A proposed conditional method for estimating ARMA(1, 1) model

Lamyaa Mohammed Ali Hameed[a,*]

[a]Department of Statistics, College of Administration and Economic, Baghdad University, Iraq

(Communicated by Madjid Eshaghi Gordji)

Abstract

This paper aims to study the parameters estimation methods of the stationary mixed model (autoregressive-moving average) of low order ARMA (1, 1) regarding to time domain analysis in univariate time series. Using the approximating methods: Back Forecasting (BF), Classical Conditional Maximum Likelihood (CC) and Proposed Conditional Maximum Likelihood(PC). A comparison is done among the three methods by Mean Squared Error (MSE) using several simulation experiments; the obtained results from the empirical analysis indicate that the accuracy of the proposed conditional method is better than the classical conditional method.

Keywords: ARMA model; Estimation; Conditional Maximum Likelihood; Back Forecasting; Sum Squared Error.

1. Introduction

The problem of parameter estimation is one of the required stages for modeling time series according to Box and Jenkins methodology[4], therefore after data representation, model selection and identification, the stage of parameters estimation is needed, so that this is the second stage of the model building stages. The accuracy of the next stages of the diagnostics and forecasting depends on the accuracy of the estimates Obtained in the appropriate method. For estimating (ARMA) (Autoregressive Moving Average) model of N observation of Z process, there are three important steps are required for this stage [5]:

Type Selection, order selection and parameter estimation. Choosing the appropriate method for estimating is not a simple matter and the ordinary methods may not be efficient and appropriate if the observations properties do not match , the assumptions that should be existing when using

these methods, and because there is a random error in ARMA model which is associated with the explanatory variable so the (Ordinary Least Square)(OLS)estimators are inconsistent , therefore the researchers have adopted other methods of estimation that are more efficient ,and are often used in the analysis of time series . Estimation methods may require conditions such as stationarity or invertibility, and the model error are Normally distributed. Most ARMA estimation methods are approximate methods; one of these methods is Conditional Maximum Likelihood Method, in this method the initial values for the function representing the model are assumed as fixed values [4] there are also Unconditional Method , in this method the initial values are predicted by Back Forecasting (BF) method, while the estimate in Exact Likelihood Method by attempt to calculate the exact likelihood function , or by using the fixed distribution in formation the likelihood function[7] , in order to obtain high level estimators of efficiency . ARMA (1,1) model was studied theoretically and empirically using Simulation Technique with respect to time domain analysis of univariate time series and using different methods of estimation : Classical Conditional maximum likelihood (CC), Unconditional (Back forecasting) Method (BF) and proposed Conditional Method(PC).The methods were compared by simulation using Mean Squared Error (MSE) criterion to achieve the best estimation method. One of the basic assumptions adopted in the simulation is that the random errors ($a_t s$) of the model are distributed normally identically and independently with zero mean and constant variance σ_a^2 , assuming the coefficient of autoregressive ϕ_1 and moving average θ_1 both of them do not depend on the time , and lays inside the stationarity rejoin for ϕ_1 for and invertibility region for θ_1 , hence this research is concerned with an important stage of building model which is estimation stage , in order to studying conditional and unconditional methods for ARMA(1,1) and propose a conditional method for model estimation and then a comparison of estimation methods is made using simulation. The first to use the method of estimating time series parameters was Durban[7] in 1960 using ordinary least squares (OLS) applied sequentially by repeating steps and compensating for preliminary estimates in the original equation. In 1970, Box and Jenkins [4] gave an appropriate method of calculating the quadratic form of ARMA process(p,q), and they obtained appropriate expressions of future values as a conditional to previous values. In 1974, Newbold[19] derived the function of the exact maximum likelihood of ARMA processes (p,q) by generalizing the Box-Jenkins method[4] in the derivation of the exact maximum likelihood function for MA processes. In 1977, Ali[1] developed appropriate methods for calculating the exact likelihood function of the univariate variable and stationarity ARMA models. In 1979, Ansley[3] used cholesky decomposition to find the exact likelihood function of for ARMA processes, and in 1980 Hannan[9] used successive estimates based on ARMA models for stationary time series and in 1984 Melard[17] wrote a quick algorithm designed to calculate the likelihood function for stationary ARMA (p,q) models, in 2000, Karanasos[11] introduced a new method of calculating the theoretical of autocovariance function of ARMA model . In 2003, Davis and et al. [5] presented a research included an estimate of the MLE for observations derived from a time series model of calculations whose conditional distribution was based on past observations which follow Poisson Distribution. In (2008)[16] Mclead and Zhang gave a new likelihood based AR approximation for ARMA model and develop a new fast Algorithm easily implemented in high level quantitative programming environments, in 2009 ,Muler[18] ,et al. introduced a new class of robust estimation for ARMA models , they are M – estimates , these are closely related to those based on robust filter. In 2015 Lei[13], developed a robust Kalman filtering for ARMA modeling method. In 2016 Lei ,et al. [14], used Whittle likelihood estimation with time series contain a serial correlation in the residuals. In (2017) Krone, et al. [12], compared five estimation methods: the r_1, C-statistic, ordinary least squares, maximum likelihood estimation and Bayesian MCMC estimation, they compared these estimators with regard to bias, variability and rejection rate. Finally, in (2019) Horner [10], et al. used expectation maximization to estimate parameter of

