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Using apgarch/avgarch models Gaussian and non-Gaussian for modeling volatility exchange rate

Suhail Najm Abdulla[a,*], Heba Dhaher Alwan[a]

[a]Department of Statistics, University of Baghdad, Baghdad, Iraq

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Abstract

This paper aims to measure the effect of the volatility on the daily closing price for the (Iraqi dinar against US dollar) from (21 July 2011) until (21 July 2021) using the models of asymmetric general autoregressive conditional heterogeneity (APGARCH and AVGARCH). The parameter estimated by Maximum Likelihood Estimation method and the error term assumed two distributional (General error distribution and Student's t distribution), the results showed that the APGARCH(1,2) with error term distributed (Student's t) distribution is the best model for the return series of the (IQ/USD) exchange rate to get the lowest value according to the information criteria for determining ranks (AIC, BIC) in addition to the presence of the asymmetric effect of the leverage, and this is evidence that negative shocks affect volatility more than positive shocks (the impact of the positive shocks is less than the impact of the negative shocks).

Keywords: Volatility, asymmetric GARCH model, exchange rate

1. Introduction

The topic of modeling the volatility of financial returns has received a great deal of interest from researchers , Many investors accept relatively low returns in order to avoid investments with high risks, , The appropriate modeling of the volatility of financial returns is able to lead to obtaining accurate predictions of volatility, so it was very important to develop suitable models for the volatility of financial returns that It takes into account the asymmetry of volatility (that is, the volatility are in a positive and negative direction), in addition to the thickness of the tails of the non- Gaussian

*Corresponding author
Email addresses: dr.suhail.najm@coadec.uobaghdad.edu.iq (Suhail Najm Abdulla), Hebadhaher.hd@gmail.com (Heba Dhaher Alwan)

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distribution . one of the most prominent of these models for capturing changes is the asymmetric (GARCH) models. In this study we using two of these models on the exchange rates of the local currency (the Iraqi dinar against the US dollar) for the period from (22/7/2011) until (22/7/2021) due to the deterioration and instability that it witnessed Currency value During the study period and through the stages of analysis of the returns series (diagnosis, estimation, determination of the rank, examination of suitability) to the selection of the appropriate model, which gives more accurate results in prediction. Time series occupies wide areas in our lives, especially the economic fields, specifically the financial ones. Hence, interest began in studying financial time series, which are often characterized by the feature of instability or volatility, meaning that there are periods of time fluctuations followed by periods of relative calm. In order to address this, it was necessary to use statistical models that take into account these fluctuations and try to explain them, and these models are non-linear (ARCH) models, which are known as autoregressive models conditioned by the heterogeneity of variance, which were proposed by the researcher (Robert. Engle, 1982)[8] in A study on the estimation of inflation variance in the United Kingdom to fill in the shortfall of the ARIMA linear models,This model is built on the basis of the autoregressive representation of the conditional variance, that is, the size of the variance of the current error term is considered dependent on the representations of the square error limits of the previous periods, and it has been relied on the hypothesis of the normal distribution of errors. In 1986 the researcher (Bollerslev) [2] proposed the generalized nonlinear ARCH model or the conditional autoregressive model of generalized variance heterogeneity (GARCH for short). Where he applied these models using the (t-student) distribution. The researchers continued to apply these models using distributions other than the normal distribution, including the researcher (Fereland, 2006), who applied these models using the (Poisson) distribution, and we also mention (Zhu & Fokianos, 2011) who employed the (Negative Binomial) distribution. Despite the importance of these models, they were subjected to many criticisms by some economists such as (Nelson, 1991)[16] and (Cao & Tsay, 1992), especially with regard to determining the relationship between the random error square and the conditional variance. This relationship is achieved only in cases where the changes of the phenomenon Studied in the same direction and the same size of impact, but in cases characterized by fluctuations in opposite directions, these models cannot take into account these fluctuations, and all these criticisms led to the emergence of many other models from GARCH that take into account the various positive and negative effects of shocks, including Asymmetric Generalized Autoregressive Conditional Heterogeneity Models and its acronym (Asymmetric GARCH), which was the beginning of transformation in the field of applied economic measurement.

This paper attempt to study the characteristics exchange rate of volatility of the daily data for (Iraqi dinar against US dollar) for the period (21/7/2011) until (21/7/2021) by studying (AVGARCH & APGARCH) models. the error term assumed tow distributions (Student-t distribution and General error distribution) .

