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Uncertain probability in information systems changed over time

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Abstract

In the information age, the decision-making process depends on data analysis. Most life problems are characterized by indeterminacy. In this research, we study data tables that change with time (changing information systems) and we use the concept of uncertain probabilities to introduce stochastic variables in data tables that changed over time. The uncertain probabilities are based on constructing neighborhoods on the set of objects in information systems and use topological approximations. Some examples are given to indicate the suggested approach. These concepts help in obtaining decisions using mathematical models in which no codes are used, or converting verbal variables into digital, and this helps to make approximations close to reality.

Keywords: Information system, uncertain probability, stochastic variables.

1. Introduction and Preliminaries

Statistical data analysis is based mostly on conclusion that use theorems and concepts from mathematical fields such as algebra, geometry, calculus and other recent branches of mathematics. In the recent era of high interaction between people and nations, most real life data are characterized by uncertainty.
Also parameters can be qualitative, consequently, geometric modeling was generalized to topological models. And theory of topological spaces is used to generalize the concepts of probabilities.
Many work have been done connecting topology to data analysis M.E.Abdel monsef et al [2, 1] applied the concept of near open sets to introduce types of generalized probability in topological spaces.

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Set valued mapping was used by S.Sedghi et al [9] to investigate rough probability. Luay .A et al [7] investigate properties of rough probability in topological spaces. In [3], the concept of stochastic information system was introduced as a generalization of information system, and a detailed example was given for reduction based on imprecise probabilities. The concept of stochastic pre topology is initiated by using a pre topological structure on information system. [10], The connection between stochastic pre topology rough sets and time series discussed in [4].
The main purpose of this work is to construct probability measures for uncertain events in information systems that changes over time, and define random variables that depends on time (stochastic variables).

The following is how the rest of the paper is organized:
Preliminaries of notions and facts used throughout the paper are described in Section 2, along with some examples. The definition of uncertain rough probability is introduced in section 3 along with some of its properties. Section 4 is reserved for case study and comparison between the suggested type of rough probability and similar concepts.

The aim of this section is to present the following concepts and facts that will be used in the paper:

Definition 1.1. *An ordered 4-tuple (U, Q, V, f) is the name given to the information system where U represent the set of objects, Q represent the set of features(attributes), $V = \cap_{q\in Q} V_q$ is the set of all possible values of features, while $f : U \times Q \to V$ is called the information function. We can say that $V_q^* = f(x, q)$ of course $f(x, q) \in V_q$*

The notation, which treats the information function as a set of functions, will be regarded as equivalent, then. The following is an example of an information system

Example 1.2. *Let us $U = \{x_1, x_2, x_3, \ldots, x_{10}\}$, set of patient (1)*
$Q = \{q_1, q_2, q_3, q_4\} = \{$temperature, headache, nausea, cough$\}$ set of 4 systems.
Where $\{$heigh, veryhiegh, normal, low$\} = \{a, c, d, e\}, V = \{v_{q_1}, v_{q_2}, v_{q_3}, v_{q_4}\}$.

Where, $f : Q \to V$ and the information function $f : U \times Q \to V$ is represented by the following table

Definition 1.3. *If (U, Q, f, V) represent an information system, $p \subset Q$ the class $[X]_{\tilde{p}}$is defined by*

$$[X]_{\tilde{p}} = \{y : f(X, q) = f(y, q), \ \ \forall q \in P\}$$

Definition 1.4. *If (U, Q, f, V) is an information system, $X \subset U, \underline{\tilde{P}}(X)$, is written as a lower approximation of X. The set $\underline{\tilde{P}}(X)$ described as follows:*

$$\underline{\tilde{P}}(X) = \{X \in U : [X]_{\tilde{P}} \subseteq X\}$$

As a result, the lower approximation of the set X is the set of the objects $X \in U$, with relation to which on the basis of values of features P, we can certainly state that they are elements of the set X.

Definition 1.5. *If (U, Q, f, V) represent an information system, $X \subset U, \overline{\tilde{P}}(X)$, is written as an upper approximation of X. The set $\overline{\tilde{P}}(X)$ described as follows:*

$$\overline{\tilde{P}}(X) = \{X \in U : [X]_{\tilde{p}} \cap X = \phi\}$$

is called $\tilde{P}$ upper approximation of the set $X \subseteq U$.

The upper approximation of the set is the set of the objects, relating to which, based on the values of features, we can't tell for sure that they aren't elements of the set.

Definition 1.6. *If (U, Q, f, V) is an information system, $X \subset U, Pos_{\tilde{p}}(X)$, The positive region of X is denoted by $\tilde{p}-$positive region of the set X is defined as*

$$Pos_{\tilde{p}}(X) = \underline{\tilde{p}}X.$$

The set X's positive region corresponds to its lower approximation..

