950446	0.965591	0.962528
		3	6.46E-02	0.935168	0.954841	0.935448	0.934665	0.94696	0.948978
		4	9.19E-02	0.906402	0.927779	0.902019	0.902078	0.924234	0.926182
		5	1.11E-01	0.890365	0.918493	0.892882	0.893165	0.90912	0.909898
	50	1	1.19E-02	0.983088	0.987525	0.979847	0.983963	0.991556	0.987878
		2	2.82E-02	0.975629	0.981662	0.973161	0.972092	0.986639	0.98674
		3	2.38E-02	0.971549	0.985106	0.968614	0.972187	0.982767	0.981509
		4	4.11E-02	0.957987	0.97396	0.959262	0.961089	0.9777	0.975903
		5	5.34E-02	0.943397	0.965642	0.948671	0.947767	0.965532	0.970319
	80	1	1.14E-02	0.997283	0.996121	0.997346	0.994699	0.991001	0.999183
		2	6.13E-03	0.987808	0.995945	0.989172	0.991278	0.992528	0.990281
		3	2.67E-02	0.966458	0.977281	0.973494	0.968125	0.980923	0.978849
		4	3.55E-02	0.957386	0.971675	0.965553	0.9592	0.97563	0.972589
		5	5.35E-02	0.950978	0.962674	0.954146	0.951518	0.963393	0.961275
0.75	30	1	3.75E-02	0.98397	0.993923	0.984467	0.989944	0.993227	0.996467
		2	5.58E-02	0.983059	0.995111	0.986239	0.986117	0.98828	0.992927
		3	8.21E-02	0.97461	0.987243	0.976012	0.975718	0.981339	0.988193
		4	1.24E-01	0.95602	0.97901	0.949731	0.95406	0.977966	0.97662
		5	1.24E-01	0.95145	0.980955	0.955207	0.957562	0.971248	0.973725
	50	1	5.08E-02	0.97918	0.984171	0.980752	0.979421	0.990989	0.985015
		2	9.95E-02	0.968337	0.967359	0.960304	0.960652	0.968854	0.967513
		3	1.54E-01	0.936237	0.947299	0.931562	0.931521	0.948997	0.944789
		4	1.73E-01	0.919365	0.936385	0.921737	0.92669	0.934228	0.934582
		5	2.07E-01	0.911511	0.918553	0.909493	0.908567	0.921784	0.918409
	80	1	3.02E-02	0.995359	0.996199	0.989409	0.992373	0.99168	0.990293
		2	6.30E-02	0.983905	0.980616	0.981466	0.977694	0.989787	0.984127
		3	1.12E-01	0.958629	0.971877	0.959205	0.96095	0.965301	0.966862
		4	1.12E-01	0.958699	0.967687	0.957604	0.956854	0.968444	0.968283
		5	1.17E-01	0.958504	0.967541	0.953373	0.958196	0.96067	0.965606

From table 5 survival estimators shows that the estimation methods give different outputs in spite of the input parameters are same and the survival estimators are decreasing depending on (t) values

From table 6 survival estimators shows that the estimation methods give different outputs depending on pollution ration change in spite of the input parameters are same and the survival estimators are decreasing depending on (t) values

Table 7: the Mean Square Error for $(Survival\ )$ estimator for each estimation method according to simulation parameters with (0.05) outlier pollution ration $(MS_1 = MSE - S_{S-UBM}, MS_2 = MSE - S_{Q-UBM}, MS_3 = MSE - S_{W-UBM}, MS_4 = MSE - S_{SEL-MCMC}, MS_5 = MSE - S_{\text{LINEX}-MCMC}, Best = Min(MS1, .., MS5))$