autoregressive exogenous and autoregressive modes subject to missing data. This paper is organized as follows: the ARMA model is introduced in section 2. The estimation methods, GLS, BF and the proposed method are in section 3. The efficient of these estimators using simulation are compared In Section 4. The conclusions are given in section 5.

2. Mixed ARMA models

The ARMA (p, q) model is given by[22]:

$$Z_t = \phi_1 Z_{t-1} + \phi_2 Z_{t-2} + \cdots \phi_p Z_{t-p} + a_t - \theta_1 a_{t-1} - \theta_2 a_{t-2} - \cdots - \theta_q a_{t-q} \tag{2.1}$$

Where, $Z_t = \mathring{Z}_t - \mu$, $a_t \sim IND(0, \sigma_a^2)$

The First Order Mixed ARMA (1, 1) Model
In equation (2.1) when p=1, and q=1, then the form of the model becomes

$$Z_t = \phi_1 Z_{t-1} + a_t - \theta_1 a_{t-1} \tag{2.2}$$

The model can be written in form of Back Shift Operator as follows:

$$(1 - \phi_1 B)Z_t = (1 - \theta_1 B)a_t \tag{2.3}$$

Rewriting the model in form of autoregressive as follows

$$Z_t = \sum_{j=1}^{\infty} \theta_1^{j-1}(\phi_1 - \theta_1)Z_{t-j} + a_t \qquad , j \geq 1 \tag{2.4}$$

Where,

$$a_t = Z_t - \sum_{j=1}^{\infty} \pi_j Z_{t-j} \tag{2.5}$$

So that

$$\pi_j = \theta^{j-1}(\phi_1 - \theta_1) \qquad , j \geq 1 \tag{2.6}$$

Rewriting the model in form of moving average as follows:

$$Z_t = a_t + (\phi_1 - \theta_1) \sum_{j=1}^{\infty} \phi_1^{j-1} a_{t-j} \tag{2.7}$$

Where

$$Z_t = \sum_{j=0}^{\infty} \psi_j a_{t-j} \tag{2.8}$$

$$\psi_0 = 1$$
$$\psi_j = (\phi_1 - \theta_1)\phi_1^{j-1} \qquad , j \geq 1 \tag{2.9}$$

3. EstimationMethods

Most estimation methods depend on the maximum likelihood formulas, most of these formulas are approximated to the conditional or unconditional least square estimation with moderate sample size, and there are two types of maximum likelihood methods estimation : the exact and the approximate methods , in exact method the likelihood function have derived ,whereas in approximate methods there are the conditional and unconditional methods, in conditional methods , the initial values of errors series a_t and the observation series Z_t as fixed values , and replaced it by unconditional expectations for a_t and Z_t . These expectations are equal to zero[4, 22], In the unconditional method , the initial values of errors series a_t and the observations series Z_t are obtained by Back Forecasting .