Definition 1.7. *If (U, Q, f, V) is an information system, $X \subset U, Bn_{\tilde{p}}(X)$, the following is how the boundary region of X*
$\tilde{P}-$boundary region of the set X is defined as

$$Bn_{\tilde{p}}(X) = \overline{\tilde{P}}X \backslash \underline{\tilde{P}}X$$

Definition 1.8. *If (U, Q, f, V) is an information system, $X \subset U, Neg_{\tilde{p}}(X)$, is written as the negative region of X*
$\tilde{P}-$ negative region of the set X is defined as

$$Neg_{\tilde{p}}(X) = U \backslash \overline{P}X$$

The negative region of the set X is the set of the objects $X \in U$, with respect to which, on the basis of function values P, We may categorically claim that they are no elements of the set X.

The examples that follow indicate the fundamental concepts of approximation of X.

2. Uncertain probability in information system method

It is possible to say that the sample space of a random experiment, take throwing a dice for example , is a topological space in which the neighborhood of any element is only itself .Where this can be seen as every element is distinguishable from all other elements. this is a very special case , science real life random experiments not generally produce outcomes which are not interrelated .But there is a type of relation between outcomes .This relation determines similarities and dissimilarities between outcomes which can be represented as objects of information systems . Many works have been done on studying uncertain probability of non-defined subset in information systems using rough set approximation (lower and upper approximation).see[9,10,2,3], stochastic connection with rough set [4,5,7].

3. Method

We study rough probability in information system change over time.
The following is medical information system (data1) represents data of 6 patients, 4 attributes {temperature, headache, nausea, cough} and Decisions for Flu (yes, no).

	Attributes				decision
Case	temperature	Headache	nausea	cough	Flu
1	a	yes	no	yes	yes
2	c	yes	yes	no	yes
3	a	yes	no	yes	yes
4	a	yes	yes	no	yes
5	c	no	yes	no	no
6	d	no	yes	no	no

$$U/R = \{\{1,2\},\{3,4\},\{5,6\}\}$$

where $\{high, veryhigh, normal\} = \{a, c, d\}$. And U/R is the classification of the six patients writing to the following systems
$(iRj) \leftrightarrow V_q^i = V_q^j$ for all $q \in Q$ attributes $\{temperature, headache, nausea, cough\}$ With represent to all elements.
Use Definition 1.1 the classification of patients U/R, we will find the probabilities of

$$U/R = \{\{1,3\},\{2,5\},\{4\},\{6\}\}$$

In the following table at time t we obtain rough for some sets in $P(U)$.
Most of these works used non changeable information systems. The purpose of this work is to study rough probability in information system change over time.
We suggest the following definition for rough probability

Definition 3.1. *If* (U, Q, f, V) *is an information system and* ACU *is a rough set then*

$$P(A) = \left[P(\underline{A}), P(A^b), P(A^n)\right].$$

Remark 3.2. *In Defintion 3.1*
$P(\underline{A})$ *The probability of the elements that surly belongs to A.*
$P(A^b)$ *The probability that elements is belonging to A and* A^c*.*
$P(A^n)$ *The probability that elements surly not belong to A.*

Table 1: Probabilities of an information system at time t

A	$\underline{A}$	$\overline{A}$	A^b	A^n	$P(\underline{A})$	$P(A^b)$	$P(A^n)$
$\{1\}$	ϕ	$\{1,3\}$	$\{1,3\}$	$\{2,4,5,6\}$	0	$\frac{2}{6}$	$\frac{4}{6}$
$\{1,2\}$	ϕ	$\{1,2,3,5\}$	$\{1,2,3,5\}$	$\{4,6\}$	0	$\frac{4}{6}$	$\frac{2}{6}$
$\{2,3,4\}$	$\{4\}$	$\{2,3,4,1,5\}$	$\{1,2,3,5\}$	$\{6\}$	$\frac{1}{6}$	$\frac{4}{6}$	$\frac{1}{6}$
$\{1,2,3,4\}$	$\{1,3,4\}$	$\{1,2,5,3,4\}$	$\{2,5\}$	$\{6\}$	$\frac{3}{6}$	$\frac{2}{6}$	$\frac{1}{6}$
$\{2,3,4,5,6\}$	$\{2,4,5,6\}$	$\{1,2,3,4,5,6\}$	$\{1,3\}$	ϕ	$\frac{4}{6}$	$\frac{2}{6}$	0

Properities:

1. $P(\underline{A}) + P(A^b) + P(A^n) = 1$
2. $P(\underline{A}) < P(A) < P(\overline{A} \cup A^b)$

Our approach use change in information system on time to get the following random variables that depend on time.
Now we will study the above in case the information system changes from time to considering that the previous information system is at a time t, where $\{high, normal, low\} = \{a, d, e\}$

	Attributes				decision
Case	temperature	Headache	nausea	cough	
1	e	yes	no	yes	yes
2	e	yes	yes	no	yes
3	a	yes	no	yes	yes
4	a	yes	yes	no	yes
5	d	no	yes	no	no
6	d	no	yes	no	no

$$U/R = \{\{1,2\},\{3,4\},\{5,6\}\}$$

By the same way that use in the pervious Table 1 we get Table 2.