2. Material and methods of analysis

2.1. Data for analysis

The research data represent the daily closing price of the (IQ/USD) index for the period from 21/7/2011 until 21/7/2021 ,That's about (3439) views . after it was transferred to the return series through the next formula :

$$z_t = \ln(p_t) - \ln(p_{t-1}) \tag{2.1}$$

Where z_t is the return series , p_t is the price of the current day and p_{t-1} is the price of the previous day.

2.2. Unit root test: Philips Peron test [17]

Approved test to determine whether the series in Equation (2.1) stationary or it has a unit root, through the following formula :

$$t^*_{\widehat{p}} = \sqrt{\mathrm{k}}\,\frac{\widehat{p}-1}{\sigma\widehat{p}} + \frac{T\,(k-1)\,\sigma_{\widehat{p}}}{\sqrt{k}} \tag{2.2}$$

and the hypotheses are given as the follows:

$$H_o : p = 0 \tag{2.3}$$

$$H_1 : p < 0 \tag{2.4}$$

Where $k = \frac{\sigma_t}{s_t^2}$, σ_t, s_t^2 is the estimation of short-term and long-term variance respectively, T is the sample size. the null hypotheses was accept if P value $> \alpha$ $(\alpha = 0.05)$.

2.3. Jarque - Bera test [9]

This test depends on calculating the difference between the coefficients of skewness and kurtosis for the time series, and its results are considered supportive of them. The null hypothesis was accept if the error distributed normally. The statistic is :

$$JK = \frac{N-K}{6}\left(S^2 + \frac{1}{4}\ (K-3)^2\right) \qquad \sim\ x^2\alpha(2) \tag{2.5}$$

Where k and S are kurtosis and skewness coefficient respectively, and the hypothesis are given as the follows :

$$H_0 : Normality \tag{2.6}$$

$$H_1 : non\ Normality \tag{2.7}$$

2.4. Ljung-Box test [13]

It is used to test the autocorrelation error in the return series , The statistic was given by :

$$Q_{(m)} = n\,(n+2)\sum_{k=1}^{m}\frac{\widehat{p}_k^2}{n-k} \quad \sim\ \mathrm{x}^2{}_{(m-p)} \tag{2.8}$$

Where n is the size of series , k is the number of time lags , $\widehat{p}_k^2$ is the residual autocorrelation and the hypothesis is :

$$H_0 : p_1 = p_2 = \ldots p_k \ldots . = p_m = 0 \qquad \forall k = 1, 2, 3, \ldots, m \tag{2.9}$$

$$H_1 : p_k \neq 0 \qquad for\ some\ value\ of\ k \tag{2.10}$$

We accept H_0 and the residual are no serial correlations if P value greater than α significant .

2.5. ARCH test [11]

We use this test to test the ARCH effect in the series ,and the statistic of this test are :

$$ARCHtest = T \times \widehat{R}^2 \ \sim \ x_p^2 \tag{2.11}$$

Where T is the total number of observation given by :

$$T = n - lag \tag{2.12}$$

and $\widehat{R}^2$ based on Regression with the formula :

$$\widehat{R}^2 = \frac{\text{SSR}}{\text{SST}} \tag{2.13}$$

The arch test hypothesis is :

$$H_0 = \alpha_i = 0 \qquad No\ ARCH\ effect \tag{2.14}$$

$$H_0 = \alpha_i \neq 0 \qquad ARCH\ \ effect \qquad i = 1, 2, 3 \ldots, q \tag{2.15}$$

2.6. Asymmetric Power GARCH (APGARCH) [10]

This model was developed by Ding & Granger (1993) when they added the power parameter instead of the square to allow to see the effect of the leverage (negative shocks) ,The conditional variance equation is :

$$\varepsilon_t = \sigma_t z_t \quad ; \qquad \sigma_t^\delta = \omega + \sum_{i=1}^{p} \alpha_i \left(|\varepsilon_{t-i}| - \gamma_i \varepsilon_{t-i}\right)^\delta + \sum_{j=1}^{q} \beta_j \sigma_{t-j}^\delta \tag{2.16}$$

where (ε_{t-i}) is an identical independent series follows Normal distribution with zero mean and One variance , (δ) is the leverage power and (γ_i) is the leverage effect and its value is $1 > \gamma_i < -1$. when this value is equal to zero then the positive and the negative shocks is the same effect.