Conditional Maximum Likelihood for ARMA (p,q) Process[22]

For the any stationary ARMA (p,q) model in (2.1)

$$Z_t = \phi_1 Z_{t-1} + ... + \phi_p Z_{t-p} + a_t - \theta_1 a_{t-1} - ... - \theta_q a_{t-q}$$

Where $Z_t = \mathring{Z}_t - \mu$ and $\{a_t\}$ are i.i.d.N $(0, \sigma_a^2)$,
The joint probability density of $a = (a_1 , a_2 , a_3 , ..., a_n ,)'$ is :

$$p(a|\phi, \mu, \theta, \sigma_a^2) = (2\pi\sigma_a^2)^{-\frac{n}{2}} \exp\left[-\frac{1}{2\sigma_a^2} \sum_{t=1}^{n} a_t^2\right] \tag{3.1}$$

Rewriting (2.1) as

$$a_t = \theta_1 a_{t-1} + ... + \theta_q a_{t-q} + Z_t - \phi_1 Z_{t-1} - ... - \phi_p Z_{t-p} \tag{3.2}$$

We can write down the likelihood function of the parameters $(\phi , \mu , \theta , \sigma_a^2)$ as follows :
Let $Z = (Z_1, Z_2, ..., Z_n)'$
And assume the initial conditions

$$Z_* = (Z_{1-p}, ..., Z_{-1}, Z_0)'$$

and

$$a_* = (a_{1-q}, ..., a_{-1}, a_0)'$$

The conditional log likelihood function is :

$$LnL_*(\phi, \mu, \theta, \sigma_a^2) = -\frac{n}{2}\ln 2\pi\sigma_a^2 - \frac{S_*(\phi, \mu, \theta,)}{2\sigma_a^2} \tag{3.3}$$

$$S_*(\phi, \mu, \theta) = \sum_{t=1}^{n} a_t^2(\phi, \mu, \theta|Z_*, a_*, Z) \tag{3.4}$$

The quantities: $\hat{\phi}, \hat{\mu}, \hat{\theta}$ which minimize (3.3) are called the conditional maximum likelihood estimators. There are some alternatives for specifying the initial conditions Z_* and a_* based on the assumptions that $\{Z_t\}$ is stationary and $\{a_t\}$ is a series of i.i.d. N(0, a_t) random variables , by replacing the unknown Z_t with the sample mean $\overline{Z}$ and the unknown a_t with its expected value of 0. For the model in (2.1) assume: $E(a_t) = \mu = 0$

$$a_p = a_{p-1} = ... = a_{p+1-q} = 0$$

And Calculate a_t for t $\geq$ (p+1) using (2.1). The conditional sum of squares function in (3.1) thus becomes

$$S_*(\phi, \mu, \theta) = \sum_{t=p+1}^{n} a_t^2(\phi, \mu, \theta|Z) \tag{3.5}$$

For ARMA (1, 1) model the conditional sum of square function becomes

$$S_*(\phi_1, \mu, \theta_1) = \sum_{t=2}^{n} a_t^2(\phi_1, \mu, \theta_1|Z)$$

The proposed Conditional Method (PC)
For the general stationary ARMA (p,q) model in (2.1) which is

$$Z_t = \phi_1 Z_{t-1} + ... + \phi_p Z_{t-p} + a_t - \theta_1 a_{t-1} - ... - \theta_q a_{t-q}$$

for specifying the initial conditions Z_* and a_* based on the same previous assumptions that $\{Z_t\}$ is stationary and $\{a_t\}$ is a white noise process of i.i.d. N(0, a_t) random variables , by replace the unknown Z_t with the Harmonic mean H_z and the unknown a_t with its Harmonic mean, and assume that:

$$a_p = a_{p-1} = ... = a_{p+1-q} \neq 0$$

And calculate a_t for all t $\geq$ (p+1) when p>q , and calculate a_t for all t $\geq$ (q+1) when q>p , and assume H is the harmonic mean is that:

$$H_z = \frac{n}{\sum_{t=1}^{n} \frac{1}{Z_t}} \tag{3.6}$$

$$H_a = \frac{n}{\sum_{t=1}^{n} \frac{1}{a_t}} \tag{3.7}$$

Where, H_Z is the Harmonic mean for the observations of the series Z_t,
And H_a is the Harmonic mean for the observations of the series a_t.
And after adding the unknown value to the series $\{a_t\}$ subtract the mean and Divided on the standard deviation to insure keep the same distribution. And when is that p>q so the function of conditional sum square using (2.1) being

$$S_*(\phi, \mu, \theta) = \sum_{t=p+1}^{n} a_t^2(\phi, \mu, \theta|Z_H) \tag{3.8}$$