Table 2: Probabilities of an information system at time t+1.

A	$\underline{A}$	$\overline{A}$	A^b	A^n	$P(\underline{A})$	$P(A^b)$	$P(A^n)$	γ
$\{1,2\}$	$\{1,2\}$	$\{1,2\}$	ϕ	$\{3,4,5,6\}$	$\frac{2}{6}$	0	$\frac{4}{6}$	1
$\{3,4,5\}$	$\{3,4\}$	$\{3,4,5,6\}$	$\{5,6\}$	$\{1,2\}$	$\frac{2}{6}$	$\frac{2}{6}$	$\frac{2}{6}$	$\frac{1}{2}$
$\{1,2,3\}$	$\{1,2\}$	$\{1,2,3,4\}$	$\{3,4\}$	$\{5,6\}$	$\frac{2}{6}$	$\frac{2}{6}$	$\frac{2}{6}$	$\frac{1}{2}$
$\{1\}$	ϕ	$\{1,2\}$	$\{1,2\}$	$\{3,4,5,6\}$	0	$\frac{2}{6}$	$\frac{4}{6}$	0
$\{2,3,4,5,6\}$	$\{3,4,5,6\}$	$\{1,2,3,4,5,6\}$	$\{1,2\}$	ϕ	$\frac{4}{6}$	$\frac{2}{6}$	0	$\frac{4}{6}$

Now we will study the above in case the information system changes from time $t+1$ to $t+2$ considering that the previous information system is at a time $t+1$.

	Attributes				decision
Case	temperature	Headache	nausea	cough	
1	high	yes	no	yes	yes
2	very high	yes	yes	no	yes
3	high	yes	no	yes	yes
4	Very high	yes	yes	no	yes
5	normal	no	yes	no	no
6	normal	no	yes	no	no

$$U/R = \{\{1,3\},\{2,4\},\{5,6\}\}$$

Table 3: Probabilities of an information system at time t+2.

A	$\underline{A}$	$\overline{A}$	A^b	A^n	$P(\underline{A})$	$P(A^b)$	$P(A^n)$	γ
$\{1,3\}$	$\{1,3\}$	$\{1,3\}$	ϕ	$\{2,4,5,6\}$	$\frac{2}{6}$	0	$\frac{4}{6}$	1
$\{4,5\}$	ϕ	$\{2,4,5,6\}$	$\{2,4,5,6\}$	$\{1,3\}$	0	$\frac{4}{6}$	$\frac{2}{6}$	0
$\{1,2,4\}$	ϕ	$\{1,3,2,4\}$	$\{1,2,3,4\}$	$\{5,6\}$	0	$\frac{4}{6}$	$\frac{2}{6}$	0
$\{3\}$	ϕ	$\{1,3\}$	$\{1,3\}$	$\{2,4,5,6\}$	0	$\frac{2}{6}$	$\frac{4}{6}$	0
$\{2,3,4,5,6\}$	$\{2,4,5,6\}$	$\{1,2,3,4,5,6\}$	$\{1,3\}$	ϕ	$\frac{4}{6}$	$\frac{2}{6}$	0	$\frac{4}{6}$

Table 4: Comparison between three cases for $\underline{A}$.

A	$\{1\}$	$\{1,2\}$	$\{2,3,4\}$	$\{1,2,3,4\}$	$\{2,3,4,5,6\}$	$\{1,3\}$
$\underline{A}$ at time t	$\{1\}$	$\{1,2\}$	$\{2,3,4\}$	$\{1,2,3,4\}$	$\{2,3,4,5,6\}$	$\{1,3\}$
$\underline{A}$ at time t+1	ϕ	$\{1,2\}$	ϕ	ϕ	ϕ	ϕ
$\underline{A}$ at time t+2	ϕ	ϕ	ϕ	ϕ	ϕ	$\{1,3\}$
$P(\underline{A})$ at time t	$\frac{1}{6}$	$\frac{2}{6}$	$\frac{3}{6}$	$\frac{4}{6}$	$\frac{5}{6}$	$\frac{2}{6}$
$P(\underline{A})$ at time t+1	0	$\frac{2}{6}$	0	0	0	0
$P(\underline{A})$ at time t+1	0	0	0	0	0	$\frac{2}{6}$

Table 5: Comparison between three cases for $\overline{A}$.