2.7. Asymmetric Absolute Value GARCH (AVGARCH) [1]

Taylor (1986) developed this model . as a tool for correlating the news impact curve with the conditional volatility of the shocks and their impact on the conditional standard deviation within the framework of an absolute value model. The conditional variance equation is:

$$\varepsilon_t = \sigma_t z_t \quad ; \qquad {\sigma_t}^2 = \omega \ + \ \sum_{i=1}^{p} \alpha_i \left(|\varepsilon_t + b| - c\left(\varepsilon_t + b\right)\right)^2 \ + \sum_{j=1}^{q} \beta_j \sigma_{t-1}^2 \tag{2.17}$$

where (b) is the parameter of asymmetry in the small shocks which allows a shift in the impact curve during rotation, and indicate to the effected of the random shock in the past period on the random shock in this period, c is the rotation parameter which allows rotation during the shift and indicated to the large shocks and ε_{t-1} is identical independent series distributed Normal with zero mean and One variance.

2.8. Distribution assumptions of error term and estimation [6]

The volatility estimated in this paper depends on APGARCH and AVGARCH models with the lower rank ($p = 1,\ q = 1, p = 1,\ q = 2, p = 2,\ q = 1, p = 2,\ q = 2$) assuming three distributions of random error (Normal , Students- t , General error distribution) and The models were estimated using Maximum Likelihood Estimation method .the mathematical formula is :

$$L(\theta) = \sum_{t=1}^{n} J_t(\theta) \tag{2.18}$$

$$\log(L\theta) = -\frac{1}{2}\sum_{t-1}^{T} \quad ln(2\pi) + ln\sigma_t + \frac{(\varepsilon_t^2)}{\sigma_t} \tag{2.19}$$

i – The log likelihood with Normal distribution is :

$$J_t\left(\theta\right) = -\frac{1}{2}\log\left(2\pi\right) - \frac{1}{2}\log\left(\sigma_t^2\right) - \frac{1}{2}\left(\frac{\varepsilon_t^2}{\sigma_t^2}\right)$$

ii –The Log Likelihood with General error distribution is

$$L\left(\theta\right) = n\left[\log\left(\frac{v}{\lambda_v}\right) - \left(1 + \frac{1}{v}\right)\log\left(2\right) - log\Gamma\left(\frac{1}{v}\right) - \frac{1}{2}\sum_{t=1}^{n}\ \log\left({\sigma_t}^2\right) - \frac{1}{2}\ \sum_{t=1}^{n}\sigma_t^{-v}\left|\frac{\varepsilon_t}{\lambda_v}\right|^v\right] \tag{2.20}$$

where $v < 2$ is the shape parameter controls the tail behavior
iii -The Log Likelihood with Student's-t distribution is :

$$\begin{aligned} L\left(\theta\right) =& n\left[\log\Gamma\left(\frac{v+1}{2}\right) - \log\Gamma\left(\frac{v}{2}\right) - \frac{1}{2}\log\pi\left(v-2\right)\right] \\ & -\frac{1}{2}\sum_{t=1}^{n}\log\left(\sigma_t^2\right) + \left(v+1\right)\log\left[1 + \frac{z_t^2}{\sigma_t^2\left(v-2\right)}\right]\Bigg] \end{aligned} \tag{2.21}$$

where $v > 2$ the tail behavior based on it.

3. Result and discussion

3.1. The data

Figure 1 represent series of daily closing price for (IQ/USD) from 21/7/2011 to 21/7/2021.

3.2. Descriptive Statistics

The statistical results in Table 1 indicate that there is a difference in the lowest and highest exchange rates for IQ/USD Return Series . The standard deviation values were close to zero, which indicates a low level of dispersion returns at the exchange rate. In addition, it has high kurtosis values in the positive direction , this indicates that the volatility increase in the positive direction .

3.3. Unit root test

We use Philips Peron test to investigate the stationary properties of the log return series . the result in the Table 2 show that all of the P value are less than the level of significance ($\alpha = 0.05$) and this indicates that the return series for exchange rates is stationary , and therefore rejects the null hypothesis that there is a unit root problem .