Where :
$Z_{H :}$ represented the new series of observations after substitute the previous values of Z_* with harmonic mean of the series observation Z_t .
when q>p the function of sum square error will be :

$$S_*(\phi, \mu, \theta) = \sum_{t=q+1}^{n} a_t^2(\phi, \mu, \theta|Z_H) \tag{3.9}$$

It is one of the measures of central tendency, which differs from the arithmetic mean, median and mode whose values are equal in the Normal distribution. Zero values are treated as missing values.

And when p=q we can use any one of the two equations, the degree of freedom of using equation (3.8) is;

$$d.f. = (n - p) - (p + q + 1) = n - (2p + q + 1)$$

the degree of freedom of using equation (3.9) is ;

$$d.f. = (n - q) - (p + q + 1) = n - (2q + p + 1)$$

For ARMA $(1, 1)$ model the sum square error function is:

$$S_*(\phi_1, \mu, \theta_1) = \sum_{t=2}^{n} a_t(\phi_1, \mu, \theta_1 | Z_H) \tag{3.10}$$

$$\hat{\sigma}_a^2 = \frac{S_*(\hat{\phi}, \hat{\mu}, \hat{\theta})}{n - 4}$$

Unconditional Backcasting Method[22]
For the back cast the unknown values

$$Z_* = (Z_{1-p}, ..., Z_{-1}, Z_0)', a_* = (a_{1-q}, ..., a_{-1}, a_0)'$$

Required the sum of squares, for any ARMA model we can written also in the backward form:

$$(1 - \phi_1 B - ... - \phi_p B^p) Z_t = (1 - \theta_1 B - ... - \theta_q B^q) a_t \tag{3.11}$$

Or in the forward form:

$$(1 - \phi_1 F - ... - \phi_p F^p) Z_t = (1 - \theta_1 F - ... - \theta_q F^q) e_t \tag{3.12}$$

Where $F^j Z_t = Z_{t+j}$
Box and Jenkins[4] introduced the following unconditional function:

$$LnL(\phi, \mu, \theta, \sigma_a^2) = -\frac{n}{2} \ln 2\pi\sigma_a^2 - \frac{S(\phi, \mu, \theta)}{2\sigma_a^2} \tag{3.13}$$

So that S (ϕ,μ,θ) is the function which means unconditional sum of square, therefore :

$$S(\phi, \mu, \theta) = \sum_{t=-\infty}^{n} [E(a_t | \phi, \mu, \theta, Z)]^2 \tag{3.14}$$

the equation (3.14) is approximated to:

$$S(\phi, \mu, \theta) = \sum_{t=-H}^{n} [E(a_t | \phi, \mu, \theta, Z)]^2 \tag{3.15}$$

Where H is large integer so that $E(Z_t | \phi, \mu, \theta, Z) \approx H$
There fore $E(a_t | \phi, \mu, \theta, Z)$ Is ignoring when t $\leq$- (H+1).

4. The simulation

Simulation experiments include writing a number of programs in Visual Basic language; using the following steps:

1. Generating random numbers that follow the continuous uniform distribution within the period (0,1) using cumulative distribution function(c.d.f.) that describes the model under study , when u_1 and u_2 are independent random numbers from a regular distribution [20],the two variables :

$$Z_1 = (-2 \log u_1)^{\frac{1}{2}} \cos(2\pi u_2)$$
$$Z_2 = (-2 \log u_1)^{\frac{1}{2}} \sin(2\pi u_2)$$

   They will be independent with a standard Normal distribution with mean zero and constant variance equal 1.so one of these variables is used as a random errors to generate the model observation .