A	$\{1\}$	$\{1,2\}$	$\{2,3,4\}$	$\{1,2,3,4\}$	$\{2,3,4,5,6\}$	$\{1,3\}$
$\overline{A}$ at time t	$\{1\}$	$\{1,2\}$	$\{2,3,4\}$	$\{1,2,3,4\}$	$\{2,3,4,5,6\}$	$\{1,3\}$
$\overline{A}$ at time t+1	$\{1,2\}$	$\{1,2,3,4\}$	$\{1,2,3,4\}$	$\{1,2,3,4\}$	$\{1,2,3,4,5,6\}$	$\{1,2,3,4\}$
$\overline{A}$ at time t+2	$\{1,3\}$	$\{1,2,3,4\}$	$\{1,2,3,4\}$	$\{1,2,3,4\}$	$\{1,2,3,4,5,6\}$	$\{1,3\}$
$P(\overline{A})$ at time t	$\frac{1}{6}$	$\frac{2}{6}$	$\frac{3}{6}$	$\frac{4}{6}$	$\frac{5}{6}$	$\frac{2}{6}$
$P(\overline{A})$ at time t+1	$\frac{2}{6}$	$\frac{4}{6}$	$\frac{4}{6}$	$\frac{4}{6}$	$\frac{6}{6}=1$	$\frac{4}{6}$
$P(\overline{A})$ at time t+2	$\frac{2}{6}$	$\frac{2}{6}$	$\frac{4}{6}$	$\frac{4}{6}$	$\frac{6}{6}=1$	$\frac{2}{6}$

Table 6: Comparison between three cases for A^b.

A	$\{1\}$	$\{1,2\}$	$\{2,3,4\}$	$\{1,2,3,4\}$	$\{2,3,4,5,6\}$	$\{1,3\}$
A^b at time t	ϕ	ϕ	ϕ	ϕ	ϕ	ϕ
A^b at time t+1	$\{1,2\}$	$\{3,4\}$	$\{1,2,3,4\}$	$\{1,2,3,4\}$	$\{1,2,3,4,5,6\}$	$\{1,2,3,4\}$
A^b at time t+2	$\{1,3\}$	$\{1,2,3,4\}$	$\{1,2,3,4\}$	$\{1,2,3,4\}$	$\{1,2,3,4,5,6\}$	ϕ
$P(A^b)$ at time t	0	0	0	0	0	0
$P(A^b)$ at time t+1	$\frac{2}{6}$	$\frac{2}{6}$	$\frac{4}{6}$	$\frac{4}{6}$	$\frac{6}{6}=1$	$\frac{4}{6}$
$P(A^b)$ at time t+2	$\frac{2}{6}$	$\frac{2}{6}$	$\frac{4}{6}$	$\frac{4}{6}$	$\frac{6}{6}=1$	0

Table 7: Comparison between three cases for A^n.

A	$\{1\}$	$\{1,2\}$	$\{2,3,4\}$	$\{1,2,3,4\}$	$\{2,3,4,5,6\}$	$\{1,3\}$
A^n at time t	$\{2,3,4,5,6\}$	$\{3,4,5,6\}$	$\{1,5,6\}$	$\{5,6\}$	$\{1\}$	$\{2,4,5,6\}$
A^n at time t+1	$\{3,4,5,6\}$	$\{5,6\}$	$\{5,6\}$	$\{5,6\}$	ϕ	$\{5,6\}$
A^n at time t+2	$\{2,4,5,6\}$	$\{5,6\}$	$\{5,6\}$	$\{5,6\}$	ϕ	$\{2,4,5,6\}$
$P(A^n)$ at time t	$\frac{5}{6}$	$\frac{4}{6}$	$\frac{3}{6}$	$\frac{2}{6}$	$\frac{1}{6}$	$\frac{4}{6}$
$P(A^n)$ at time t+1	$\frac{4}{6}$	$\frac{2}{6}$	$\frac{2}{6}$	$\frac{2}{6}$	0	$\frac{2}{6}$
$P(A^n)$ at time t+2	$\frac{4}{6}$	$\frac{2}{6}$	$\frac{2}{6}$	$\frac{2}{6}$	0	$\frac{4}{6}$

4. Conclusion and discussion

The upper and lower probabilities of unspecified events in information systems are considered the starting point for studying statistical measures in the light of impartiality and studying stochastic variables dependent on time whether verbal or quantitative.

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