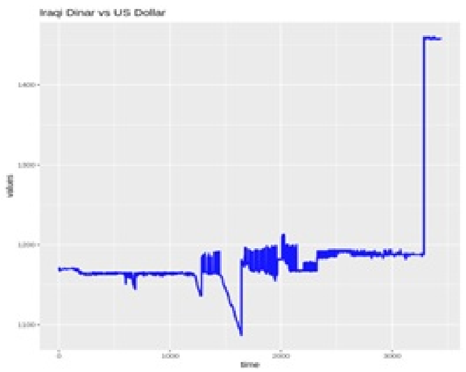

Figure 1: Daily closing price for (IQ/USD)

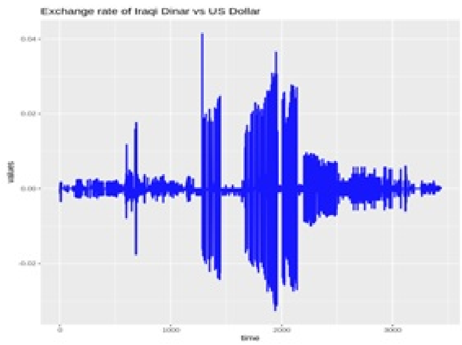

Figure 2: Daily return series for (IQ/USD)

Table 1: Descriptive Statistics

Descriptive Statistics	
42631	Jarque- Bera
2.2e-16	Prob.
0.005	Std.Dev
0.157	Skeweness
20.253	Kurtosis
0.000	Mean
-0.003	Min
0.000	Median
0.041	Max

Table 2: Unit root test- Philips Peron test

ARCH-LM test		
Lags	P value	Chi squared–Statistics
5	2.2e-16	630.27
15	2.2e-16	919.85
20	2.2e-16	958.35
25	2.2e-16	991.78
30	2.2e-16	1024

Table 3: ARCH-LM test for return series of (IQ/USD) exchange rate

Test	Type of model	Test statistics	P value
Philips Peron	No drift no trend	-3667	0.01
	With drift no trend	-3665	0.01
	With drift and trend	-3665	0.01

3.4. Testing for ARCH effects

The result of using ARCH-LM test in the Table 3 show that the P value is less than 0.05 . This indicates that there is a serial correlation in the return series residual and thus the existence of an ARCH effect in the return series residual.

3.5. *Estimation result*

The Tables 4, 5 shows the result estimation for AVGARCH and APGARCH models assuming three distributions of the error term (Normal , Students- t , General error distribution) for return series of IQ/USD or return series of turn series table exchange rate.

Table 4: estimation result for AVGARCH model

Distribution	Order	Coefficient of AVGARCH										
		ω	σ	α_1	α_2	β_1	β_2	c_1	c_2	b_1	b_2	v
Normal	(1,1)	0	0	0.331	–	0.786	–	0.570	-	-1.225	-	-
	P value	1e-06	0e+00	0e+00	-	0e+00	-	0e+00	-	0e+00	-	-
	(1,2)	0	0	0.453	–	0.055	0.643	0.526	-	-1.112	-	-
	P value	0.00018	0.000	0.000	–	0.000	0.00	0.000	–	0.00	–	–
	(2,1)	0	0	0.270	0.102	0.417	–	0.182	0.608	-1.069	1.806	–
	P value	0.03475	0.0000	0.00000	0.00000	0.00000	-	0.00000	0.0000	0.00000	0.0000	-
	(2,2)	0	0	0.233	0.100	0.421	0	0.053	0.744	-0.984	1.791	-
	p value	0.04025	0	0	0	0	1	0.01766	0	0	0	-
GED	(1,1)	0	0	0.050	-	0.9	-	0.02	-	0.05	-	2
	p value	0.44382*	0	0	0	0	-	0	-	0	-0	
	(1,2)	0	0	0.05	-	0.450	0.450	0.450	-	0.05	-	2
	p value	0	0.5173*	0	-	0	-	0	-	0	-	0
	(2,1)	0	0	0.025	0.025	0.9	-	0.010	0.010	0.025	0.025	2
	p value	0	0.6017*	0	0	0	-	0	0	0	0	0
	(2,2)	0	0	0.025	0.025	0.450	0.450	0.010	0.010	0.025	0.025	2
	p value	0	0.589*	0	0	0	0	0	0	0	0	0
Student t	(1,1)	0	0	0.599	-	0.158	-	-0.284	-	0.468	-	2.140
	p value	0.98362*	0.9730*	0	-	0	-	0	-	0	-	0
	(1,2)	0	0	0.7	-	0.136	0.087	-0.510	-	0.571	-	2.168
	p value	0.98671*	0.9827*	0	-	0	0	0	-	0	-	0
	(2,1)	0	0	0.795	0.006	0.221	-0.373	0.175	0.045	-9.374	-	2.121
	p value	0.9909*	0.9645*	0	0	0	0	0	0.00001	0.01157	-	0
	(2,2)	0	0	0.958	0.006	0.204	0.072	-0.186	-0.464	0	5.337	2.128
	p value	0.9905*	0.9639*	0	0	0	0.00273	0	0	0.8563*	0	0