2. Compensate the values a_t which was generated by the form of ARMA (1, 1), to obtain the values of $\{Z_t\}$ series,

3. Suppose $Z_0 = a_0 = 0$.

4. Choice three sample size, n= (25, 50,150).

5. Using the three methods to obtain the parameters of ARMA (1, 1) model.

Using Mean Squared Error (MSE)[2] for comparison among the methods

$$MSE(\hat{\phi}) = \frac{1}{R} \sum_{i=1}^{R} (\hat{\phi}_i - \phi)^2$$

Results Discussion of Simulation
The default values for ARMA (1, 1) coefficients are: $(\phi_1, \theta_1) = \{(0.5, 0.6), (-0.5, 0.5), (0.6, 0.5), (0.2, 0.8), (- 0.2, -0.8), (0.9, 0.1)\}$
Sum Squared Error (SSE)
The lowest value of the sum squared error within the boundaries of the stationarity rejoin for ϕ_1 and the invertibility region for θ_1 , we find it when the value of the parameters (ϕ_1 and θ_1) is equal to ($\mp 0.9, \mp 0.9$).The highest value of the sum squared error we find it nearby the value in which ϕ_1 is equal to θ_1 , so when ϕ_1 is equal to θ_1 ,The model becomes a White Noise, therefore the sum of squared error equal to the sample size (n). Thus, the values above and below sample size appear the highest values . The following table shows some values of ϕ_1 and θ_1 at a sample size of 300.

Table 1: sum squares error of ARMA (1, 1) model for some values of (ϕ_1, θ_1) **n=300**

Method (ϕ_1, θ_1)	(CC)	(PC)	Method (ϕ_1, θ_1)	(CC)	(PC)
(0.1, −0.9)	142.47110	142.41252	**(−0.9, −0.1)**	69.04746	69.12391
(0.1, 0.5)	265.89560	265.89524	**(−0.9, 0.5)**	26.66932	25.88614
(0.1, 0.9)	189.57190	189.55236	**(−0.9, 0.9)**	16.73522	16.73692
(0.2, −0.9)	125.68150	125.59239	**(−0.8, −0.9)**	293.29320	293.29271
(0.2, 0.5)	281.05480	281.05453	**(−0.8, 0.5)**	49.88706	49.88752
(0.2, 0.9)	206.27340	205.22462	**(−0.8, 0.9)**	31.44528	31.44526
(0.3, −0.9)	109.09150	108.80731	**(−0.7, −0.9)**	279.77270	279.77296
(0.3, 0.5)	292.60930	292.60915	**(−0.7, 0.5)**	75.82188	75.82193
(0.3, 0.9)	222.64960	222.56546	**(−0.7, 0.9)**	48.13568	48.13580
(0.4, −0.9)	92.73707	92.73224	**(−0.6, −0.9)**	263.55870	262.66862
(0.4, 0.5)	299.45770	299.45769	**(−0.6, 0.5)**	102.95650	102.83429
(0.4, 0.9)	238.61930	237.35246	**(−0.6, 0.9)**	65.90971	64.99726
(0.5, −0.9)	76.67810	76.08848	**(−0.5, −0.9)**	246.35180	245.38848
(0.5, 0.1)	236.34590	236.23237	**(−0.5, 0.5)**	130.02550	129.84316
(0.5, 0.9)	254.04050	253.89418	**(−0.5, 0.9)**	84.03494	83.90942
(0.6, −0.9)	60.99521	59.88416	**(−0.4, −0.9)**	228.85120	228.85936
(0.6, 0.5)	291.87580	290.61642	**(−0.4, 0.5)**	156.33770	155.84217
(0.6, 0.9)	268.68410	268.68545	**(−0.4, 0.9)**	102.14470	102.32938
(0.7, −0.9)	45.75372	45.75423	**(−0.3, −0.9)**	211.32730	211.22469
(0.7, 0.5)	271.42190	271.42198	**(−0.3, 0.5)**	181.53530	181.46219
(0.7, 0.9)	282.19310	282.09297	**(−0.3, 0.9)**	120.09570	120.13925
(0.8, −0.9)	30.89482	30.89446	**(−0.2, −0.9)**	193.88860	193.72137
(0.8, 0.5)	232.05710	232.05744	**(−0.2, 0.5)**	205.36950	205.32469
(0.8, 0.9)	293.84040	293.74171	**(−0.2, 0.9)**	137.83490	136.64113
(0.9, −0.9)	16.20783	16.10663	**(−0.1, −0.9)**	176.58530	176.56989
(0.9, 0.5)	159.51560	159.51659	**(−0.1, 0.5)**	227.58970	227.58989