α	n	MS_1	MS_2	MS_3	MS_4	MS_5	$Best$	$Index$
0.25	30	9.68E-03	3.24E-07	1.63E-06	2.18E-03	2.05E-03	3.24E-07	2
	50	2.08E-03	4.04E-09	4.53E-09	1.65E-03	1.67E-03	4.04E-09	2
	80	1.07E-03	7.19E-11	6.57E-10	7.49E-04	7.48E-04	7.19E-11	2
0.50	30	6.62E-03	1.05E-07	5.77E-09	4.39E-03	4.26E-03	5.77E-09	3
	50	1.60E-04	1.73E-09	3.62E-09	9.45E-05	9.47E-05	1.73E-09	2
	80	1.37E-04	1.26E-11	2.22E-11	8.19E-05	8.21E-05	1.26E-11	2
0.75	30	5.49E-03	1.34E-08	1.29E-07	3.73E-03	3.77E-03	1.34E-08	2
	50	1.26E-03	2.25E-09	1.88E-10	1.17E-03	1.16E-03	1.88E-10	3
	80	9.74E-04	5.46E-11	1.06E-10	8.68E-04	8.68E-04	5.46E-11	2

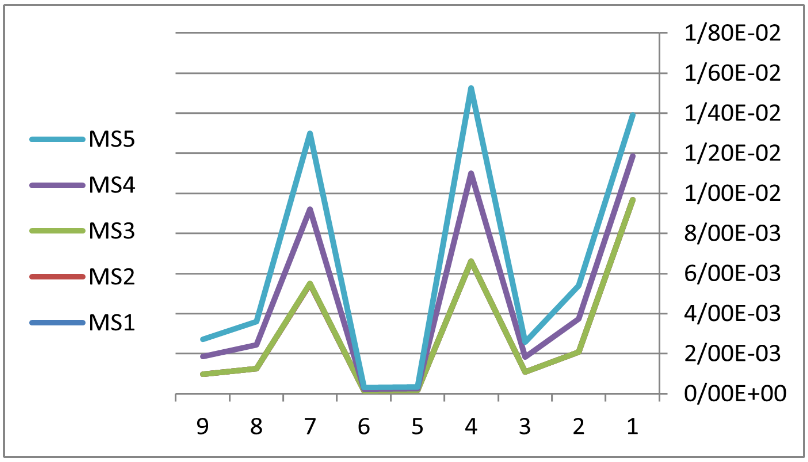

Figure 3: the Mean Square Error for $(Survival)$ estimator for each estimation method according to simulation parameters with (0.05) pollution and $MS_1 = MSE - S_{S-UBM}$, $MS_2 = MSE - S_{Q-UBM}, MS_3 = MSE - S_{W-UBM}, MS_4 = MSE - S_{SEL-MCMC}, MS_5 = MSE - S_{\text{LINX}-MCMC}, Best = Min(MS1, .., MS5)$

From table 7 and fig 3 results shows that the best estimation method for each simulation experiment are deferent and depend on (input parameter, sample size) and the best estimation methods was $(MSE_2,\ MSE_3)$

Table 8: the Mean Square Error for ($Survival$) estimator for each estimation method according to simulation parameters with (0.10) outlier pollution ration ($MS_1 = MSE - S_{S-UBM}$, $MS_2 = MSE - S_{Q-UBM}, MS_3 = MSE - S_{W-UBM}, MS_4 = MSE - S_{SEL-MCMC}, MS_5 = MSE - S_{\text{LINEX}-MCMC}, Best = Min(MS1, .., MS5)$)

α	n	MS_1	MS_2	MS_3	MS_4	MS_5	$Best$	$Index$
0.25	30	1.55E-02	1.49E-03	5.07E-03	9.57E-03	9.85E-03	1.49E-03	2
	50	3.22E-03	6.57E-03	2.90E-03	7.11E-03	6.27E-03	2.90E-03	3
	80	6.01E-03	8.94E-03	8.67E-03	2.25E-03	6.80E-03	2.25E-03	4
0.50	30	8.36E-03	9.71E-03	2.51E-03	1.23E-02	8.39E-03	2.51E-03	3
	50	6.23E-03	2.31E-04	5.29E-03	6.68E-03	4.36E-03	2.31E-04	2
	80	6.05E-03	2.14E-03	4.55E-03	2.28E-03	8.91E-03	2.14E-03	2
0.75	30	1.17E-02	1.88E-03	8.79E-03	4.95E-03	8.84E-03	1.88E-03	2
	50	6.71E-03	5.93E-03	2.04E-03	4.10E-03	5.71E-03	2.04E-03	3
	80	2.78E-03	4.12E-03	8.83E-03	7.65E-03	1.07E-02	2.78E-03	1