*Indicates of significance at level 0.05.

From the Table 4, all the parameter of AVGARCH for Normal distribution are significant, for General error distribution we note all the parameter are significant except the parameter ω in the rank (1,1) and the parameter σ in the ranks $[(1,2),(2,1),(2,2)]$, for the Student t note that all parameter are significant except ω and σ for all ranks and (b_1) in the order (2,2) , finally, the asymmetric effect appears clearly in all distributions and ranks .

Table 5: estimation result for APGARCH model

Distribution	Order	Coefficient									
		ω	σ	α_1	α_2	β_1	β_2	γ_1	γ_2	δ	v
Normal	(1,1)	0	0	0.66	–	0.916	–	-0.193	-	2.297	-
	P value	0.9626*	0.9959*	0	-	0	-	0.00004	-	0	-
	(1,2)	0	0	0.107	–	0.357	0.510	-0.116	-	2.272	-
	P value	0.8937*	0.9530*	0	–	0	0	0.00046	–	0	–
	(2,1)	0	0	0.042	0.018	-	0.905	-0.163	0.862	2.232	-
	P value0.3923*	0.9802*		0	0	-	0	0	0	0	-
	(2,2)	0	0	0.060	0.035	0.361	0.507	-0.239	0.567	2.208	-
	p value	0.00812	0.9971*	0	0	0	0	0.00003	0	0	-
GED	(1,1)	0	0	0.050	-	0.9	-	0.05	-	2	2
	p value	0	0.4901*	0	-	0	-	0	-	0	0
	(1,2)	0	0	0.05	-	0.450	0.450	0.05	-	2	2
	p value	0	0.5618*	0	-	0	0	0	-	0	0
	(2,1)	0	0	0.025	0.025	0.9	-	0.025	0.025	2	2
	p value	0	0.5779*	0	0	0	-	0	0	0	0
	(2,2)	0	0	0.025	0.025	0.450	0.450	0.025	0.025	2	2
	p value	0	0.5964*	0	0	0	0	0	0	0	0
Student t	(1,1)	0	0	0.593	-	0.413	-	-0.360	-0.360	0.957	2.193
	p value	0.9679*	0.9992*	0	-	0	-	0	0	0	0
	(1,2)	0	0	0.778	-	0.114	0.099	-0.182	-0.182	0.919	2.173
	p value	0.98671*	0.9827*	0	-	0	0	0	0	0	0
	(2,1)	0	0	0.617	0.038	0.366	-	-0.346	0.326	1.038	2.132
	p value	0.9909*	0.9645*	0	0.000038	0	-	0	0.2999*	0	0
	(2,2)	0	0	0.680	0.040	0.138	0.140	-0.233	0.226	0.999	2.149
	p value	0.9905*	0.9639*	0	0	0.00001	0	0	0..00195	0	0

* Indicates of significance at level 0.05

From the Table 5, all the parameters of APGARCH model are significant for Normal distribution except the parameters ω and σ are not significant, in addition to the appearance of the leverage effect , for General error distribution we also noticed that the Presence of leverage effect and all the parameters was significant except the parameters σ, finally for the Student t distribution all the parameters significant except the parameters ω and σ in the all ranks , In addition to the parameter γ_2 in the rank (2,1) it's also not significant .

3.6. Selecting the best model [4]

The best model is chosen to be predicted based on (Akaike , BIC , Drapers) information criteria and the logarithm of the function of the Maximum Likelihood , as the best rank corresponds to the lowest value among the criteria and the largest value for the Log Likelihood. the results in the Table 6 shows that the best model is APGARCH with rank (1,2) as it has obtained the lowest value for the information criteria (AIC,BIC) and the largest value for the Log Likelihood based on (Student t distribution), followed by AVGARCH with rank (2,2) according to (DIC) information criteria and the Log Likelihood based on Student- t too.