Comparison among the methods using MSE

Table 2: Mean Square Error (MSE) of the estimated ϕ_1 and θ_1 for ARMA(1,1)model of different values of ϕ_1 and θ_1 in all samples for the experiment repeated 1000 times.

N	True value	ϕ_1	θ_1	ϕ_1	θ_1	ϕ_1	θ_1
	Method	**0.5**	**0.6**	**-0.5**	**0.5**	**0.6**	**0.5**
	CC	0.010570	0.005540	0.003295	0.032616	0.880970	0.700402
	PC	0.005055	0.008501	0.002190	0.044352	0.208940	0.170817
	BF	0.000160	0.154610	0.018961	0.027523	0.040361	0.019937
...	True value	ϕ_1	θ_1	ϕ_1	θ_1	ϕ_1	θ_1
	Method	**0.2**	**0.8**	**-0.2**	**-0.8**	**0.9**	**0.1**
	CC	0.016589	0.001391	0.018279	0.002294	0.194128	0.003540
	PC	0.008354	0.016693	0.012634	0.014787	0.127235	0.000906
	BF	0.023994	0.038967	0.062800	0.025825	0.000005	0.003528
	True value	ϕ_1	θ_1	ϕ_1	θ_1	ϕ_1	θ_1
	Method	**0.5**	**0.6**	**-0.5**	**0.5**	**0.6**	**0.5**
	CC	0.035570	0.035680	0.001011	0.019099	0.317420	0.290521
550	PC	0.019770	0.038180	0.000548	0.026994	0.081682	0.090420
	BF	0.000090	0.022980	0.004942	0.070225	0.000117	0.000253
	True value	ϕ_1	θ_1	ϕ_1	θ_1	ϕ_1	θ_1
	Method	**0.2**	**0.8**	**-0.2**	**-0.8**	**0.9**	**0.1**
	CC	0.000142	0.010221	0.000016	0.017083	0.014908	0.007140
	PC	0.000031	0.021550	0.000024	0.027225	0.012634	0.006131
	BF	0.001884	0.039085	0.018961	0.005868	0.006147	0.001706
	True value	ϕ_1	θ_1	ϕ_1	θ_1	ϕ_1	θ_1
	Method	**0.5**	**0.6**	**-0.5**	**0.5**	**0.6**	**0.5**
	CC	0.018710	0.018440	0.000123	0.026929	0.168921	0.103362
1150	PC	0.010920	0.017190	8.28E-05	0.031364	0.056692	0.033452
	BF	0.001150	0.012280	0.002683	0.007604	0.000001	0.000650
	True value	ϕ_1	θ_1	ϕ_1	θ_1	ϕ_1	θ_1
	Method	**0.2**	**0.8**	**-0.2**	**0.8**	**0.9**	**0.1**
	CC	0.002043	0.009139	0.001747	0.010141	0.001498	0.004058
	PC	0.001673	0.015006	0.001421	0.016384	0.001444	0.004264
	BF	0.000876	0.008668	0.001731	0.000256	0.003058	0.004409

5. Conclusions

1. Through simulation experiments we find that the BF method is the better at samples 150 and 50 , whereas , the proposed conditional method (PC) is in the second order at the two sample sizes 150 and 50 , so , (PC) method is the best of all the methods at sample size (3.15) , while , the classical conditional method (CC) was in the last order at all sample sizes .

2. The MSE values of the two methods CC and PC were concentrated at moving average parameter θ_1 for the two sample sizes 150 and 50 , while, the value of MSE was concentrated at the autoregressive parameter ϕ_1 at sample size 25 . As for the back forecasting method, the value of MSE was concentrated in moving average parameter θ_1 for all sample sizes.