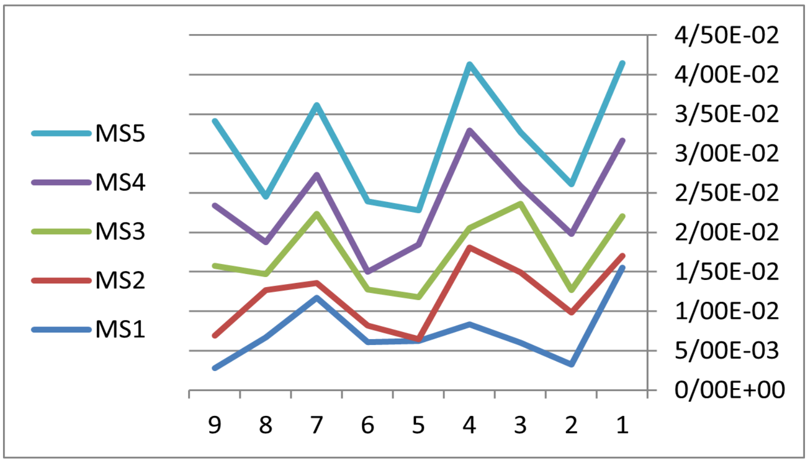

Figure 4: the Mean Square Error for ($Survival$) estimator for each estimation method according to simulation parameters with(0.10) pollution and $MS_1 = MSE - S_{S-UBM}$, $MS_2 = MSE - S_{Q-UBM}, MS_3 = MSE - S_{W-UBM}, MS_4 = MSE - S_{SEL-MCMC}, MS_5 = MSE - S_{\text{LINEX}-MCMC}, Best = Min(MS1, .., MS5)$

From table 8 and fig 4 results shows that the best estimation method for each simulation experiment are deferent and depend on (input parameter, sample size) and the best estimation methods was (MSE_2, MSE_3, MSE_4)

8. Experimental results

Real data representing (60) patients with infected with (COVID-19). The time of infection was taken from first infection observation until death, the survival time for patients appeared in the Burr-X distribution and according to the Kolmogorov–Smirnov test, estimation methods were applied and the results appeared according to the following tables

Table 9: the 1^{st} (10) Survival estimator for each estimation method according to real sample with $(t = the\ reall\ time\ , S = true\ , S_1 = S_{S-UBM}, S_2 = S_{Q-UBM}, S_3 = S_{W-UBM}, S_4 = S_{SEL-MCMC}, S_5 = S_{\text{LINEX}-MCMC})$

i	$t(i)$	S_{real}	s_1	s_2	s_3	s_4	s_5
1	8.72E-03	0.999966	0.999935	0.999966	0.999965	0.999963	0.999961
2	9.90E-03	0.999956	0.999916	0.999956	0.999955	0.999953	0.99995
3	3.22E-02	0.999534	0.99911	0.999534	0.999527	0.999501	0.999469
4	3.24E-02	0.999528	0.999097	0.999527	0.99952	0.999493	0.999461
5	3.87E-02	0.999328	0.998715	0.999327	0.999317	0.999279	0.999233
6	4.09E-02	0.999249	0.998565	0.999248	0.999237	0.999195	0.999143
7	0.077588	0.997295	0.994836	0.997292	0.997252	0.997099	0.996915
8	8.02E-02	0.997113	0.994489	0.99711	0.997067	0.996904	0.996707
9	8.31E-02	0.9969	0.994083	0.996897	0.996851	0.996675	0.996464
10	0.13149	0.99225	0.98524	0.992243	0.992128	0.99169	0.991164

Table 10: the estimated distribution parameter for real sample $(t = the\ reall\ time\ , S = true, S_1 = S_{S-UBM}, S_2 = S_{Q-UBM}, S_3 = S_{W-UBM}, S_4 = S_{SEL-MCMC}, S_5 = S_{\text{LINEX}-MCMC})$

S_{S-UBM}	S_{Q-UBM}	S_{W-UBM}	$S_{SEL-MCMC}$	$S_{\text{LINEX}-MCMC}$
0.560073	0.450413	0.457127	0.482621	0.513318

Tables 9 and 10 show the Bayesian estimation methods give a suitable parameter and survival estimators.