Table 6: information criteria to select the best model for return series of (IQ/USD) exchange rate

		AIC	BIC	DIC	LLH	AIC	BIC	DIC	LLH	AIC	BIC	DIC	LLH
AVGARCH	(1,1)	-9.565	-9.555	-20.582	16439.5	-7.070	-7.058	-19.923	12154.08	-11.492	-11.479	-24.345	19750.18
	(1,2)	-9.610	-9.598	-22.463	16517.81	-6.995	-6.981	-21.684	12025.8	-11.541	-11.527	-26.230	19836.28
	(2,1)	-9.651	-9.635	-26.176	16589.49	-7.196	-7.178	-25.557	12373.15	-11.668	-11.651	-30.029	20056.4
	(2,2)	-9.650	-9.632	-28.011	16588.56	-6.835	-6.815	-27.032	11753.03	-11.730	-11.710	-31.927	20162.51
APGARCH	(1,1)	-9.569	-9.558	-20.585	16444.89	-7.157	-7.147	-20.012	12306.99	-11.618	-11.606	-24.471	19967.16
	(1,2)	-9.593	-9.580	-22.445	16486.93	-6.208	-6.194	-20.897	10674.18	-11.770	-11.756	-26.459	20229.54
	(2,1)	-9.562	-9.547	-24.250	16434.99	-7.155	-7.139	-23.680	12301.41	-11.687	-11.671	-28.212	20086.94
	(2,2)	-9.585	-9.569	-26.110	16476.02	-6.206	-6.188	-24.567	12306.99	-11.637	-11.619	-29.998	20002.2

3.7. Diagnostic checking

Diagnostic tests for the models are based on standard residual of the return series. Table 7 shows that all P value more than 0.05 and that's mean there's no autocorrelation in the standard residual of the return series for (IQ/USD) exchange rate .

Table 7: Ljung- Box test for the best models

Model	AVGARCH		APGARCH	
Lags	P value	Test statistics	P value	Test statistics
5	0.9999998	0.01707289	0.9999998	0.007313824
15	1	0.02198543	1	0.0220046
20	1	0.02899410	1	0.02921324
25	1	0.03480457	1	0.03573311
30	1	0.04175330	1	0.04297038

Table 8 shows the ARCH (L M) test ,we note that P value greater than 0.05 that's refers to there is no heterogeneity in the standard residual of the return series for (IQ/USD) exchange rate .

Table 8: ARCH (LM) test for the ARCH effect to the best model

Model	AVGARCH		APGARCH	
Lags	P value	Test statistics	P value	Test statistics
5	1	0.007311309	1	0.010268
15	1	0.022118	1	0.022137
20	1	0.030673	1	0.029549
25	1	0.03519	1	0.036143
30	1	0.042313	1	0.043573

3.8. Forecasting

For the forecasting in the period of the (IQ/USD) exchange rate we employ the dynamic method of APGARCH (1,1) and AVGARCH (2,2) with Students t distribution using statistic forecast (MSE, MAE, RMSE). The best model has the lowest value of the test values.

Table 9: statistic forecast for best model

Models	MSE	MAE	RMSE
AVGARCH (2,2)	5.767507e-10	2.76847e-06	2.400813e-05
APGARCH (1,2)	2.022716e-10	1.555608e-06	1.421987e-05

Table 9 show that APGARCH (1,2) with students t distribution has the lowest value in the all tests , which indicates the preference of the model APGARCH(1,2) over the model AVGARCH (2,2).

4. Conclusion

This study focuses on the modeling for the volatility in the return series of the exchange rate for the Iraqi dinar against US dollar, and it concluded that the return series not following the normal distribution, the model APGARCH (1,2) with error term Students' t distribution is the best model as it obtained the lowest values according to the information criteria (AIC & BIC) and the greatest value of the (MLE), followed by the AVGARCH (2,2) according to (DIC) information criteria .The presence of the effect of financial leverage has also appeared and that's mean the stock price effected by the negative shocks more than positive shocks, this indicates the existence of the asymmetric effect and that volatility are affected by negative shocks more than positive shocks (the impact of the positive shocks is less than the impact of the negative shocks).

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