3. The lowest value of the sum squared error shows when the value of the parameters $(\phi_1, \theta_1) = (\mp 0.9, \mp 0.9)$. The highest value of the sum squared error we find it nearby the value in which ϕ_1 is equal to θ_1 .

References

[1] M. M. Ali,*Analysis of Autoregressive- Moving Average Models: Estimation and Prediction*, Biometrica, 64 (1977) 535-45.

[2] D. J. Allcroft, *A Spectral Estimator of ARMA Parameters from Thresholded Data*, 2000 , `http://bioss.ac.uk`.

[3] C. F. Ansley, *An Algorithm for the Exact Likelihood of a Mixed Autoregressive Moving Average Process*, Biometrrka, 66(1) (1979) 59-65.

[4] G. E. P. Box and G.M.Jenkins, *Time Series Analysis forecasting and Control* , 2nd ed., Holden- Day, San Francisco, 1976.

[5] R. A. Davis, T. M. Dunsmuir and S. B. Streett, *Maximum Likelihood Estimation for an Observation Driven Model for Poisson Counts*, National Center for Atmospheric Research, 2003.

[6] S. De Waele & Broersen *Modeling Radar Data with Time Series Models*, Dept. of Applied Physics, Delft University of Technology, 2001, E-mail: S.dewaele@tn.tudelft.nl.

[7] J. Durban, *Estimation of Parameters in Time- Series Regression Models*, J. Royal. Stat. Soc., Series B, 22(1960) 139-153.

[8] J. D. Hamilton, *Time Series Analysis* ,Princeton University Press Princeton, New Jersey, 1994.

[9] E. J. Hannan, *Recursive Estimation Based on ARMA Models* , Ann. Stat., 8 (4) (1980) 762-777.

[10] M. Horner,, S. N. Pakzad and N. S. Gulgec, , *Parameter Estimation of Autoregressive-Exogenous and Autoregressive Models Subject to Missing Data Using Expectation Maximization*, Front. Built Environ., 13 (2019) 2019, `https://doi.org/10.3389/fbuil.2019.00109`

[11] M. Karanasos, *A New Method for Obtaining the Autocovariance of an ARMA Model: An Exact Form Solution*, Econom. Theory, 16 (2)(2000) 280-282.

[12] T. Krone, C. J. AlbersMarieke and M. E. Timmerman, *A comparative simulation study of AR(1) estimators in short time series*, Qual. Quantity, 51(1)(2017) 1-21.

[13] H. Lei, *Auto Regressive Moving Average (ARMA) Modeling Method for Gyro Random Noise Using a Robust Kalman Filter*, Sensors, 15(10)(2015) 25277-25286 ,`https://doi.org/10.3390/s151025277`

[14] H. Lei, Y. Xia and X.Qin, *Estimation of semi varying coefficient time series models with ARMA errors*, Ann. stat. ,V 44(4)(2016) 1618-1660 .

[15] S. Makridakis, S. C. Wheelwright and R. J. Hyndman , *Forecasting Methods and Applications*, 3rd ed, John Wiley & Sons, 1998.

[16] A. I. Mcleod and Y. Zhang, *Faster ARMA maximum likelihood estimation* , Comput. Stat. Data Anal., 52(4) (2008) 2166-2176.

[17] G. Melard, *A Fast Algorithm for the Exact Likelihood of Autoregressive- Moving Average Models*, Appl. Stat., 33(1)(1984) 104-114.

[18] N. Muler, D. Pena and V. J. Yohai, *Robust estimation for ARMA models*, Ann. stat. 37(2)(2009) 816-840.

[19] P. Newbold,*The Exact Likelihood Function For a Mixed Autoregressive Moving Average Process*, Biometrika, 61(39)(1974) 423- 426.

[20] R. Paul and D.Balmer, *Simulation Modeling*, Chartwell-Bratt (Publishing and Training) Ltd, 1993.

[21] D. S. G Pollock, *Time-series analysis, signal processing and dynamics*, Academic Press, 1999.

[22] W. W. S. Wei, *Time Series Analysis Univariate and Multivariate Methods*, Addison- Wesley Publishing Company, Inc, 1990.