9. Conclustions and Suggestions

1- Bayesian estimation methods give best estimators in most simulation experiments
2-Changing sample size changing estimators and give difference mean square error and increasing sample size gives minimum mean square error
3-High outlier pollution ratio ($S_{SEL-MCMC}$) can gives best simulation estimators
4-Chaging parameter values can gives different mean square error values
5-Simulation experiments results affected with (parameter values, outlier ratios, and sample size)
6- Change distribution parameter values may give another mean square error results
7-Aplying the Bayesian estimation methods on other distributions (pareto and weibull) distributions can give another mean square error values .

References

[1] M. T. Ahmed, M. A. Khaleel, P. E. Oguntunde and M. K. Abdal-Hammed , editors, *A New Version of the Exponentiated Burr X distribution*, J. Phys.: Conf. Ser., 2021, IOP Publishing.

[2] F. G. Akgül and B. Şenoğlu, *Inferences for stress–strength reliability of Burr Type X distributions based on ranked set sampling*, Commun. Stat.- Simul. Comput., (2020) 1-17.

[3] S. Bals , S. Van Aert , G. Van Tendeloo and D. Avila-Brande, *Statistical estimation of atomic positions from exit wave reconstruction with a precision in the picometer range*, Phys. Rev. Lett., 96(9)(2006) 096106.

[4] M. Eliwa, Z. Alhussain, E. Ahmed, M. Salah, H. Ahmed and M. El-Morshedy, *Bivariate Gompertz generator of distributions: statistical properties and estimation with application to model football data*, J. National Sci. Found. Sri Lanka., 48(2)(2020).

[5] J. Fan and W. Zhang, *Statistical estimation in varying coefficient models*, The Ann. Stat., 27(5)(1999) 1491-518.

[6] A. S. Hassan, A. M. Abd-Elfattah, M. M. Hassan , *Bayesian analysis for mixture of Burr XII and Burr X distributions*, Far East J. Math. Sci. , 103(6) (2018) 1031-1041.

[7] N. A. Ibrahim, M. A. Khaleel, F. Merovci, A. Kilicman and M. Shitan , *Weibull Burr X distribution properties and application*, Pak. J. Stat.,33(5)(2017) 315-336.

[8] T. Koopmans, *Statistical estimation of simultaneous economic relations*, J. Am. Stat. Assoc., 40(232)(1945) 448-466.

[9] B. Legried, E. K. Molloy,T. Warnow and S. Roch, *Polynomial-time statistical estimation of species trees under gene duplication and loss*. J. Comput. Biol., 28(5)(2021) 452-468.

[10] A. K. Mahto, Y. M. Tripathi and S-J. Wu, *Statistical inference based on progressively type-II censored data from the Burr X distribution under progressive-stress accelerated life test*, J. Stat. Comput. Simul., 91(2)(2021) 368-382.

[11] I. A. Noaman and A. H. A. Abd Al-Ameer , *Comparison of Classical and Bayesian methods to Estimate the shape parameter and Reliability function in Burr type X or two parameter of exponential Rayleigh distribution under different Loss function*, J. Administration Econ., 42 (119) (2019) 42-58.

[12] A. Rabie and J. Li, *E-Bayesian estimation for Burr-X distribution based on generalized type-I hybrid censoring scheme*, Am. J. Math. Management Sci., 39(1)(2020) 41-55.

[13] M. Ravikumar and R. Kantam, *Two step estimation in burr type X distribution*, Int. J. Stat. Appl. Math., 1(4)(2016) 37-41.

[14] Z. Yang, N. Goldman and A. Friday, *Maximum likelihood trees from DNA sequences: a peculiar statistical estimation problem*, Syst. Biol., 44(3)(1995) 384-99.

[15] G. Yari and Z. Tondpour, *The new Burr distribution and its application*, Math. Sci., 11(1)(2017) 47-54.

[16] S. Zhou, Y. Chen and J. Shi, *Statistical estimation and testing for variation root-cause identification of multistage manufacturing processes*. IEEE Trans. Autom. Sci. Eng., 1(1)(2004) 73-83.